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# Modelling of Flow around Two Aligned Cylinders

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**Summary** Flow around two cylinders is considered, where closed form solutions are compared to numerical results in order to justify the practical use of the theoretical solutions when the flow in front of the cylinders is analysed. For a relatively highly mutual distance between the cylinders the numerical and analytical results are comparable. Opposite, when the cylinders are closely placed the potential flow solutions become inadequate compared to the numerical findings.

## Introduction

Modelling of flow around cylinders is of paramount importance in many engineering applications. Wind flow around wind turbine towers is one example. The presence of a wind turbine influences the wind flow locally which may develop into a turbulent flow. The turbulence in the wake behind the turbine is significantly different from the turbulence in front of the wind turbine. The wake effects behind the turbine is in particular important for the design of off-shore wind turbine farms [1] (annex D). The wind velocities at the blades depend on the position of the rotor plane i.e. if it is placed upwind or downwind. Tower shadow is another aerodynamic disturbance of the wind flow due to the presence of the tower [2]. During tower passage the blades hit a zone of stagnating and deflected mean wind velocities. The characteristics of the shadow zone is different in case of a downwind and upwind wind turbine. The complexity of these aerodynamic phenomena is further increased if the mono tower is thought replaced by a tripod configuration. Firstly, the question is if the disturbance from the interaction of the three cylinders is significant for the wind velocities. A second question is to what extent the analytical solutions properly estimate the wind velocities when compared to numerical results, where turbulence and wake effects are included. Therefore, in this paper we explore a closed form solution of the wind velocities around two aligned cylinders and compare the results to the wind velocities obtained from a computational fluid dynamic (CFD) model.

## Closed form solution

The wind velocity components upstream in the vicinity of a single cylinder are determined from classical potential flow theory [5]. The wind velocity components for flow around multiple cylinders can be found from [3], where conformal mapping techniques are adopted. Here, we determine the velocity components based on the work by Alassar [4], where the problem is formulated in a bipolar coordinate system $(\xi, \eta)$. The stream function for two cylinders reads

$$\psi = 2bU_0 \left(\psi_x \cos\gamma + \psi_y \sin\gamma\right), \tag{1}$$

where $U_0$ is the mean wind velocity, $b$ is the mutual distance between the cylinders in the bipolar coordinate system and $\gamma$ sets the direction of the incoming wind with respect to the cylinders, see Figure 1. The individual stream functions in equation 1 are

$$\psi_x = -\frac{\sinh(\xi)}{2(\cosh\xi - \cos\eta)} + \sum_{n=1}^{\infty}\frac{\cos(n\eta)\exp(-n\xi_2)\sinh n(\xi-\xi_1)}{\sinh n(\xi_2-\xi_1)} + \sum_{n=1}^{\infty}\frac{\cos(n\eta)\exp(+n\xi_1)\sinh n(\xi-\xi_2)}{\sinh n(\xi_2-\xi_1)} + \sum_{n=1}^{\infty}\frac{\sinh n(\xi_2+\xi_1)}{\sinh n(\xi_2-\xi_1)},$$

$$\psi_y = \frac{\sin(\eta)}{2(\cosh\xi - \cos\eta)} - \sum_{n=1}^{\infty}\frac{\sin(n\eta)\exp(-n\xi_2)\sinh n(\xi-\xi_1)}{\sinh n(\xi_2-\xi_1)} + \sum_{n=1}^{\infty}\frac{\sin(n\eta)\exp(+n\xi_1)\sinh n(\xi-\xi_2)}{\sinh n(\xi_2-\xi_1)},$$

where indices $x$ and $y$ indicate flow parallel and perpendicular to the tripod legs, respectively, see Figure 1(left). In the following equations the radii $R$ of the cylinders are identical. The distance

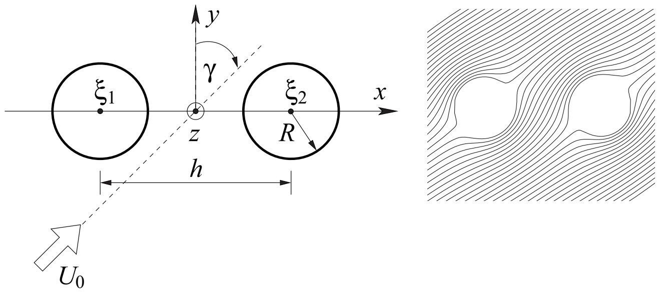


Figure 1: Orientation of free mean wind $U_0$ is controlled by the angle $\gamma$ and the center of the cylinders is expressed in the bipolar coordinates $\xi$, whereas the physical distance between the cylinders is denoted $h$ in Cartesian coordinates and $b$ in the bipolar domain and the stream function $\psi$ from equation (1) with $\gamma = \pi/6$.

