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KEYWORDS

System identification, ARMA models, modal parameters, offshore structures, random waves, wave loads, experiment.

ABSTRACT

In this paper modal parameter as well as wave load identification by calibration of ARMA models is considered for a simple offshore structure. The theory of identification by ARMA calibration is presented as an identification technique in the time domain which can be applied for white noise excited systems. The technique is generalized also to include the case of ambient excitation processes such as wave excitation which are non-white processes. Due to those results a simple but effective approach for identification of the load process is proposed. Finally the theoretical presentation is illustrated by an experimental example of a monopile model excited by random waves. The identification results show that the approach is able to give very reliable estimates of the modal parameters. Furthermore, a comparison of the identified wave load process and the calculated load process based on the Morison equation shows that important information can be obtained about the wave loading by this approach.

INTRODUCTION

In this paper a procedure for evaluating the modal parameters and the wave load will be presented for the case of a monopile platform excited by random waves.

The problem of system identification of ambient excited structures such as offshore structures is that the load process is difficult to measure and it will often be necessary to apply the classical white noise assumption, for a review see Jensen (1990). This approach leads to estimates of the modal parameters. However, from observation of the response it is clear that the response also contains information about the wave load process, and the purpose of this paper is to quantify this information so that modal as well as wave load identification will be the result of the response analysis.

A simple offshore structure such as a monopile platform, see (e.g. Cook 1982) can be assumed to be described by a vertical cylinder with diameter $D(z)$ and length L excited by the forces of inertia due to a unidirectional wave excitation with the horizontal particle acceleration $\ddot{u}(t,z)$ where z is the vertical coordinate (positive directions upwards with zero in the mean water level). This means that the load on the structure per unit length is assumed to be:

$$p(t,z) = \rho\frac{\pi}{4}D(z)^2\,C_M\,\ddot{u}(t,z) - \rho\frac{\pi}{4}D(z)^2\,[C_M - 1]\,\ddot{x}(t,z) \qquad (1)$$

where the structural response $\ddot{x}(t,z)$ has been taken into account and where the coeficient of inertia C_M is assumed to be independent of z and the density of the fluid is given by ρ (e.g. Sarpkaya and Isaacson 1981).

The structural model of the cylinder is described by a continuous beam with distributed mass $m(z)$. The beam is clamped at the foundation and the topside is modelled by a discrete mass M_{top}. At a water depth d this means with a contribution from a finite number of modes n that the structural model is given by the set of equations (e.g. Lin 1967):

$$\ddot{q}_i(t) + \frac{1}{M_i}\sum_{j=1}^{n}\int_{-d}^{0}\rho\frac{\pi D(z)^2\,(z)}{4}(C_M - 1)\phi^{(i)}(z)\phi^{(j)}(z)\,dz\,\ddot{q}_j(t) + 2\omega_i\zeta_i\dot{q}_i(t)$$

$$+\omega_i^2 q_i(t) = \frac{1}{M_1}\int_{-d}^{0}\phi^{(i)}(z)\rho\frac{\pi D(z)^2\,(z)}{4}C_M\,\ddot{u}(t,z)\,dz \quad i,j = 1,2,\ldots n \quad (2)$$

where:

$M_i = \int_{-d}^{L-d}\phi^{(i)}(z)^2 m(z)dz + \phi^{(i)}(L-d)^2 M_{top}$

$\phi^{(i)}(z) =$ the ith eigenmode

$x(z,t) = \sum_{i=1}^{n}\phi^{(i)}(z)q_i(t)$

$\omega_i =$ the ith eigenfrequency

$\zeta_i =$ the ith damping ratio

In the frequency domain the modal load spectrum $S_{\Gamma_i\Gamma_j}(\omega)$ due to a wave excitation can be given by:

$$S_{\Gamma_i\Gamma_j}(\omega) = \int_{-d}^{0}\int_{-d}^{0}\phi^{(i)}(z_1)\phi^{(j)}(z_2)S_{pp}(z_1,z_2,\omega)dz_1dz_2 \qquad (3)$$

where the load spectrum $S_{pp}(z_1,z_2,\omega)$ is derived from the linear wave theory (e.g. Sarpkaya and Isaacson 1981):

$$S_{pp}(z_1,z_2,\omega) = g^2\mathbf{k}^2(\omega)\left(\rho\frac{\pi D(z)^2}{4}C_M\right)^2 S_{\eta\eta}(\omega) \frac{\cosh\big(\mathbf{k}(\omega)(z_1+d)\big)\cosh\big(\mathbf{k}(\omega)(z_2+d)\big)}{\cosh^2\big(\mathbf{k}(\omega)d\big)} \qquad (4)$$

where $S_{\eta\eta}(\omega)$ is the wave elevation spectrum and the wave number function $\mathbf{k}(\omega)$ can be evaluated by the dispersion equation $\mathbf{k}(\omega) = \frac{\omega^2}{g}\coth(\mathbf{k}(\omega)d)$.

IDENTIFICATION BY ARMA MODELS

Since information about the wave excitation is measured not to be avaiable, the identification method has to be based on the measured response. Furthermore it is in principle necessary to assume that the excitation is white noise if information about the excitation process is not to be included in the analysis.

