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STRUCTURAL RELIABILITY THEORY
PAPER NO. 155

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# STOCHASTIC FINITE ELEMENTS IN RELIABILITY-BASED STRUCTURAL OPTIMIZATION

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## ABSTRACT

Application of stochastic finite elements in structural optimization is considered. It is shown how stochastic fields modelling e.g. the modulus of elasticity can be discretized in stochastic variables and how a sensitivity analysis of the reliability of a structural system with respect to optimization variables can be performed. A computer implementation is described and an illustrative example is given.

**Key Words:** Reliability analysis; Stochastic finite elements; Structural optimization; Stochastic fields.

## 1. INTRODUCTION

The development of effective First Order Reliability Methods (FORM), see (Madsen et al., 1986) and optimization algorithms in combination with sensitivity analysis methods has implied that a large number of decison problems in structural engineering can now be solved with a reasonable amount of resources. Examples are optimal reliability-based design, inspection and maintenance planning with respect to cracks in steel structures and optimal reliability-based experiment planning. These decision problems can be solved on the basis of classical decision theory using Bayesian statistics, see e.g. (Sørensen et al., 1995) and (Kroon et al., 1995).

Uncertain quantities are generally modelled by stochastic variables and the reliability is estimated using FORM. However, uncertainties related to structural material characteristics in general have to be modelled by stochastic fields. Using stochastic finite elements the stochastic fields can be discretized. In relation to optimization of structural systems the discretized stochastic fields have until now been characterized in an approximate way by stochastic variables related to the midpoint or the average value of the stochastic field in a given stochastic finite element discretization, see (Brenner, 1991) and (Der Kiureghian et al., 1988). However, a newly developed method, see e.g. (Deodatis, 1991), makes it possible to formulate 'exact' stochastic stiffness matrices by the so-called weighted integral method.

The purpose of the present paper is to demonstrate how reliability-based optimization problems involving random spatial fluctuations of material properties can be solved using FORM analysis and weighted integrals in stochastic finite elements.

The solution of this optimization problem can be divided into four main tasks, namely (stochastic) finite element analyses, sensitivity analyses, reliability analyses and application of an optimization algorithm. It is discussed how these four tasks can be linked effectively, especially the implementation of stochastic finite elements in such a system is considered. The suggested methology is demonstrated by an illustrative example.

## 2. RELIABILITY-BASED STRUCTURAL OPTIMIZATION

As described in (Enevoldsen et al., 1994) an optimization (design) problem in *reliability-based structural optimization* can be formulated by

$$\min \ C(\mathbf{b}) \tag{1}$$

$$s.t. \ \ \beta_i(\mathbf{b}) \geq \beta_i^{\min} \qquad ,i = 1,\ldots,M \tag{2}$$

$$B_i(\mathbf{b}) \geq 0 \qquad ,i = 1,\ldots,m \tag{3}$$

$$b_i^l \leq b_i \leq b_i^u \qquad ,i = 1,\ldots,N \tag{4}$$

where $\mathbf{b} = (b_1,\ldots,b_N)$ are the design/optimization variables. The optimization variables are assumed to be related to size and shape parameters. The objective function $C$ is often chosen as the structural weight.

The constraints in (2) are related to the reliability of single elements of the structural system. $\beta_i, i = 1,...,M$ are reliability indices for $M$ failure modes. $\beta_i^{\min}, i = 1,...,M$ are the corresponding lower limits on the reliabilities. The deterministic inequality constraints in (3) ensure that response characteristics such as displacements and stresses do not exceed codified critical values. The inequality constraints can also include general design requirements for the design variables. The constraints in (4) are simple bounds.

Alternatively, the optimization problem (1) - (4) can be formulated as a *system reliability-based optimization* problem if the constraint (2) is exchanged with a constraint on the system reliability index $\beta^s$ estimated on the basis of the $M$ failure modes in (2). Further, a more general optimization problem can be formulated if the objective function in (1) is exchanged with the total expected costs in the design lifetime, see (Sørensen et al., 1994).

