Risk-based Optimization and Reliability Levels of Coastal Structures

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Risk-based optimization and reliability levels of coastal structures

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Abstract

Identification of optimum reliability levels for coastal structures is considered. A class of breakwaters is considered where no human injuries can be expected in cases of failure. The optimum reliability level is identified by minimizing the total costs over the service life of the structure, including building costs, maintenance and repair costs, downtime costs and decommission costs. Different formulations are considered. Stochastic models are presented for the main failure modes for rubble mound breakwaters without superstructures, typically used for outer protection of basins. The influence on the minimum-cost reliability levels is investigated for different values of the real rate of interest, the service lifetime, the downtime costs due to malfunction and the decommission costs.

Keywords: Coastal structures, optimum reliability level, total cost model, stochastic models

1 Introduction

For civil engineering structures reliability levels are defined dependent on consequences of failure. In JCSS [1] three consequence classes are defined dependent on risk to life and economic consequences. Similar in ISO 2394 [2] four consequence classes are defined (small, some, moderate and great). In both [1] and [2] the tentative reliability level is also dependent on the relative cost of safety measures. The lowest consequence classes in both [1] and [2] assume small or negligible risk to life. In [3] and [4] four safety classes are suggested for coastal structures assuming no risk of human injury in the lowest safety classes. In this paper indicative values of optimum annual probabilities of failure and the importance of the relative costs of safety measures are investigated for a class of coastal structures, namely rubble mound breakwaters without superstructures, typically used for outer protection of basins. This class of strictures can be characterized by negligible risks of human injury in case of failure.

In most standards and recommendations safety in breakwater design is introduced by use of overall safety factors combined with a specific return period sea state. Reliability analysis of structures designed in this way shows generally relatively low reliability levels compared to other civil engineering structures. In [5] tentative reliability levels which depend on the functional and economic importance of the breakwater are specified.

For the class of breakwaters considered it can be assumed that humans spent little time in the vicinity of the structures. Therefore it is reasonable to determine the reliability level by life-cycle cost optimization where the total costs over the service life of the structure are minimized, including building costs, maintenance and repair costs, downtime costs and decommission costs.

Three types of limit states are used, see [6]: ULS: ultimate limit state, SLS: serviceability limit state, and RLS: repairable limit state, where maintenance is performed allowing the structure to fulfil its main function. Three representative case structures defined in [6] are considered and based on identified cost-optimal designs indicative optimum levels of reliability are obtained. The present paper deals with the same case studies as in [6] but provides more detailed information related to the formulation of reliability-based optimization, the applied stochastic models and the influence of decommission costs.

2 Formulation of reliability-based optimisation problems for breakwaters

Reliability based optimization problems can be formulated in different ways based on the general Bayesian decision theoretical framework in e.g. [7] and [8]. In section 2.1 the case is considered where one single breakwater is considered and the breakwater is systematically reconstructed in case of failure. Next, in section 2.2 it is assumed that the breakwater is not systematically reconstructed in case of failure. This could e.g. be the case if new design methods, materials techniques have been developed and rebuilding of the same type of breakwater is not expected to be profitable.

2.1 Systematical rebuilding in case of failure

The following assumptions are made:

- one breakwater is considered
- the breakwater is assumed to be systematically rebuild in case of failure
- the main design variables are $\mathbf{z} = (z_1, ..., z_N)$, e.g. weight of armour blocks
- the initial (building) costs are $C_I(\mathbf{z})$
- the direct failure costs are C_F
- the benefits per year are b
- the real rate of interest is r
- failure events are assumed to be modeled by a Poisson process with rate λ. The probability of failure is $P_F(\mathbf{z})$.

The optimal design is determined from the following optimization problem, see e.g. [9]:

$$\begin{aligned} \max_{\mathbf{z}} \quad & W(\mathbf{z}) = \frac{b}{r\,C_0} - \frac{C_I(\mathbf{z})}{C_0} - \left(\frac{C_I(\mathbf{z})}{C_0} + \frac{C_F}{C_0} \right) \frac{\lambda P_F(\mathbf{z})}{r + \lambda P_F(\mathbf{z})} \\ \text{s.t.} \quad & z_i^l \le z_i \le z_i^u \quad , i = 1, ..., N \\ & P_F(\mathbf{z}) \le P_F^{\max} \end{aligned} \tag{1}$$

where $\mathbf{z}^l$ and $\mathbf{z}^u$ are lower and upper bounds on the design variables. C_0 is the reference initial cost of corresponding to a reference design $\mathbf{z}_0$. $P_F^{\max}$ is the maximum acceptable probability of failure e.g. with a reference time of one year. This type of constraint is typically required by regulators. The optimal design $\mathbf{z}^*$ is determined by solution of (1). If the constraint on the maximum acceptable probability of failure is omitted, then the corresponding value $P_F(\mathbf{z}^*)$ can be considered as the optimal probability of failure related to the failure event and the actual cost-benefit ratios used.