between the cylinders in the bipolar coordinate system is defined as $b = R\sinh(\xi_2)$. The coordinate $\xi_2$ express the center of the second cylinder and is determined from $\xi_2 = \mathrm{arccosh}\frac{\mathrm{h}}{2\mathrm{R}}$. in which $h$ is the physical distance between the cylinders and $\xi_1 = -\xi_2$ is the coordinate to the first cylinder. The stream function is illustrated in Figure 1(right), where $\gamma = \pi/6$ rad and $R = 1$ m. The relations between the bipolar and Cartesian coordinates are defined as [6]

$$x = \frac{b\sinh\xi}{\cosh\xi - \cos\eta}, \quad y = \frac{b\sin\eta}{\cosh\xi - \cos\eta}, \tag{2}$$

and after mathematically manipulations the inverse relations can be derived as

$$\xi = \mathrm{arccoth}\left(\frac{\mathrm{b}^2+\mathrm{x}^2+\mathrm{y}^2}{2\mathrm{bx}}\right), \quad \eta = \mathrm{arccot}\left(\frac{-\mathrm{b}^2+\mathrm{x}^2+\mathrm{y}^2}{2\mathrm{by}}\right). \tag{3}$$

The wind velocities are determined from

$$\begin{Bmatrix} \frac{\partial\psi}{\partial x} \\ \\ \frac{\partial\psi}{\partial y} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial\xi} & \frac{\partial y}{\partial\xi} \\ \\ \frac{\partial x}{\partial\eta} & \frac{\partial x}{\partial\eta} \end{bmatrix}^{-1} \begin{Bmatrix} \frac{\partial\psi}{\partial\xi} \\ \\ \frac{\partial\psi}{\partial\eta} \end{Bmatrix}. \tag{4}$$

### Numerical solution

The flow field is modelled with the incompressible Reynolds Averaged Navier-Stokes (RANS) equations with the $k - \epsilon$ turbulence model in a transient solution in ANSYS Flotran. A Courant number lower than one is ensured with a time step size $\Delta t = 0.05$ s, minimum element size 0.12 m and a maximum wind velocity 2 m/s.

### Example with flow perpendicular to the cylinders ($\gamma = 0$)

The control domain is $(x, y) = [-15, 15]$ m and the centers of the two cylinders with radii $R = 1$ m are placed at the line $y = 0$ with a mutual distance $h = [3\,6\,9]$ m. The free mean wind velocity is $U_0 = 1$ m/s. The Reynolds number is $\text{Re} \approx 1.3 \cdot 10^6$ with a kinematic viscosity $\nu = 15.0 \cdot 10^{-6}$ m$^2$/s, which means the separation at the boundary layer remains laminar with a vortex shedding behind the cylinders. The theoretical and numerical wind velocity component