The reliability indices in (2) are assumed to be determined on the basis of limit state functions which can be written

$$g_i(\mathbf{b}, \mathbf{x}(\mathbf{b}), \ \mathbf{y}(\mathbf{b}, \mathbf{x}(\mathbf{b}))) = 0 \qquad i = 1,...,M \tag{5}$$

where $\mathbf{x}$ are realizations of stochastic variables $\mathbf{X}$. Realizations $\mathbf{x}$ where $g \leq 0$ corresponds to failure states, while realizations $\mathbf{x}$ where $g > 0$ corresponds to safe states. $\mathbf{y}$ are performances such as displacements and stresses.

The stochastic variables $\mathbf{X}$ can depend on the optimization variables $\mathbf{b}$, for example if the expected value $\mu_{X_i}$ of $X_i$ or the standard deviation $\sigma_{X_i}$ of $X_i$ are related to an optimization variable by e.g. $b_j = \mu_{X_i}$.

The deterministic constraints in (3) are assumed to be related to the limit state functions $g_i$ by

$$B_i = g_i(\mathbf{b}, \mathbf{x}^d(\mathbf{b}),\ \mathbf{y}(\mathbf{b}, \mathbf{x}^d(\mathbf{b}))) \qquad i = 1, ..., m \tag{6}$$

where $\mathbf{x}^d$ are design values calculated from, e.g. $x_j^d = \gamma_j(\mu_{X_j} + k_j\sigma_{X_j})$, where $k_j$ is a factor defining the characteristic value and $\gamma_j$ is a partial safety factor.

## 3. STOCHASTIC FINITE ELEMENTS

If the properties of the structural elements are uncertain and depend on the spatial coordinates then they have to be modelled by stochastic fields. In the following it is shown how stochastic fields can be discretized in combination with structural finite elements.

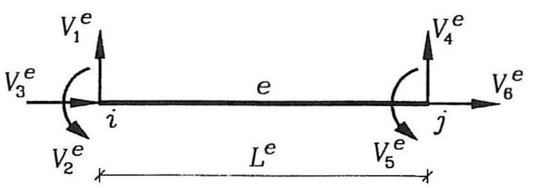


Figure 1. Two-dimensional beam element.

As an example a structural system modelled by two-dimensional beam elements is considered. The presentation in this section can easily be extended to truss and three-dimensional beam elements. Element $e$ has the length $L^e$, the displacement vector is denoted $\mathbf{v}^e = (v_1^e, ..., v_6^e)$ and the corresponding nodal forces $\mathbf{p} = (Q_i^e, M_i^e, N_i^e, Q_j^e, M_j^e, N_j^e)$, see figure 1.

Based on the flexibility method of structural analysis the local stiffness matrix $\mathbf{k}^e$ for element $e$ can be obtained from

$$\mathbf{k}^e = \mathbf{H}^e \mathbf{F}^{e^{-1}} \mathbf{H}^{e^T} \tag{7}$$

where $\mathbf{H}^e$ is the compatibility matrix. The flexibility matrix $\mathbf{F}^e$ is related to the internal forces $M_i^e, M_j^e$ and $N_j^e$ by

$$\mathbf{F}^e = \begin{bmatrix} \int_0^{L^e} \frac{x-L^e}{L^e}\frac{x-L^e}{L^e}\frac{1}{I^e E^e(x)}dx & \int_0^{L^e} \frac{x}{L^e}\frac{x-L^e}{L^e}\frac{1}{I^e E^e(x)}dx & 0 \\ \int_0^{L^e} \frac{x}{L^e}\frac{x-L^e}{L^e}\frac{1}{I^e E^e(x)}dx & \int_0^{L^e} \frac{x}{L^e}\frac{x}{L^e}\frac{1}{I^e E^e(x)}dx & 0 \\ 0 & 0 & \int_0^{L^e} \frac{1}{A^e E^e(x)}dx \end{bmatrix} \tag{8}$$