The failure rate λ and probability of failure can be estimated for the considered failure event, if a limit state equation, $g(X_1,...,X_n,\mathbf{z})$ and a stochastic model for the stochastic variables, $(X_1,...,X_n)$ are established. If more than one failure event is critical, then a series-parallel system model of the relevant failure modes can be used.

2.2 No rebuilding in case of failure

The assumptions are the same as in section 2.1 except:

- the breakwater is assumed not to be rebuild in case of failure
- the design lifetime is T_L
- the probability of failure in the time interval $[0,T]$ is denoted $P_F(T,\mathbf{z})$. The annual probability of failure is $\Delta P_F(T,\mathbf{z}) = P_F(T,\mathbf{z}) - P_F(T-1,\mathbf{z})$; T in [years].

The optimal design is determined from the following optimization problem:

$$\begin{aligned}\max_{\mathbf{z}} \quad & W(\mathbf{z}) = \sum_{i=1}^{T_L} \frac{b}{C_0}\left(1-P_F(i,\mathbf{z})\right)\frac{1}{(1+r)^i} - \frac{C_I(\mathbf{z})}{C_0} - \sum_{i=1}^{T_L}\frac{C_F}{C_0}\Delta P_F(i,\mathbf{z})\frac{1}{(1+r)^i} \\ & \qquad + C_D\left(1-P_F(T_L)\right)\frac{1}{(1+r)^{T_L}} \qquad (2) \\ \text{s.t.} \quad & z_i^l \le z_i \le z_i^u \quad , i=1,...,N \\ & \Delta P_F(i,\mathbf{z}) \le \Delta P_F^{\max}\end{aligned}$$

where $\mathbf{z}^l$ and $\mathbf{z}^u$ are lower and upper bounds on the design variables. $\Delta P_F^{\max}$ is the maximum acceptable annual probability of failure. $C_D(T_L)$ is the cost of decommissioning at end of service lifetime. The corresponding minimum annual reliability index is $\Delta\beta^{\min}$. It is noted that in (2) the annual benefits and failure costs are added.

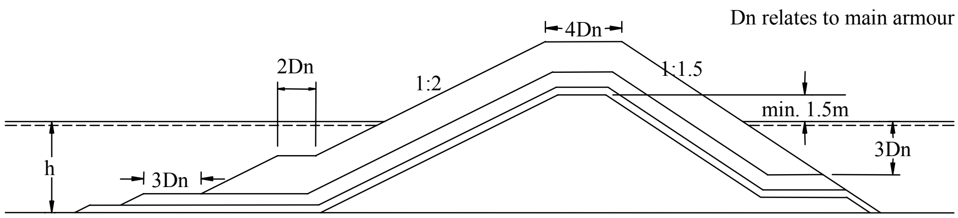
The optimal design $\mathbf{z}^*$ is determined by solution of (2). If the constraint on the maximum acceptable annual probability of failure is omitted, then as in section 2.1 the corresponding value $\max_t \Delta P_F(t,\mathbf{z}^*)$ can be considered as the optimal annual probability of failure related to the failure event and the actual cost-benefit ratios used.

In general, a simulation procedure can be used to identify the cost optimum reliability level. First the type of breakwater, service lifetime T_L, water depth and long-term wave statistics have to be identified. The main design parameter is usually related to the design values of significant wave height H_S^T and the weight of armour units $z = W = \rho D_n^3$ where D_n is a nominal dimension of the armour unit size and ρ is the armour mass density. For deep water conditions $D_n = cH_S^T$ where c is a constant. Given the design values of significant wave height H_S^T the structure is designed with conventional deterministic methods, and the construction costs are calculated. Given a repair strategy chosen at the design stage, realizations of the sea states during the service lifetime are simulated, and probabilities of minor and major repairs, and of ultimate failure are determined. The total expected costs during the design lifetime can then be calculated. Finally the design wave height resulting in the lowest total expected costs is identified. The corresponding probabilities of minor and major repairs, and of ultimate failure are then the optimum reliability levels.

