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Predicting Statistical Distributions of Footbridge Vibrations

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Summary The paper considers vibration response of footbridges to pedestrian loading. Employing Newmark and Monte Carlo simulation methods, a statistical distribution of bridge vibration levels is calculated modelling walking parameters such as step frequency and stride length as random variables. The importance of modelling stride length and walking speed as random variables is evaluated, and results suggest that it is not necessary to model all parameters stochastically to produce fair response estimates.

Introduction

Vertical vibrations in footbridges generated by pedestrians are of concern as they may reach levels rendering bridges unfit for their intended use [1]. Basically vibrations may be perceived as unacceptable by bridge users. For evaluating the vibration serviceability limit state it is useful to employ numerical methods as, hereby, vibration levels may be predicted already at the design stage. As an add-on to this it is considered sensible to adapt a stochastic approach to modelling some of the walking parameters (parameters of the walking load model). Basically parameters such as step frequency and stride length (step length) may change from one pedestrian to the next and since there are proposals in the literature on the stochastic nature of these walking parameters [2], [3], they may be incorporated into numerical calculations predicting bridge vibrations to pedestrian loading. With this randomness implemented, the results of such calculations provide a statistical distribution of bridge vibration levels, and the paper provides an example of such result. As will be discussed in the paper this is a more refined and useful approach than the approach suggested in some current codes (for the British Standard [4]) addressing the serviceability limit state related to actions of walking.

Generally it might not be necessary to model the stochastic nature of all walking parameters for obtaining a fair estimate of the statistical distribution of bridge vibration levels. As an example illustrating this point, various approaches to modelling walking speed is considered in the paper. The walking speed depends on both step frequency and stride length, which are random variables. However, a deterministic model for walking speed is introduced, and the statistical distribution of bridge vibration levels calculated on this assumption is compared with that obtained by modelling step frequency and stride length as random variables.

In order to examine and illustrate the points outlined above, it is considered sufficient to employ a quite simple bridge model (SDOF model representing the first vertical bending mode of a pin-supported bridge) and to study only the response to single-person pedestrian loading.

Assumptions for the studies and study approaches

Bridge model

The modal characteristics of the bridge considered for the present studies are shown in Table 1.

f	M	ζ
2.00 Hz	39.500 kg	0.3%

Table 1: Dynamic characteristics.

The frequency of the bridge (f) is chosen such that it represents a bridge prone to react lively to actions of walking. The bridge damping ratio (ζ) is quite low, but yet realistic for bridges with low damping. The modal mass (M) is also believed to be quite realistic considering the frequency of the bridge, as is the length of the bridge, L, which is assumed to be 43 m (distance between the two pin supports).

Walking load

For the paper (and as often done for modelling the vertical excitation generated by a pedestrian [5]), the dynamic load acting on the bridge, $f(t)$, is modelled as shown in equation (1).

$$f(t) = \alpha W \cos(2\pi f_s t) \tag{1}$$

It is a harmonic load with a frequency, f_s, representing the step frequency of walking. The step frequency is assumed constant during the locomotion of the pedestrian whilst crossing the bridge. The amplitude of the load is αW, where α is the dynamic load factor where W is the static weight of the pedestrian.

Assuming that the mode space function of the first vertical bending mode of the bridge corresponds to a half-sine, it can be shown that the modal load on the bridge (first vertical bending mode) may be computed using either equation (2) or equation (3):

$$q(t) = \alpha W \cos(2\pi f_s t) \sin(\pi \frac{f_s l_s}{L} t) \tag{2}$$

$$q(t) = \alpha W \cos(2\pi f_s t) \sin(\pi \frac{v}{L} t) \tag{3}$$

In equation (2), l_s is the stride length (or step length) of the pedestrian. For a given pedestrian (bridge crossing) l_s is assumed to be a constant as is the step frequency, f_s. Hereby the walking velocity, v, is also a constant, and it can be computed from the equation $v = f_s l_s$. This relationship is used when setting up equation (2). Equation (3) represents a simplification of the load in which it is assumed that any pedestrian traverses the bridge using a constant and the same walking speed v. Such restricting assumption is not made in the load model in equation (2).

Approaches to predicting bridge response

For simulating load action two different approaches are considered:

Approach 1: In this approach it is assumed that the value of f_s will change from one pedestrian to the next and that also the value of l_s will change from one pedestrian to the next. The two parameters are modelled as independent random variables employing Gaussian distributions.

Proposals as for the mean value, μ, and standard deviation, σ, are available in literature. In this approach walking speed is a random variable. When simulating loads, equation (2) is employed. How α and W are modelled are explained later.