Therefore as a start the structure is assumed to be excited by a Gaussian white noise vector process. The response measured at a single point is assumed to be sampled at discrete time instants $x_t = x(t\Delta)$ $t = 1, 2, 3, \ldots T$, where Δ is the sampling interval. From the discrete response record, an ARMA$(2n, 2n-1)$ model can be estimated. The ARMA model is given by:

$$x_t = \underbrace{-\sum_{i=1}^{2n} \Phi_i x_{t-i}}_{AR-part} + a_t + \underbrace{\sum_{i=1}^{2n-1} \Theta_i a_{t-i}}_{MA-part} \tag{5}$$

where n is the number of degrees of freedom of the structure, see (2), Φ_i and Θ_i are the unknown parameters characterizing the ARMA model, and the discrete time series a_t is the residual process which is restricted to be a discrete Gaussian white noise process. The residual process a_t is estimated together with the unknown parameters. Different estimation approaches have been reviewed in Ljung (1987). The model order $(2n, 2n-1)$ can be shown to be a consistent choice since it gives the ARMA model of the discretisized response the same statistical properties characterized by the 1st and 2nd moment as the random process in the continuous time domain, see Natke and Kozin (1986).

(5) can be rewritten by the polynomial form if the backwards shift operator q^i defined by $x_{t-i} = q^i x_t$ is introduced:

$$\Phi(q)x_t = \Theta(q)a_t \tag{6}$$

where:

$$\Phi(q) = 1 + \Phi_1 q + \Phi_2 q^2 + \cdots + \Phi_{2n} q^{2n} \tag{7}$$

$$\Theta(q) = 1 + \Theta_1 q + \Theta_2 q^2 + \cdots + \Theta_{2n-1} q^{2n-1} \tag{8}$$

The ARMA model can be reformulated as a transfer function which can be transformed into the frequency domain by the Z-transform. The Z-transform is defined by: $X(Z) = \sum_{t=-\infty}^{+\infty} x_t Z^t$, (e.g. Rabiner and Gold 1975). An expression in the frequency domain is obtained by the substitution $Z = e^{i\omega}$ which gives:

$$H(\omega) = \frac{\Theta(e^{i\omega})}{\Phi(e^{i\omega})} = e^{i\omega} \frac{((e^{-i\omega} - \nu_1)(e^{-i\omega} - \nu_2) \cdots (e^{-i\omega} - \nu_{2n-1}))}{((e^{-i\omega} - \lambda_1)(e^{-i\omega} - \lambda_2) \cdots (e^{-i\omega} - \lambda_{2n}))} \tag{9}$$

where λ_j and ν_j are the jth roots of the characteristic equations of (7) and (8), respectively. The roots of (7) correspond to the poles of the transfer function and the roots of (8) correspond to the zeroes of the transfer function. Generally, a root will be a complex number. The modal parameters are found from the $2n$ roots λ_i of (7). In e.g. (Pi and Mickleborough 1989) it is shown that the roots are related to the modal parameters through the $2n$ relations :

$$\begin{aligned} (\,\lambda_i\,) &= (\,\exp(\mu_i \Delta)\,) \\ \mu_{(i)12} &= -\omega_i \zeta_i \pm \mathrm{i}\omega_i \sqrt{1-\zeta_i^2} \end{aligned} \tag{10}$$

The index (12) refers here to the fact that the λ_i-values are found as complex conjugated pairs if the modes are underdamped. It is seen that this set of equations gives the relation between the estimated AR parameters $\overline{\Phi}^T = (\,\Phi_1 \quad \Phi_2 \quad \ldots \quad \Phi_{2n}\,)$ and the modal parameters $\overline{P}^T = (f_1, f_2, \ldots, f_n, \zeta_1, \zeta_2, \ldots, \zeta_n)$.

As shown in Ljung (1987), the covariance matrix of the ARMA parameters follows from the estimation of the ARMA model. Therefore it is quite straightforward to obtain the uncertainty of the modal parameters by linearization (Gersh 1974, Jensen et al. 1990).

Non-White Excitation

In the application of ARMA models it was assumed that the excitation process acting on the underlying system ideally was a white noise process. However, in practice the wave excitation process will always be a coloured process.

Thus if an ndof system excited by the coloured excitation $v(t)$ is considered:

$$\Phi(q)x_t = \Theta(q)v_t \tag{11}$$

the violated assumptions will mean that the estimated parameters of the ARMA model will be biased (Ljung 1987). However, the bias problem can be overcome by increasing the model order until the non-whiteness of the excitation has been built into the model. Such an ARMA model which exceeds the expected order of the underlying system will be called an oversized model.

This ARMA model of the measured discrete response record will not only contain information about the structure but also information about the excitation. This means that the proper ARMA model of an ndof vibrating system will be of the order $(2n+l, 2n-1+m)$ exceeding the ideal model order $(2n, 2n-1)$:

$$\Psi(q)\Phi(q)x_t = \Lambda(q)\Theta(q)a_t \tag{12}$$

where the polynomial $\Psi(q)$ and $\Lambda(q)$ are of order l and m, respectively and where the non-white excitation has been modelled by an ARMA(l, m) model:

$$\Psi(q)v_t = \Lambda(q)a_t \tag{13}$$

see figure 1. Thus the oversized ARMA model of the response may be considered as a result of a two-stage ARMA modelling.

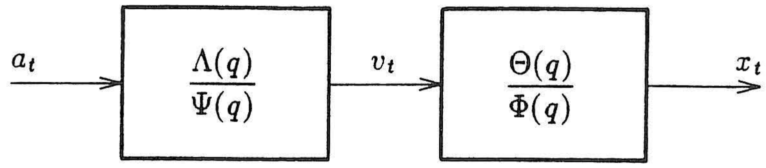


Figure 1. Two-stage ARMA model of a non-white excited system.

Hence the problem is to choose the additional orders l and m and obvivously this choice may include investigation of a large amount of oppotunities. In practice it is therefore necessary to choose a model strategy and to investigate the models included to find a proper model. Pandit and Wu (1983) have shown that the order of the ARMA model should be of the form $(2n + l, 2n + l - 1)$ where l is an even number.

