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Cognitive Multiple-Antenna Network in Outage-Restricted Primary System

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***Abstract*— In the commons model for the spectrum sharing, cognitive users can access the spectrum as long as the target performance in the legitimate primary system is not violated. In this paper, we consider a downlink primary multiple-input-single-output (MISO) system which operates under a controlled interference from the downlink MISO cognitive radio, also called secondary system. We derive *exact* expressions for outage probability of the primary user under Rayleigh fading, when the primary system is exposed to interference from a secondary base station. Moreover, in high-SNR scenario, a closed-form asymptotic formula for the outage probability is derived, which shows that the primary receiver achieves full spatial diversity under given interference from the secondary user. Next, the optimum transmit power in the secondary system is investigated for maximizing the ergodic capacity when there is an outage constraint at the primary system, and a simple solution is proposed. Finally, the analytical results are confirmed by simulations, in which we analyze the impact of different parameters, such as the number of antennas and the amount of the interference on the system performance; these could be used as system design guidelines.**

I. Introduction

Cognitive radio is a promising method to solve the spectrum scarcity problem [1], [2]. Certain radio resources could be employed by cognitive radio network, i. e. the secondary system, provided that it does not cause an adverse interference to the primary system, a.k.a. spectrum owner or licensee. In this paper, we focus on concurrent cognitive radio network (or commons spectrum usage model [3]), in which secondary users are allowed to use the spectrum even when the primary system is active, provided that the amount of interference to the primary receiver is kept below a particular threshold.

Various works [4]–[7] have discussed achievable rates in cognitive radio from the viewpoint of information theory. In [6], [8] the achievable rate region for cognitive Gaussian multiple-access channels (MACs) has been characterized. As it can be seen from [9, Eq. (6.29)], the boundary on sum-rate capacity of downlink Gaussian network has inter-user interference terms, and thus, its optimization is not straightforward. To avoid inter-user interference, here we assume that the primary/secondary system use an orthogonal multiple access, such as time-division multiple access (TDMA) or orthogonal frequency division multiple access (OFDMA). At each given time each of the systems serves one user. Due to the assumed lack of transmit Channel State Information (CSI) in the paper, such a user is selected in a round-robin manner. Thus, we can effectively treat the primary/secondary system as being a single-user system.

This paper investigates the problem of power control in a multiple-input-single-output (MISO) cognitive radio network in which the secondary user coexists with a outage-restricted single-cell MISO primary system. Note that [10] and [11] studied the same scenario when the instantaneous interference power at the primary receiver should be less than a threshold. However, in this work, instead of limiting the interference power, it is assumed that the outage probability of the primary receiver should be kept below a certain threshold. This implies that the secondary receiver need not to know the instantaneous CSI towards the primary receiver. We therefore propose power controlled space-time coded transmission at the secondary transmitter, in which the secondary transmitter uses the knowledge of the CSI statistics.

Our main contributions can be summarized as follows:

- We derive a closed-form expression for the outage probability of MISO primary system in presence of interference from the multiple-antenna secondary transmitter over Rayleigh fading channels. The simplicity of the obtained expression allows insights on the system performance and its optimization. We analyze the diversity order of the outage-restricted MISO primary system based on the asymptotic behavior of outage probability. It is shown that in a primary system with M_p transmit antenna, full spatial diversity is still achievable in presence of interference from the space-time coded secondary user.
- We formulate the problem of maximizing the secondary downlink ergodic capacity for an outage–restricted primary system under the the assumption of no instantaneous CSI knowledge at the secondary transmitter. We propose a simple power control scheme to maximize the secondary downlink capacity given the outage probability constraint at the primary receiver.