Approach 2: In this approach only f_s is modelled as a random variable (in the same way as in approach 1). For any pedestrian the walking speed, v, is set to 1.413 m/s. This is value in close proximity of 1.5 m/s, which is a figure often referred to as the walking speed associated with normal walk. The value 1.413 m/s is obtained using the equation $v = f_s\, l_s$ employing mean values of the random variables f_s and l_s. When simulating loads, equation (3) is employed. The parameters α and W are modelled in the same way as in approach 1.

Employing Monte Carlo simulation methods and a Newmark time integration scheme, bridge acceleration time-histories for a large amount of simulations (bridge crossings) are calculated; specifically, the vertical bridge acceleration at midspan. For every bridge crossing, the peak acceleration level, a, encountered at this position is extracted and this procedure provides a basis for calculating the probability distribution function for peak accelerations.

Inputs for calculations

Table 2 outlines the mean values (μ) and standard deviations (σ) employed for calculations.

Variable	Unit	μ	σ	Reference
f_s	Hz	1.99	0.173	[2]
l_s	m	0.71	0.071	[3]

Table 2:   Assumptions for walking parameters.

The static weight of the pedestrian, W, is set to 750 N. The dynamic load factor, α, is modelled to be conditioned on f_s, as results of measurements reported in [6] clearly reveal a relationship with f_s. The modelled relationship is:

$$\alpha = af_s^3 + bf_s^2 + cf_s + d \tag{4}$$

where

$$a = -0.2649 \quad b = 1.3208 \quad c = -1.7597 \quad d = 0.7613. \tag{5}$$

In equation (4), the unit Hz is to be used for f_s.

Results

Calculated statistical distributions of peak midspan accelerations are presented in Fig. 1.

From such distribution it may for instance be identified that there is a 5 % probability of reaching vibration levels above 0.54 m/s^2 (using approach 1 for this particular bridge). Hence for 1 out of 20 crossings an acceleration level above 0.54 m/s^2 is expected to occur. If employing the British Standard [4] for computing vertical bridge vibrations generated by a pedestrian, you would by default assume resonant action, and would not be provided with information on the probability of encountering the acceleration level calculated. The strength of modelling the stochastic nature of walking parameters (which is not done in [4]) in thus that it gives a more refined understanding of the likelihood of encountering various bridge vibration levels.
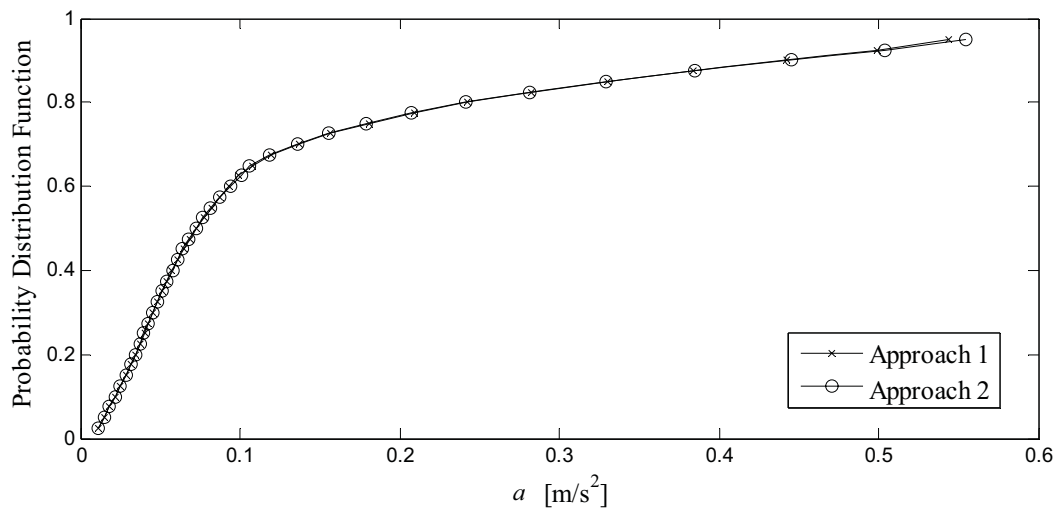
Figure 1: Statistical distributions of bridge vibration levels.

However, it might not be necessary to mode each and every parameter of the load model stochastically for obtaining a fair estimate of the statistical distribution of bridge response. The results in Fig. 1 suggest that the estimate obtained assuming the same walking velocity for every pedestrian provides a result which is in fair agreement with the result obtained when modelling walking speed as a random variable. In some way this suggests that it is not that important to model the stochastic nature of stride length. This conclusion is drawn based on examining only a single bridge (and using a quite simple bridge model). Additionally, multi-person pedestrian loading is a matter of concern, and it is not addressed or considered in this paper. Hence, care should be taken to generalise the conclusion.

Concluding remarks

The paper has illustrated the usefulness of employing a walking load model in which parameters are treated as random variables. Furthermore is has been shown that it is quite likely that not every single parameter of the load model need be modelled stochastically for obtaining a far estimate of the statistical distribution of bridge vibration levels.

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