The determination of the proper total model order can e.g. be based on the following two criteria:

- The residual process a_t should be white noise.
- The decrease in Akaike's final prediction error defined by:

$$(FPE) = \frac{1 + (4n - 1)/T}{1 - (4n - 1)/T}\,\sigma_a^2 \tag{14}$$

should be approximately zero if the model order was increased once more. σ_a^2 denotes the variance of the residual process a_t ($n/T << 1 \rightarrow (FPE) \approx \sigma_a^2$). This criterion may be explained by the fact that as long as an increase in the model order leads to a decrease in σ_a^2 it will mean that the ARMA is able to give a deterministic description of a larger portion of the measured time series.

Furthermore, it may be noticed that if the moder order has been chosen too large this may be detected by inspection of the poles and zeroes given by (7) and (8). If a zero lies in the area of the confidence range of a pole or vice versa this might be a set of poles and zeroes of the model which could be removed from the model without changing the properties of the model.

From (12) it is seen that the oversized ARMA model will contain poles and zeroes which will be characerizing the structure, the load process and potentional additional measurement noise. Thus, to estimate the eigenfrequencies and the damping ratios it will be necessary to determine which poles are associated with the structure. This can be done by inspection of the poles based on a priori knowledge:

• The damping ratio associated with structural modes will lie in the range below 0.10 while modes associated with noise and excitation process typically will have considerably large damping ratios.
• A priori knowledge on the source of an eigenfrequency associated with a pole will exist.
• The uncertainty of the roots and poles associated with the structure will typically be low compared with those caused by the excitation and measurement noise.

Wave Load Identification

Since the modal parameters can be obtained as shown in the previous section it would be tempting also to try to quantify the information about the coloured load process. This could e.g. be done by determing the pure structural ARMA($2n, 2n-1$) model from the modal parameters and then filter the measured response by a filter corresponding to the inverse ARMA($2n, 2n-1$) model:

$$v_t = \frac{\Phi(q)}{\Theta(q)} x_t \tag{15}$$

Thus the point would be that the filter contains all the characteristics of the response except those associated with the load process. In the case of noise present in the measurements, this means that the inverse ARMA model should contain not only the structural poles and zeros but also those associated with the measurement noise. This possibility of including the noise characteristics is a very important aspect of the approach.

Just like the identification of the structural roots of the ARMA models the identification of the roots associated with the load process has to rely on inspection of the roots based on the a priori knowledge. The difficulty in the approach lies in determining the zeroes of the oversized ARMA model which are associated with the structure. The zeroes are associated with the weigthing of the different modes.

In the case of displacement or velocity response of an sdof system excited by white noise it is possible to identify an eigenfrequency and a damping ratio, and it is straightforward to estimate the ARMA(2,1) model since the exact expressions exist (e.g. Pandit and Wu 1983). This approach is not possible when acceleration data are applied. The reason is that the variance of the acceleration response due to a white noise excitation will be unbounded (e.g. Lin 1967) This means that the invertibility condition of the ARMA model cannot be fulfilled, see Pandit and Wu (1983) and consequently the zeroes cannot be evaluated directly. Instead it is suggested that the real zero of the oversized ARMA model with the smallest uncertainty is chosen, or alternatively general approach proposed below is applied.

In the general case of an ndof system it is possible to obtain estimates/a priori knowledge of the zeroes by a curvefit to the measured response spectrum of the calculated response spectrum given the residual spectrum $S_{aa}(\omega)$, and the transfer function of the ARMA($2n, 2n-1$) model given by (9) with the identified structural poles inserted:

$$\min_{w.r.t.\Theta} V$$
$$V = \sum_{\omega_i \in \Omega} \left(S_{xx}(\omega_i) - S_{aa}(\omega_i) \left| H(\omega_i, \Phi, \Theta) \right|^2 \right)^2 \tag{16}$$

The frequency range Ω should be the range where the structural resonance is thought to dominate the measured response.

EXPERIMENTAL EXAMPLE: WAVE EXCITED MONOPILE MODEL

An experiment has been performed with a vertical pile excited by random waves in a wave bassin as shown in figure 2. The pile was made of a PVC pipe and stifftened inside such that only the 1st bending mode in the wave direction would dominate the response. The pile was tested with and without a top mass of 7 kg corresponding to approximately a first eigenfrequency $f_1 = 2.0$ Hz and $f_1 = 3.25$ Hz, respectively.

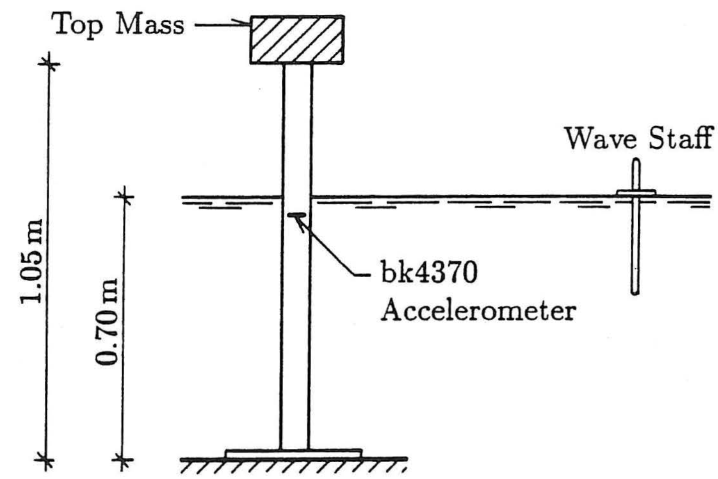


Figure 2. The experimental set-up.

