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Holm-Jørgensen, Kristian; Nielsen, Søren R.K.

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# **Nonlinear Displacements of a Wind Turbine Blade based on a Multibody Formulation with a Local Observer Frame**

**K. Holm-Jørgensen and S.R.K. Nielsen**

Department of Civil Engineering  
Aalborg University, Aalborg, Denmark  
e-mail: [soren.nielsen@civil.aau.dk](mailto:soren.nielsen@civil.aau.dk)

**Summary** The paper deals with different updating algorithms of the moving frame of reference parameters in a multibody formulation for flexible structures. The updating algorithms are based on the motion of one or two beam element nodes in the belonging substructure. An example of a clamped wind turbine demonstrates that the updating algorithm by the use of two beam element nodes is far superior.

## **Introduction**

The basic idea of flexible multibody dynamics is to introduce a moving frame of reference to each substructure. Relative to the moving frame elastic displacements are relatively small, rendering linear analysis possible. This moving frame is defined by a position vector and a parameter vector defining the origin and rotation of the moving frame relative to a fixed frame of reference. In the floating frame of reference formulation these referential coordinates describe the rigid body translation and rotation of the structure and become a part of the degrees of freedom in the system vector of the multibody system, see e.g. Shabana [1]. The use of such a mixed set of referential and elastic coordinates leads to highly non-linear system equations. To circumvent these difficulties Kawamoto et al. [2, 3, 4, 5] suggested to let the moving frame of reference float in a controlled way relative to the moving substructure, so these are always sufficiently close to each other, in order for the small displacement assumption to be fulfilled. They named this type of moving frame a Local Observer Frame. Hereby, the system matrices do not depend on the degrees of freedom in the system vector by explicitly predicting the motion of the moving frame. To reduce or eliminate the gap between the predicted and actual motion, it is necessary to regularly update the motion of the moving frame of reference as demonstrated in Kawamoto et al. [5]. In Kawamoto et al. [2] the updating scheme is originally described, where the orientation, angular velocity, and angular acceleration of the moving frame are updated based on a local triad linked to four nodes in the body. The updating scheme of the moving frame of reference in the present paper follows the same principles as described in Kawamoto et al. [3]. A small change when updating the moving frame is presented, where the orientation of the moving frame is updated based on either the motion of one or two beam element nodes.

## **Multibody Formulation with a Local Observer Frame**

The idea is to describe the motion of a substructure in a  $(x_1, x_2, x_3)$ -coordinate system, which is freely moving in the vicinity of the substructure. Further, a fixed  $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ -coordinate system is introduced common for all substructures. The origin of the moving frame is described by a position vector with the global components  $\bar{\mathbf{x}}_c$ , and its rotation is determined by the parameter vector (or pseudo vector)  $\boldsymbol{\theta}$ . In dynamic simulations the substructure may drift away from the moving frame, which requires sequential updating of the position, velocity and acceleration of the moving frame origin together with the rotation, angular velocity and angular acceleration vectors. In this paper only static simulations are in focus where the moving frames are updated to reduce the displacements of the substructure from the belonging moving frame in order for the small

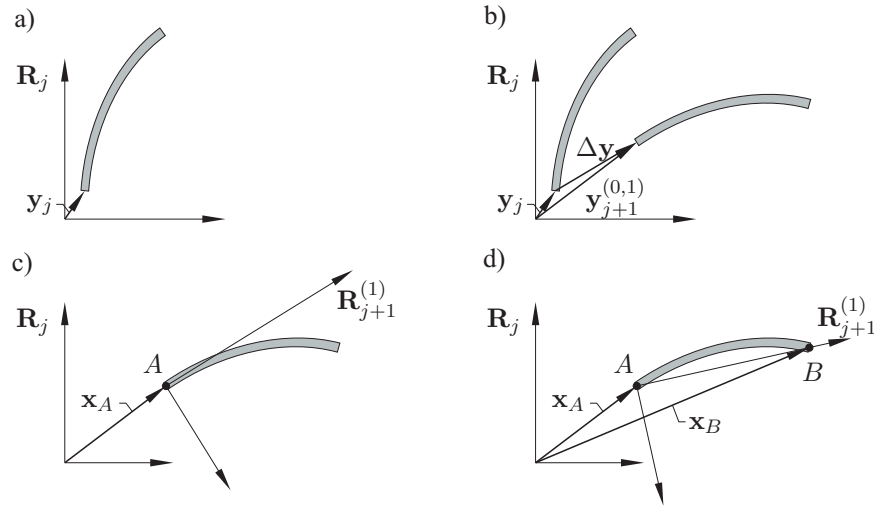


Figure 1: a) Moving frame and substructure at the load step  $j$ . b) Iterate the system vector for the load step  $j + 1$ . c) Updating of the moving frame based on the motion of the node at the origin. d) Updating of the moving frame based on the motion of two end nodes.

displacement assumption to be fulfilled. The equations of motion and updating algorithms for dynamic simulations are described in [6], which reduce to the static case when mass and damping terms are disregarded.

