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# Sparse Channel Estimation Including the Impact of the Transceiver Filters with Application to OFDM

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**Abstract**—Traditionally, the dictionary matrices used in sparse wireless channel estimation have been based on the discrete Fourier transform, following the assumption that the channel frequency response (CFR) can be approximated as a linear combination of a small number of multipath components, each one being contributed by a specific propagation path. In practical communication systems, however, the channel response experienced by the receiver includes additional effects to those induced by the propagation channel. This composite channel embodies, in particular, the impact of the transmit (shaping) and receive (demodulation) filters. Hence, the assumption of the CFR being sparse in the canonical Fourier dictionary may no longer hold. In this work, we derive a signal model and subsequently a novel dictionary matrix for sparse estimation that account for the impact of transceiver filters. Numerical results obtained in an OFDM transmission scenario demonstrate the superior accuracy of a sparse estimator that uses our proposed dictionary rather than the classical Fourier dictionary, and its robustness against a mismatch in the assumed transmit filter characteristics.

## I. INTRODUCTION

Many channel models proposed for wireless communication systems characterize the impulse response of the radio channel as the sum of a few dominant multipath components, each associated with a delay and a complex gain [1]. As a result, the channel frequency response (CFR), defined as the Fourier transform of the channel impulse response (CIR), admits a sparse representation in a specific Fourier dictionary. Methods from compressed sensing (CS) and sparse channel representations have been proposed to devise estimators of the radio channel responses that exploit this property [2]–[5].

However, the receiver of a wireless communication system observes a composite channel response that includes the impact of the propagation channel together with other effects, such as those induced by antenna or transceiver filter<sup>1</sup> responses. The combination of these effects results in a composite CFR that exhibits sparsity in a different (a priori unknown) dictionary. This naturally raises two questions: (i) can a CS-based estimator using classical Fourier dictionaries still yield precise sparse estimates of the composite channel?, and (ii) if this is not the case, which dictionary should the estimator use in order to produce accurate sparse estimates of the composite channel response?

The authors of [6] showed that the performance of CS algorithms is highly sensitive to mismatches in the used

dictionary matrix. Such a situation is encountered when CS-based channel estimators are applied to a signal model which incorrectly assumes perfect low-pass transceiver filters. To the authors' knowledge, only a few contributions have explored the effects of transceiver filters on sparse estimators before. In [3], the authors apply CS techniques to estimate the channel in a multicarrier system using perfect low-pass filters in highly mobile setups; they observe that the discrete delay-Doppler spreading function is approximately sparse. In this contribution we are, however, interested in the limitations that the practical implementation of such transceiver filters impose on the accuracy that sparse (or CS-based) estimators can attain. In [7], an OFDM system with transceiver filters is also analyzed. The author claims that, under the conditions of a sufficiently large bandwidth (e.g. 256 MHz), the resulting composite channel response appears approximately sparse. It is, however, unclear whether this conclusion holds for systems employing a smaller bandwidth as, e.g., an LTE system [1].

In this article we derive a model for the received OFDM signal with a dictionary explicitly accounting for the distortion introduced by transceiver filters. We then apply a CS-based channel estimator to this signal model and to the classical model which neglects this distortion [2], [4], [5]. Numerical investigations are conducted considering an LTE system as use-case. They reveal that the performance of the sparse estimator, measured in terms of mean squared error (MSE) of the CFR estimates, is significantly improved when it is applied in combination with our proposed dictionary, compared to when it is applied in combination with the classical Fourier dictionary. These investigations also demonstrate that the sparse estimator used in combination with the former dictionary is robust towards mismatches between the true and assumed characteristics of the filters, and that it performs well even in scenarios where the channel response exhibits a large number of multipath components.

The remainder of this paper is organized as follows. In Section II we derive an OFDM received signal model which includes the effects of the transceiver filters. Based on this, we propose in Section III a novel design of the dictionary used by sparse channel estimators. In Section IV we test the performance of the aforementioned estimators. In Section V we sum up the observations and conclude the paper.