The waves were generated by a wave machine controlled by a computer loaded with a set of input parameters corresponding to the desired wave elevation spectrum which was a Jonswap spectrum modified to the Danish part of the North Sea, see DS449 (1983). The spectrum was characterized by a peak enhancement constant $\gamma = 3.3$ and a peak frequency $f_p = 0.69$ Hz while the significant wave height was varyed with $H_s = 0.05, 0.10$ and 0.15 m. With a water depth of 0.70 m it was reasonable to assume the first order wave theory to be valid. The waves corresponded to long crested deep water waves and with a diameter of the cylinder $D = 0.07$ m, the forces of inertia were dominating in the fluid-structure problem with a Keulegan-Carpenter Number of 6-7 and a Reynolds Number of 330000. Any contributions from drag forces were neglegted since forces of inertia were dominating. A typical wave and response spectre is shown in figure 3.

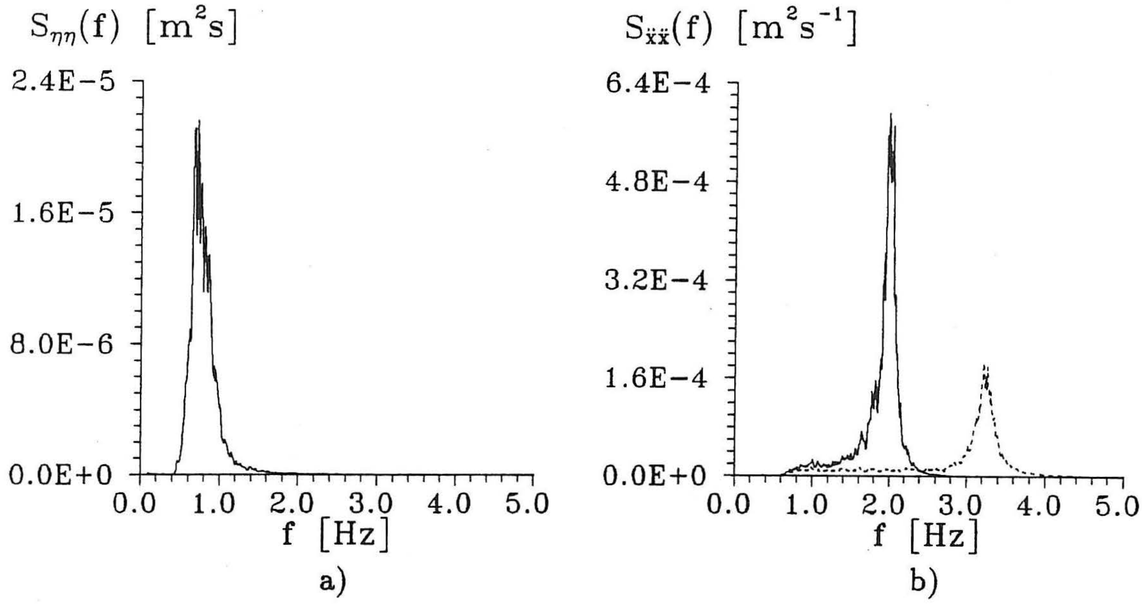


Figure 3. a) Wave elevation spectrum for $H_s = 0.10$ m, b) Corresponding response spectrum with and without top mass.

Modal Identification

It was found that an ARMA(8, 7) model would be a proper model of all the measured records. An example of the (FPE) as a function of the model order is shown in figure 4a. It is seen that the (FPE) is significantly reduced until an ARMA(6, 5) model has been reached. However, an ARMA(8, 7) model was chosen to ensure the whiteness of the residual process a_t. The autospectrum of the residual process is shown in figure 4b. It is seen that the autospectrum of the residual process is quite broadbanded.

With this ARMA model the modal parameters have been estimated together with their coefficients of variation, see table 1. Generally, it is seen that the results are very satisfactory indeed with coefficient of variation of less than 0.2% and 3-5% in magnitude for the eigenfrequency and the damping ratio, respectively. The exception is the results in the first row of the table where the uncertainty is quite large. However, this was found to be due to clipping in the measuring equipment so those results should not be weighted too much and have only been included for the sake of completeness.

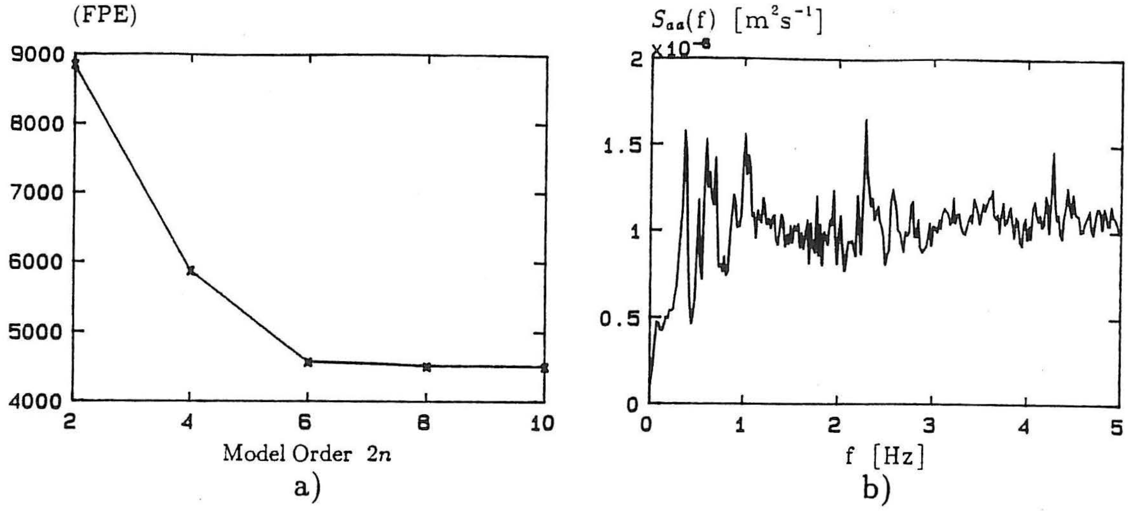


Figure 4. a:(FPE) versus model order, b: Autospectrum of the residual process of an ARMA(8, 7) model. ($H_s = 0.05$ m and $M_{top} = 0$ kg).