where it is assumed that the moment of inertia $I^e$ and the cross-sectional area $A^e$ are constant in element $e$. $E^e$ denotes the modulus of elasticity. It is assumed that $E^e$ is uncertain and that it can be modelled by a stochastic field $\{E^e(x), x \in L_\Omega\}$ where $L_\Omega$ denotes the total length of all beam elements in the structure. The stochastic field $\{E^e(x)\}$ is modelled by

$$\frac{1}{E^e(x)} = \frac{1}{E_0^e}(1 + f_E(x)) \tag{9}$$

where $E_0^e$ is the expected value of $\{E^e(x)\}$ and $\{f_E(x)\}$ is a homogeneous Gaussian stochastic field with zero mean and covariance function

$$C_{f_E}(x) = \int_{-\infty}^{\infty} S_{f_E}(\omega)e^{i\omega x}d\omega \tag{10}$$

$S_{f_E}(\omega)$ is the spectral density function of $\{f_E(x)\}$.

If (9) is introduced in (8) the flexibility matrix can be written

$$\mathbf{F}^e = \mathbf{F}_0^e + X_1^e\mathbf{F}_1^e + X_2^e\mathbf{F}_2^e + X_3^e\mathbf{F}_3^e \tag{11}$$

where $X_1^e$, $X_2^e$ and $X_3^e$ are zero mean normally distributed stochastic variables defined by

$$X_i^e = \int_0^{L^e} x^{i-1} f_E(x)dx \quad i = 1,2,3 \tag{12}$$

and

$$\mathbf{F}_0^e = \begin{bmatrix} \frac{L^e}{3E_0^e I^e} & -\frac{L^e}{6E_0^e I_0^e} & 0 \\ -\frac{L^e}{6E_0^e I^e} & \frac{L^e}{3E_0^e I_0^e} & 0 \\ 0 & 0 & \frac{L^e}{E_0^e A^e} \end{bmatrix} \qquad \mathbf{F}_1^e = \begin{bmatrix} -\frac{1}{E_0^e I^e} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{E_0^e A^e} \end{bmatrix}$$

$$\mathbf{F}_2^e = \begin{bmatrix} \frac{2}{L^e E_0^e I^e} & \frac{1}{L^e E_0^e I_0^e} & 0 \\ \frac{1}{L^e E_0^e I^e} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{F}_3^e = \begin{bmatrix} -\frac{1}{L^{e2} E_0^e I^e} & -\frac{1}{L^{e2} E_0^e I_0^e} & 0 \\ -\frac{1}{L^{e2} E_0^e I^e} & -\frac{1}{L^{e2} E_0^e I_0^e} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{13}$$

If two stochastic variables $X_i^e$ and $X_j^e$ are related to the same finite element the covariances become

$$\mathrm{Cov}[X_i^e, X_j^e] = \int_0^{L^e} \int_0^{L^e} x_1^{i-1} x_2^{j-1} C_{f_E}(x_1 - x_2) dx_1 dx_2 \tag{14}$$

If $X_i^e$ and $X_j^e$ are related to finite elements in two different elements (14) can easily be generalized such that $\mathrm{Cov}[X_i^e, X_j^e]$ can be estimated.

The global stiffness matrix $\mathbf{K}$ is obtained from

$$\mathbf{K} = \sum_{e=1}^{N_e} \mathbf{T}^{e^T} \mathbf{k}^e \mathbf{T}^e = \sum_{e=1}^{N_e} \mathbf{T}^{e^T} \mathbf{H}^e \mathbf{F}^{e^{-1}} \mathbf{H}^{e^T} \mathbf{T}^e \tag{15}$$

where $N_e$ is the number of finite elements and $\mathbf{T}^e$ is the geometric transformation matrix from the local coordinates to global coordinates.

If $\mathbf{P}$ denotes the global nodal load vector, the global displacements $\mathbf{v}$ are obtained from the global finite element equations $\mathbf{Kv} = \mathbf{P}$. If distributed loads are applied then $\mathbf{P}$ becomes dependent on the stochastic variables $X_1^e, ..., X_3^e$, see (Deodatis, 1991).

