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The Robust Control Mixer Module Method for Control Reconfiguration

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Abstract

The *Control Mixer* concept proposed in [8] is efficient in improving an ordinary control system into a fault tolerant one, especially for these control systems of which the real-time and on-line redesign of the control laws is very difficult. In order to consider the stability, performance and robustness of the reconfigurated system simultaneously, and to deal with a more general controller reconfiguration than the static feedback mechanism by using the control mixer approach, the robust control mixer module method is proposed in this paper. The form of the control mixer module extends from a static gain matrix into a LTI dynamical system, and furthermore multiple dynamical control mixer modules can be employed in our consideration. The H_∞ control theory is used for the analysis and design of the robust control mixer modules. Finally, one practical robot arm (ERA) system as benchmark is used to test the proposed method.

1 Introduction

The critical idea of the *control mixer* concept firstly proposed in [8] is that the baseline (nominal) control law is still under operation when some fault happens in the controlled system, alternatively, some extra gain matrices, referred to as *control mixer modules*, will be inserted into the faulty closed-loop control systems. The control mixer modules redistribute the signals in the closed-loop systems so as to preserve the closed-loop system functionality as much as possible.

This method is directly motivated by the research of self-repairing flight control systems [2, 8, 10, 11, 17], where the control mixer module is used to distribute the forces and moments of the failed surfaces to the remaining healthy control surfaces. The existing digital flight control laws are mainly designed using the classical control methods iteratively and loop-by-loop, furthermore adjusted through extensive experimentation besides a lot of heuristic expert knowledge [1, 15], so it is impossible (or very difficult) to redesign this kind of control laws in an on-line and systematic way. While the control mixer method avoids the redesign problem of the operating control law, alternatively, the reconfiguration task is to redesign the control mixer module according to different fault conditions provided by the FDI mechanism.

The control mixer method can be implemented as off-line (pre-storing a set of modules for anticipated faults) or on-line (depending on real-time FDI mechanism) forms [2]. With respect to the possible unanticipated faults, in this paper we consider the on-line control mixer module design problem[1]. The on-line control mixer module method is a restructurable control approach [2, 7, 15].

Within all related previous work [2, 8, 10, 11, 17], the control mixer modules are designed as a matrix (gain) form based on the Pseudo-Inverse techniques [6]. The designed control mixer matrix (gain) minimizes the Frobenius-norm between the original and reconfigured actuator matrices [2] or closed-loop system transition matrices [4]. But the stability of the reconfigured system can not be guaranteed by this design approach. Gao and P.J. Antsaklis proposed a Modified Pseudo-Inverse Method in [4] with respect to the stability constraint, it can be extended to deal with the control mixer design with the state feedback information [18], but their method loses the optimal sense for the general MIMO control systems. Furthermore, the Pseudo-Inverse based techniques require that the nominal and faulty control input matrices (for actuator faults) or system matrices (for general faults) must be known perfectly, if there is some false information in the FDI provided information, the reconfigurated controller designed by this approach maybe lead to a disaster consequence in the system. So the robustness of control mixer should be guaranteed not only to the ordinary exogenous disturbance and uncertainties, but also to the uncertainty of fault information provided by FDI systems. From the system configuration point of view, the Pseudo-Inverse based approaches are only suitable for the closed-loop control systems with static feedback mechanism, they are not suitable for the closed-loop control systems where the baseline (nominal) controller is a dynamical system form.

In order to overcome above mentioned drawbacks of Pseudo-Inverse based approaches for the control mixer module design, a novel design approach, called the *robust control mixer module method*, is proposed in this paper. Comparing with previous Pseudo-Inverse based methods, here

- The form of the control mixer module extends from a static matrix form into a linear time-invariant dynamical system;
- Instead of using a single gain matrix, multiple dynamical compensating filter modules can be employed in our consideration;
- Matching the closed-loop transfer function matrices of the nominal system and reconfigurated faulty system in the H_∞-norm sense.

The H_∞ optimization theory [3, 5, 9, 13, 14] is used for the robust control mixer module design after augmenting the optimal control mixer design problem into a standard robust control problem.

The robust control mixer method has more design flexibility and extensive applicable range comparing with the Pseudo-Inverse based methods, and furthmore, multiple objectives, such as stability, performance and robustness of the reconfigurated control systems, can be considered simultaneously and systematically in the design procedure. The rest of the paper is organized as: section 2 formulates the robust control mixer design problem; Section 3 outlines the usage of the H_∞ control theory to design or cooperatively design the dynamical control mixer modules,

[1] Which can also be used directly for the off-line design.

and some sufficient conditions for the existence of the optimal modules are obtained; In section 4 one subsystem of the European Robot Arm (ERA) system [12] as the benchmark is employed to test the proposed method; Finally, section 5 is the conclusions.

2 Problem Formulation

In the following, we restrict our discussion to a class of continuous time LTI control systems, and the faults considered are abrupt actuator, sensor and component faults.

Consider a class of continuous time LTI control systems with plant input disturbances, where the plant P_n and controller C_n have the following nominal state space forms:

$$P_n : \begin{cases} \dot{x}_p(t) = A_p x_p(t) + B_p(\omega(t) + u(t)), \quad t \geq t_0 \\ y(t) = C_p x_p(t) + D_p(\omega(t) + u(t)), \end{cases} \tag{1}$$

$$C_n : \begin{cases} \dot{x}_c(t) = A_c x_c(t) + B_c u_c(t), \quad t \geq t_0 \\ y_c(t) = C_c x_c(t) + D_c u_c(t), \end{cases} \tag{2}$$

where $x_p \in R^{n_p}$ ($x_c \in R^{n_c}$) is the plant (controller) state vector, $u \in R^{m_p}$ ($u_c \in R^{m_c}$) is the plant control (controller input) vector, $y \in R^{r_p}$ ($y_c \in R^{r_c}$) is the plant (controller) output vector, $\omega = [\omega_i^T \ \omega_s^T \ \omega_o^T]^T \in R^{n_i+n_s+n_o}$ is the stack of plant external signals, which includes the reference input $\omega_i \in R^{n_i}$, process noise input $\omega_s \in R^{n_s}$, and measurement noise input $\omega_o \in R^{n_o}$. Assume $\|\omega\|_\infty \leq 1$ [2]. The plant and controller connect with each other into a closed-loop system through the relationship:

$$u(t) = -y_c(t), \quad \text{and} \quad u_c(t) = y(t). \tag{3}$$

Equations (1),(2) and (3) define the *nominal closed-loop control system* in the following analysis.

When a fault occurs, the plant P_n changes abruptly to the following state space form:

$$P_f : \begin{cases} \dot{x}_f(t) = A_f x_f(t) + B_f(\omega(t) + u(t)), \quad t \geq t_f \geq t_0 \\ y(t) = C_f x_f(t) + D_f(\omega(t) + u(t)), \end{cases} \tag{4}$$

Equations (4),(2) and (3) define the *faulty closed-loop control system.*

The class of faults to be considered is defined by the following relationships between the nominal and faulty system matrices of the plant:

$$\begin{array}{ll} A_f = A_p + \Delta_A, & B_f = B_p + \Delta_B, \\ C_f = C_p + \Delta_C, & D_f = D_p + \Delta_D, \end{array} \tag{5}$$

The FDI system should supply the faulty system matrices A_f, B_f, C_f and D_f, in addition to the estimated fault occurring time t_f.