II. System Model and Protocol Description

We consider a cellular based primary network coexisting with a secondary network, both of them operating in downlink mode. A primary base station (BS) is transmitting to a primary mobile station (MS) and there is a infrastructure-based secondary network. The secondary BS transmits to the secondary MS by using primary frequencies without license. The primary BS and the secondary BS have M_p

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and M_s antennas, respectively. Each primary/secondary MS is equipped with a single antenna. Due to the lack of CSI, it is assumed that the M_s antennas equally share the power, which results in equal contribution to the SNR at the primary receiver. It is assumed that $\boldsymbol{g}_p = [g_{p,1}, g_{p,2}, \ldots, g_{p,M_p}]$ is the vector of channel coefficients from the multiple-antenna primary BS to primary MS, and $\boldsymbol{g} = [g_1, g_2, \ldots, g_{M_p}]$ is the vector of channel coefficients of the interference link from the multiple-antenna primary BS to the secondary MS. In addition, $\boldsymbol{h} = [h_1, h_2, \ldots, h_{M_s}]$, is the vector channel coefficients from the multiple-antenna secondary BS to the secondary MS and $\boldsymbol{h}_p = [h_{p,1}, h_{p,2}, \ldots, h_{p,M_s}]$ is the vector of interference link from the multiple-antenna secondary BS to the primary MS. Throughout this paper, we assume that all channels experience independent Rayleigh fading. The average power of each antenna of the primary BS and the secondary BS are assumed to be P_g and P_s, respectively. We also assume that the instantaneous channel state information (CSI) is not available at both the primary BS and secondary BS, and thus it is optimal to use full-rate full-diversity space-time codes [9].

In the presence of interference from the secondary network, the received signal at the primary MS can be represented as

$$y_p = \sqrt{P_g} \sum_{n=1}^{M_p} g_{p,n}\, x_{p,n} + \sqrt{P_s} \sum_{m=1}^{M_s} h_{p,m}\, x_m + v_p, \quad (1)$$

where $x_{p,n}$ is the signal sent by the n-th antenna of primary BS, normalized as $\mathbb{E}\{|x_{p,n}|^2\} = 1$, for $n = 1, 2, \ldots, M_p$, is constant during the whole packet transmission and v_p is the Gaussian noise at the primary MS with variance $\mathcal{N}_p$. Moreover, we assume that the signal transmitted from the m-th antenna's of the secondary BS is $\sqrt{P_s}\, x_m$, where $\mathbb{E}\{|x_m|^2\} = 1$, for $m = 1, 2, \ldots, M_s$.

The primary BS uses fixed transmission rate R_p in the downlink. The minimum SNR to support the rate R_p is denoted by $\gamma_{\text{th}} = 2^{R_p} - 1$. If the achievable rate is lower than R_p, then outage occurs. Let the maximal allowed outage probability be $\rho_{\max}$. If a primary MS has a outage probability of $\rho_m < \rho_{\max}$, then the receiver has an *outage margin* and additional interference can be received from the secondary BS without violating the target operation regime of the primary system. Considering normalized bandwidth and space-time coded transmission for both primary and secondary systems, the achievable instantaneous rate of the primary user is

$$r_p = \log_2 \left(1 + \frac{P_g \sum_{n=1}^{M_p} |g_{p,n}|^2}{\mathcal{N}_p + P_s \sum_{m=1}^{M_s} |h_{p,m}|^2} \right). \quad (2)$$

For the secondary system, we assume that the signals are simultaneously transmitted from all the antennas on the same resource (frequency and time). Thus, the received signal at the secondary BS is given as

$$y_s = \sqrt{P_s} \sum_{m=1}^{M_s} h_m x_m + \sqrt{P_g} \sum_{n=1}^{M_p} g_n\, x_{p,n} + v_s, \quad (3)$$

where v_s is the Gaussian noise at the secondary BS with variance $\mathcal{N}_s$.

III. Permissible Power Allocation with Unknown CSI at Transmitters

In this section, we investigate the permissible power level of the secondary system such that the primary system operates below a certain outage threshold. Moreover, as mentioned earlier, we assume that instantaneous CSI is not available at the primary and secondary transmitters. In the following, we derive the outage probability for the space-time coded MISO primary system when we have an interference margin at the receiver. Then, the results are utilized to find the permissible transmit power for the secondary system.