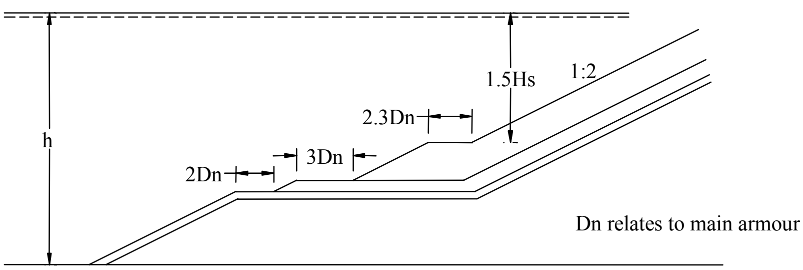
3 Cost functions for breakwater design

Given the design values of significant wave height, H_S^T, the cross section of the breakwater is obtained for shallow and deep water conditions, see Figure 1 and the description in [6] .

Figure 1 Shallow and deep water cross sections, [6]



Shallow water cross section: h < 1.5 H_S + 2.7 · D_n



Deep water cross section: h ≥ 1.5 H_S + 2.7 D_n

Repair is related to main armour damage given by the relative number of displaced units, D, as shown in Table 1, see [5]. The damage parameter is $S = A_e/D_{n50}^2$, where A_e is the cross sectional eroded area, and D_{n50} = (mean armour unit volume)$^{1/3}$. N_{od} is the number of displaced units within a strip with width D_n.

Table 1 Repair policy as function of damage levels, [6].

Damage levels	S (rock)	N_{od} (cubes)	Estimated D	Repair policy
Initial	2	0	2 %	no repair
Serviceability (minor damage, only to armour)	5	0.8	5 %	repair armour
Repairable (major damage, armour + filter 1)	8	2.0	15 %	repair armour + filter 1
Ultimate (failure)	13	3.0	30 %	repair armour + filter 1 and 2

The cost of repairs are estimated as follows, see [6]:

Serviceability: Cost of repair of minor damage

$$C_{R1} = (1+K)\, D\, C_{I,\,armour}\, R \tag{3}$$

where $C_{I,\,armour}$ is the initial construction cost of the main armour layer, R = 1.5 is a factor signifying high cost of repairs, and K = 0.3 is a factor signifying mobilization costs. The chosen values of R and K are estimates, but can vary considerably from case to case.

Repairable: Cost of repair of major damage

$$C_{R2} = D\,(C_{I,\,armour} + C_{I,\,filter\ 1})\, R + K\, D\, C_{I,\,armour}\, R \tag{4}$$

where $C_{I,\,filter\ 1}$ is the initial construction cost of filter 1.

Ultimate: Cost of repair after a failure

$$C_{R3} = D\,(C_{I,\,armour} + C_{I,\,filter\ 1} + C_{I,\,filter\ 2})\, R + K\, D\, C_{I,\,armour}\, R \tag{5}$$

where $C_{I,\,filter\ 2}$ is the initial construction cost of filter 2.

Repairs are assumed to take place immediately after the damage limit for repair is exceeded. Calculations are performed for a structure length of 1 km and damage is assumed to take place over the whole length of the breakwater.

The influence of downtime costs is analysed by inclusion of downtime costs of 200.000 EUR/day in 3 months, when $D \geq 15\%$. The downtime costs are related to 1 km length of breakwater.

Simulations are performed with and without damage accumulation. Several models for damage accumulation exist. However, it is assumed that the following model can be used to indicate the effect of damage accumulation, see [6]. The duration of each storm is set to 1,000 waves. No damage is assumed to occur in storms with H_S-values below a critical value $H_{S,C}$ corresponding to damage levels S = 1 and N_{od} = 0.002 for 1000 waves in the van der Meer stability formulae, [10] and [11], for rock and cubes, respectively. Damage accumulation takes place only when the next storm has a higher H_S-value than the preceding value. The relative decrease in damage with the number of waves inherent in the stability formulae is taken care of by keeping track solely of the number of waves, which contribute to damage.