Consider the general closed-loop configuration of Fig. 1, in the nominal case all the dashed boxes (control mixer modules) are identity matrices. $\mathcal{P}$ ($\mathcal{P}_f$) denotes the transfer function matrix of the nominal (faulty) controlled plant, and $\mathcal{G}$ denotes the transfer function matrix of the nominal controller. Let the nominal closed-loop transfer function matrix from $\omega(t)$ to $y(t)$

[2] For the general case, proper weighting function can be selected to make this assumption satisfied.

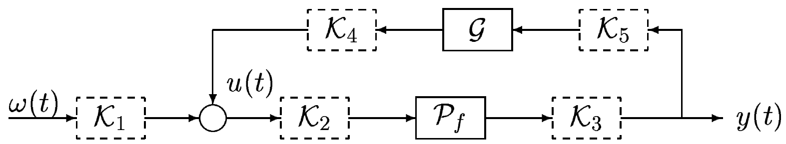


Figure 1: The General Control System Structure with Possible Control Mixer Modules

be denoted as $\mathcal{F}$, and let $\mathcal{F}_f(\mathcal{K})$ be the closed-loop transfer function matrix for the case a fault has occurred and a number of the compensating systems (robust control mixer modules) are used, which are linear, time-invariant and different from the identity matrix case. Then the robust control mixer module (RCMM) design problem is defined as:

1. Select a subset of the compensating systems $\mathcal{K}_i$ $(i = 1, \cdots, 5)$ from Fig. 1, which can be different from the identity matrix, denote the selected subset by $\mathcal{K} = \{\mathcal{K}_i\}$;

2. Design the compensating system $\mathcal{K}_i$ in 1. by solving the optimization problem

$$\min_{\mathcal{K}} \|(\mathcal{F} - \mathcal{F}_f(\mathcal{K}))\mathcal{W}\|_\infty \tag{6}$$

under the condition that the reconfigurated closed-loop control system is internal stable, where $\mathcal{W}$ is a weighting function.

It is obvious that the RCMM design problem is a two-degree-of-freedom design problem, i.e., selection of control mixer modules and design of each selected module, the two sub- design problems influence each other, so that which should be cooperatively designed and synthesized in an integrated way.

3 Design of the Robust Control Mixer Modules

The RCMM design problem not only relates to the concrete forms of the nominal plant, faulty plant and controller systems, but also relates to the whole network structure of the closed loop systems. In the following analysis, we assume the considered system has the general control system structure as shown in Fig.1. Firstly, we consider the individual design of the compensating filters $\mathcal{K}_1$ and $\mathcal{K}_4$, after that, the necessarity for the cooperative design by the H_∞ control design approach for $\mathcal{K}_1$ and $\mathcal{K}_4$ is explored.

3.1 Design of Compensator $\mathcal{K}_1$

According to the robust control mixer design problem formulation, the design problem of using compensator $\mathcal{K}_1$ simplifies to

$$\min_{\mathcal{K}_1} \|(\mathcal{F} - \mathcal{F}_f(\mathcal{K}_1))\mathcal{W}\|_\infty. \tag{7}$$

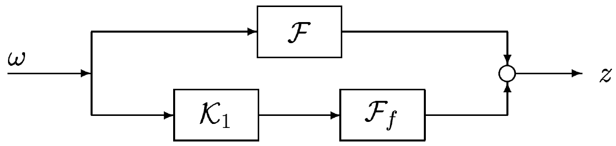


Figure 2: The Formulation of Designing $\mathcal{K}_1$

under the condition that the reconfigurated closed-loop system is stable. This problem can be directly regarded as a model-matching problem [3] as shown in Fig. 2, where $\mathcal{F}_f(\mathcal{K}_1) = \mathcal{F}_f(s)\mathcal{K}_1(s)$.

Under the assumption that the weighting function $\mathcal{W}$ is absorbed into $\mathcal{F}$ and $\mathcal{F}_f$, and the nominal closed-loop system $\mathcal{F}$ is stable and detectable, then it follows directly from ([3], Chapt 6):

Lemma 1: The optimal control mixer module $\mathcal{K}_1$ exists if the faulty closed-loop system is still stable, i.e., $\mathcal{F}_f \in \mathcal{RH}_\infty$, and has no zeros and poles on the imaginary axis.

Considering that the fault information provided by FDI system has a state space forms (5), so here we also use the system state space form for further exploration of the $\mathcal{K}_1$ design. Assume that the weighting function has been added in the plant and controller modules, and denote the nominal closed-loop control system as state space form $[A_{pc}, B_{pc}, C_{pc}, D_{pc}]$, and the faulty closed-loop system as $[A_{fc}, B_{fc}, C_{fc}, D_{fc}]$. With respect to the requirement (7), combine the nominal closed-loop system with the reconfigurated closed-loop system into an augmented system as shown in Fig. 2, then we can augment the optimal $\mathcal{K}_1$ design problem (7) into a standard H_∞ control problem as shown in Fig. 3. According to the H_∞ theory [5], we have

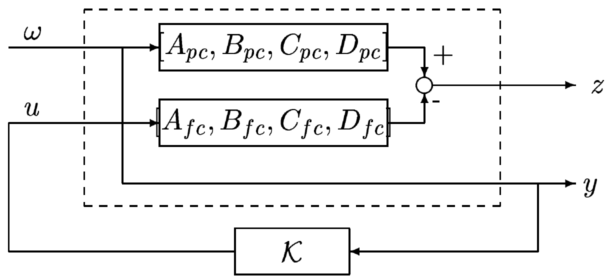


Figure 3: The Augmented Reconfigurated System with $\mathcal{K}$ ($\mathcal{K}_1$)

Lemma 2: Under the assumption that the nominal closed-loop control system $[A_{pc}, B_{pc}, C_{pc}, D_{pc}]$ is stable and detectable, the solution $\mathcal{K}$ for the H_∞ optimization problem (Fig. 3) exists if

- A_{pc} of the nominal closed-loop system has no eigenvalues on the imaginary axis;
- A_{fc} of the faulty closed-loop system is stable and has no eigenvalues on the imaginary

axis; and

- D_{fc} of the faulty closed-loop system is full column rank.

Theorem 1: When the nominal and faulty closed-loop systems of the optimal $\mathcal{K}_1$ design problem (7) satisfy the conditions proposed in Lemma 2, the solution $\mathcal{K}$ of the H$_\infty$ optimization problem (Fig. 3) is also a solution of the optimal $\mathcal{K}_1$ design problem (7).

Note 1: The conditions in Lemma 1 and 2 are only sufficient. Once the faulty closed-loop system is unstable, there is still possible that a controller to stabilize the faulty system and make (7) satisfied.

3.2 Design of Compensator $\mathcal{K}_4$

Consider that there is only $\mathcal{K}_4$ is selected from Fig. 1, then the system configuration of Fig. 1 simplifies to Fig. 4.