A. Outage Probability at the Primary Receiver

As stated above, the interference from the secondary BS should be kept below a level in order to coexist with the primary system. Thus, the secondary BS chooses the transmit power P_s such that the outage performance for the primary system is not violated.

We derive the outage probability $\rho_{\text{out}} \triangleq \Pr\{r_p < R_p\}$ of the primary MS as the probability that the transmit rate R_p is larger than the supported rate r_p in (2). This probability, expressed as cumulative distribution function (CDF), depends on the transmission parameters and the channel conditions in both the primary and the secondary system. By defining $\gamma_{\text{th}} \triangleq (2^{R_p} - 1)$, and using (2), the outage probability at the primary user can be represented as

$$\rho_{\text{out}}^{\text{ST}} = \Pr\left\{ \frac{P_g \sum_{n=1}^{M_p} |g_{p,n}|^2}{\mathcal{N}_p + P_s \sum_{m=1}^{M_s} |h_{p,m}|^2} < \gamma_{\text{th}} \right\}. \quad (4)$$

Proposition *1:* Consider a finite set of independent random variables $\mathcal{X} = \{X_1, \ldots, X_{M_p}\}$ and $\mathcal{Y} = \{Y_1, \ldots, Y_{M_s}\}$ with exponential distribution and mean of σ_x^2 and σ_y^2, respectively. The CDF of

$$\text{SINR}_{\text{ST}} = \frac{\sum_{n=1}^{M_p} X_n}{1 + \sum_{m=1}^{M_s} Y_m}$$

can be calculated as

$$\Pr\{\text{SINR}_{\text{ST}} < \gamma\} = 1 - \sum_{n=0}^{M_p-1} \frac{\gamma^n e^{-\frac{\gamma}{\sigma_x^2}}}{\sigma_x^{2n}\, n!\, (M_s-1)!} \sum_{i=0}^{n} \binom{n}{i} \frac{(i+M_s-1)!\, \sigma_y^{2i}}{\left(1 + \frac{\sigma_y^2}{\sigma_x^2}\gamma\right)^{i+M_s}}. \quad (5)$$

Proof: The proof is given in Appendix I. ■

To guarantee that the interference from secondary system does not reduce the quality of primary MS, the outage probability at the primary receiver should be lower than the maximal allowed. From Proposition 1 and by defining $X_n = \frac{P_g |g_{p,n}|^2}{\mathcal{N}_p}$, $n = 1, \ldots, M_p$, and $Y_m = \frac{P_s |h_{p,m}|^2}{\mathcal{N}_p}$, $m = 1, \ldots, M_s$, the outage probability in (4) can be written as

$$\rho_{\text{out}}^{\text{ST}} = 1 - \sum_{n=0}^{M_p-1} \frac{\gamma_{\text{th}}^n \mathcal{N}_p^n e^{-\frac{\gamma_{\text{th}} \mathcal{N}_p}{P_g \sigma_{g_p}^2}}}{(P_g \sigma_{g_p}^2)^n\, n!\, (M_s-1)!} \times \sum_{i=0}^{n} \binom{n}{i} \frac{(i+M_s-1)!\, (P_s \sigma_{h_p}^2)^i}{\left(1 + \frac{P_s \sigma_{h_p}^2}{P_g \sigma_{g_p}^2}\gamma_{\text{th}}\right)^{i+M_s} \mathcal{N}_p^i}, \quad (6)$$

where $\sigma_{g_p}^2$ and $\sigma_{h_p}^2$ are the means of the channel coefficients $|g_{p,n}|^2$, $n = 1, \ldots, M_p$ and $|h_{p,m}|^2$, $m = 1, \ldots, M_s$, respectively.