The optimal design is determined using the optimization problem formulated in section 2.2 assuming no rebuilding in case of failure. No benefits and costs related to loss of life are included. The optimization problem is written:

$$\min_T \quad C(T) = C_I(T) + \sum_{t=1}^{T_L} \left\{ C_{R_1}(T) P_{R_1}(t) + C_{R_2}(T) P_{R_2}(t) + C_F(T) P_F(t) \right\} \frac{1}{(1+r)^t} + C_D \left(1 - P_F(T_L)\right) \frac{1}{(1+r)^{T_L}} \tag{6}$$

where

T return period used for deterministic design

$C_{R_1}(T)$ cost of repair for minor damage

$P_{R_1}(t)$ probability of minor damage in year t

$C_{R_2}(T)$ cost of repair for major damage

$P_{R_2}(t)$ probability of major damage in year t

$C_F(T)$ cost of failure including downtime costs

$P_F(t)$ probability of failure t

$C_D(T_L)$ cost of decommissioning at end of service lifetime

Two types of rock armours are considered, namely rock armour and concrete cubes. The failure modes are as follows:

Rock armour, slope 1:2

The van der Meer formula is used, see [10], and the limit state equation is written:

$$g = S - \left(\frac{X_{H_S} H_S}{Z\, 7.42\, \Delta\, D_n\, N_z^{-0.1}\, S_m^{0.25}} \right)^5 \tag{7}$$

where the parameters are described in table 2.

Cubes, slope 1:2

The van der Meer formula is used, see van der Meer [10], but modified to slope 1:2. The limit state equation is written:

$$g = N_{od} - \left(\left(Z 0.136 \frac{X_{H_S} H_S}{\Delta\, D_n} S_m^{0.1} - 0.15 \right) N_z^{0.3} \right)^{2.5} \tag{8}$$

where the parameters are described in table 3.

Table 2 Stochastic model: Rock armour 1:2.

	Description	Distribution	Expected value	Standard deviation
S	critical damage level		see table 1	
H_S	annual maximum significant wave height	Weibull, see table 4		
X_{H_S}	model uncertainty wave height	Normal	1	0.1
Z	model uncertainty	Normal	1	0.0645
Δ	model parameter	Normal	1.57	0.06
N_z	number of waves in one storm		1000	
S_m	wave steepness	Normal	0.025	0.005
D_n	armour size	Normal	$0.35\,H_S^T$	COV=0.05
H_S^T	design wave height with return period T years			
$W=\rho D_n^3$	weight of armours			
ρ	armour density		2.65 ton/m^3	

Table 3 Stochastic model: Cubes 1:2.

		distribution	Expected value	Standard deviation
S	critical damage level		see table 1	
H_S	annual maximum significant wave height	Weibull, see table 4		
X_{H_S}	model uncertainty wave height	Normal	1	0.1
Z	model uncertainty	Normal	1	0.0645
Δ	model parameter	Normal	1.33	0.03
N_z	number of waves in one storm		1000	
S_m	wave steepness	Normal	0.025	0.005
D_n	armour size	Normal	$0.28\,H_S^T$	COV=0.01
H_S^T	design wave height with return period T years			
$W=\rho D_n^3$	weight of armours			
ρ	armour density		2.4 ton/m^3	

5 Case studies

The main data for three cases are given in Table 4. The basic data are the same as those used in [6], but the decommission costs are included in the following results as 50% of the initial costs if the structure survives to the end of the design lifetime.

Table 4 Case study data, [6]

Case	Water depth	Armour Density	Waves Origin $H_{S,o}^{100y}$	Distribution $H_{S,o}^{400y}$	Built-in unit prices core / filter 1 / filter 2/ armour in EURO/m^3
A	10 m	Rock 2.65 t/m^3	Follonica 5.64 m	Weibull 6.20 m	10 / 16 / 20 / 40
B	15 m	Cube 2.40 t/m^3	Follonica 5.64 m	Weibull 6.20 m	10 / 16 / 20 / 40
C	30 m	Cube 2.40 t/m^3	Sines 13.2 m	Weibull 14.2 m	5 / 10 / 25 / 35

The applied long-term wave statistics are based on fitting of 3-parameter Weibull distributions to field wave data from Follonica and Sines, which represent shallow and deep-water conditions. The statistical uncertainties on the distribution parameters are included in the analyses. The 100 and 400 years return period expectation values of the deep-water significant wave height H_s are given in Table 4. Details are given in [12].