The performances $y_i$ in (5) are assumed to be either a displacement or a stress component in a finite element. Therefore, in general

$$y_i = y_i(\mathbf{b}, \mathbf{X}(\mathbf{b}), \mathbf{v}(\mathbf{b}, \mathbf{X}(\mathbf{b}))) \tag{16}$$

## 4. SENSITIVITY ANALYSIS

In First Order Reliability Methods (FORM) the generally correlated and non-normally distributed stochastic variables $\mathbf{X}$ are transformed into standardized and normally distributed stochastic variables $\mathbf{U} = (U_1, U_2, \ldots, U_n) : \mathbf{X} = \mathbf{T}(\mathbf{U}, \mathbf{b})$, see e.g. (Madsen et al., 1986). In the $u$-space the reliability index $\beta$ is defined by

$$\min_{\mathbf{u}} \ \beta = (\mathbf{u}^T \mathbf{u})^{\frac{1}{2}} \tag{17}$$

$$s.t. \ \ g(\mathbf{b}, \mathbf{x}(\mathbf{b}, \mathbf{u}), \ y_i(\mathbf{b}, \mathbf{X}(\mathbf{b}), \mathbf{v}(\mathbf{b}, \mathbf{X}(\mathbf{b})))) = 0 \tag{18}$$

The solution point $\mathbf{u}^*$ is denoted the $\beta$-point. If the failure function is approximately linear the probability of failure $P_f$ of the failure mode can be determined with good accuracy from $P_f \approx \Phi(-\beta)$ where $\Phi$ is the standard normal distribution function.

*Gradients with respect to* **u**

In iteration algorithms used to solve the optimization problem (17) - (18) the gradient $\nabla_{\mathbf{u}} g$ of the failure function $g$ with respect to $u_1, ..., u_n$ is needed. $\nabla_{\mathbf{u}} g$ is obtained from

$$\nabla_{\mathbf{u}}^T g = \nabla_{\mathbf{x}}^T g \mathbf{J}_{\mathbf{xu}} + \nabla_{\mathbf{y}}^T g \mathbf{J}_{\mathbf{yx}} \mathbf{J}_{\mathbf{xu}} \tag{19}$$

where $T$ signifies the transpose and $\mathbf{J}_{\mathbf{xu}}$ and $\mathbf{J}_{\mathbf{yx}}$ indicate Jacobian matrices. $\nabla_{\mathbf{y}} g$, $\nabla_{\mathbf{x}} g$ and $\mathbf{J}_{\mathbf{xu}}$ can usually be estimated analytically or numerically without significant errors and with relatively low cost within reliability programmes. $\mathbf{J}_{\mathbf{yx}}$ can be obtained as described below.

*Gradients with respect to* **b**

The reliability-based optimization problems in (1) - (4) can usually be solved using first order optimization algorithms, such as the NLPQL algorithm, see (Schittkowski, 1986). These methods require the gradient of objective function and of all constraints.

The gradients of the reliability indices $\beta_i$ with respect to the design variables **b** are obtained from, see (Madsen et al., 1986)

$$\nabla_{\mathbf{b}}^T \beta_i = \frac{1}{|\nabla_u g_i|} [\nabla_{\mathbf{b}}^T g + \nabla_{\mathbf{x}}^T g \mathbf{J}_{\mathbf{xb}} + \nabla_{\mathbf{y}}^T g (\mathbf{J}_{\mathbf{yb}} + \mathbf{J}_{\mathbf{yx}} \mathbf{J}_{\mathbf{xb}} + \mathbf{J}_{\mathbf{yv}} (\mathbf{J}_{\mathbf{vb}} + \mathbf{J}_{\mathbf{vx}} \mathbf{J}_{\mathbf{xb}}))] \tag{20}$$

where the gradients $\nabla_{\mathbf{b}}^T g, \nabla_{\mathbf{x}}^T g, \nabla_{\mathbf{y}}^T g$ and the Jacobians $\mathbf{J}_{\mathbf{yb}}, \mathbf{J}_{\mathbf{yx}}$ and $\mathbf{J}_{\mathbf{yv}}$ can be estimated analytically or numerically without significant errors and with relatively low cost. The elements in $\mathbf{J}_{\mathbf{xb}}$ are determined as $\frac{\partial x_i}{\partial b_k} = \frac{\partial T_j(\mathbf{u}_i^*, \mathbf{b})}{\partial b_k}$, where $\mathbf{u}_i^*$ is the $\beta$-point for the $i$th limit state function.

