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ON THE FICTITIOUS CRACK MODEL OF CONCRETE FRACTURE

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On the Fictitious Crack Model of Concrete Fracture.

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ABSTRACT.

In this paper the sub-structure method introduced by Petersson [4] is reformulated for the three point bending specimen in order to show how complete load-displacement relations without truncation of significance can be obtained. The problem of instability caused by the linearization of the softening in the fracture zone is discussed, and an alternative energy formulation is given and makes it possible to distinguish between stable and unstable situations. The reformulated sub-structure method is implemented on computer in a way that makes it possible to use a multilinear stress-crack-opening-displacement relation for the material in the fracture zone, and some qualitative results are given.

1. INTRODUCTION.

The fracture mechanical properties of concrete in tension has been an important subject in concrete research during the last decade. The fracture properties have been studied both experimentally and theoretically by using fracture mechanical concepts.

Several models based on fracture mechanical ideas have been established to describe the fracture of concrete in tension, among those the Fictitious Crack Model (FC-Model) formulated by Hillerborg and co-workers [1], [2], [3], [4], [6] and [7] is one of the most well-known.

In the FC-Model material point on the crack extension path is assumed to be in one of three possible states, an undisturbed elastic state (no fracture, no lack of compatibility), a fracture state where the material is softened by microcracking (the fictitious crack), and a state of no stress transmission, the point lies on a free surface.

The elastic state of all points in the body excluding those in the FC-zone is described by the linear theory of elasticity. The separation of points in the FC-zone is described by a special constitutive relation, the so-called stress-crack-opening-displacement relation (σ-w relation) given by the function $f(.)$ defined in figure 1.

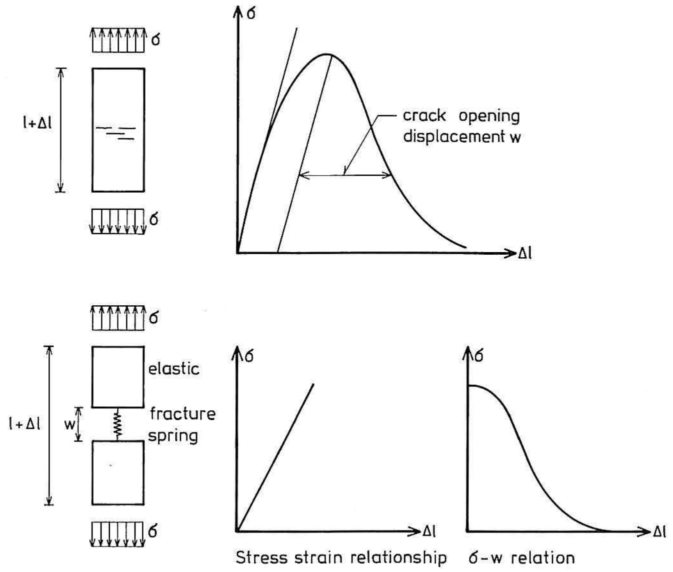

Figure 1. The stress-crack-opening-displacement relation for the material in the FC-zone.

Petersson [4] implemented the FC-model on computer using the so-called sub-structure method. He virtually cut the body partially through along the crack extension path, discretized the problem by defining a finite number of points (the nodes) in which he satisfied the compatibility conditions in the elastic part of the body, and the fracture conditions in the FC-zone, expressing the conditions by a set of linear equations. However, he did not virtually cut the body into two pieces allowing the crack to extend to ultimate fracture. This leads to a significant truncation of the calculated force-displacement relation which is difficult to remove without introducing an unacceptable large number of nodes.

In paragraph 2 it will be shown how the problem af truncation of the force-displacement relation can be overcomed by virtually cutting the body into two sub-bodies using a displacement boundary value technique to solve the problem.

In paragraph 3 an alternative method based on minimizing the total potential energy of the system is given leading to a formulation where it is possible in a simple way to check for stability of the system in each incremental step of the calculation of the force-displacement relation. Finally in paragraph 4 the reformulated sub-structure method is implemented on computer in a way that makes it possible to use a multilinear σ-w relation in the FC-zone, and some qualitative results are given.

2. THE DIRECT SUB-STRUCTURE METHOD.

A simply supported beam with length l width d and height h loaded by a force F_1 in the middle like the standard test specimen proposed by RILEM [8] is considered, see figure 2.a. A crack with length a is assumed to be present in the tension side of the beam just beneath the applied load. When the load is applied on the beam, the crack extends, and a fracture zone with length c is developed in front of the crack tip.