**Notation:** Boldface uppercase and lowercase letters designate matrices and vectors, respectively. The diagonal matrix

<sup>1</sup>Henceforth, we use the term “transceiver filter” to designate either the transmit (shaping) or the receive (demodulation) filters.

$\mathbf{A} = \text{diag}(\mathbf{a})$  has the entries of the vector  $\mathbf{a}$  as diagonal elements. We denote by  $[\mathbf{A}]_{i,j}$  the  $(i,j)$ th element of the matrix  $\mathbf{A}$ . We define the  $N \times N$  discrete Fourier transform (DFT) matrix with  $\mathbf{F} \in \mathbb{C}^{N \times N}$ ,  $[\mathbf{F}]_{m,n} = 1/\sqrt{N}e^{-j2\pi mn/N}$ ,  $m, n \in [0 : N-1]$ ;  $\mathbf{I}$  is the identity matrix. A function  $f$  which maps the set  $\mathcal{E}$  to the set  $\mathcal{F}$  is denoted as  $f : \mathcal{E} \rightarrow \mathcal{F}$  and its support is  $\text{supp}(f) = \{x \in \mathcal{E} \mid f(x) \neq 0\}$ ; the notation  $|\mathcal{F}|$  denotes the cardinality of  $\mathcal{F}$ . We represent the convolution of two functions  $f$  and  $g$  as  $f * g$ ;  $\delta(\cdot)$  is the Dirac delta function. The notation  $[P_1 : P_2]$  denotes the set  $\{p \in \mathbb{N} \mid P_1 \leq p \leq P_2\}$ . The superscripts  $(\cdot)^T$  and  $(\cdot)^H$  denote transposition and Hermitian transposition respectively. The notation  $\|\mathbf{a}\|_0$  denotes the number of non-zero entries of  $\mathbf{a}$ .

## II. SIGNAL MODEL

We consider a single-input single-output (SISO) OFDM system model. By contrast to the traditional approach [5], we account for the response of the transceiver filters in the derivation of the model. The message consists of a vector  $\mathbf{u} = [u_0, \dots, u_{N_B-1}]^T$  of information bits which are encoded with a code rate  $R = N_B/N_C$  and interleaved to yield the vector  $\mathbf{c} = [c_0, \dots, c_{N_C-1}]^T$ . The code vector is modulated onto a vector of complex symbols that are multiplexed with the pilot symbols producing the symbol vector  $\mathbf{x} = [x_0, \dots, x_{N-1}]^T$ . The symbol  $x_i$  is a pilot symbol if  $i \in \mathcal{P}$  or a data symbol if  $i \in \mathcal{D}$ , where  $\mathcal{P}$  and  $\mathcal{D}$  represent the subsets of pilot and data indices respectively, so that  $\mathcal{P} \cup \mathcal{D} = \{0, \dots, N-1\}$ ,  $\mathcal{P} \cap \mathcal{D} = \emptyset$ ,  $|\mathcal{P}| = N_P$  and  $|\mathcal{D}| = N_D$ . We refer to  $\mathcal{P}$  as the pilot pattern. The symbol vector  $\mathbf{x}$  is passed through an inverse DFT block, yielding  $\mathbf{s} = [s_0, \dots, s_{N-1}]^T = \mathbf{F}^H \mathbf{x}$ . Next,  $\mathbf{s}$  is appended a  $\mu$ -sample long cyclic prefix (CP) and the entries of the resulting vector are modulated using a transmit shaping filter with impulse response  $\psi_{\text{tx}}(t)$  to produce the continuous-time OFDM signal

$$s(t) = \sum_{n=-\mu}^{N-1} s_n \psi_{\text{tx}}(t - nT_s), \quad t \in [-\mu T_s, NT_s] \quad (1)$$