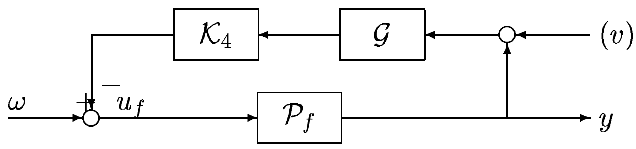


Figure 4: The structure of reconfigurated closed-loop system using $\mathcal{K}_4$

The design problem of using compensator $\mathcal{K}_4$ reduces to solve the optimal problem:

$$\min_{\mathcal{K}_4} \|(\mathcal{F} - \mathcal{F}_f(\mathcal{K}_4))\mathcal{W}\|_\infty, \tag{8}$$

under the condition that the reconfigurated closed-loop system is internal stable.

On the basis of requirement (8), we construct an augmented control system as shown in Fig. 5. Once we regard the parts included in the dash-box in Fig. 5 as a controlled plant with input vectors ω and u, and output vectors z_1 and z_2, then the design of controller $\mathcal{K}$ of this augmented control system becomes a standard H$_\infty$ optimal design problem, then we have

Lemma 3: The augmented control system (Fig. 5) can be stabilized by a real rational and proper controller $\mathcal{K}$ if and only if $\mathcal{K}$ can stabilize the $\mathcal{G}\mathcal{P}_f$.

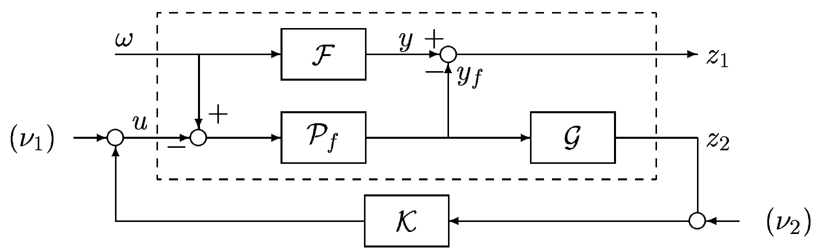


Figure 5: The Augmented Control System with Controller $\mathcal{K}$ ($\mathcal{K}_4$)

Proof: The plant outputs of the augmented control system are z_1 and z_2, where:

$$z_1 = y - y_f = \mathcal{F}\omega - \mathcal{P}_f(\omega - u), \quad z_2 = \mathcal{G}y_f = \mathcal{G}\mathcal{P}_f(\omega - u).$$

So the plant of the augmented control system can be expressed as a transfer function matrix form:

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \mathcal{F} - \mathcal{P}_f & \mathcal{P}_f \\ \mathcal{G}\mathcal{P}_f & -\mathcal{G}\mathcal{P}_f \end{bmatrix} \begin{bmatrix} \omega \\ u \end{bmatrix} \hat{=} \begin{bmatrix} \mathcal{G}_{11} & \mathcal{G}_{12} \\ \mathcal{G}_{21} & \mathcal{G}_{22} \end{bmatrix} \begin{bmatrix} \omega \\ u \end{bmatrix}. \tag{9}$$

Then according to the H_∞ theory [3], the controller $\mathcal{K}$ can stabilize the plant if and only if $\mathcal{K}$ can stabilize $\mathcal{G}_{22} = \mathcal{G}\mathcal{P}_f$.

Lemma 4: The optimal solution $\mathcal{K}$ for the augmented H_∞ optimization problem exists if $\mathcal{G}\mathcal{P}_f$ is stabilizable and $\mathcal{P}_f M_2$ and $\overline{N}_2$ has no zeros and poles on the imaginary axis. Specially for the SISO systems, the condition is that the $\mathcal{P}_f M_2 \overline{N}_2$ ($\mathcal{P}_f N_2 \overline{M}_2$) has no zeros and poles on the imaginary axis. Where $M_2,\ N_2,\ \overline{M}_2,\ \overline{N}_2$ are the components of doubly-coprime factorization of $\mathcal{G}\mathcal{P}_f$.

Proof: According to the H_∞ theory [3], bring a doubly-coprime factorization of $\mathcal{G}_{22}$ as

$$\mathcal{G}_{22} \hat{=} \mathcal{G}\mathcal{P}_f = N_2 M_2^{-1} = \overline{M}_2^{-1} \overline{N}_2, \text{ and}$$

$$\begin{bmatrix} \overline{X}_2 & -\overline{Y}_2 \\ -\overline{N}_2 & \overline{M}_2 \end{bmatrix} \begin{bmatrix} M_2 & Y_2 \\ N_2 & X_2 \end{bmatrix} = I$$

where the eight matrices $M_2, N_2, X_2, Y_2, \overline{M}_2, \overline{N}_2, \overline{X}_2, \overline{Y}_2$ are all belong to $\mathcal{RH}_\infty$.

Then the controller $\mathcal{K}$ stabilizing the augmented plant (9) can be parameterized as

$$\mathcal{K} = (Y_2 - M_2\mathcal{Q})(X_2 - N_2\mathcal{Q})^{-1} = (\overline{X}_2 - \mathcal{Q}\overline{N}_2)^{-1}(\overline{Y}_2 - \mathcal{Q}\overline{M}_2) \tag{10}$$

where $\mathcal{Q} \in \mathcal{RH}_\infty$. Define

$$\begin{array}{l} \mathcal{T}_1 = \mathcal{G}_{11} + \mathcal{G}_{12} M_2 \overline{Y}_2 \mathcal{G}_{21} = \mathcal{F} - \mathcal{P}_f + \mathcal{P}_f M_2 \overline{Y}_2 \mathcal{G}\mathcal{P}_f \\ \mathcal{T}_2 = \mathcal{G}_{12} M_2 = \mathcal{P}_f M_2 \\ \mathcal{T}_3 = \overline{M}_2 \mathcal{G}_{21} = \overline{M}_2 \mathcal{G}\mathcal{P}_f = \overline{N}_2 \end{array} \tag{11}$$

According to [3], once the transfer functions $\mathcal{P}_f$, $\mathcal{G}$ and $\mathcal{F}$ are all real-rational and proper, there is $\mathcal{T}_i \in \mathcal{RH}_\infty,\ i = 1, 2, 3$, and the transfer function matrix $\mathcal{T}_{\omega z_1}$ of the closed -loop system from ω to z_1 equals $\mathcal{T}_1 - \mathcal{T}_2\mathcal{Q}\mathcal{T}_3$. Then the H_∞ optimization problem of the augmented control system

$$\min_{\mathcal{K}} \|\mathcal{T}_{\omega z_1}\|_\infty, \tag{12}$$

under the condition that the closed-loop system is internal stable, transfers into a model-matching problem

$$\min_{\mathcal{Q} \in \mathcal{RH}_\infty} \|(\mathcal{T}_1 - \mathcal{T}_2\mathcal{Q}\mathcal{T}_3)\|_\infty. \tag{13}$$

With respect to the Theorem in Chapt 6 [3] (pp.62), the optimal solution for the model-following problem (13) exists if the ranks of $\mathcal{T}_2(j\omega) \hat{=} -\mathcal{P}_f M_2(j\omega)$ and $\mathcal{T}_3(j\omega) \hat{=} \overline{N}_2(j\omega)$ are constant for all $0 \le \omega \le \infty$.