Instead of doing like Petersson [4], who solved the problem like a contact problem cutting the beam partially through at the mid-section, the beam is departed into two separate sub-bodies or sub-structures, A and B as shown in figure 2.b. The original problem is a stress boundary value problem, since the boundary conditions are given in terms of stresses, but the new problem formed by separating the original structure into two sub-structures A and B cannot be formulated as a stress boundary value problem since the two sub-structures are geometrically indeterminated. Instead of the stress condition given by F_1 a displacement condition δ_1 is applied at the middle, and a displacement condition δ_2 is applied at the right hand support. In this formulation δ_1 is given beforehand and the load F_1 and the displacement δ_2 are unknowns that are to be determined by the analysis. In this case the stress solution will be symmetric, and advantage could be taken of that by considering only one of the sub-structures. However, in order to illustrate the applicability of this method on non-symmetric problems the whole body will be considered.

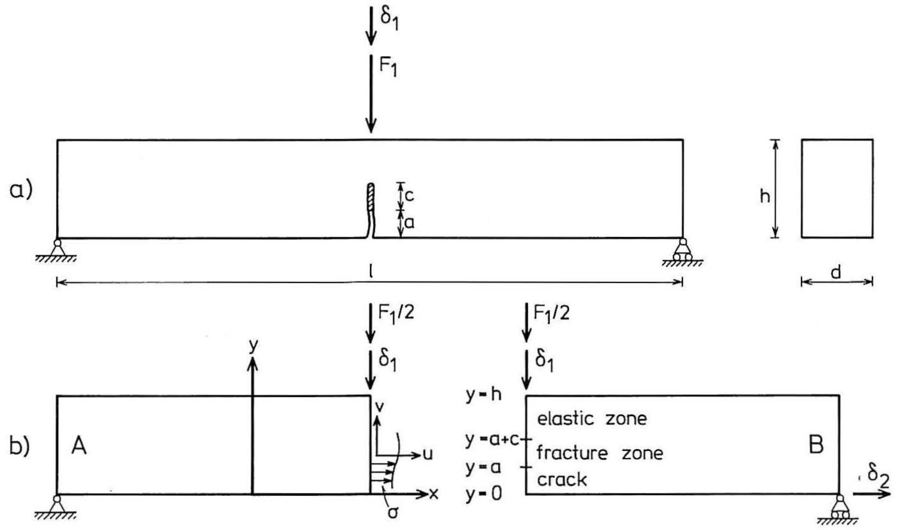

Figure 2. 2.a: The considered beam. 2.b: Part A and B.

2.1. Integral Equations.

Points and displacements are measured in the (x, y) and (u, v) coordinate systems respectively, see figure 2.b, the x-axis lies underneath the beam. Assuming small displacements the rigid-body displacements can be expressed as

$$u_0^A = 2\frac{\delta_1}{l}y \; ; \quad u_0^B = \delta_2 - 2\frac{\delta_1}{l}y \tag{1}$$

where u_0^A and u_0^B are the displacements on the virtual surfaces created by departing the original body into two new bodies. Let some stresses $\sigma(y)$ be applied on the virtual surfaces. The displacements u_σ^A and u_σ^B caused by these stresses are given by

$$\begin{aligned} u_\sigma^A(y) &= \int_a^h \sigma(y')g(y,y')dy' \\ u_\sigma^B(y) &= -\int_a^h \sigma(y')g(y,y')dy' \; ; \quad 0 \le y \le h \end{aligned} \tag{2}$$

where $g(y, y')$ is Greens function for the considered displacements. The total displacement fields of the two bodies A and B are then given by

$$u^A = u_\sigma^A + u_0^A \ ; \ \ u^B = u_\sigma^B + u_0^B \tag{3}$$

which yield the crack-opening-displacement

$$\begin{aligned} w(y) &= u^B - u^A \\ &= -2\int_a^h \sigma(y')g(y,y')dy' + \delta_2 - 4\frac{\delta_1}{l}y \end{aligned} \tag{4}$$

The conditions we have to satisfy are the compatibility condition

$$\begin{aligned} w(y) = 0 \ \Rightarrow \\ 2\int_a^h \sigma(y')g(y,y')dy' - \delta_2 + 4\frac{\delta_1}{l} = 0 \ ; \ \ a + c \le y \le h \end{aligned} \tag{5}$$

the fracture condition

$$\begin{aligned} f(w(y)) = \sigma(y) \ \Rightarrow \\ f(-2\int_a^h \sigma(y')g(y,y')dy' + \delta_2 - 4\frac{\delta_1}{l}y) = \sigma(y) \ ; \ \ a \le y \le a + c \end{aligned} \tag{6}$$

which determines the unknown stresses $\sigma(y)$, and finally the equilibrium condition

$$\int_a^h \sigma(y)dy = 0 \tag{7}$$

which determines the unknown displacement δ_2.