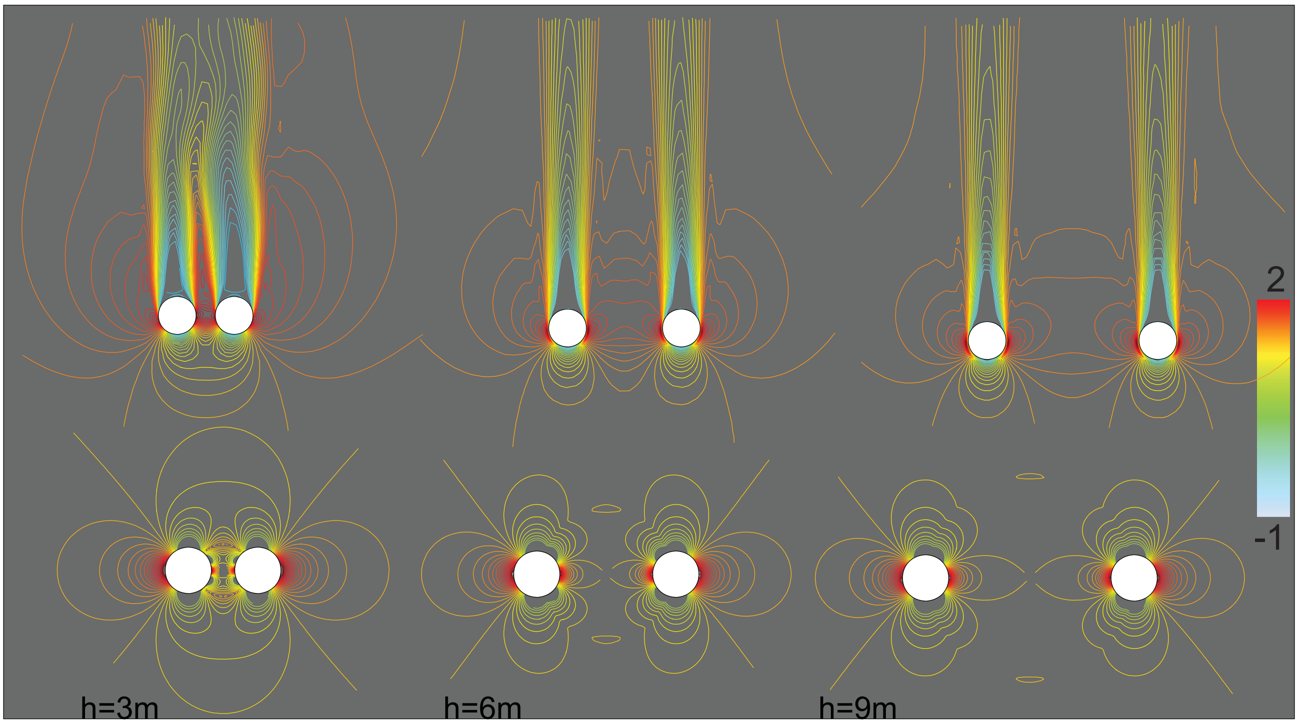


Figure 2: Contour plot of velocity component $U_y$ [m/s] obtained from CFD computations (top) and analytical solution (bottom) with $\gamma = 0$ and different distances $h$ between the cylinders.

$U_y(x, y)$ for $y = [-2, 2]$ m are presented in Figure 3, whereas Figure 2 shows the contour plots of $U_y$. We observe that the shadow zone (decreased velocity) is increased as the mutual distance of the cylinders is decreased. The potential flow theory underestimates the wind velocity in between the cylinders when compared to the numerical findings. Locally in front of the cylinders and at the exterior sides the velocity profiles are fairly comparable. However, behind the cylinders the potential flow theory is invalid as expected and negative velocities are found due to wake effects in the numerical simulations.

### Concluding remarks

Wind flow around two aligned cylinders with arbitrary distance is considered. The purpose is to justify the use of potential flow theory to determine the wind velocities in front of the cylinders

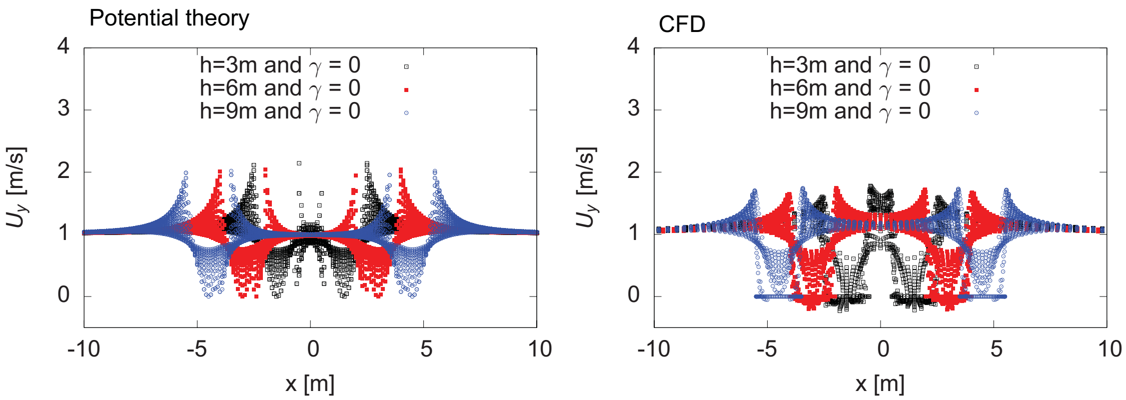


Figure 3: Wind velocity $U_y(x, y)$ ($y = [-2, 2]$ m) for $\gamma = 0$ coordinate and the distance $h$ between the cylinders.

when compared to more realistic CFD computations.

The potential flow theory predicts the velocities well in front of the cylinders when compared to CFD simulations with turbulence included. However, in between the cylinders the theoretical velocities become inadequate when the mutual distance is lower than 6 m. Clearly, behind the cylinders the potential flow theory is invalid. Therefore, in case of a down wind turbine with a tripod tower configuration [2], the idea is to tabulate the wind velocities obtained from various CFD computations with flow passing multiple cylinders from different directions. With this database, more realistic wind velocities are imposed to the wind turbine blade and an accurate estimate of the fatigue life is possible.

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