where  $T_s$  is the sampling period. We assume that  $\text{supp}(\psi_{\text{tx}}) = [0, T]$ , with  $T = aT_s$ ,  $a > 0$ . The signal  $s(t)$  is sent across a wireless channel with CIR  $g(\tau)$  modeled as the sum of  $L$  (specular) multipath components, with the complex gains  $\beta = [\beta_0, \dots, \beta_{L-1}]^T$  and delays  $\tau = [\tau_0, \dots, \tau_{L-1}]^T$ :

$$g(\tau) = \sum_{l=0}^{L-1} \beta_l \delta(\tau - \tau_l). \quad (2)$$

We assume that  $g(\tau)$  remains invariant over the duration of one OFDM symbol. At the reception, the signal appears as the convolution of the transmitted signal (1) and the CIR (2) corrupted by additive white Gaussian noise  $n(t)$  with spectral height  $\sigma^2$ , i.e.

$$z(t) = (s * g)(t) + n(t). \quad (3)$$

The received signal is next passed through a receive demodulation filter with response  $\psi_{\text{rx}}(t)$ ,<sup>2</sup>  $\text{supp}(\psi_{\text{rx}}) = [0, T]$ , producing the output

$$r(t) = (z * \psi_{\text{rx}})(t) = \sum_{n=-\mu}^{N-1} s_n (\psi_{\text{tx}} * g * \psi_{\text{rx}})(t - nT_s) + \nu(t) \quad (4)$$

where  $\nu(t) = (\psi_{\text{rx}} * n)(t)$ . The output signal  $r(t)$  is sampled and the CP is discarded, yielding the vector  $\mathbf{r} = [r_0, \dots, r_{N-1}]^T$  with the entries

$$r_k = r(kT_s) = \sum_{n=-\mu}^{N-1} s_n q((k-n)T_s) + \nu(kT_s), \quad (5)$$

$k \in [0 : N-1]$ . In the above expression, we defined the composite channel response  $q(t) = (g * \psi_{\text{tx}} * \psi_{\text{rx}})(t) = (g * \phi)(t)$ ,  $\text{supp}(q) \subseteq [0, \tau_{L-1} + 2T]$ , with  $\phi(t) = (\psi_{\text{tx}} * \psi_{\text{rx}})(t)$ ,  $\text{supp}(\phi) = [0, 2T]$ . The noise samples in (5) form a circularly-symmetric complex Gaussian process with variance  $\lambda^{-1}$ ,  $\lambda \geq 0$  that is uncorrelated when the autocorrelation of  $\psi_{\text{rx}}(t)$  satisfies the Nyquist criterion.<sup>3</sup>

We observe that decreasing the system bandwidth, i.e. increasing the sampling period  $T_s$ , results in widening the convolved response of the transceiver filters  $\phi(t)$ . As a result, for large bandwidths (small  $T_s$ ) the composite response exhibits an approximately specular behavior as the response  $\phi(t)$  decays fast, which justifies disregarding the filters' effects [7]. Conversely, when employing a smaller bandwidth (larger  $T_s$ ) as e.g. in 20 MHz LTE systems, each multipath component in (2) is convolved with the slow-decaying response  $\phi(t)$ . Under such conditions,  $q(t)$  is not well approximated by a specular response anymore.

In order to avoid inter-symbol interference, it must be ensured that  $r_k = 0$  for  $k > N + \mu$ , which implies that  $q((k-n)T_s) = 0$  for  $k-n \geq \mu+1$ . When this condition is satisfied, the signal  $\mathbf{y} = [y_0, \dots, y_{N-1}]^T$  observed after the DFT processing at the receiver reads

$$\mathbf{y} = \mathbf{F}\mathbf{r} = \mathbf{X}\mathbf{M}\beta + \xi \quad (6)$$

where  $\mathbf{X} = \text{diag}(x_0, \dots, x_{N-1})$ ,  $\mathbf{M} = \sqrt{N}\mathbf{F}\Phi$ ,  $\beta$  is defined before (2),  $\xi = \mathbf{F}\nu$ ,  $\nu = [\nu(0T_s), \dots, \nu((N-1)T_s)]^T \in \mathbb{C}^N$ , and  $\Phi \in \mathbb{R}^{N \times L}$ ,  $[\Phi]_{n,l} = \phi(nT_s - \tau_l)$ ,  $n \in [0 : N-1]$ ,  $l \in [0 : L-1]$ .