Table 5, 6 and 7 and figures 3, 4 and 5 show results where damage accumulation is not included.

Table 5 and figure 3 show some results for case A with shallow water conditions and thus depth limited waves. The optimization is performed directly on the armour unit mass. It is seen that if downtime costs are included, then the optimum armour unit mass increases.

Figure 3 Case A. Total costs in 50 years lifetime. No damage accumulation included. Left figure: no downtime costs included; right figure: downtime costs included.

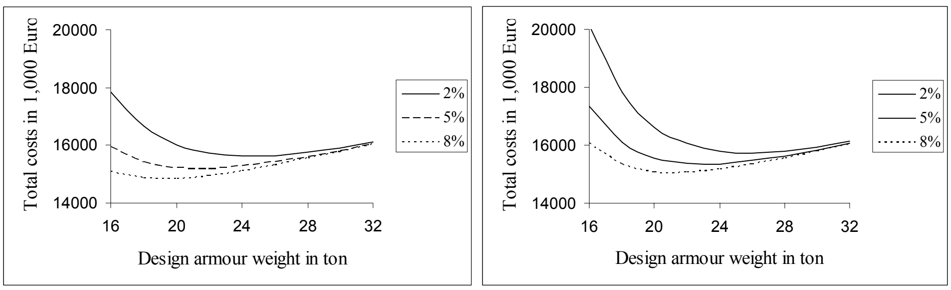


Table 5 Case A. Optimum reliability levels for rock armoured outer breakwater. 50 years service lifetime. No damage accumulation included. Costs in 1,000,000 Euro for 1 km length.

Real interest rate (%)	Downtime costs	Optimum armour unit mass W_{50} (t)	Optimum limit state average number of events within service lifetime SLS	RLS	ULS	Construction costs	Total lifetime costs
2	None	20	0.29	0.049	0.0059	14.2	17.5
5		18	0.51	0.10	0.015	13.8	15.1
8		26	0.88	0.21	0.036	13.4	14.4
2	0.2 per day in 3 months	22	0.17	0.026	0.0031	14.5	17.9
5		20	0.29	0.049	0.0059	14.2	15.6
8		18	0.51	0.10	0.015	13.8	15.0

Table 6 and figure 4 show some results for case B. The optimization is performed with respect to the design return period T (and the corresponding armour unit mass is determined given the return period and associated significant wave height H_s^T). It is seen again that if downtime costs are included, then the optimum design return period T and armour unit mass increase.

Table 6 Case B. Optimum reliability levels for concrete cube armoured breakwater. 50 years service lifetime. No damage accumulation included. Costs in 1,000,000 Euro for 1 km length.

Real interest rate (%)	Downtime Costs	Optimum T (years)	H_s^T (m)	W_{50} (t)	Optimum limit state average number of events within service lifetime SLS	RLS	ULS	Construction costs	Total lifetime costs
2	None	200	5.92	10.9	0.49	0.039	0.011	16.2	20.1
5		100	5.64	9.5	0.88	0.079	0.027	15.8	17.4
8		50	5.36	8.1	1.52	0.15	0.060	15.3	16.6
2	0.2 per day in 3 months	400	6.20	12.5	0.29	0.027	0.0034	16.7	20.6
5		200	5.92	10.9	0.49	0.039	0.011	16.2	17.8
8		200	5.92	10.9	0.49	0.039	0.011	16.2	17.0

Figure 4 Case B. Total costs in 50 years lifetime. No damage accumulation included. Left figure: no downtime costs included; right figure: downtime costs included.