When $\mathcal{P}_f M_2$ and $\overline{N}_2$ has no zeros and poles on the imaginary axis, and with respect to Lemma 3, we know the optimal solution $\mathcal{K}$ exists for the H$_\infty$ optimization problem (12).

Specially, when the considered system is an SISO system, then the functions $\mathcal{T}_2$ and $\mathcal{T}_3$ can be combined together, i.e.,

$$\begin{aligned}\mathcal{T}_2\mathcal{T}_3 &= \mathcal{G}_{12}M_2\overline{M}_2\mathcal{G}_{21} = -\mathcal{P}_f M_2\overline{M}_2\mathcal{G}_y\mathcal{P}_f \\ &= \begin{cases} -\mathcal{P}_f M_2\overline{M}_2 N_2 M_2^{-1}, & \text{or} \\ -\mathcal{P}_f M_2\overline{M}_2\overline{M}_2^{-1}\overline{N}_2 \end{cases} = \begin{cases} -\mathcal{P}_f N_2\overline{M}_2, & \text{or} \\ -\mathcal{P}_f M_2\overline{N}_2. \end{cases}\end{aligned}$$

So the optimal solution $\mathcal{K}$ for the augmented H$_\infty$ problem exists of if $\mathcal{P}_f M_2\overline{N}_2$ ($\mathcal{P}_f N_2\overline{M}_2$) has no zeros and poles on the imaginary axis besides $\mathcal{G}\mathcal{P}_f$ is stabilizable.

Similarly as the discussion of the compensator $\mathcal{K}_1$ design, we also further explore the optimal $\mathcal{K}$ design problem in the state space form. Denote the faulty plant as a state space form $[A_f, B_f, C_f, D_f]$, the nominal closed-loop system as $[A_{pc}, B_{pc}, C_{pc}, D_{pc}]$, and the nominal controller as $[A_c, B_c, C_c, D_c]$, then according to the H$_\infty$ theory [5], under assumption that the weighting function $\mathcal{W}$ has been absorbed into $\mathcal{F}$ and $\mathcal{P}_f$, we have

Lemma 5: The optimal solution $\mathcal{K}$ for the H$_\infty$ optimization problem (12) exists, if

- The systems (A_f, B_f) and $(A_c, B_c D_f)$ are stabilizable;
- D_f is full column rank, and $D_c D_f$ is full row rank; and
- A_{pc} and A_c have no eigenvalues on the imaginary axis, and $\begin{bmatrix} A_f - j\omega & -B_f \\ -C_f & D_f \end{bmatrix}$ is full column rank; $\begin{bmatrix} A_f - j\omega & -B_f \\ -D_c C_f & D_c D_f \end{bmatrix}$ is full row rank.

Proof: The controlled plant for the H$_\infty$ problem (12) has the state space form

$$\begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \hat{=} \begin{bmatrix} \begin{bmatrix} A_{pc} & 0 & 0 \\ 0 & A_f & 0 \\ 0 & B_c C_f & A_c \end{bmatrix} & \begin{bmatrix} B_{pc} \\ B_f \\ B_c D_f \end{bmatrix} & \begin{bmatrix} 0 \\ -B_f \\ -B_c D_f \end{bmatrix} \\ \begin{bmatrix} C_{pc} & -C_f & 0 \end{bmatrix} & D_{pc} - D_f & D_f \\ \begin{bmatrix} 0 & D_c C_f & C_c \end{bmatrix} & D_c D_f & -D_c D_f \end{bmatrix}$$

Similarly as the proof for Lemma 2, we can check all conditions proposed in Lemma 5 make the four preconditions in [5] satisfied.

Theorem 2: When the nominal closed-loop system, nominal controller and faulty plant of the optimal $\mathcal{K}_4$ design problem (8) satisfy the conditions proposed in Lemma 4 or 5, the solution $\mathcal{K}$ of the H$_\infty$ optimization problem (12) is also a solution of the optimal $\mathcal{K}_4$ design problem (8).

Proof: With respect to the definition of H$_\infty$ optimization problem, the proper and real-rational optimal controller $\mathcal{K}$ of the problem (12) satisfies:

1. making the transfer function matrix of system Fig. 5 from ω to z_1 satisfies

$$\min_{\mathcal{K}} \|\mathcal{T}_{\omega z_1}\|_\infty = \min_{\mathcal{K}} \|\mathcal{F} - (I + \mathcal{P}_f \mathcal{K} \mathcal{G})^{-1} \mathcal{P}_f\|_\infty$$

2. making the closed-loop system internal stable, i.e., the nine transfer function matrices of system Fig. 5 from ω, ν_1 and ν_2 to z_1, z_2 and u are all stable, where

$$\begin{array}{lll} \mathcal{T}_{\omega z_1} = \mathcal{F} - (I + \mathcal{P}_f \mathcal{K} \mathcal{G})^{-1} \mathcal{P}_f, & \mathcal{T}_{\omega z_2} = (I + \mathcal{G} \mathcal{P}_f \mathcal{K})^{-1} \mathcal{G} \mathcal{P}_f, & \mathcal{T}_{\omega u} = (I + \mathcal{K} \mathcal{G} \mathcal{P}_f)^{-1}, \\ \mathcal{T}_{\nu_1 z_1} = (I + \mathcal{P}_f \mathcal{K} \mathcal{G})^{-1} \mathcal{P}_f, & \mathcal{T}_{\nu_1 z_2} = -(I + \mathcal{G} \mathcal{P}_f \mathcal{K})^{-1} \mathcal{G} \mathcal{P}_f, & \mathcal{T}_{\nu_1 u} = (I + \mathcal{K} \mathcal{G} \mathcal{P}_f)^{-1}, \\ \mathcal{T}_{\nu_2 z_1} = -(I + \mathcal{P}_f \mathcal{K} \mathcal{G})^{-1} \mathcal{P}_f \mathcal{K}, & \mathcal{T}_{\nu_2 z_2} = -(I + \mathcal{G} \mathcal{P}_f \mathcal{K})^{-1} \mathcal{G} \mathcal{P}_f \mathcal{K}, & \mathcal{T}_{\nu_2 u} = (I + \mathcal{K} \mathcal{G} \mathcal{P}_f)^{-1} \mathcal{K} \end{array} \tag{14}$$

While the optimal compensator $\mathcal{K}_4$ design problem requires (8) and the following four transfer function matrices are stable (as shown in Fig. 4), i.e,

$$\begin{array}{ll} \mathcal{T}^4_{\omega y} = (I + \mathcal{P}_f \mathcal{K}_4 \mathcal{G})^{-1} \mathcal{P}_f, & \mathcal{T}^4_{\omega u_f} = (I + \mathcal{K}_4 \mathcal{G} \mathcal{P}_f)^{-1}, \\ \mathcal{T}^4_{\nu y} = -(I + \mathcal{P}_f \mathcal{K}_4 \mathcal{G})^{-1} \mathcal{P}_f \mathcal{K}_4 \mathcal{G}, & \mathcal{T}^4_{\nu u_f} = -(I + \mathcal{K}_4 \mathcal{G} \mathcal{P}_f)^{-1} \mathcal{K}_4 \mathcal{G} \end{array} \tag{15}$$

The following relationships can be noted from (14) and (15) if we take $\mathcal{K} \equiv \mathcal{K}_4$:

$$\begin{array}{ll} \mathcal{T}^4_{\omega y} \equiv \mathcal{T}_{\nu_1 z_1}, & \mathcal{T}^4_{\omega u_f} \equiv \mathcal{T}_{\nu_1 u}, \\ \mathcal{T}^4_{\nu y} \equiv \mathcal{T}_{\nu_2 z_1} \mathcal{G}, & \mathcal{T}^4_{\nu u_f} \equiv -\mathcal{T}_{\nu_2 u} \mathcal{G} \end{array} \tag{16}$$

When we assume the transfer function matrix $\mathcal{G}$ of the nominal controller is stable, then it is obvious that if the optimal solution for H_∞ optimization problem (12) which satisfies the conditions in Lemma 4 or 5 exists, it is also a solution of the optimal compensator $\mathcal{K}_4$ design problem (8).