In order to estimate the CFR at all subcarriers, i.e.  $\mathbf{h} = \mathbf{M}\beta$ , we use the  $N_P$  observations corresponding to the pilot subcarriers given by the pattern  $\mathcal{P}$ . The received signal observed at each pilot subcarrier  $\mathbf{y}^{(\mathcal{P})}$  is divided by the corresponding known transmitted symbol. We note that the superindex  $(\cdot)^{(\mathcal{P})}$  applied to a matrix  $\mathbf{A}$  denotes a matrix  $\mathbf{A}^{(\mathcal{P})}$  that contains the rows of  $\mathbf{A}$  corresponding to the pattern  $\mathcal{P}$ . The vector of observations used for estimating the channel vector reads

$$\mathbf{t} = [\mathbf{X}^{(\mathcal{P})}]^{-1} \mathbf{y}^{(\mathcal{P})} = \mathbf{M}^{(\mathcal{P})} \beta + [\mathbf{X}^{(\mathcal{P})}]^{-1} \xi^{(\mathcal{P})}. \quad (7)$$

<sup>2</sup>Without loss of generality, we assume  $\psi_{\text{rx}}(t)$  and  $\psi_{\text{tx}}(t)$  have energy one.

<sup>3</sup>The receive filter's autocorrelation  $A_{\text{rx}}(t) = \psi_{\text{rx}}(t) * \psi_{\text{rx}}(-t)$  satisfies the condition  $A_{\text{rx}}(kT_s) = 0, \forall k \neq 0$ .

Thus, the observation  $\mathbf{t}$  contains the samples of the CFR at the pilot subcarrier frequencies corrupted by noise. By contrast, the traditional observation model [2], [5] disregards the effects of transceiver filters. In this case,

$$\mathbf{t} = \mathbf{T}^{(P)}\boldsymbol{\beta} + [\mathbf{X}^{(P)}]^{-1}\boldsymbol{\xi}^{(P)} \quad (8)$$

where  $\mathbf{T} \in \mathbb{C}^{N \times L}$  has the entries  $[\mathbf{T}]_{n,l} = e^{-j2\pi \frac{n}{NT_s} \tau_l}$ ,  $n \in [0 : N-1]$ ,  $l \in [0, L-1]$  and  $\tau_l \in \boldsymbol{\tau}$ . We will next discuss the effects of using the model (7) instead of (8) in the context of sparse channel estimation.

### III. COMPRESSED SENSING INFERENCE FOR CHANNEL ESTIMATION

If the matrix  $\mathbf{M}$ —and, hence,  $\mathbf{M}^{(P)}$ —was known, estimating  $\mathbf{h}$  would be equivalent to estimating the entries of the vector of complex channel gains  $\boldsymbol{\beta}$ . Unfortunately, neither the dimensions of  $\boldsymbol{\beta}$  and  $\boldsymbol{\tau}$  (i.e. the number of multipath components in (2)) nor the entries of  $\boldsymbol{\tau}$ , which the matrix  $\mathbf{M}$  depends on, are known. In order to overcome this limitation we employ methods from CS to estimate the CIR and consequently the CFR  $\mathbf{h}$ . To that end, a discretized version of the CIR in (2) is used:

$$\bar{g}(\tau) = \sum_{k=0}^{K-1} \alpha_k \delta(\tau - \bar{\tau}_k) \quad (9)$$

where  $\bar{\tau}_k = k\Delta_\tau$ ,  $k \in [0 : K-1]$  and we define  $\boldsymbol{\alpha} = [\alpha_0, \dots, \alpha_{K-1}]^T$  and  $\bar{\boldsymbol{\tau}} = [\bar{\tau}_0, \dots, \bar{\tau}_{K-1}]^T$ .