To compare the derived outage probability with the outage probability in absence of interference, i.e., ρ_0, we have

$$\begin{aligned}\rho_0 &= \Pr\left\{\frac{P_g \sum_{n=1}^{M_p} |g_{p,n}|^2}{\mathcal{N}_p} < \gamma_{\text{th}}\right\} \\ &= 1 - \sum_{n=0}^{M_p-1} \frac{1}{n!}\left(\frac{\gamma_{\text{th}}\mathcal{N}_p}{P_g\sigma_{g_p}^2}\right)^n e^{-\frac{\gamma_{\text{th}}\mathcal{N}_p}{P_g\sigma_{g_p}^2}} \\ &= \frac{\gamma_{\text{inc}}\left(\frac{\gamma_{\text{th}}\mathcal{N}_p}{P_g\sigma_{g_p}^2}, M_p\right)}{(M_p-1)!} \end{aligned} \tag{7}$$

where $\gamma_{\text{inc}}(x,k)$ is incomplete gamma function of order k [12].

Corollary *1:* In the MISO primary link with space-time coding, the minimum outage margin requirement for the primary receiver which allows cognitive system to operate is given in (7), i.e., $\rho_m > \rho_0$.

Furthermore, from (6) and (7), the target outage probability at the primary receiver can be expressed in terms of ρ_0 as

$$\begin{aligned}\rho_m =& 1 - \sum_{n=0}^{M_p-1} \frac{\gamma_{\text{inc}}^{-n}((M_p-1)!\rho_0, M_p) e^{-\gamma_{\text{inc}}^{-1}((M_p-1)!\rho_0, M_p)}}{n!\,(M_s-1)!} \\ &\times \sum_{i=0}^{n} \binom{n}{i} \frac{(i+M_s-1)!\,\frac{\text{SNR}_{\text{sp}}^i}{M_s^i}}{\left(1+\frac{\text{SNR}_{\text{sp}}}{M_s}\gamma_{\text{inc}}^{-1}((M_p-1)!\rho_0, M_p)\right)^{i+M_s}}, \end{aligned} \tag{8}$$

where $\text{SNR}_{\text{sp}} = \frac{M_s P_s \sigma_{h_p}^2}{\mathcal{N}_p}$ is the average interference power received from the secondary BS, and $\gamma_{\text{inc}}^{-1}(\cdot,k)$ denotes the inverse function of $\gamma_{\text{inc}}(\cdot,k)$, which for example, could be found by the built-in function "gammaincinv(x,k)" in MATLAB.

Proposition *2:* In high SNR_p scenario, for the space-time coded primary system over Rayleigh fading channels, the outage probability of the primary receiver in presence of space-time coded secondary system can be stated as

$$\rho_{\text{out}}^{\text{ST}} \approx \frac{\Delta}{\text{SNR}_p^{M_p}} \tag{9}$$

where

$$\Delta = \frac{\gamma_{\text{th}}^{M_p} M_p^{M_p}}{M_p!\,(M_s-1)!} \sum_{n=0}^{M_p} \binom{M_p}{n} (n+M_s-1)! \left(\frac{\text{SNR}_{\text{sp}}}{M_s}\right)^n, \tag{10}$$

where $\text{SNR}_p = \frac{M_p P_g \sigma_{g_p}^2}{\mathcal{N}_p}$ is the received SNR of the primary link.

Proof: The proof is given in Appendix II. ■

From Proposition 2, and by using the definition of diversity order $G_d = \lim_{\text{SNR}\to\infty} \frac{-\log(\rho_{\text{out}})}{\log(\text{SNR})}$ [13, Eq. (1.19)], we have the following corollary:

Corollary *2:* A space-time coded primary link with a transmitter equipped with M_p antennas retains the full diversity order M_p even under interference from a secondary transmission.