H_s [m]	M_{top} [Kg]	T	f_1 [Hz]	δ_{f_1} [%]	ζ_1 [%]	δ_{ζ_1} [%]
0.05	7	36000	2.004	1.0	4.04	25.2
0.10	7	27000	2.008	0.2	4.55	4.6
0.15	7	36000	1.964	0.1	4.17	3.3
0.05	0	36000	3.258	0.2	3.86	4.0
0.10	0	36000	3.252	0.1	3.99	2.8
0.15	0	36000	3.201	0.1	4.14	3.1

Table 1. Estimated modal parameters for ARMA(8, 7) models obtained from acceleration records lowpass filtered with a cut-off frequency of 5 Hz and sampled with a frequency of 10 Hz.

It is also noticed that the modal estimates seem to be independent of the applied H_s even though the eigenfrequency shows a tendency to drop for the most severe sea state. With regard to the damping ratio, it seems to be somewhat smaller than the damping ratio estimated in connection with previously performed free decay experiments in air and in still water. For the case with a top mass, in air the damping was found to be in the range of $\zeta_1 = 2.7\%$ while in water the range is $\zeta_1 = 4.8\%$. Without top mass in air $\zeta_1 = 2.3\%$ was found. The increase in damping due to the water is likely to be due to wave radiation and hydrodynamic damping, (Cook and Vandiver 1982 and Vandiver 1980).

Results of Wave Load Identification

The modal load process was identified from the measured response due to the principles outlined earlier. Examples of the poles and zeroes of the oversized ARMA model which was inspected for determination of the inverse ARMA models are shown in figure 5. The rectangular boxes are the calculated 95%-confidence areas.

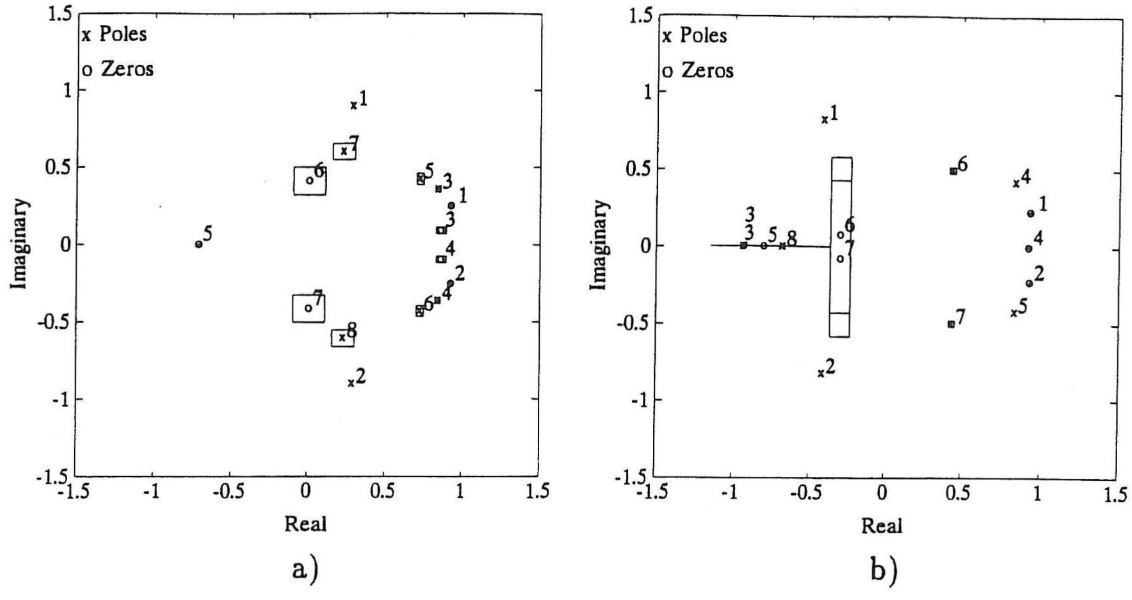


Fig 5. Examples of poles and zeroes of oversized ARMA model for $H_s = 0.10$m. a) $M_{top} = 7$ kg. b) $M_{top} = 0$ kg.

The inverse ARMA$(2,1)$ models were determined by choosing the poles corresponding to the eigenfrequency, and the zero was chosen to be the real zero with the smallest coefficient of variation. Then the response records were filtered by the inverse ARMA$(2,1)$ models and compared with the corresponding modal load proces which could be estimated according to (3)-(4). This was possible since the wave elevation had been measured. The spectre of the wave elevation and the filtered response was obtained by FFT applying the Welch method with 50% overlap with time segments of 512 points and a Hanning window. Since the important point is the shape of the load process spectrum the modal load spectrum was calculated by a mean value of C_M corresponding to the best fit in the peak area. The results are shown in figure 6. In the calculation the mode shape and the mass distribution were known by a priori knowledge.

It is seen from figure 6a that a sensible agreement is found for $M_{top} = 0$ kg ($f_1 = 3.25$ Hz) since the peak of the wave excitation is clearly identified with good agreement between (3)-(4) based on the Morison equation and the filtered response. However, it is seen that (3)-(4) give a much more narrow force peak than identified by the inverse filtering, which might be due to the fluid loading mechanism in and above the mean sea level which has not been taken into account by the application of the linear wave theory and the Morison equation. Beyond the peak area the agreement is very poor which is partly due to a low signal noise ratio on the measured wave elevation signal, and partly due to the limitations of the Morison equation.