#### A. Canonical compressed channel sensing model

Making use of (9), the canonical linear model used in compressed channel sensing [5], [8] approximates the observation model by:

$$\mathbf{t} \approx \mathbf{H}^{(P)}\boldsymbol{\alpha} + \mathbf{w} \quad (10)$$

where  $\mathbf{w} \in \mathbb{C}^{N_P}$  is circularly-symmetric Gaussian distributed with zero-mean and covariance matrix  $\lambda^{-1}\mathbf{I}$ , and the matrix  $\mathbf{H} \in \mathbb{C}^{N \times K}$  has entries

$$[\mathbf{H}]_{n,k} = e^{-j2\pi \frac{n}{NT_s} \bar{\tau}_k}, \quad (11)$$

$n \in [0 : N-1]$ ,  $k \in [0 : K-1]$ . By choosing a sufficiently small sampling interval  $\Delta_\tau$  and sufficiently large  $K \gg L$ , many entries of  $\boldsymbol{\alpha}$  are expected to be either zero or close to zero, i.e.  $\boldsymbol{\alpha}$  is expected to be approximately sparse.

Various CS methods [2], [5] have been proposed to compute sparse estimates of  $\boldsymbol{\alpha}$  in (10). Once an estimate  $\hat{\boldsymbol{\alpha}}$  has been obtained, the estimated CFR is computed as  $\hat{\mathbf{h}} = \mathbf{H}\hat{\boldsymbol{\alpha}}$ .

#### B. Novel compressed channel sensing model

We note that the model (10) is based on two drastic approximations of the true observation model (7): (i) the approximated observation model (8) that disregards the transceiver filters' effects, and (ii) the discretized CIR (9). While the approximation (ii) is necessary, we amend the approximation (i) and recast the CS model to

$$\mathbf{t} \approx \mathbf{H}_\phi^{(P)}\boldsymbol{\alpha} + \mathbf{w} \quad (12)$$

with  $\mathbf{H}_\phi \in \mathbb{C}^{N \times K}$  defined as

$$[\mathbf{H}_\phi]_{n,k} = \sqrt{N} \sum_{m=0}^{N-1} [\mathbf{F}]_{n,m} \phi(mT_s - \bar{\tau}_k), \quad (13)$$

$n \in [0 : N-1]$ ,  $k \in [0 : K-1]$ . We then apply CS techniques to the model (13) and obtain sparse estimates  $\tilde{\boldsymbol{\alpha}}$ ; similarly, we compute the estimated CFR  $\tilde{\mathbf{h}}_\phi = \mathbf{H}_\phi \tilde{\boldsymbol{\alpha}}$ .

We note that the CFR  $\mathbf{h}$  is  $L$ -sparse<sup>4</sup> in the dictionary  $\mathbf{M}$  defined in (6). However, since  $\mathbf{M}$  is unknown a priori, we employ the two approximate dictionaries  $\mathbf{H}_\phi$  and  $\mathbf{H}$  and consequently assume that  $\mathbf{h}$  is approximately sparse in  $\mathbf{H}_\phi$  and  $\mathbf{H}$ . In CS, the usage of approximate dictionaries is typically referred to as dictionary mismatch [6]. In the case of the dictionary  $\mathbf{H}_\phi$ , the mismatch is caused by the discretization of the delay domain carried out in (9); for the dictionary  $\mathbf{H}$ , an additional source of mismatch is present due to the neglect of the transceiver filter responses. Hence, we conjecture that a sparse estimator employing (12) instead of (10) will provide a more accurate estimate of  $\mathbf{h}$ . This conjecture is based on the fact that, when using the dictionary from (13),  $\tilde{\boldsymbol{\alpha}}$  would correspond to an estimate of the CIR  $g$  in (2), while utilizing (11) would yield an estimate  $\hat{\boldsymbol{\alpha}}$  of the composite channel  $q$  defined after (5). For small-to-medium bandwidths, the latter approach will result in estimates of  $\boldsymbol{\alpha}$  with more non-zero entries than those obtained with the former approach, due to the effect of the filters response  $\phi$  on the composite response  $q$ . Thus, a less accurate reconstruction of the CFR is expected.