2.2. The System of Linear Equations.

The first step in setting up the system of linear equations is to discretize the integral equations, i.e. the coordinate y is restricted to attain the discrete values y_i, $i = 1, 2, ..., n$, and consequently the stresses $\sigma(y)$ and Greens function $g(y, y')$ are expressed in terms of the node forces s_i and the Greens matrix $g_{ij} = g(y_i, y_j)$ respectively. The spacing a_0 between nodes is assumed to be constant.

The conditions (5), (6), and (7) can now be expressed as the sums

$$\sum_{j=k}^{n} g_{ij}s_j - \frac{\delta_2}{2} + 2\frac{\delta_1}{l}y_i = 0 \; ; \quad m \le i \le n \tag{8}$$

$$f(-2\sum_{j=k}^{n} g_{ij}s_j + \delta_2 - 4\frac{\delta_1}{l}y_i) = s_i \; ; \quad k \le i < m \tag{9}$$

$$\sum_{j=k}^{n} s_j = 0 \; ; \quad k \le i \le n \tag{10}$$

where k is the first node of the fracture zone, $y_k = a$, and m is the first node of the elastic zone, $y_m = a + c$. Taking $f(.)$ as a linear function

$$f(w) = f_0 + \alpha w \; ; \quad \alpha \le 0 \tag{11}$$

equation (9) yields

$$f_0 - 2\alpha\sum_{j=k}^{n} g_{ij}s_j + \alpha\delta_2 - 4\alpha\frac{\delta_1}{l}y_i - s_i = 0 \; ; \quad k \le i < m \tag{12}$$

If $f(.)$ is taken not as a linear function, but as a piecewise linear function to approximate a more general σ-w relation, the problem can still be expressed as a system of linear equations formed by equation (8), (10), and (12). In this case different α-values have to be used for the nodes according to where they are situated on the f-curve. The system of linear equations can then be written

$$\overline{\overline{A}}\,\overline{x} = \overline{b} \tag{13}$$

where the coefficient matrix $\overline{\overline{A}}$ is given by

$$\overline{\overline{A}} = \begin{pmatrix} 2\alpha_k g_{k,k}+1 & 2\alpha_k g_{k,k+1} & \cdot\;\cdot & 2\alpha_k g_{k,n} & -\alpha_k \\ 2\alpha_{k+1} g_{k+1,k} & 2\alpha_{k+1} g_{k+1,k+1}+1 & \cdot\;\cdot & 2\alpha_{k+1} g_{k+1,n} & -\alpha_{k+1} \\ \cdot & & & & \\ \cdot & & & & \\ 2\alpha_{m-1} g_{m-1,k} & 2\alpha_{m-1} g_{m-1,k+1} & \cdot\;\cdot & 2\alpha_{m-1} g_{m-1,n} & -\alpha_{m-1} \\ -g_{m,k} & -g_{m,k+1} & \cdot\;\cdot & -g_{m,n} & 0.5 \\ \cdot & & & & \\ \cdot & & & & \\ -g_{n,k} & -g_{n,k+1} & \cdot\;\cdot & -g_{n,n} & 0.5 \\ 1 & 1 & \cdot\;\cdot & 1 & 0 \end{pmatrix} \tag{14}$$

and where $\overline{x}$ and $\overline{b}$ are given by

$$\overline{x} = \begin{pmatrix} s_k \\ s_{k+1} \\ \cdot \\ \cdot \\ s_{m-1} \\ s_m \\ \cdot \\ \cdot \\ s_n \\ \delta_2 \end{pmatrix} \; ; \; \overline{b} = \begin{pmatrix} f_{0,k} - 4\alpha_k \frac{\delta_1}{l} y_k \\ f_{0,k+1} - 4\alpha_{k+1} \frac{\delta_1}{l} y_{k+1} \\ \cdot \\ \cdot \\ f_{0,m-1} - 4\alpha_{m-1} \frac{\delta_1}{l} y_{m-1} \\ 2\frac{\delta_1}{l} y_m \\ \cdot \\ \cdot \\ 2\frac{\delta_1}{l} y_n \\ 0 \end{pmatrix} \tag{15}$$

The non-linearity of the problem is then introduced by updating the coefficient matrix $\overline{\overline{A}}$ and the right-hand side $\overline{b}$ according to the movements of the nodes on the f-curve. The problem of keeping track of where the nodes are situated, and the problem of updating $\overline{\overline{A}}$ and $\overline{b}$ are treated in paragraph 4. It must be noted, however, that eq. (11) expresses the crack-opening-displacement relations for the node forces and not for the stresses. This means that the constitutive parameters f_0 and α for the first node $i = 1$ and the last node $i = n$ have to be multiplied by a factor 0.5 to correct for the smaller areas corresponding to these nodes.