From figure 6b it is seen that it becomes difficult to estimate the load process when its peak frequency is closer to the eigenfrequency of the structure ($f_1 = 2.0$ Hz).

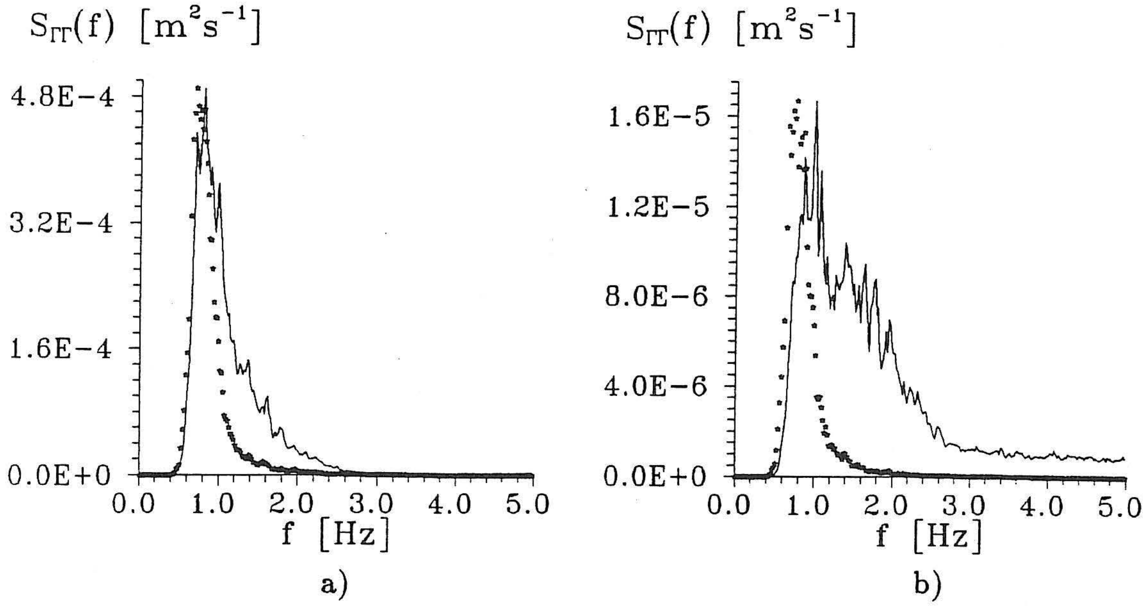
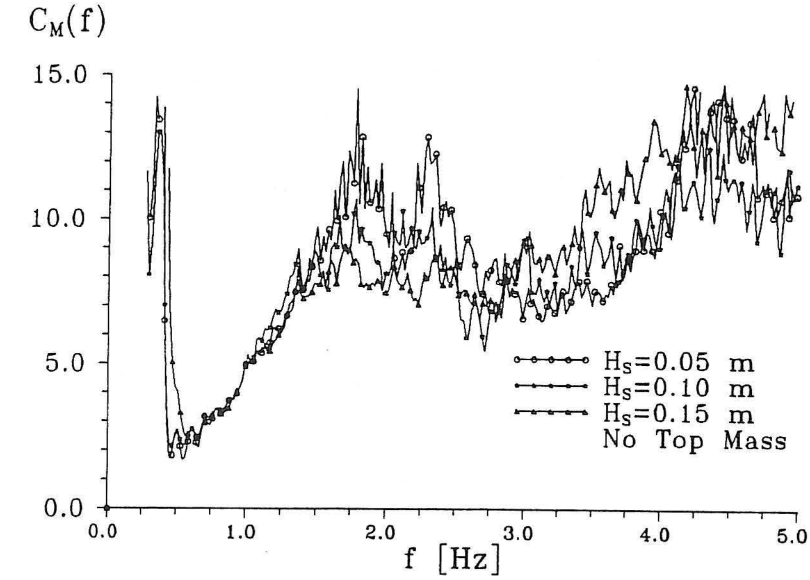


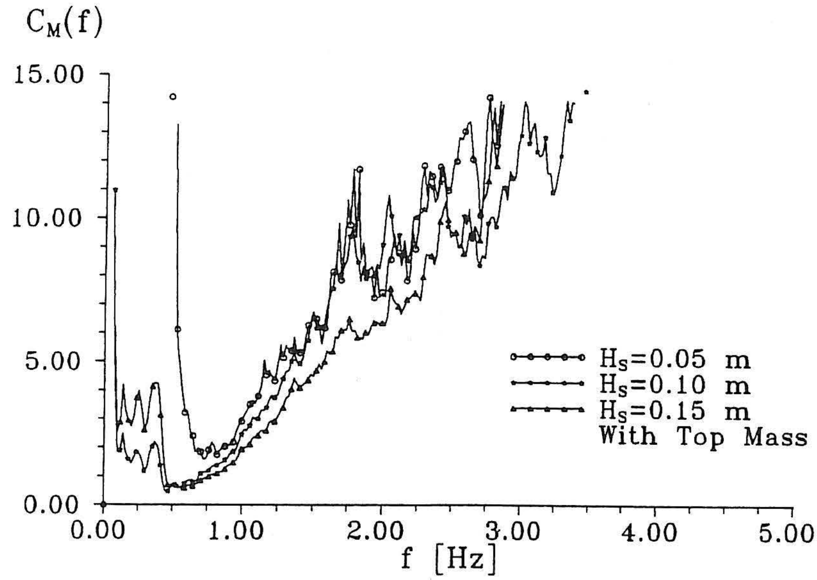
Figure 6. Comparison of normalized modal load spectrum acting on pile for $H_s =$ 0.10 m: a) $M_{top} = 0$ kg, b) $M_{top} = 7$ kg.