#### C. Sparse channel estimation using sparse Bayesian learning

We use a Bayesian inference method commonly referred to as sparse Bayesian learning (SBL) to obtain sparse estimates of  $\boldsymbol{\alpha}$  in the CS models in (10) and (12). SBL makes use of models of the prior pdf  $p(\boldsymbol{\alpha})$  that strongly penalize non-sparse estimates in maximum-a-posteriori based estimators [9], [10]. In this work, we have selected the prior model proposed in [2], where the prior pdf of  $\boldsymbol{\alpha}$  is formulated as

$$p(\boldsymbol{\alpha}) = p(\boldsymbol{\alpha}; \epsilon, \eta) = \int_0^\infty p(\boldsymbol{\alpha}|\boldsymbol{\gamma})p(\boldsymbol{\gamma}; \epsilon, \eta)d\boldsymbol{\gamma} \quad (14)$$

with  $p(\boldsymbol{\alpha}|\boldsymbol{\gamma}) = \prod_{k=0}^{K-1} p(\alpha_k|\gamma_k)$ ,  $p(\boldsymbol{\gamma}; \epsilon, \eta) = \prod_{k=0}^{K-1} p(\gamma_k; \epsilon, \eta)$ , where  $p(\alpha_k|\gamma_k)$  is a Gaussian pdf with zero-mean and variance  $\gamma_k$ , and  $p(\gamma_k; \epsilon, \eta)$  is a Gamma pdf with shape and rate parameters  $\epsilon$  and  $\eta$ , respectively. Using the above prior model, the estimation algorithm presented in [2] is applied to the observation models (10) and (12).

## IV. PERFORMANCE EVALUATION

### A. Setup

In this section we study the performance of the SBL channel estimator using our proposed dictionary matrix design in a SISO LTE OFDM setup [1], with the settings specified in

<sup>4</sup>A signal  $\mathbf{a}$  is  $L$ -sparse in a dictionary  $\mathbf{A}$  if a vector  $\mathbf{b}$  exists with  $\|\mathbf{b}\|_0 = L$  s.t.  $\mathbf{a} = \mathbf{A}\mathbf{b}$ .

TABLE I  
PARAMETER SETTINGS

Sampling time $T_s$	32.55 ns
Bandwidth $B$	20 MHz
CP length	144 $T_s$
Modulation	64 QAM
Turbo-encoder rate	948/1024
Decoder	BCJR [12]
Number of subcarriers $N$	1200
Number of pilots/time slot $N_P$	400

Table I. The pilots are arranged according to the pattern specified in [11]. Both transmit and receive filters are truncated square-root raised cosine filters with roll-offs  $r_{TX}$  and  $r_{RX}$  respectively, and duration  $T = 3T_s$ .

We employ two different versions of the SBL estimator proposed in [2]. One version of the estimator, which we coin SE, uses the classical dictionary matrix design in (11). The second version, referred to as SE(F) henceforth, uses our proposed dictionary matrix design in (13), which accounts for the responses of the transceiver filters. For both estimators, we use the parameter setting ( $\epsilon = 0.5, \eta = 1, K = 500, \Delta_\tau = 10\text{ns}$ ), see (9) and (14). For benchmarking purposes, we consider two additional estimators: (i) a genie-aided estimator (GAE) [13] that assumes perfect knowledge of multipath components delays, i.e. uses the dictionary  $\mathbf{M}$  from (6), and (ii) a robust design of the classical Wiener filter estimator, which we refer to as robust Wiener filter (RWF). The latter estimator is designed assuming that the channel has a maximum excess delay of  $5 \mu\text{s}$  and a robust covariance matrix following [14]. Note that this assumption is also implicitly made in the design of SE and SE(F) via the aforementioned parameter setting.