When the node-forces are determined by solving the system of linear equations described above, the total force F_1 is obtained from the equilibrium condition

$$\frac{1}{4} F_1 l + \sum_{j=k}^{n} s_j y_j = 0 \tag{16}$$

and the crack-opening-displacements w_i are given by

$$w_i = -2 \sum_{j=k}^{n} g_{ij} s_j + \delta_2 - 4\frac{\delta_1}{l} y_i \; ; \quad k \leq i < m \tag{17}$$

3. THE ENERGY SUB-STRUCTURE METHOD.

In the preceeding section a system of linear equations was set up by forming a coefficient matrix $\overline{\overline{A}}$ containing essentially the Greens coefficients g_{ij} and the discrete spring constants α_i describing the local proporties of the crack-opening-displacement relation. In this case where the spring stiffnesses α are less than zero, it might occour that the total energy becomes non positive definite i.e., the solutions found by solving the system (13) do not correspond to a true minimum for the potential energy. In this case the system is unstable. The standard way to check for positive definiteness of the system is to require for all $\overline{x}$ that $\overline{x}^t\overline{\overline{A}}\overline{x} \geq 0$, i.e. the matrix $\overline{\overline{A}}$ has to be positive definite. However, this is only meaningful if $\overline{x}^t\overline{\overline{A}}\,\overline{x}$ can be interpretated as the quadratic part of the total potential energy of the system. In this case, where the sequence of the equations (rows can be interchanged) and the sign of the coefficients of a given row is arbitrary, this is clearly not so. Using the formulation given above there is no simple way to check for positive definiteness of the energy. However, if the signs of the matrix coefficients are chosen as shown in eq. (14) then the sign of the determinant, $det\overline{\overline{A}}$, has to be unchanged through all fracture states. This means that if the determinant $det\overline{\overline{A}}$ changes sign then the energy is no longer positive definite. However, the constant sign of the determinant is only a necessary but not sufficient requirement for the energy to be positive definite.

If a safe way to check for positive definiteness of the energy is neaded, another formulation has to be given. One method is to express the total potential energy of the system and then obtain the solution by requiring that the potential energy is minimum. In that case a system of linear equations is obtained for which we only have to require that the corresponding system matrix is positive definite. Here we will not consider the whole beam, but take advantage of the symmetry, and therefore only part A of the original beam is considered as shown in figure 3.

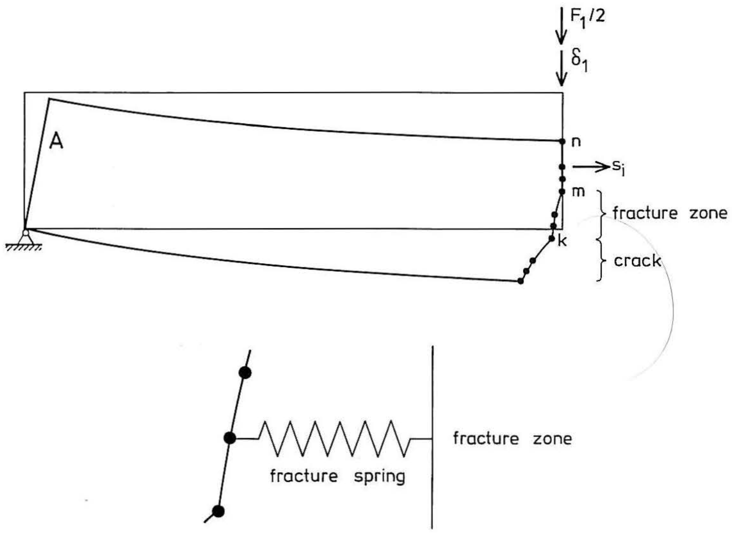

Figure 3. The beam considered for the energy analysis.