Example: Consider a plant has the LTI form [16]:

$$\begin{cases} \dot{x}_p \hat{=} A_p x_p + B_p u_p = \begin{bmatrix} 0.15 & -2.45 \\ 0.45 & -0.9 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_p; \\ y \hat{=} C_p x_p + D_p u_p = \begin{bmatrix} -0.35 & 0.55 \end{bmatrix} x_p + 0.01 u_p, \end{cases} \tag{17}$$

A control law for output tracking is given as [18]:

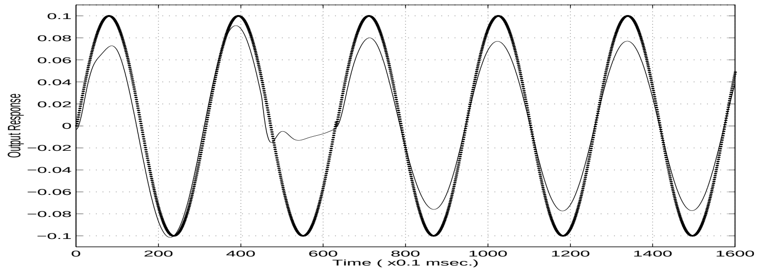
$$\begin{cases} \dot{x}_c \hat{=} A_c x_c + B_c u_c = \begin{bmatrix} 0.3597 & -1.0039 \\ 0.4605 & -0.5279 \end{bmatrix} x_c + \begin{bmatrix} 0.0829 \\ 0.113 \end{bmatrix} u_c; \\ y_c \hat{=} C_c x_c + D_c u_c = \begin{bmatrix} -0.5925 & 0.959 \end{bmatrix} x_c + 0.01 u_c, \end{cases} \tag{18}$$

The plant and controller connect through $u_c(t) = y(t)$ and $u_p(t) = \omega(t) - y_c(t)$, where $\omega(t)$ is the reference signal. The system parameters B_f, C_f, D_f of the faulty plant are expressed as: $B_f = F_B B_p$, $C_f = F_C C_p$, $D_f = F_D D_p$. It should be noted that the Pseudo-Inverse based approaches for the control mixer design are unsuitable for this system.

When multiple faults happen simultaneously in the plant system, denoted as $F_B = 0.1$, $F_C = 0.5$ and $F_D = 1.2$, and we add $\pm 5\%$ FDI estimator errors, i.e., assume the FDI estimated parameters are

$$\hat{F}_B = 0.105, \quad \hat{F}_C = 0.52, \quad \textit{and } \hat{F}_D = 1.18. \tag{19}$$

The simulation results of using control mixer modules $\mathcal{K}_1$ and $\mathcal{K}_4$ individually is shown in Fig. 6 and Fig. 7 respectively. The fault happens at $t_f = 45msec.$ and control mixer is switched into operation at $t_c = 63msec.$



(Boldface line: reference signal; Thin line: output response)

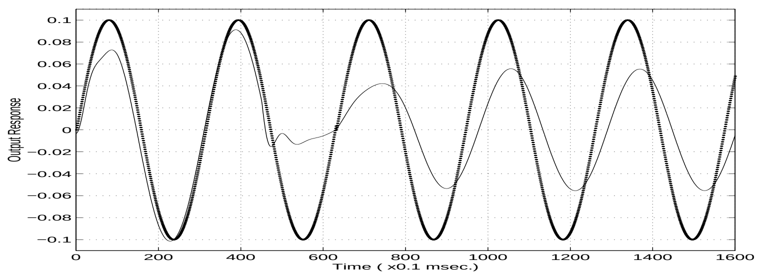
Figure 6: The Control Reconfiguration Using RCMM $\mathcal{K}_1$

This fault (19) makes the faulty closed-loop system lose the tracking ability (during [45, 63]), while the robust control mixer method makes the reconfigurated closed-loop system recover the tracking ability partially. For this specific fault case, it can be seen that $\mathcal{K}_1$ plays a better function than $\mathcal{K}_4$, since the H_∞ norm of the transfer function from ω to z of the closed-loop augmented control system corresponding to the usage of $\mathcal{K}_1$ is smaller than that of the usage of $\mathcal{K}_4$, i.e., $(\|\mathcal{T}^1_{\omega z}\|_\infty = 0.7335) < (\|\mathcal{T}^4_{\omega z}\|_\infty = 3.3239)$.

Up till now, we just discussed the independent design of each separate control mixer module, in the following we will explore the necessity of cooperation design of multiple robust control mixer modules.

3.3 Cooperative Design

As the RCMM design problem formulation, more than one control mixer modules $\mathcal{K}_i$ $(i = 1, 2, \cdots, 5)$ can be employed simultaneously for the control reconfiguration if it's necessary to improve the reconfiguration quality. So the functional redundancy of possible control mixer modules should be explored before doing the cooperative design. The redundancy of the control



(Boldface line: reference signal; Thin line: output response)

Figure 7: The Control Reconfiguration Using RCMM $\mathcal{K}_4$

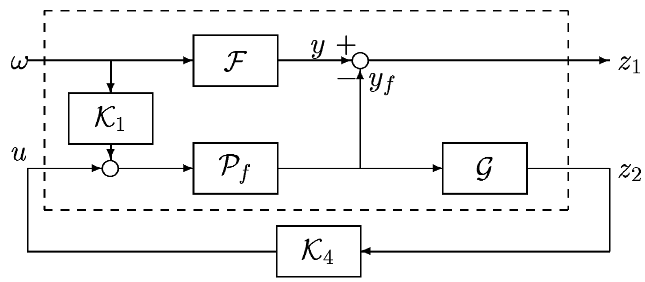


Figure 8: The Augmented Reconfigurated System with $\mathcal{K}_1$ and $\mathcal{K}_4$

mixer modules reflects that the functions of control mixer modules at some specific locations in the closed loop system can be substituted completely by the functions of proper control mixer modules at other locations. From the system structure shown in Fig.1, it can be seen that in the local component level, the control mixers in different locations have different functions, but in the global system level, some positions are redundant in the functional point of view. By utilizing the equivalent relationship of transfer function (matrices) approach, we can get:

Lemma 6: For the specific systems with different numbers of inputs and outputs, there are