### Update Algorithms for Static Analysis

In this section it is described how the moving frames are updated for use in static simulations. The reason for updating the moving frames in static analysis is to account for large nonlinear displacements. In Figure 1 a series of sketches are shown to illustrate the procedure when updating the moving frames in a static simulation. The lower index ' $j$ ' indicates a load step and an upper index ( $k_1$ ) is used to specify the updating step of the moving frame of reference parameters within the load step. Similarly, an upper index ( $k_1, k_2$ ) is used for the system vector, where  $k_2$  indicates the iteration step of the system vector within the present updating step  $k_1$  of the moving frame of reference. When determining the motion of the multibody system it is necessary that both the moving frame parameters and system vector have the same upper index  $k_1$ . In Figure 1a the moving frame and substructure are shown for the converged solution at load step  $j$ . In the next load step  $j + 1$  the exterior load is changed, and the substructure is iterated to a new position within the moving frame, see Figure 1b. Due to the nonlinear rotational constraints several iterations may be necessary to obtain a residual which is within the specified convergency limits. When the solution has converged it is chosen to update the moving frame. In the present situation two methods are possible. In Figure 1c the node at the origin of the substructure is used to update the moving frame. Hereby, the moving frame obtains the same position and orientation as this node. Another possibility is demonstrated in Figure 1d, where the motion of the node at the origin and an arbitrary point, here the end node, are used to update the moving frame. At this point the updated moving frame and displacement vector do not correspond and it is therefore necessary to iterate the position of the substructure within the updated frame, similarly to Figure 1b.

### Tip Displacement of a Clamped Wind Turbine Blade

In this section the accuracy of the updating methods for the multibody formulation are further investigated. A co-rotational beam formulation with 20 elements is used as the reference model,

which has been implemented by use of Krenk [7]. This type of formulation corresponds to having a moving frame for each beam element which is updated based on the motion of the end nodes in the respective elements. The examples are based on a clamped wind turbine blade where prismatic elements are used based on the mean value of the cross section parameters at the end points in the respective beam elements. The blade is discretized by a total of 20 elements with the same reference length. The total referential length of the blade is  $L = 44.8$  m. The numeration of the nodes is chronological from the root to the tip. An exterior tip load in the flapwise  $\bar{x}_1$ -direction is applied so the tip displacement is approximately 20% of the undeformed blade length.

### Convergency of Updating Algorithms

In this section the convergency of the two updating algorithms from section is investigated by increasing the number of substructures in the blade. In this example a constant reference length is used for each substructure. Because a total of 20 elements of equal reference length are used in the discretization of the blade the number of substructures become  $n_{sub} = [1, 2, 4, 5, 10, 20]$ . The tip position of the blade after deformation is shown in Figure 2 based on the two updating algorithms and the different number of multibodes. In Figure 2 it is shown that the updating algorithm based