The total potential energy can be written as

$$U = U_F + U_e + U_s \tag{18}$$

where U_F is the the potential of the load F_1, U_e is the strain energy of the body, and U_s is the strain energy of the springs describing the crack-opening in the fracture zone. Using the equilibrium equation (16) the potential of the load F_1 is given by

$$\begin{aligned} U_F &= -\frac{1}{2}F_1\delta_1 \\ &= 2\frac{\delta_1}{l}\sum_{j=k}^{n} s_j y_j \end{aligned} \tag{19}$$

and the strain energy of the body is

$$U_e = \frac{1}{2}\sum_{i=k}^{n} s_i \sum_{j=k}^{n} g_{ij} s_j \tag{20}$$

The strain energy of the fracture springs is slightly more difficult to determine. Eq. (11) can be written

$$s_i = \alpha_i(w_i - w_{0i}) \; ; \quad w_{0i} = -\frac{f_{0i}}{\alpha_i} \tag{21}$$

which yields the strain energy

$$U_s = \frac{1}{2}\sum_{j=k}^{m-1} \alpha_j(w_j - w_{0j})^2 \tag{22}$$

Now substituting the expressions for w_j and w_{0j} into eq. (22) yields

$$U_s = \frac{1}{2}\sum_{i=k}^{m-1} \alpha_i(\sum_{j=k}^{n} g_{ij}s_j + \frac{f_{0i}}{\alpha_i})^2 \tag{23}$$

and the total potential energy then becomes

$$U = 2\frac{\delta_1}{l}\sum_{i=k}^{n} s_i y_i + \frac{1}{2}\sum_{i=k}^{n} s_i \sum_{j=k}^{n} g_{ij}s_j + \frac{1}{2}\sum_{i=k}^{m-1} \alpha_i(\sum_{j=k}^{n} g_{ij}s_j + \frac{f_{0i}}{\alpha_i})^2 \tag{24}$$

The system of linear equations is now obtained by requiring that $\partial U/\partial s_q = 0$ for $k \leq q \leq n$

$$\begin{aligned} 2\frac{\delta_1}{l}y_q + \sum_{j=k}^{n} g_{qj}s_j + \sum_{i=k}^{m-1} \alpha_i(\sum_{j=k}^{n} g_{ij}s_j + \frac{f_{0i}}{\alpha_i})g_{iq} = 0 \; ; \quad k \leq q < m \\ 2\frac{\delta_1}{l}y_q + \sum_{j=k}^{n} g_{qj}s_j = 0 \; ; \quad m \leq q \leq n \end{aligned} \tag{25}$$

which yield the matrix equation

$$\overline{\overline{C}}\,\overline{s} = \overline{d} \tag{26}$$

where the vector $\overline{s}$ contains the node forces $s_i, i = 1, 2, ...n$, and where the matrix elements c_{ij} and the elements of the left hand side d_i are given by

$$c_{ij} = \begin{cases} g_{i+k,j+k} + \sum_{p=k}^{m-1} g_{i+k,p}\alpha_p g_{p,j+k} \; ; & 0 \leq i, j < m-k \\ g_{i+k,j+k} \; ; & m-k \leq i, j < n-k \end{cases} \tag{27}$$

and

$$d_i = \begin{cases} -2\frac{\delta_1}{l}y_i - \sum_{j=k}^{m-1} g_{ij} f_{0i} \; ; & 0 \leq i < m-k \\ -2\frac{\delta_1}{l}y_i \; ; & m-k \leq i < n-k \end{cases} \tag{28}$$

Here the matrix elements c_{ij} and the elements of the right hand side d_i are numbered c_{00}, c_{01}, ... and d_0, d_1, ... respectively. It is easy to see that the flexibility matrix $\overline{\overline{C}}$ is symmetric and that $\overline{s}^t\overline{\overline{C}}\overline{s}$ represents the quadratic part of the potential energy. Therefore using the energy sub-structure method one only have to show that the flexibility matrix $\overline{\overline{C}}$ is positive definite to be sure that the system is stable.

It should be noted that the fracture spring stiffness coefficients α_i must be multiplied by an additional factor 2 since the length of the fracture springs in this formulation is only half the real length.

4. IMPLEMENTATION.

Only the direct sub-structure method has been implemented, but it might as well have been the energy sub-structure method.

In the following the incremental step used for calculation of a typical point on the load-deflection relation for the considered beam will be explained. The points in the FC-zone are all lying on the σ-w relation as shown in figure 4, denoting the node forces and node crack-opening-displacements by s_j and w_j respectively, $k \leq j < m$.