Instead of comparing the modal load spectrum directly, a socalled coefficient of inertia has been considered as a function of frequency as shown in figure 7. Some people claim that a simplified expression of C_M can be applied in the frequency domain even though everybody knows that the Morison equation cannot describe the fluid-structure interaction in random waves with structural, dynamic amplification. For the most simple case, a harmonic wave C_M would vary along the structure and with the frequency of the wave.

From figur 7 it is seen that sensible but frequency dependent values are found from 0.5 Hz to just above 1 Hz while the values increase out of range for increasing frequency. It is noticed that $C_M(f)$ tends to become smaller for increasing H_s which is in agreement with general observations (e.g. Sarpkaya and Isaacson 1981). On the other hand, it is seen that $C_M(f)$ seems to increase with increasing frequency which is just the opposite of the suggested simplifications of Karadeniz (1985) in which $C_M(f)$ decreases almost exponentially from a maximum value of 2 in the low frequency region. However, most of this disagreement might again be explained by the low signal/noise ratio in the higher frequency regions.



a)



b)

Figure 7. C_M as a function of frequency: a) Without top mass, b) with top mass.

Conclusion

This paper has shown that identification with ARMA models gives reliable modal estimates not only for simulated cases but also for real experiments. The reliability of the estimates can be directly quantified by coefficients of variation of the modal parameters.

Furthermore, the paper has shown that the ARMA model of a response record will also contain information about the load process when the process is coloured. This will lead to an oversized ARMA model which means that information about the load process can and should be extracted from the ARMA model. The inverse ARMA model can be found from the oversized ARMA model by inspection of the poles and zeroes based on the available a priori knowledge of the structure and the excitation process.

For an sdof structural system it will often be sufficient to apply an inverse ARMA$(2,1)$ model when the eigenfrequency and peak frequency of the load process are well seperated. When they become closly spaced problems may arise which is a subject which should be studied further.

The presented appoach of extracting information about the load process should be followed by studies when the structural system has several excited eigenmodes and also by cases where the drag mechanism is significant. If further reasonable success is obtained the approach will be useful in the analysis of the response records of simple offshore structures such a monotowers and structures in general. Furthermore, the approach may be useful in laboratories within hydrodynamics where estimation of the wave loads is an important research topic.

Appendix. References

Cook, M.F., and J.K. Vandiver (1982), "Measured and Predicted Dynamic Response of a Single Pile Platform to Random Excitation", Proceeding, pp 637-645, Offshore Technology Conference, Houston.

DS411 (1983), "Code of Practice for the Design and Construction of Pile Supported Offshore Steel Structures", DS449, The Danish Society for Engineers.

Gersch, W. (1974) "On the Achievable Accuracy of Structural System Parameter Estimates", J. Sound of Vibrations, 34(1), pp 63-79.

Jensen, J. L. (1990), "System Identification of Offshore Platforms", Ph.D. Thesis, University of Aalborg, Denmark.

Jensen, J. L., R. Brincker and A. Rytter (1990), "Uncertainty of Modal Parameters Estimated by ARMA Models", Proceedings of the 9th International Conference on Experimental Mechanics, 20-24 August 1990, The Technical University of Denmark.

Karadeniz, H. (1985), "Stochastic Analysis Program for Offshore Structures", Report Dept. of Civil Engineering, Delft University of Technology, Delft, Netherlands.

Kozin, F. and H. G. Natke (1986), "System Identification Techniques", Structural Safety, 3 (1986) pp 269-316, Elsevier Science Publishers B.V.

Lin, Y.K. (1967), "Probilistic Theory of Structural Dynamics", McGraw-Hill

Book Company.

Ljung, L. (1987), "System Identification - Theory for the User", Prentice-Hall.

Pandit, S. M. and S.-M. Wu (1983), "Time Series and System Analysis with Applications", John Wiley & Sons.

Pi, Y. L. and N. C. Mickleborough (1989), "Modal Identification of Vibrating Structures using ARMA Model" J. Engineering Mechanics, Vol. 115, No. 10, Oct., American Society of Civil Engineers.

Rabiner, L. R. and B. Gold (1975), "Theory and Application of Digital Signal Processing", Prentice-Hall.

Sarpkaya T. and M. Isaacson (1981), "Mechanics of Wave Forces on Offshore Structures", Van Nostrand Reinhold Company Inc.

Vandiver, J.K. (1980), "Prediction of the Damping-Controlled Response of Offshore Structures to Random Wave Excitation", Society of Petroleum Engineers Journal, Feb., 1980, pp 5-14

Appendix. Notation

a_t	Residual process.
C_M	Coefficient of inertia.
D	Diameter of cylinder.
d	Water depth.
f	Frequency.
f_i	The ith eigenfrequency.
g	gravity.
H_s	Significant wave height.
$H(q)$	System transfer function.
$\mathbf{k}(\omega)$	Frequency dependent wave number.
L	Length of structure (cylinder).
l	Model order.
M_{top}	Mass at cylinder top.
m	Distributed mass per unit length.
m	Increased model order.
n	Number of degrees of freedom.
$p(t,z)$	Load process at given z.
q_i	Modal coordinate of response.
$S_{pp}(z_1, z_2, \omega)$	Load spectrum.
$S_{\eta\eta}(\omega)$	Wave elevation spectrum.
$S_{xx}(\omega)$	Response autospectrum for given z.
$S_{\Gamma_i\Gamma_j}(\omega)$	Modal load spectrum. of the ith and jth modes.
T	Number of sampled points.
t	Time.
V	Error function.
v_t	Load process.
$X(Z)$	Z-transformed.
$x(t,z)$	Displacement response of structure.
$\ddot{x}(t,z)$	Acceleration response.
x_t	Discrete time series of $x(t,z)$ for given z.
z	Cylinder coordinate.
δ	Coefficient of variation.
ζ_i	The ith damping ratio.
$\eta(t)$	Wave elevation process.
λ_i	Pole/root of polynomial.
$\mu_{(i)12}$	Complex eigenvalue.
ν_i	Zero/root of polynomial.
ρ	Density of fluid.
$\phi^{(i)}$	The ith eigenmode.
ω_i	The ith eigenfrequency in [rad/s].
ω	The frequency in [rad/s].
$\Gamma_i(t)$	ith modal load process.
Δ	Sampling interval.
$\Psi(q)$, $\Lambda(q)$	Polynomials.
$\Theta(q)$	MA polynomial.
Θ_i	MA parameter.
$\Phi(q)$	AR polynomial.
Φ_i	AR parameter.
Ω	Frequency range.