B. Power Control at the Secondary System

Since we assume instantaneous channels are unknown at the secondary transmitter, we use ergodic capacity as an objective function for our power allocation scheme which is independent of the instantaneous channel variations. For the ergodic capacity of the secondary system, given as $\overline{C}_{\text{s}} = \mathbb{E}\{C_{\text{s}}\}$, where $\mathbb{E}\{\cdot\}$ denotes the expectation operation, from (3) we have

$$\overline{C}_{\text{s}} = \mathbb{E}\left\{\log_2\left(1 + \frac{P_s}{\mathcal{N}_{s,I}} \sum_{m=1}^{M_s} |h_m|^2\right)\right\}, \tag{11}$$

where P_s is the transmit power per antenna and $\mathcal{N}_{s,I} = \mathcal{N}_s$ when the primary signal is decoded and removed at the secondary user and $\mathcal{N}_{s,I} = \mathcal{N}_s + P_g \sum_{n=1}^{M_p} |g_n|^2$ if the primary signal is treated as noise at the secondary receiver.

Since $|h_m|^2$ are i.i.d. random variables, a closed-form lower-bound for the expression in (11) is given by [14, Proposition 1]

$$\overline{C}_{\text{s}} \geq \log_2\left(1 + \frac{P_s\,\sigma_h^2}{\mathcal{N}_{\text{ST}}} \exp\left(\sum_{m=1}^{M_s-1} \frac{1}{m} - \kappa\right)\right) \tag{12}$$

where $\kappa \approx 0.577$ is Euler's constant, σ_h^2 is the mean of the channel coefficients $|h_m|^2$, $m = 1, \ldots, M_s$, and $\mathcal{N}_{\text{ST}} = \mathcal{N}_s$ when the primary signal is decoded at the secondary user and $\mathcal{N}_{\text{ST}} = \mathcal{N}_s + M_p P_g \sigma_g^2$ if the primary signal is treated as noise at the secondary receiver.

Now, using (6) and (12), we formulate the problem of power control in cognitive MISO system (or downlink cognitive network). Therefore, the power allocation problem, which has a constraint on the outage probability at the primary receiver node (MS), can be formulated as

$$\begin{aligned}&\max_{P_s}\ \log_2\left(1 + \frac{P_s\,\sigma_h^2}{\mathcal{N}_{\text{ST}}} \exp\left(\sum_{m=1}^{M_s-1} \frac{1}{m} - \kappa\right)\right), \\ &\text{s.t.}\quad \rho_{\text{out}}^{\text{ST}} \leq \rho_m, \quad P_s \geq 0. \end{aligned} \tag{13}$$

where $\rho_{\text{out}}^{\text{ST}}$ is defined in (6). Since both the objective function in (13) and $\rho_{\text{out}}^{\text{ST}}$ are increasing functions of the power coefficient P_s for $P_s \geq 0$, to find the optimal value of the problem in (13), the first constraint is turned into the equality. Thus, the single positive root of the increasing function $\rho_{\text{out}}^{\text{ST}}(P_s) - \rho_m = 0$ should be calculated. Hence, for a given initial value, P_s^* can be calculated using the following recursive equation:

$$P_s^{(t+1)} = \frac{P_g\sigma_{g_p}^2}{\sigma_{h_p}^2\gamma_{\text{th}}} \left[\left(\frac{e^{-\frac{\gamma_{\text{th}}\mathcal{N}_p}{P_g\sigma_{g_p}^2}}}{\Phi(P_s^{(t)})}\right)^{1/M_s} - 1\right], \tag{14}$$

where

$$\begin{aligned}\Phi(P_s^{(t)}) =& 1 - \rho_m - \sum_{n=1}^{M_p-1} \frac{\gamma_{\text{th}}^n \mathcal{N}_p^n e^{-\frac{\gamma_{\text{th}}\mathcal{N}_p}{P_g\sigma_{g_p}^2}}}{(P_g\sigma_{g_p}^2)^n\, n!\,(M_s-1)!} \\ &\times \sum_{i=0}^{n} \binom{n}{i} \frac{(i+M_s-1)!\,(P_s^{(t)}\sigma_{h_p}^2)^i}{\left(1+\frac{P_s^{(t)}\sigma_{h_p}^2}{P_g\sigma_{g_p}^2}\gamma_{\text{th}}\right)^{i+M_s} \mathcal{N}_p^i}, \end{aligned} \tag{15}$$

and $P_s^{(t)}$ is the updated version of the power coefficient in the t-th iteration. For the case of single-antenna primary BS, the closed-form solution is obtained in the following proposition.