We assess the accuracy of the investigated estimators by evaluating the performance, in terms of MSE of the CFR estimates and coded bit-error rate (BER), of a receiver employing them. The receiver is evaluated in two different propagation scenarios, each characterized by a specific channel model. In both scenarios, block fading is assumed. Scenario A employs a sparse 3GPP-like channel, modeled as specified in [1]: the CIR consists of five multipath components, with associated delays drawn independently from a uniform distribution with a 10 ns range, centered around the delays specified in Table II.<sup>5</sup> Scenario B employs the model introduced in [15], and later studied in [16]. The CIRs generated from the model exhibit a number of clustered multipath components which varies over different realizations, with cluster and within-cluster delays following Poisson arrival processes with rates  $\Lambda$  and  $\lambda$ , respectively. The conditional second moments of the channel gains are modeled by the power-delay constants  $(\Gamma, \gamma)$ . We set  $[1/\Lambda, 1/\lambda] [\mu\text{s}] = [0.3, 5]$  and  $[\Gamma, \gamma] [\text{ns}] = [600, 200]$ , leading to channel realizations that contain, in average, fifteen multipath components. Scenario A enables us to determine whether the filters' effects impair the receiver

TABLE II  
SCENARIO A. CHANNEL POWER DELAY PROFILE

Delays $[\mu\text{s}]$	0	0.5	1.6	2.3	3.3
Power $[\text{dB}]$	-1	0	-3	-5	-7

performance when they are not accounted for. Scenario B allows the total number of multipath components to vary over different realizations in order to account for the variability of scatterers in the environment, introducing therefore an additional degree of freedom compared with the channel in Scenario A. The harsher channel conditions of Scenario B allow us to draw further conclusions on the performance of the studied estimators.

### B. Numerical Results

In Fig. 1 we depict the performance of the sparse estimators in terms of MSE of the CFR in Scenarios A and B respectively when the transmit and receive filters are perfectly matched ( $r_{TX} = r_{RX} = 0.5$ ). In both scenarios, we observe that the mismatched dictionary matrix used in SE degrades the estimator's performance as the SNR increases. Since it accounts for the filter's responses in the dictionary matrix, SE(F) performs closely to GAE for all SNR values; the slight performance degradation that SE(F) suffers in Scenario B at high SNR is caused by the presence of numerous multipath components. However, in both scenarios SE(F) outperforms SE and RWF.

In practical situations, the characteristics of the transmitter's radio frequency front end may be unknown by the receiver. As a result, the receiver often possesses incomplete information for computing the dictionary matrix. This provides an incentive to study how the mismatch between the transmit and receive filters affects the accuracy of the estimator. To conduct this investigation, we fix the receive filter roll-off to  $r_{RX} = 0.5$ , and vary the transmit filter roll-off  $r_{TX}$ . The GAE and SE(F) estimators assume  $r_{TX} = r_{RX}$ , regardless of the actual value of  $r_{TX}$ . The resulting MSE is depicted in Fig. 2 for the two scenarios at 30 dB SNR. As expected, SE(F) achieves its best performance when the roll-off factor of the transmit and receive filters coincide ( $r_{TX} = r_{RX} = 0.5$ ). However, even in the case of a roll-off mismatch, SE(F) always outperforms SE. Hence, SE(F) is robust against mismatches of the assumed filter roll-off parameters.

Fig. 3 depicts the BER performance of a receiver employing the investigated channel estimators. We have selected as an ideal reference a receiver which has knowledge of the true CFR coefficients. We observe that, in Scenario A, SE(F) performs almost as well as the ideal reference, with a gain of up to 1 dB with respect to SE. The harsher channel conditions in Scenario B lead to a less significant benefit of using SE(F). Nonetheless, a receiver employing SE(F) always performs better than a receiver employing SE.