The Jacobians $\mathbf{J}_{\mathbf{vb}}$ and $\mathbf{J}_{\mathbf{vx}}$ can be obtained by quasi-analytical sensitivity analysis. If **P** does not depend on **b** then

$$\frac{d\mathbf{v}}{db_j} = -\mathbf{K}^{-1} \frac{d\mathbf{K}}{db_j} \mathbf{v} \tag{21}$$

For size design variables $\mathbf{H}^e$, $X_1^e$, $X_2^e$ and $X_3^e$ will not be dependent on **b**. Therefore

$$\frac{d\mathbf{K}}{db_j} = -\sum_{e=1}^{N_e} \mathbf{T}^{e^T} \mathbf{H}^e \mathbf{F}^{e^{-1}} \left( \frac{d\mathbf{F}_0^e}{db_j} + x_1^e \frac{d\mathbf{F}_1^e}{db_j} + x_2^e \frac{d\mathbf{F}_2^e}{db_j} + x_3^e \frac{d\mathbf{F}_3^e}{db_j} \right) \mathbf{F}^{e^{-1}} \mathbf{H}^{e^T} \mathbf{T}^e \tag{22}$$

where $\frac{d\mathbf{F}_0^e}{db_j}$, $\frac{d\mathbf{F}_1^e}{db_j}$, $\frac{d\mathbf{F}_2^e}{db_j}$ and $\frac{d\mathbf{F}_3^e}{db_j}$ are obtained from (13). If e.g. $x_j = x_2^e$ then

$$\frac{d\mathbf{K}}{dx_j} = -\mathbf{T}^{e^T} \mathbf{H}^e \mathbf{F}^{e^{-1}} \mathbf{F}_2^e \mathbf{F}^{e^{-1}} \mathbf{H}^{e^T} \mathbf{T}^e \tag{23}$$

## 5. ALGORITHMS

The above sensitivity analysis is implemented in a computer programme which can be used to solve the design problem in (1) -(4). The computer programme consists of the following modules: • OPTI: A general optimization algorithm where the optimization problem in (1)-(4) is programmed, e.g. NLPQL, (Schittkowski, 1986). • RELI: A general reliability programme (e.g. PRADSS (Sørensen, 1991)) which calculates the reliability indices $\beta$ and sensitivities $\frac{d\beta}{db_j}$. • GFUN: Limit state functions • STAT: Calculation of covariances of stochastic variables defined by (12). • FEM: A general linear finite element programme. • SFEM: modifies the local stiffness matrices in FEM according to realizations of $\mathbf{X}$ and $\mathbf{b}$ using (7) and (11). • SENS: evaluates the sensitivity coefficients in (20)-(23).

## 6. EXAMPLE

The structure shown in figure 2 is considered. All structural elements are assumed to have a rectangular cross-section. The elements are divided into four groups, see table 1. The $N = 9$ design variables have initial values, lower and upper bounds: $b_i = 0.5$ m, $b_i^l = 0.3$ m, $b_i^u = 0.6$ m, i=1,3,5,7,8 and $b_i = 0.02$ m, $b_i^l = 0.01$ m, $b_i^u = 0.04$ m, i=2,4,6,9.

| group | elements | width | height | thickness |
|---|---|---|---|---|
| 1 | 1,4,7,10 | $b_1$ | $b_1$ | $b_2$ |
| 2 | 2,5,8,11 | $b_3$ | $b_3$ | $b_4$ |
| 3 | 3,6,9,12 | $b_5$ | $b_5$ | $b_6$ |
| 4 | 13-21 | $b_7$ | $b_8$ | $b_9$ |

Table 1. Structural elements.