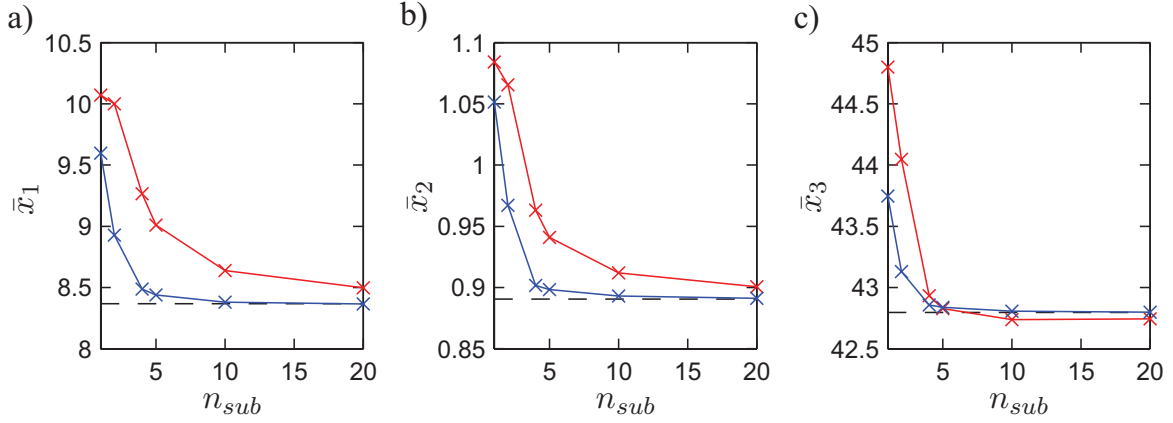


Figure 2: Tip position of the blade by use of 20 elements of equal reference length which are divided into a number of substructures  $n_{sub}$  of equal reference length. a) Tip position in  $\bar{x}_1$  (flapwise). b) Tip position in  $\bar{x}_2$  (edgewise). c) Tip position in  $\bar{x}_3$  (spanwise). ( — ) Update based on node at origin. ( — ) Update based on end nodes. ( — — ) Co-rotating formulation by use of 20 elements.

on the motion of both end points in the substructure converges much faster than by only using the motion of one end point. 4 substructures updated based on the end points give similar results as by use of 20 substructures updated based on the motion of the node at the origin of the substructures. A total of 168 and 360 degrees of freedom, respectively, are used in these two cases. Moreover, far fewer moving frames need to be updated when only 4 substructures are present instead of 20 substructures.

### Wind Turbine Blade Modelled by Two Substructures

In this section two substructures are used to model the blade. For both substructures the updating algorithm based on the position of the nodes at the ends of each substructure is used. It is examined how the best results are obtained by splitting the blade into the two substructures at different nodes throughout the blade. Because at least one element is necessary in each substructure it can not be split at node 1 and node 21. The results of the tip position by splitting the blade into two substructures at different nodes are shown in Figure 3. Here, the best results are obtained by

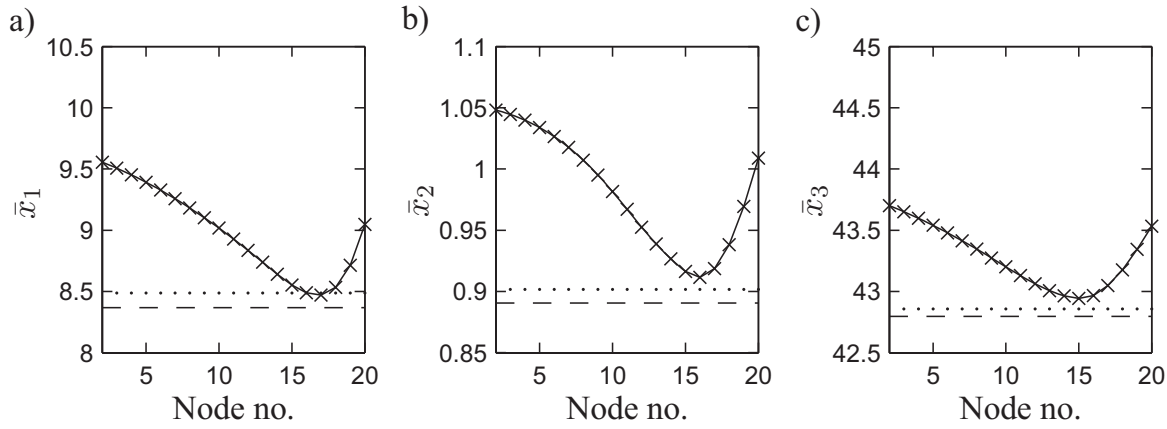


Figure 3: Position of blade tip when split into 2 substructures at different nodes throughout the blade. Both moving frames are updated based on the end nodes in the respective substructure. a) Tip position in  $\bar{x}_1$ . b) Tip position in  $\bar{x}_2$ . c) Tip position in  $\bar{x}_3$ . ( — ) 2 substructures. ( ..... ) 4 substructures of equal reference length. ( — — ) Co-rotating formulation by use of 20 elements.

splitting the blade into two substructures at node 16. It is also shown that the results by use of these two substructures are almost identical to the co-rotating formulation and the case where four substructures of equal reference length are used.

### Concluding Remarks

It can be concluded that the updating methods for the present multibody formulation and the co-rotating formulation both converge towards the same results. It is demonstrated that by updating the moving frame based on the motion of the end nodes in the substructure is far superior to just using the node at the origin of the substructure. For the clamped wind turbine blade it is demonstrated that by use of two substructures of unequal reference length makes it possible to absorb the non-linearities in an efficient way, which otherwise would require four substructures of equal reference length.

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