Proposition *3:* The optimum transmit power P_s^* from each secondary BS antenna for maximizing the secondary capacity given an outage probability requirement ρ_m at the primary MS, when $M_p = 1$, can be expressed as

$$P_s^* = \frac{P_g\,\sigma_{g_p}^2}{\sigma_{h_p}^2\,\gamma_{\text{th}}}\left[e^{-\frac{\gamma_{\text{th}}\,\mathcal{N}_p}{M_s\,P_g\,\sigma_{g_p}^2}}\left(1-\rho_m\right)^{-1/M_s}-1\right], \qquad (16)$$

Proof: The proof is obvious from (14) and (15). ■

Note that the second constraint in P_s can be also modified to support the limited power scenario. That is, by assuming that there is a power constraint at the secondary BS, i.e., $P_s \leq P_s^{\max}$ where $P_s^{\max}$ is the maximum power budget of cognitive BS. Thus, the optimization problem in (13) can be rewritten as

$$\max_{P_s}\ \log_2\left(1+\frac{P_s\,\sigma_h^2}{\mathcal{N}_{\text{AS}}}\exp\left(\sum_{m=1}^{M_s-1}\frac{1}{m}-\kappa\right)\right),$$
$$\text{s.t.}\quad \rho_{\text{out}}^{\text{ST}} \leq \rho_m,\quad 0 \leq P_s \leq P_s^{\max}. \qquad (17)$$

It is easy to show that by using the Lagrangian multiplier method, and satisfying the KKT conditions [15], the solution to the problem stated in (17) can be given as

$$P_s^* = P_s^{\max} - \left[P_s^{\max} - P_s\right]^+ \qquad (18)$$

where P_s is obtained from the recursive equation in (14)-(15), and $[x]^+$ denotes $[x]^+ = \max\{0, x\}$.

IV. Numerical Analysis

In this section, the analytical results are compared with simulations. It is assumed that the channels are independent Rayleigh distributed with normalized variance. In addition, the primary BS uses the fixed transmission rate of $R_p = 1$ bits/sec/Hz.

Fig. 1 confirms that the analytical expressions in Subsection III-A for finding the average outage probability have similar performance as simulation results. We consider a cognitive network with $M_s = 2$ and the primary links with $M_p = 2, 4$. One observe that the closed-form analytical results based on (6) are similar to the the simulated results. Furthermore, we have sketched the asymptotic outage probability derived in Proposition 2. It can be seen that the asymptotic expression well approximates the simulations in high SNR conditions. In addition, one can observe that Fig. 1 shows that for obtaining an outage probability of 10^{-5} at the primary MS, around 4 dB more power is required when $\text{SNR}_{\text{sp}} = 0$ dB and $M_p = 4$, compared to interference-free case. Moreover, from observing the behavior of curves in high SNR conditions, it can be observed that full-spatial diversity is achievable in presence of the secondary interference in expense of less coding gain, which confirms the result stated in Corollary 2.

Fig. 2 compares the target outage probability at primary user in presence of the secondary transmission versus the outage probability in absence of the secondary system, i.e., ρ_0, for different values of average interference SNR at primary receiver, i.e., $\text{SNR}_{\text{sp}} = -10$ dB, 0 dB, and different number of primary BS antenna $M_p = 1, 4$ and secondary BS antenna $M_s = 2, 10$. It can be seen that as interference parameter SNR_{sp} or the number of primary BS antenna M_p go down, the outage probability gets closer to ρ_0. Moreover, it is also observable that the relationship between ρ_m and ρ_0 is not sensitive to the number of secondary BS antenna M_s, especially for at lower secondary interference.