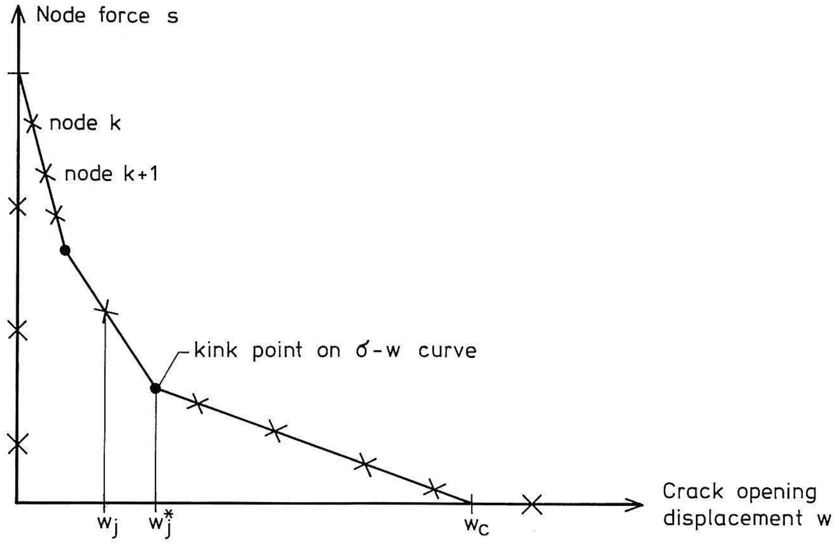

Figure 4. The location of the nodes on the stress-crack-opening relation.

The parameter controlling the problem is, as mentioned earlier, given by the beam deflection δ_1. If the beam deflection is increased, all points lying on the σ-w relation will be moving to the right, i.e towards larger w_j values. The problem is to determine which point is the first to reach a kink on the σ-w relation, because when a point is crossing a kink point on the multilinear σ-w relation, the system matrix and the right hand side of the system of linear equations, see eq. (14) and (15), have to be updated, and the system is said to change the state of fracture.

Each of the points lying on the fracture relation is having a nearest kink point when the point is moving to the right on the curve. The crack-opening-displacements of these nearest kink points are denoted w_j^*, see figure 4. In order to determine which point is the first to cross a kink on the fracture relation, the beam deflection is given a small increment $d\delta_1$, and the corresponding increments dw_j for all the nodes lying on the fracture relation are determined by solving the system of linear equations using the system matrix and right hand side corresponding to the present state of fracture. Then the sensitivities

$$\mu_j = \frac{dw_j}{w_j^* - w_j} \tag{29}$$

can be calculated. For the first elastic node, $j = k$, the sensitivity is calculated in a similar way using node forces in stead of crack-opening-displacements

$$\mu_k = \frac{ds_k}{s_k^* - s_k} \tag{29}$$

Here the kink point corresponds to the beginning of the fracture relation, and s_k^* is therefore equal to the tension strength of the nodes. The point j^* having the largest sensitivity μ^* is then moved to the nearest kink point by solving the system of linear equations using the beam deflection increment

$$\Delta\delta_1^* = \frac{d\delta_1}{\mu^*} \tag{30}$$

and using the system matrix and right hand side for the present state of fracture. Then the system matrix and right hand side are updated, and the next point on the force displacement curve can be calculated. If the crack opening displacement for a node exceeds the ultimate crackwidth w_c, the node is removed from the set of nodes in the fracture zone, i.e. there is a real crack and no stress can be transmitted.

This procedure continues until there is only one elastic node left and that node is the next to be moved into the fracture zone. The algorithm outlined above was implemented on a Personal Computer, and some qualitative relationships were investigated.

5. RESULTS.

Two problems were investigated; the problem of sensitivity to the number of nodes across the beam section, and the problem of sensitivity to the degree of approximation for the stress-crack-opening-displacement relation.

The sensitivity problems were analysed using rather small size beams. The geometry of the beams and the material properties are listed in table 1.

Height (mm)	80
Width (mm)	40
Length (mm)	400
Modulus of Elasticity E $(\frac{N}{mm^2})$	32,550
Fracture energy G_f $(\frac{N}{m})$	109.6
Tension strength f_t $(\frac{N}{mm^2})$	2.86

Table 1. Geometry and material proporties for analysed beams.

The material properties are taken as the average properties for the concretes tested by Wolinski et al. [5]. The stress-crack-opening relation measured by Wolinski et al. is shown in figure 5 together with the approximations used in the sensitivity analysis.

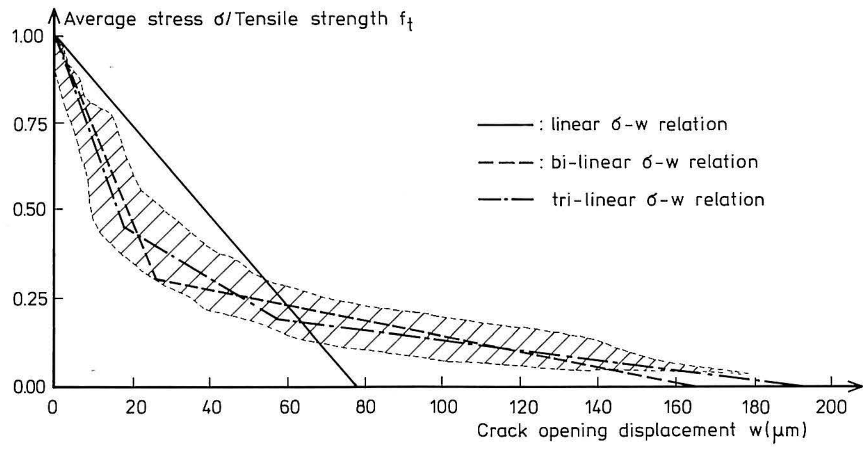

Figure 5. The stress-crack-opening relation, taken from [5].

