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# Discussion: “Model reduction of large-scale discrete plants with specified frequency-domain balanced structure”

(A. Zadegan and A. Zilouchian, *ASME J. Dyn. Sys. Meas. Contr.* , Volume 127, Issue 3, pp. 486-498, 2005)

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**Abstract:** This work presents a commentary of the article published by A. Zadegan and A. Zilouchian (*ASME J. Dyn. Sys. Meas. Contr.* , Volume 127, Issue 3, pp. 486-498, 2005). We show their order reduction method is not always true and may leads to inaccurate results and is therefore erroneous. A framework for solving the problem is also suggested .

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## 1. DISCUSSION

Model reduction of systems with specified frequency domain balanced structure is a reduction technique which is an attempt for increasing the accuracy of approximation by looking at reduction problem within specified frequency bound instead of the whole frequency domain. In this method it is not required to keep the approximation good outside the specified frequency bound of operation, the accuracy of approximation can be increased comparing to approximation results by applying well-known ordinary balanced reduction method. In this method continues time controllability and observability Grammians in terms of  $\omega$  over a frequency bound  $[\omega_1, \omega_2]$  are defined as[1-7]:

$$\begin{aligned} W_{cf} &\triangleq \frac{1}{2\pi} \int_{\omega_2}^{\omega_1} (Ij\omega - A)^{-1} B B^* (-Ij\omega - A^*)^{-1} d\omega \\ W_{of} &\triangleq \frac{1}{2\pi} \int_{\omega_2}^{\omega_1} (-Ij\omega - A^*)^{-1} C^* C (Ij\omega - A)^{-1} d\omega \end{aligned} \quad (1)$$

Similarly, for discrete time cases, Gramminans are defined as[1-7]:

$$\begin{aligned}
W_{cf} &\triangleq \frac{1}{2\pi} \int_{w_2}^{w_1} (Ie^{jw} - A)^{-1} B B^* (Ie^{-jw} - A^*)^{-1} dw \\
W_{of} &\triangleq \frac{1}{2\pi} \int_{w_2}^{w_1} (Ie^{-jw} - A^*)^{-1} C^* C (Ie^{jw} - A)^{-1} dw
\end{aligned} \tag{2}$$

This model reduction technique is based on ordinary balanced model reduction method that first proposed by B. C. Moore [8] and then improved and developed in different directions [10]. The philosophy of the model reduction method proposed by Zadegan [1-7] is very similar to the one presented by Enns [9], but it is not always true and may leads to inaccurate results. In what follows we discuss the problem of the method in more details.

In the first step of the aforementioned model reduction technique the original system should be transformed to the specified frequency domain balanced structure i.e. the controllability and observability Grammians of the transformed system should be equal and diagonal. The second step of the reduction procedure consists of partitioning and applying the generalized singular perturbation approximation to the system with specified frequency domain balanced structure.

The problem which arises in the practical implementation of the reduction technique is the infeasibility of the balancing algorithms for finding an appropriate similarity transform which should transform the original system into the frequency domain balanced structure. In order to find an appropriate similarity transform the authors of [1,2,7], have suggested to use one of the well-known numerical algorithms which was proposed by Laub for the first time[7]. In this algorithm we should apply the Cholesky factorization to the Grammians obtained from (1) or (2). Because the aforementioned Grammians are not real, we can not apply the Cholesky factorization and the overall Laub algorithm is not applicable then. If we use  $W_{cf} + \text{Conj}(W_{cf})$  and  $W_{of} + \text{Conj}(W_{of})$  instead of  $W_{cf}$  and  $W_{of}$  respectively, as what the authors of [1,2,7] have done in their works, the Laub algorithm can be applied to them but the structure which the original system is transformed to is no longer the frequency domain balanced structure. In the frequency domain balanced structure we should have the equal and diagonal Grammians but the similarity transform obtained from the aforementioned procedure can only transform the system to the structure in which the real part of the Grammians are equal and diagonal.

In order to overcome the problem, one can use input-output weights and make the dynamic system to work just within the frequency bound of interest. The frequency weighed dynamic system can be reduced successfully. In this case Plancherel's theorem can guarantee the trueness method.

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