### V. CONCLUSION

In this paper we have analyzed the effect that transceiver filters have on the accuracy of selected state-of-art sparse channel estimation techniques. Traditional CS techniques for

<sup>5</sup>We ensure in this way that the true delays are not integer multiples of the delay resolution we have selected for the sparse estimator - see (9).

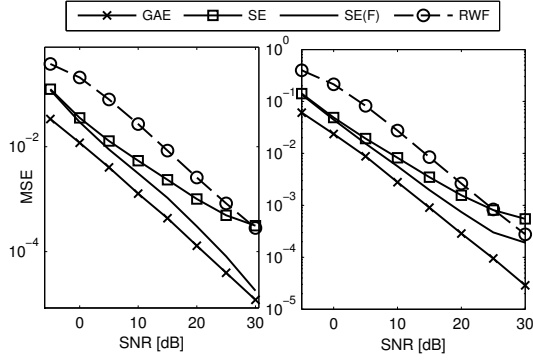


Fig. 1. Comparison of the MSE performance of the investigated channel estimators in Scenario A (left) and Scenario B (right).

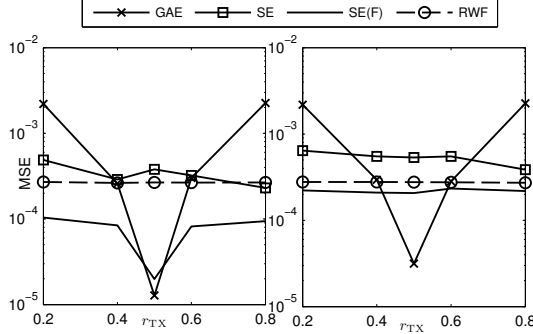


Fig. 2. Comparison of the robustness of the investigated estimators towards mismatched filter parameters ( $r_{RX} = 0.5$ ,  $r_{TX} \in [0.2 \ 0.8]$ ) in Scenario A (left) and Scenario B (right). The estimators assume  $r_{TX} = r_{RX} = 0.5$ .

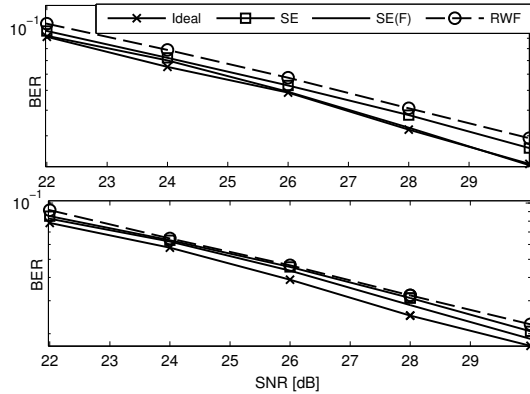


Fig. 3. Comparison of the BER performance of a receiver using SE, SE(F), and RWF in Scenario A (left) and Scenario B (right).

channel estimation employ Fourier dictionaries, which fail to embed the transceiver filters' responses. As a result, the CS-based channel estimators operate with mismatched dictionaries which degrade their estimation accuracy. To overcome this limitation, we have proposed a novel design of the dictionary matrix which accounts for the filters' responses, allowing thus for sparser representations of the channel response.

To evaluate the validity of the proposed solution, we applied an SBL estimator that includes either this new dictionary or the traditional Fourier dictionary to an OFDM communication system. Numerical results illustrated that the SBL estimator

employing our dictionary design always performs better than when it uses the classical dictionary. Additionally, we observed that our proposed dictionary matrix is especially advantageous in scenarios in which the channel exhibits a high degree of sparsity. Furthermore, even when the receiver possesses imperfect information about the filters' responses, the proposed dictionary yields a robust behavior of the sparse estimator.

Finally, we point out that, even though this study has been restricted to a particular choice of estimator, the proposed dictionary can be applied to any sparse channel estimator derived within the CS framework. Hence, we conclude that the dictionary matrix design proposed in this work will be a valuable tool to enable robust and accurate CS-based channel estimation in future generations of wireless receivers.

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