The reason for analysing small size beams is the problems of stability discussed in paragraph 3. Because of the steep fracture relation shown in figure 5, it was necessary to use small beams in order to ensure that all the cases could be analysed on the same beam size. It is not difficult to see that the problem of stability increases with the size of the beam.

First it is clear that the steeper the fracture relation the larger the risk for facing the stability problem. The dependence on the size of the beam can be obtained by simple dimensional analysis. The physical quantities influencing the stability of the beam are assumed to be the size l, the tension strength f_t, the elastic properties given by g_{ij} and the fracture properties of the material in the fracture zone described by f_0 and α. Accepting these assumptions it is easy to see, that the problem is described by three dimensionless products, and that these could be chosen as

$$\pi_1 = g_{ij} f_t l \ ; \quad \pi_2 = \frac{\alpha l}{f_t} \ ; \quad \pi_3 = \frac{f_0}{f_t} \tag{31}$$

Now let us consider another beam with the corresponding quantities l', f_t', g_{ij}', α', f_0'. From the model invariance of the π-products it is seen that the similitude requirements for the beam in the case of same tension strength, $f_t' = f_t$ are given by

$$\begin{aligned} f_0' &= f_0 \\ g_{ij}' &= g_{ij} \frac{l}{l'} \\ \alpha' &= \alpha \frac{l}{l'} \end{aligned} \tag{32}$$

From these results it is seen that if $l' > l$ then the slope on the fracture curve α' must decrease, i.e. the material has to be tougher in order to ensure the same behaviour of the two considered beams, see figure 6.

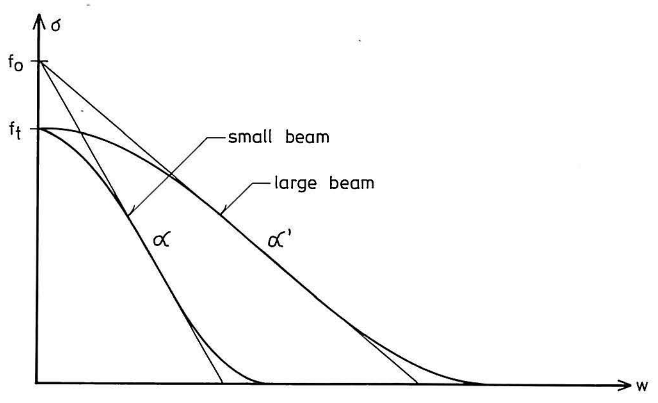

Figure 6. The fracture model law.

This means, that if the fracture proporties of the material are kept constantly and the beam size is decreased then the risk for facing the stability problem becomes smaller.

In the investigation of sensitivity to the number of nodes, the cross section was divided into 5, 10, 15 and 19 nodes, and the influence coefficients g_{ij} were determined by linear finite element analysis using four-node elements (LST-elements). The load-displacement relations were calculated using a linear stress-crack-opening-displacement relation and the results for the four cases are shown in figure 7.