- For the SISO system, the transfer function from ω to y with minimum number of robust control mixer modules can be expressed as: $\mathcal{F}_{siso} = (I + \hat{\mathcal{K}}_2\mathcal{P}_f\mathcal{G})^{-1}\mathcal{P}_f\hat{\mathcal{K}}_1$, where the $\hat{\mathcal{K}}_1$ and $\hat{\mathcal{K}}_2$ is the module combination as: (1,2), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), or (3,5).
- For the SIMO system, the transfer matrix from ω to y with minimum number of robust control mixer modules can be expressed as: $\mathcal{F}_{simo} = (I + \mathcal{K}_3\mathcal{P}_f\mathcal{G}\mathcal{K}_5)^{-1}\mathcal{K}_3\mathcal{P}_f\mathcal{K}_1$;
- For the MISO system, the transfer matrix from ω to y with minimum number of robust control mixer modules can be expressed as: $\mathcal{F}_{miso} = (I + \mathcal{P}_f\mathcal{K}_4\mathcal{G})^{-1}\mathcal{P}_f\mathcal{K}_1$;
- For the MIMO system, the transfer matrix from ω to y with minimum number of robust control mixer modules can be expressed as: $\mathcal{F}_{mimo} = (I + \mathcal{K}_3\mathcal{P}\mathcal{K}_4\mathcal{G}\mathcal{K}_5)^{-1}\mathcal{K}_3\mathcal{P}\mathcal{K}_1$.

It can be noted that the number of selected control mixer modules for the cooperative design is no larger than the numbers specified by Lemma 6. In the following we explore the benefit of designing multiple compensating filters $\mathcal{K}_1$ and $\mathcal{K}_4$ as shown in Fig. 1.

The cooperative design problem of $\mathcal{K}_1$ and $\mathcal{K}_4$ is formulated as to determine compensators $\mathcal{K}_1$ and $\mathcal{K}_4$, such that

$$\min_{\mathcal{K}_1,\mathcal{K}_4} \|\mathcal{F} - \mathcal{F}_f(\mathcal{K}_1,\mathcal{K}_4)\|_\infty \tag{20}$$

under the condition that the reconfigurated closed-loop system is internal stable.

Theorem 3: The cooperative design of $\mathcal{K}_1$ and $\mathcal{K}_4$ is necessary when $\mathcal{G}\mathcal{P}_f$ is stabilizable and $\overline{N}_2$ has zero(s) or pole(s) on the imaginary axis. Specially for the SISO systems, the condition is that the $\mathcal{P}_f M_2\overline{N}_2$ ($\mathcal{P}_f N_2\overline{M}_2$) has zero(s) or pole(s) on the imaginary axis. Where M_2, N_2, $\overline{M}_2$, $\overline{N}_2$ are the components of doubly-coprime factorization of $\mathcal{G}\mathcal{P}_f$.

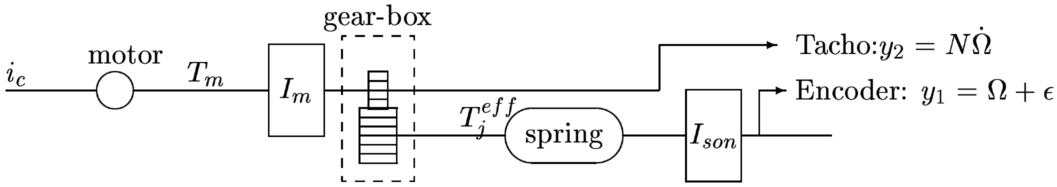


Figure 9: The Basic Structure of one joint of ERA

Proof: When the modules $\mathcal{K}_1$ and $\mathcal{K}_4$ are both employed in the reconfigurated system, similarly as the discussion for the individual $\mathcal{K}_4$ design case, we can construct an augmented system as shown in Fig. 8. The plant of this augmented control system is expressed as a transfer function matrix form:

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} (\mathcal{F} - \mathcal{P}_f)\mathcal{K}_1 & \mathcal{P}_f \\ \mathcal{G}\mathcal{P}_f\mathcal{K}_1 & \mathcal{G}\mathcal{P}_f \end{bmatrix} \begin{bmatrix} \omega \\ u \end{bmatrix} \hat{=} \begin{bmatrix} \mathcal{G}'_{11} & \mathcal{G}_{12} \\ \mathcal{G}'_{21} & \mathcal{G}_{22} \end{bmatrix} \begin{bmatrix} \omega \\ u \end{bmatrix}. \tag{21}$$

where the elements $\mathcal{G}_{12}$ and $\mathcal{G}_{22}$ are the same as those in (9).

Like the proof for lemma 4, once $\mathcal{G}\mathcal{P}_f$ is stabilizable, i.e., the controller $\mathcal{K}_4$ to stabilize the augmented plant exists, then the H$_\infty$ design problem based on (21) is reduced into a model-matching problem as (13), where

$$\begin{array}{l} \mathcal{T}'_1 = \mathcal{G}'_{11} + \mathcal{G}_{12}M_2\overline{Y}_2\mathcal{G}'_{21} = (\mathcal{F} - \mathcal{P}_f)\mathcal{K}_1 + \mathcal{P}_f M_2\overline{Y}_2\mathcal{G}\mathcal{P}_f\mathcal{K}_1 \\ \mathcal{T}_2 = \mathcal{G}_{12}M_2 = \mathcal{P}_f M_2 \\ \mathcal{T}'_3 = \overline{M}_2\mathcal{G}'_{21} = \overline{N}_2\mathcal{K}_1 \end{array} \tag{22}$$

Here the matrix $\mathcal{T}'_3$ of (22) is different from that in (11) as: $\mathcal{T}'_3 = \mathcal{T}_3\mathcal{K}_1$. The function of $\mathcal{K}_1$ is to keep the rank of matrix $\mathcal{T}'_3(j\omega)$ constant for all $0 \leq \omega \leq \infty$, i.e., when $\overline{N}_2$ has zero(s) or pole(s) on the imaginary axis, a proper design of compensator $\mathcal{K}_1$ is necessary to cancel this (these) unexpected zero(s) or pole(s).

Specially, when the considered system is an SISO system, then $\mathcal{T}_2$ and $\mathcal{T}'_3$ can be combined together as

$$\mathcal{T}_2\mathcal{T}'_3 = \mathcal{P}_f M_2\overline{N}_2\mathcal{K}_1 = \mathcal{P}_f N_2\overline{M}_2\mathcal{K}_1, \tag{23}$$

When $\mathcal{P}_f M_2\overline{N}_2$ or $\mathcal{P}_f N_2\overline{M}_2$ has zero(s) or pole(s) on the imaginary axis, then we can design the compensator $\mathcal{K}_1$ to cancel these zero(s) or pole(s) through the cross canceling of the poles and zeros, so that the rank of $\mathcal{P}_f M_2\overline{N}_2\mathcal{K}_1(j\omega)$ ($\mathcal{P}_f N_2\overline{M}_2\mathcal{K}_1(j\omega)$) is constant for all $0 \leq \omega \leq \infty$.