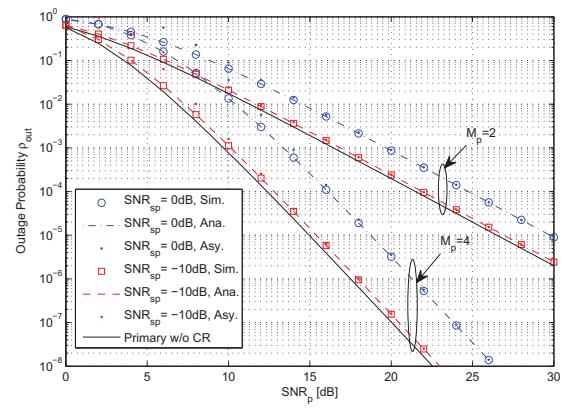

Fig. 1. Outage probability in the primary system as a function of the received primary SNR, i.e., SNR_p when $M_s = 2$ and $R_p = 1$ bits/s/Hz. The systems with different number of M_p and SNR_{sp} are compared.

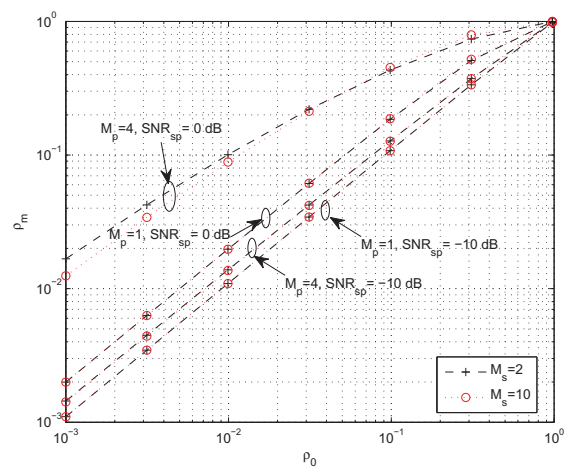

Fig. 2. The target outage probability at primary node ρ_m in presence of cognitive radio versus the outage probability in absence of the secondary system for different values of average interference SNR at primary receiver and different number of primary BS and secondary BS antennas M_p and M_s, respectively.

V. Conclusion

In this paper, we consider the concurrent downlink cognitive radio network with multiple antenna transmitters. We investigate the outage probability at the primary receiver as well as the permissible power level in the secondary system. In particular, we have derived a simple closed-form expression

for the outage probability at the primary system when there is an interference from a cognitive multiple antenna system. Furthermore, by analyzing the system performance in high SNR regime, it is shown that full spatial diversity is achievable in the primary system under Rayleigh fading and in presence of the transmission from the secondary system. We formulated the problem of finding the maximum transmit power at the multiple antenna secondary system when there is an outage probability target at the primary receiver. Simulations were in accordance with the analytic results. Furthermore, numerical results showed that the performance of the system is not very sensitive to the number of secondary antenna M_s, while the number of primary transmit antenna M_p could have a strong impact on the system performance. An interesting tops for future work is the statistics of the interference that a multi-antenna secondary transmitter causes at the primary receiver when intra-secondary system CSI is available and the power allocation across the secondary antennas is not uniform.