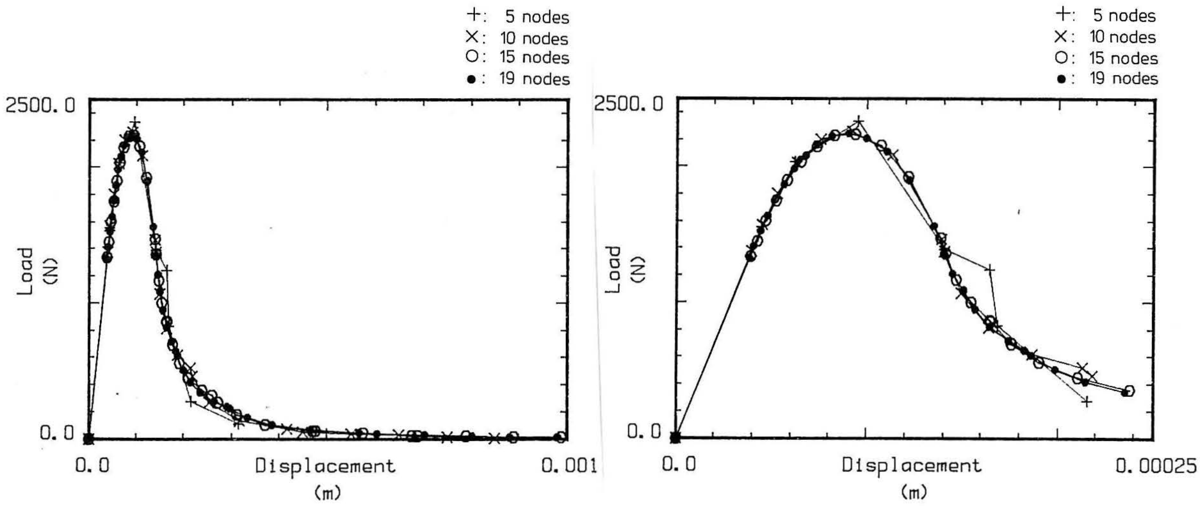

Figure 7. Sensitivity to the number of nodes.

From these results it is seen that only the curve for the rough mesh containing only 5 nodes differs significantly from the others indicating that the method is not very sensitive to the chosen number of nodes.

Three different approximations have been used to evaluate how sensitive the method is to the degree of approximation of the stress-crack-opening-displacement relation. The measured σ-w relation and the three different approximations are shown in figure 5. The results from the sensitivity analysis are shown in figure 8.

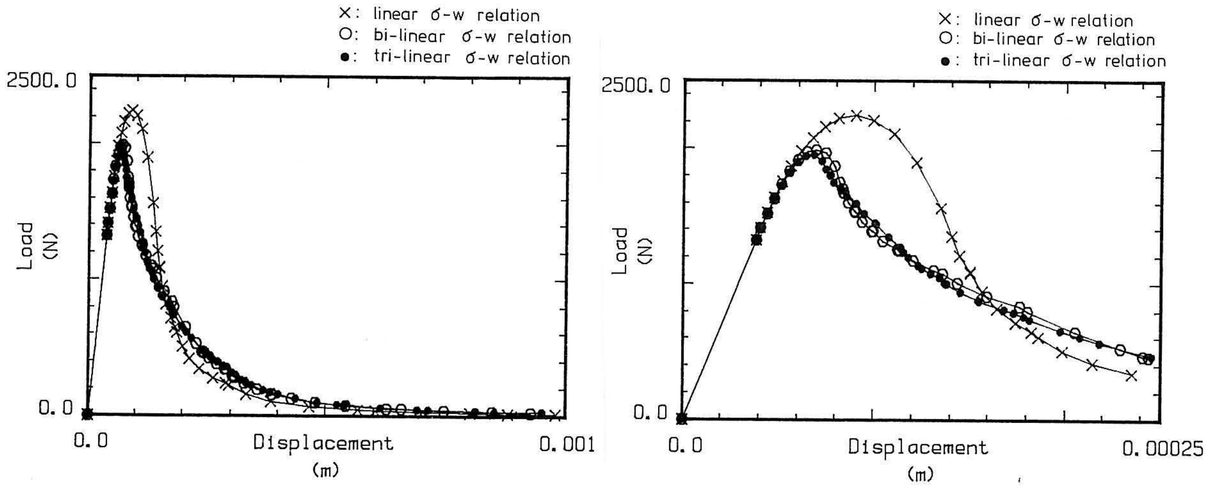

Figure 8. Sensitivity to the degree of approximation of the stress-crack-opening-displacement relation.

It is evident that the shape of the stress-crack-opening-displacement relation has great influence on the results given by the load displacement curve. Improving the approximation from a linear (no kinks) to a bilinear (one kink) stress-crack-opening-displacement relation causes a drop in the calculated ultimate load of approximately 10 %, and the shape of the load displacement relation is changed significantly. The approximation by a trilinear (two kinks) fracture relation, however, does not seem to change the results significantly indicating the sufficiency of the bilinear approximation.

5. CONCLUSION.

On the basis of the experience with the reformulated sub-structure method the following conclusions can be drawn:

1. The method is able to simulate crack growth far beyond the limits of the known sub-structure method revealing results without truncations of significance on the load displacement relation.

2. The results are not very dependent upon the number of nodes in the crack extension path, and a relatively rough discretizing can be used.

3. It is important for the results that the shape of the stress-crack-opening relation is modelled approximately correct. However, it is not necessary to use a multilinear relation. A bilinear relation seems to be sufficient.

4. It is a serious problem using the sub-structure method that the system too easy becomes unstable due to the simple local linearization of the stress-crack-opening-displacement relation. This is a problem that has to be dealt with in future research.

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