4 A Benchmark Study

In this section, the European Robot Arm (ERA) monitoring system [12] is utilized as a practical benchmark to test the robust control mixer module method for control reconfiguration. Here we consider the linear model of one joint of ERA system. A schematic representation of the joint of ERA is shown as:

The system parameters and values are:

Symbol	Notes	Symbol	Notes
$N=-260.6$	gear-box ratio	$I_m = 0.0011$	inertia of the input axis
Ω	joint angle of the internal axis	$I_{son} = 400$	inertia of the output axis
T_j^{eff}	effective joint input torque	ϵ	joint angle of output axis
$K_t = 0.6$	motor torque constant	i_c	motor current
$\beta = 0.4$	the damping coefficient	$c = 130000$	spring constant
T_{def}	deformation torque of the gear-box	T_m	motor torque

The equations of motion of the robot arm joint with spring damping are as follows:

$$N^2 I_m \ddot{\Omega} + I_{son}(\ddot{\Omega} + \ddot{\epsilon}) + \beta(\dot{\Omega} + \dot{\epsilon}) = T_j^{eff} \tag{24}$$

$$I_{son}(\ddot{\Omega} + \ddot{\epsilon}) + \beta(\dot{\Omega} + \dot{\epsilon}) = -T_{def} \tag{25}$$

The actuator model of the motor plus the gear-box is:

$$T_j^{eff} = NT_m, \quad T_m = K_t i_c, \tag{26}$$

and the deformation torque T_{def} is described as:

$$T_{def} = c\epsilon. \tag{27}$$

Denote $x_p \hat{=} [\Omega, \dot{\Omega}, \epsilon, \dot{\epsilon}]^T$, $y_p \hat{=} \begin{bmatrix} \Omega + \epsilon \\ N\dot{\Omega} \end{bmatrix}$, and $u_p = i_c$ as the input, the state space model of this system is:

$$\dot{x}_p \hat{=} A_p x_p + \overline{B}_p u_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{c}{N^2 I_m} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{\beta}{I_{son}} & -(\frac{c}{N^2 I_m} + \frac{c}{I_{son}}) & -\frac{\beta}{I_{son}} \end{bmatrix} x_p + \begin{bmatrix} 0 \\ \frac{1}{N^2 I_m} \\ 0 \\ -\frac{1}{N^2 I_m} \end{bmatrix} NK_t i_c \tag{28}$$

Substitute the values of each parameters into (28), the matrices of the system are:

$$A_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1740.2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.001 & -2065.2 & 0.001 \end{bmatrix}, \ \overline{B}_p = \begin{bmatrix} 0 \\ -2.0931 \\ 0 \\ 2.0931 \end{bmatrix},$$

and

$$C_p = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -260.6 & 0 & 0 \end{bmatrix}.$$

In this actual system, the controllable variable is the motor current i_c, and the measurable signals of the system are the encoder output $\Theta = \Omega + \epsilon$ and the tachometer output $N\dot{\Omega}$.

In [18] an LQG controller for the tracking problem is designed as shown in Fig.10 in order to test our presented reconfiguration method. The extended plant system model can be described by

$$\begin{cases} \dot{x}_p(t) = A_p x_p(t) + B_p(u_p(t) + \nu(t)) \\ y_p(t) = C_p x_p(t) + D_p(u_p(t) + \nu(t)) + \omega_o(t) \end{cases} \tag{29}$$

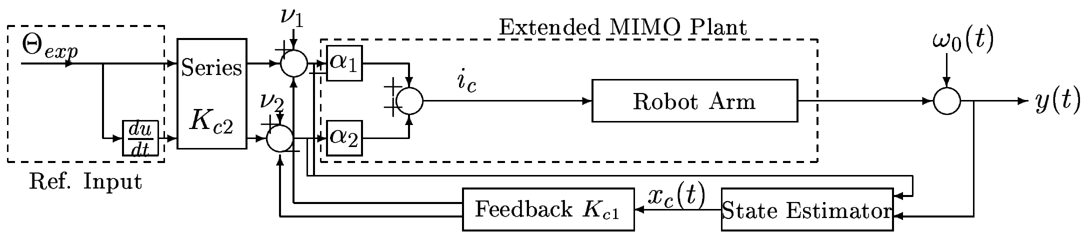


Figure 10: Modified Model of Robot Arm System with LQG Controller

where $B_p = \overline{B}_p[\alpha_1 \ \alpha_2]$, and $u_p(t) = [u_{p1}(t) \ u_{p2}(t)]^T$, process noise $\nu(t) = [\nu_1(t) \ \nu_2(t)]^T$ and measured noise $\omega_o(t) = [\omega_1(t) \ \omega_2(t)]^T$, D_p is a small diagonal matrix in order to solve the H$_\infty$ design, here we assign $D_p = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$. The modified plant system is a two-input-two-output (MIMO) system. The designed LQG controller can also be expressed as an LTI system with the state space form:

$$\begin{cases} \dot{x}_c = (A_p + K_e C_p)x_c + \begin{bmatrix} (B_p + KeD_p) & -K_e \end{bmatrix} \begin{bmatrix} u_p \\ y_p \end{bmatrix} \\ y_c = K_{c1}x_c + D_c u_p \end{cases} \tag{30}$$

Where $\alpha_1 = \alpha_2 = 0.5$, the series and compensator matrices K_{c1} and K_{c2}, and the filter matrix K_e are:

$$K_{c1} = \begin{bmatrix} -707.1068 & -205.4797 & 205.7443 & -18.8559 \\ -707.1068 & -205.4797 & 205.7443 & -18.8559 \end{bmatrix},$$

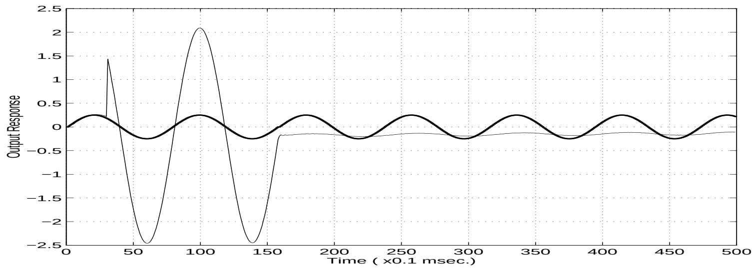
$$K_{c2} = \begin{bmatrix} -707.1068 & 0 \\ -707.1068 & 0 \end{bmatrix}, \quad K_e = \begin{bmatrix} -0.0038 & 0.0038 \\ -0.0000 & 1.4800 \\ -0.0000 & -0.0000 \\ -0.0000 & -1.4800 \end{bmatrix}.$$

Where D_c is a small diagonal matrix in order to keep the solubility of the H$_\infty$ design problem, here we assign $D_c = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$. The nominal extended closed-loop control ERA system is required to track the reference signals:

$$\begin{cases} \Omega_{exp}(t) = 0.25sin(0.8t), \\ \dot{\Omega}_{exp}(t) = 0.2cos(0.8t), \end{cases}$$

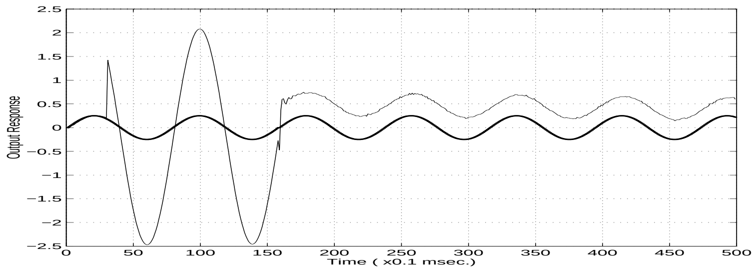
The input noise is $\nu(t) = 0.001\delta_1(t)$ and output noise is $\omega_o(t) = 0.001\delta_2(t)$, where $\delta_1(t)$ and $\delta_2(t)$ are uncorrelated continuous time white noises with uniform covariance. The system faults are regarded by a derivation of the nominal parameter values, such as the gear-box ratio becomes $N_f = F_n N$ and the motor torque constant becomes $K_{tf} = F_{kt} K_t$, where the parameters $F_n \in R$ and $F_{kt} \in R$ represent the fault levels of corresponding system parameters.