The load $P$ is assumed to be Weibull distributed with expected value = 0.25 MN and standard deviation = 0.0375 MN. The modulus of elasticity is modelled as a stochastic field with expected value $2.1 \cdot 10^5$ MPa and spectral density $S_{f_E}(\omega) = \frac{1}{4}\sigma_f^2 b^3 \omega^2 \exp(-b|\omega|)$, where $\sigma_f = 0.1$ and $b = 1$ m. The corresponding zero mean normally distributed stochastic variables $X_1$ - $X_{63}$ are defined by (12). The covariances are obtained from (14). It is assumed that the stochastic field $\{f_E(x)\}$ is independent between different structural elements.

A Serviceability Limit State (SLS) function is defined by

$$g_1 = v_{\max} - v_{4,hor} \tag{24}$$

where $v_{\max} = 0.05$ m is the maximum allowable displacement and $v_{4,hor}$ is the horisontal displacement of node 4. The minimum acceptable reliability index is $\beta_1^{\min} = 3.0$.

Ultimate Limit State (ULS) functions are defined by

$$g_i = \sigma_F - |\sigma_j| \qquad i = 2,43 \tag{25}$$

where $\sigma_F$ is the yield stress which is assumed to be Log-Normal distributed with expected value = 360 MPa and standard deviation = 36 MPa. $\sigma_j$ is the maximum stress in the considered element. The minimum acceptable reliability index is $\beta_i^{\min} = 4.7$.

The objective function is the volume of the structure. The optimization problem is solved using the modules described above. The result is $b_1 = 0.600$ m, $b_2 = 0.0298$ m, $b_3 = 0.600$ m, $b_4 = 0.0229$ m, $b_5 = 0.600$ m, $b_6 = 0.0210$ m, $b_7 = 0.600$ m, $b_8 = 0.600$ m, $b_9 = 0.0251$ m with volume 4.66 m$^3$. The active constraints are ULS in elements 4, 5, 6 and 13.

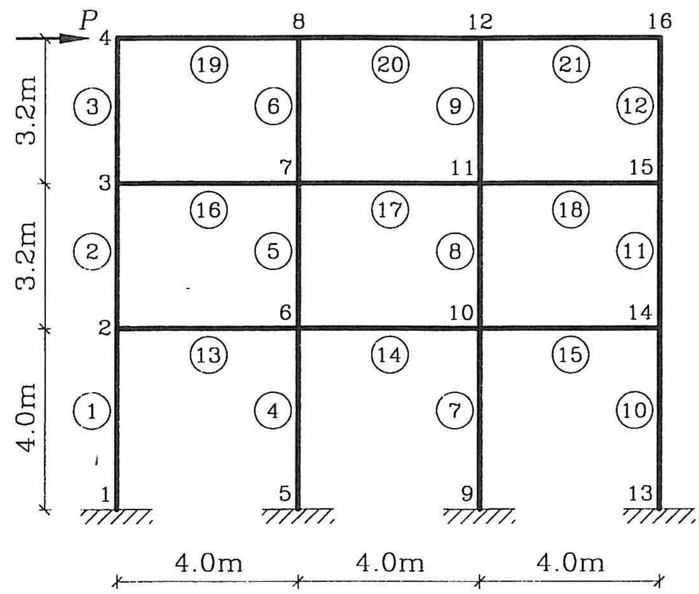


Figure 2.

## 7. CONCLUSIONS

It is shown how stochastic fields modelling the modulus of elasticity can be discretized in stochastic variables and how the reliability of a structural system can be estimated. Reliability-based optimization problems are formulated and sensitivity analyses are performed. A computer implementation is described where standard programmes for optimization, reliability analysis and structural analysis are coupled with special routines for stochastic finite elements and sensitivity analysis. An illustrative example is given.

## 8. ACKNOWLEDGEMENTS

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