Appendix I
Proof of Proposition 1

We define $Y = \sum_{m=1}^{M_s} Y_m$ which has a gamma distribution M_s degrees of freedom with PDF $p_y(y) = \frac{y^{M_s-1}}{\sigma_y^{2M_s}(M_s-1)!} e^{-\frac{y}{\sigma_y^2}}$. Moreover, the random variable $X = \sum_{n=1}^{M_p} X_n$ is defined which has Erlang distribution with CDF of $\Pr\{X < x\} = 1 - \sum_{n=0}^{M_p-1} \frac{x^n}{\sigma_x^{2n}\, n!} e^{-\frac{x}{\sigma_x^2}}$. By marginalizing over the random variable Y, the CDF of the $\mathrm{SINR_{ST}} = \frac{X}{1+Y}$ can be calculated as

$$\begin{aligned}
\Pr\{\mathrm{SINR_{ST}} < \gamma\} &= \int_0^\infty \Pr\{X < \gamma(1+y)\}\, p_y(y)\, dy \\
&= 1 - \sum_{n=0}^{M_p-1} \int_0^\infty \frac{(\gamma + \gamma\, y)^n}{\sigma_x^{2n}\, n!} e^{-\frac{\gamma(1+y)}{\sigma_x^2}} \frac{y^{M_s-1}}{\sigma_y^{2M_s}(M_s-1)!} e^{-\frac{y}{\sigma_y^2}}\, dy \\
&= 1 - \sum_{n=0}^{M_p-1} \frac{\gamma^n e^{-\frac{\gamma}{\sigma_x^2}}}{\sigma_x^{2n}\, n!\, \sigma_y^{2M_s}(M_s-1)!} \\
&\quad \times \int_0^\infty (1+y)^n e^{-\frac{\gamma y}{\sigma_x^2}}\, y^{M_s-1} e^{-\frac{y}{\sigma_y^2}}\, dy. \qquad (19)
\end{aligned}$$

Using Taylor series for expansion of $(1+y)^n$, the closed-form solution for integral in (19) is obtained as (5).

Appendix II
Proof of Proposition 2

We express the CDF of $X = \sum_{n=1}^{M_p} X_n$ in Proposition 1 in terms of incomplete gamma function as

$$\Pr\{X < x\} = \frac{\gamma_{\mathrm{inc}}\left(\frac{x}{\sigma_x^2}, M_p\right)}{(M_p-1)!}.$$

From [16, Eq. (8.7.1)], the series expansion of $\gamma_{\mathrm{inc}}(x, M_p)$ is given by

$$\gamma_{\mathrm{inc}}\left(\frac{x}{\sigma_x^2}, M_p\right) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(M_p+k)\, k!} \left(\frac{x}{\sigma_x^2}\right)^{k+M_p},$$

and thus, for a given x and $\sigma_x^2 \gg 1$, we have

$$\Pr\{X < x\} \approx \frac{1}{M_p!} \left(\frac{x}{\sigma_x^2}\right)^{M_p}. \qquad (20)$$

By marginalizing over the random variable Y and using (20), the CDF of the $\mathrm{SINR_{ST}} = \frac{X}{1+Y}$ in (19) can be rewritten as

$$\begin{aligned}
\Pr\{\mathrm{SINR_{ST}} < \gamma\} &= \int_0^\infty \Pr\{X < \gamma(1+y)\}\, p_y(y)\, dy \\
&\approx \int_0^\infty \frac{\gamma^{M_p}(1+y)^{M_p}}{\sigma_x^{2M_p}\, M_p!} \frac{y^{M_s-1}}{\sigma_y^{2M_s}(M_s-1)!} e^{-\frac{y}{\sigma_y^2}}\, dy \\
&= \frac{\gamma^{M_p}}{\sigma_x^{2M_p}\, M_p!\, \sigma_x^{2M_p}(M_s-1)!} \sum_{n=0}^{M_p} \binom{M_p}{n} (n+M_s-1)!\, \sigma_y^{2n}, \qquad (21)
\end{aligned}$$

where in the third equality, the binomial series expansion of $(1+y)^n$ is used and the closed-form solution for integral is obtained. Then, by the fact that $\sigma_x^2 \to \infty$ is equivalent to $\mathrm{SNR}_p \to \infty$, the result in (9) is obtained.

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