(a) System fault Case: When we consider the multiple-simultaneous fault case that $F_n = 0.5$ and $F_{kt} = 1.2$, the simulation results of using control mixer modules $\mathcal{K}_1$ and $\mathcal{K}_4$ individually are



(Boldface line: reference signal; Thin line: output response)

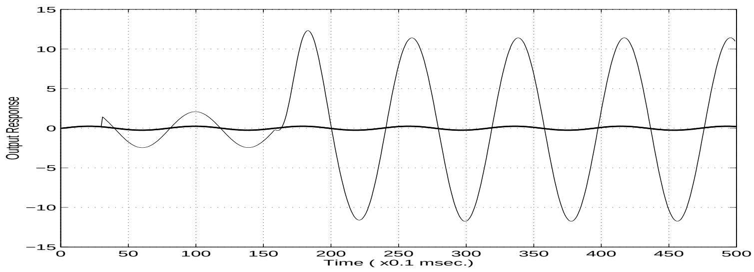
Figure 11: Fault Case (a) with RCMM $\mathcal{K}_1$ Reconfiguration



(Boldface line: reference signal; Thin line: output response)

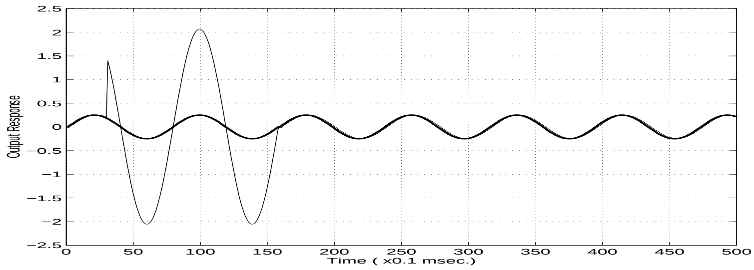
Figure 12: Fault Case (a) with RCMM $\mathcal{K}_4$ Reconfiguration

shown in Fig. 11 and Fig. 12 respectively. Under this case, the robust control mixer modules are 16th order LTI systems. The fault happens at $t = 3$msec., and the control mixer module is switched into operation at $t = 15.7$msec. The H_∞ norm of the transfer matrix from ω to z of the augmented control system corresponding to the usage of $\mathcal{K}_1$ is $\|\mathcal{T}_{\omega z}^{K_1}\|_\infty = 5.1261 \times 10^4$, which of the usage of $\mathcal{K}_4$ is $\|\mathcal{T}_{\omega z}^{K_4}\|_\infty = 9.3097 \times 10^5$. But from the simulation it is noted that the system using $\mathcal{K}_4$ has better tracking ability recovery comparing with the $\mathcal{K}_1$ case. If the matrix-form control mixer module method is used for this fault case, from the simulation result in Fig. 13, it is obvious that the reconfiguration performance of the robust control mixer modules is much better than that of the static (matrix) control mixer module.



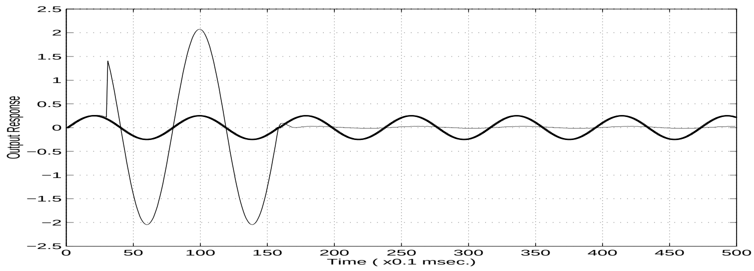
(Boldface line: reference signal; Thin line: output response)

Figure 13: Fault Case (a) with Static Control Mixer Reconfiguration



(Boldface line: reference signal; Thin line: output response)

Figure 14: Fault Case (b) with Static Control Mixer Reconfiguration Designed by (**??**)

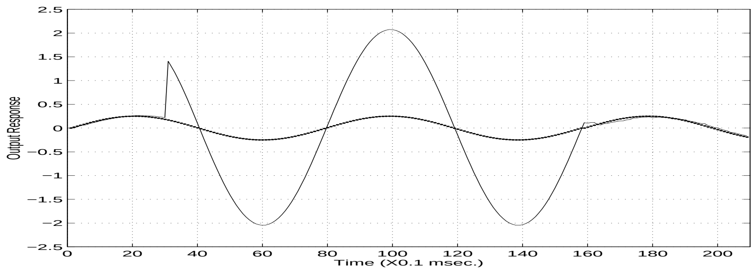


(Boldface line: reference signal; Thin line: output response)

Figure 15: Fault Case (b) with RCMM $\mathcal{K}_1$ Reconfiguration

(b) Actuator Fault Case: When we consider about the actuator fault case - $F_{kt} = 0.12$. The control reconfiguration simulation using matrix-form control mixer method with $K_f = 8.333$, is shown in the Fig. 14. This control mixer recovers the faulty system performance completely, since the ERA system is actually a SIMO system. The simulation using robust control mixer $\mathcal{K}_1$ is shown in the Fig. 15, this control mixer just keeps the faulty system stable, there is a little performance recovery. The simulation using robust control mixer $\mathcal{K}_4$ is shown in the Fig.16, this control mixer not only keeps the faulty system stable, but also has a good performance recovery.

From the simulations, it can be noticed that the static (matrix) control mixer method and robust control mixer method have distinct characters. In general, the latter has more extensive applicable range and design flexibility than the former, but the complexity of reconfigurated



(Boldface line: reference signal; Thin line: output response)

Fig.16 Fault Case (b) with RCMM $\mathcal{K}_4$ Reconfiguration system using the robust method is much high than that of using static method.

5 Conclusions

In order to meet the simultaneous consideration of stability, performance and robustness, and deal with more general control configurations by using the control mixer concept, the robust control mixer module method is proposed in this paper. The robust control mixer module design problem consists of selection of control mixer modules and design of each selected control mixer module. The H_∞ control technique is used for the robust control mixer module design after augmenting the optimal design problem into a standard robust design problem. Employed one practical robot system as one benchmark system to test our methods, the simulation results show that the robust control mixer method has more extensive applicable range and design flexibility than the (Pseudo-Inverse design based) static methods.

From the analysis of theorem 1 and 2, it can be noted that the H_∞ control design provides the conservative solutions for the robust control mixer problem, the design method for a less conservative solution for this problem is still open. Moreover, the ERA case study shows that the performance recovery measurement (3.6) is not enough for the quantitative evaluation of control reconfiguration, improvement of this method is one of our future work.

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