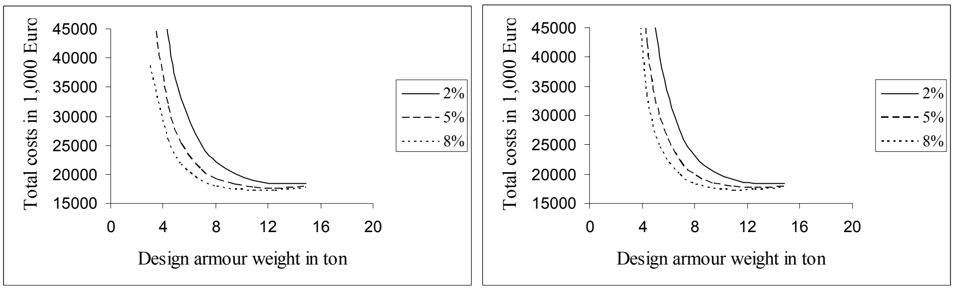


Table 7 and figure 5 show some results for case C with deep water conditions. The optimization is as for case B performed with respect to the design return period T . The design wave heights and armour unit masses are much higher than in cases A and B due to the more severe wave conditions and the higher water depth.

The above results show that when the real rate of interest increases the optimum design wave height and the armour unit mass decrease – the influence is seen to be significant.

With decreasing armour unit mass the probability of minor and major repairs and of failure increase, but due to the capitalization factor the effect of these costs at some time in the design lifetime is less important for higher rates of interest.

Table 7 Case C. Optimum reliability levels for concrete cube armoured breakwater. 50 years lifetime. No damage accumulation included. Costs in 1,000,000 Euro for 1 km length.

Real interest rate (%)	Downtime Costs	Optimum			Optimum limit state average number of events within service lifetime			Con-struc-tion costs	Total lifetime costs
		T (years)	H_s^T (m)	W_{50} (t)	SLS	RLS	ULS		
2	None	200	13.7	136	0.75	0.047	0.011	71.2	91.0
5		100	13.2	122	1.15	0.082	0.023	68.6	77.6
8		50	12.7	108	1.76	0.15	0.43	65.9	72.8
2	0.2 per day in 3 months	200	13.7	136	0.75	0.047	0.011	71.2	91.6
5		100	13.2	122	1.15	0.082	0.023	68.6	78.3
8		50	12.7	108	1.76	0.15	0.43	65.9	73.8

Figure 5 Case C. Total costs in 50 years. No damage accumulation included. Left figure: no downtime costs included; right figure: downtime costs included.

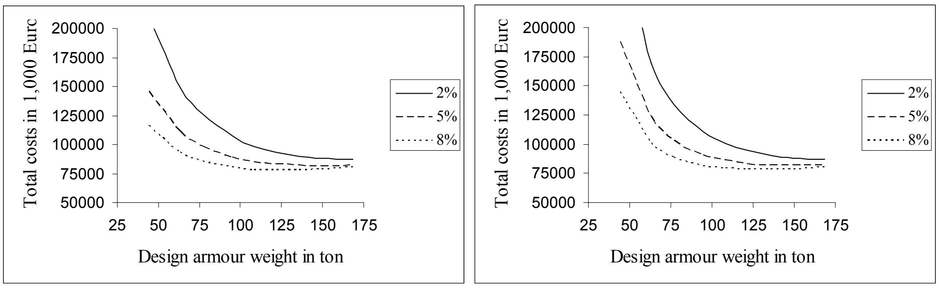


Table 8 and figure 6 show some results for case C with the service lifetime T_L equal to 25, 50 and 100 years. Decommission costs and downtime costs are not included. Further damage accumulation is included / not included.

The results show that increasing service lifetime, as expected, results in increased design return period, and thus higher armour unit masses.

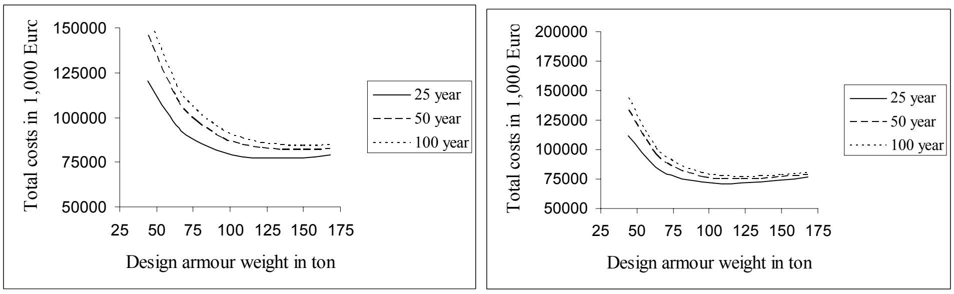
If damage accumulation is assumed then the optimum armour unit mass increases – the effect is seen to be significant. With damage accumulation the probability of minor repairs decrease, whereas (due to the fewer minor repairs) the probabilities of major repairs and failure increase relatively. The effect has in comparison with other damage accumulation models (detailed results not shown) seen to be very dependent on the model for damage accumulation, and the results thus underline the importance of choosing a correct model for damage accumulation.