Appendix. Additional Results

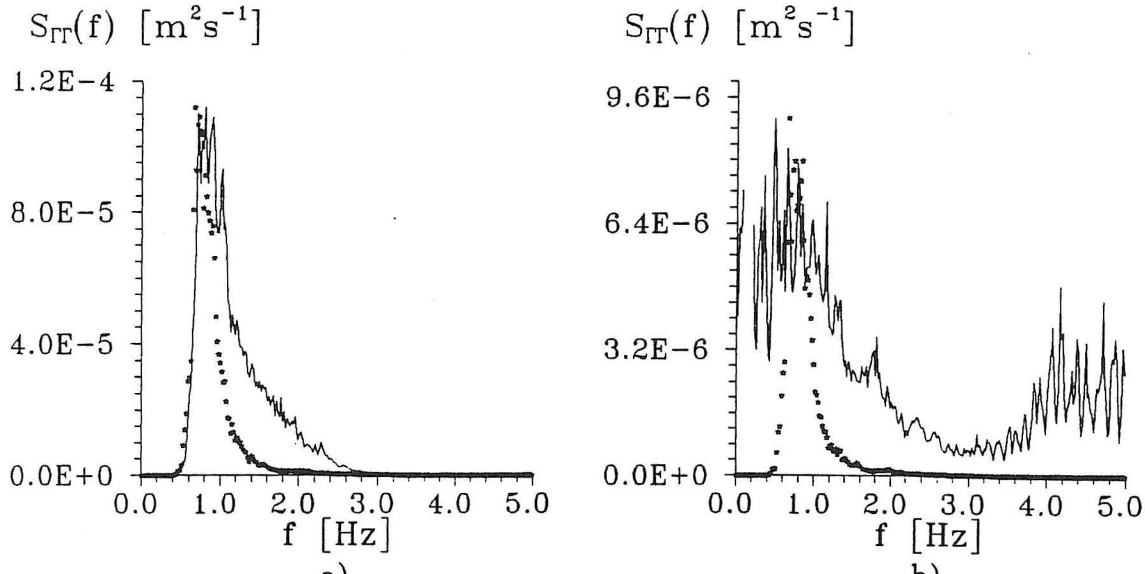


Figure A. Comparison of normalized modal load spectrum acting on pile for $H_s = 0.05$ m: a) $M_{top} = 0$ kg, b) $M_{top} = 7$ kg.

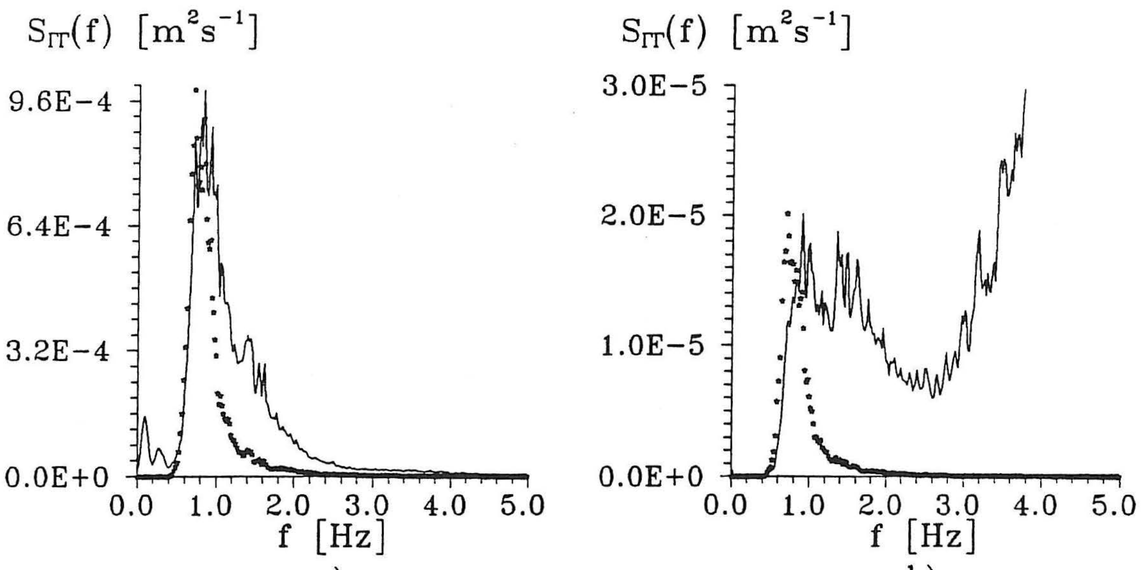


Figure B. Comparison of normalized modal load spectrum acting on pile for $H_s = 0.15$ m: a) $M_{top} = 0$ kg, b) $M_{top} = 7$ kg.

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