Further it is seen with comparison with the results in table 7 that the effect of decommission costs is marginal for the chosen level of decommission costs. This is also to be expected due to the relatively low optimal ultimate failure probability.

Table 8 Case C. Optimum reliability levels for concrete cube armoured breakwater. 5% real rate of interest. No downtime costs. Costs in 1,000,000 Euro for 1 km length.

Service lifetime (years)	Damage accumulation	Optimum			Optimum limit state average number of events within service lifetime			Construction costs	Total lifetime costs
		T (years)	H_s^T (m)	W_{50} (t)	SLS	RLS	ULS		
25	no	50	12,7	108	0,88	0.074	0.022	64,4	71,2
50		100	13,2	122	1.15	0.082	0.023	68,6	74,6
100		100	13,2	122	2,28	0.17	0.043	70,1	76,7
25	yes	200	13,7	136	1,12	0.014	0.0033	69,7	77,1
50		400	14,2	150	1,82	0.015	0.0025	73,7	81,6
100		400	14,2	150	3,91	0.030	0.0056	75,2	84,1

Figure 6 Case C. Optimum reliability levels for concrete cube armoured breakwater. 5% real rate of interest. No downtime costs. Costs in 1,000,000 Euro for 1 km length. Left figure: no damage accumulation; right figure: with damage accumulation.



The above results are based on the assumption that no systematic reconstruction is performed in case of ultimate failure. If formulation based on systematic reconstruction is used then the optimum design return period T is relatively lower.

The figures also show flat minima of total costs as function of armour unit mass. Thus it is less important to identify the exact optimum failure probability because the lifetime costs are practically independent of the design reliability level within a wide range. This is because the larger building costs of an initial safer structure are almost balanced by smaller repair and maintenance costs. Therefore it is generally preferable to choose a conservative design in order to reduce the political and financial inconveniences related to repairs. This is in accordance with the recommendations in [1] and [2] to choose the target reliability level dependent on the relative costs of safety measures.

The identified optimum reliability levels can be used to identify optimum annual reliability levels. For real rate of interests in the range 2-5 % indicative optimum annual reliability levels are: SLS: 0.01 – 0.05, RLS: 0.001 – 0.005 and ULS: 0.0001 – 0.0005. These reliability levels are closely related to the stochastic models used in the reliability-based cost optimization, and therefore these models should be used together with the indicative reliability levels.

7 Summary of main conclusions

For a class of coastal structures where the risk of human injury is negligible, basic formulations are presented for identification of the optimum reliability level. The class of structures considered is rubble mound breakwaters without superstructures, typically used for outer protection of basins. Formulations with and without systematic reconstruction are described. The most significant failure modes are presented together with stochastic models. Case studies are performed for a class of structures which is characterized by marginal risk of human injuries in case of failure, i.e. low reliability class. The results show that for the investigated type of breakwaters the critical design limit state corresponds to the Seviceability Limit State (SLS) defined by moderate damage to the armour layer, since the expected lifetime costs associated with this limit state are much higher than for the other limit states.

The identified optimum reliability levels can be used to identify optimum annual reliability levels. For real rate of interests in the range 2-5 % optimum annual reliability levels are roughly:

- SLS: 0.01 – 0.05
- RLS: 0.001 – 0.005
- ULS: 0.0001 – 0.0005

It is noted that these reliability levels are closely related to the stochastic models used in the reliability-based cost optimization, and therefore these models should be used together with the indicative reliability levels.

These results are based on the assumption that no systematic reconstruction is performed in case of ultimate failure. If formulation based on systematic reconstruction is used then the optimum reliability level is relatively lower. Further, the effect of including damage accumulation is seen to be important. The relations between total lifetime costs and the reliability levels show flat minima as function of the main design variable - the armour unit mass. Therefore conservative designs involving fewer repairs are only slightly more expensive than cost optimised designs. These conclusions are similar to those given in [6].

The effect of decommission costs is found to be marginal for the chosen level of decommission costs. This could also be expected due to the relatively low optimal ultimate failure probability.

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