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P.Thoft-Christensen*, F.M.Jensen**. C.R.Middleton*** & A.Blackmore***
*Aalborg University, Aalborg, Denmark, **CSRconsult, Aalborg, Denmark,
***Cambridge University, Cambridge, UK

1. Introduction

This paper is based on research performed for the Highways Agency, London, UK under the project DPU/9/44 *"Revision of Bridge Assessment Rules Based on Whole Life Performance: Concrete Bridges"*. It contains details of a methodology which can be used to generate Whole Life (WL) reliability profiles.

2. Limit States

Two limit states are selected for the reliability analysis:
- an ultimate limit state (ULS): collapse limit state (using yield line analysis)
- a serviceability limit state (SLS): crack width limit state (using linear elastic analysis)

2.1 Collapse (yield line) limit state

The following safety margin is used

$$Z = VE_D - W_D \tag{1}$$

where V is a model uncertainty variable, E_D is the energy dissipated in yield lines, and W_D is the work done by the applied loads.

The plastic collapse analysis and estimation of the load are performed using the COBRAS program, see [1]. The reliability analysis (element and system) is done using programs RELIAB01 and RELIAB02, see [2,3]. The RELIAB and COBRAS programs have been interfaced and include an optimisation algorithm to determine the optimal yield line pattern for each iteration of the reliability analysis. In [4] it is shown that using a fixed deterministic yield line pattern in the reliability analysis may lead to erroneous results. The estimation of the deterioration of the steel reinforcement is based on the program CORROSION, see [5].

The basic variables used in the yield line ULS are: thickness of slab, cube strength of concrete, density of concrete, depth of reinforcement, yield strength of reinforcement, and two load parameters.

2.2 Crack width limit state

Cracking shall be limited to a level that will not impair the proper functioning of the structure or cause its appearance to be unacceptable. In the absence of specific requirements (e.g. water tightness), it may be assumed that limitation of the maximum design crack width to about 0.3 will generally be satisfactory for reinforced concrete members with respect to appearance and durability.

The design crack width may be obtained from (see [6])

$$w_k = \beta s_{rm} \varepsilon_{sm} \tag{2}$$

where w_k is the design crack width, s_{rm} is the average final spacing, ε_{sm} is the mean strain allowing, under the relevant combination of loads, for the effects of tension stiffening, shrinkage, etc., and β is a coefficient relating the average crack width to the design value. For load induced cracking $\beta = 1.7$. The value of ε_{sm} may be calculated from the relation

$$\varepsilon_{sm} = \frac{\sigma_s}{E_s}(1 - \beta_1\beta_2(\frac{\sigma_{sr}}{\sigma_s})^2) \tag{3}$$

where σ_s is the stress in the reinforcement calculated on the basis of a cracked section, σ_{sr} is the stress in the reinforcement calculated on the basis of a cracked section under the loading conditions causing first cracking.

β_1 is a coefficient which takes account of the bond properties of the bars. It is = 1.0 for high bond bars, and = 0.5 for plain bars. β_2 is a coefficient which takes account of the duration of the loading or of repeated loading. It is = 1.0 for single, short term loading, and = 1.5 for a sustained load or for many cycles of repeated loading.

The average final crack spacing (in mm) for members subjected dominantly to flexure or tension can be calculated from the equation

$$s_{rm} = 50 + 0.25 k_1 k_2 \phi / \rho_r \tag{4}$$

where ϕ is the bar size in use (or the average bar size). ρ_r is the effective reinforcement ratio, $A_s / A_{c.eff}$, where A_s is the area of reinforcement contained within the effective tension area, $A_{c.eff}$. k_1 is a coefficient which takes account of the bond properties of the bar. It is = 0.8 for high bond bars and = 1.6 for plain bond bars. k_2 is a coefficient which takes account of the strain distribution. It is = 0.5 for bending and = 1.0 for pure tension.

The crack width limit state can then be formulated by

$$g(\cdot) = w_{max} - z_c w_k \tag{5}$$

where z_c is a model uncertainty stochastic variable.

The stochastic variables used in the crack SLS are: concrete cover, distance between reinforcement bars, diameter of reinforcement bars, thickness of slab, elastic modulus of reinforcement bars, tensile strength of concrete, external bending moment, and one model uncertainty variable.

3. Deterioration modelling

Several models can be used to model the deterioration of reinforcement steel in concrete slabs. However, there is a general agreement that the model presented below is acceptable in most cases.

Corrosion initiation period refers to the time during which the passivation of steel is destroyed and the reinforcement starts to corrode actively. Practical experience of bridges in wetter countries like UK shows that chloride ingress is far bigger a problem that carbonation.

The rate of chloride penetration into concrete, as a function of depth from the concrete surface and time, can be represented by Fick's law of diffusion as follows:

$$\frac{\delta c}{\delta t} = D_C \frac{\delta^2 c}{\delta x^2} \tag{6}$$

where c is the chloride ion concentration, as % of the weight of cement, at distance x cm from the concrete surface after t seconds of exposure to the chloride source. D_C is the chloride diffusion coefficient expressed in cm²/sec. The solution of the differential equation (6) is

$$C(x,t) = C_0 \left\{ 1 - \text{erf}\left(\frac{x}{2\sqrt{D_C \cdot t}} \right) \right\} \tag{7}$$

where C_0 is the equilibrium chloride concentration on the concrete surface, as % of the weight of cement, x is the distance from the concrete surface in cm, t is the time in sec, erf is the error function, D_C is the diffusion coefficient in cm²/sec and $C(x,t)$ is the chloride concentration at any position x at time t. In a real structure, if $C(x,t)$ is assumed to be the chloride corrosion threshold and x is the thickness of concrete cover, then the corrosion initiation period, T_I, can be calculated based on a knowledge of the parameters C_0 and D_C. For bridge decks under de-icing conditions C_0=1.6, as % of cement weight, is often used.

The time T_I to initiation of reinforcement corrosion is

$$T_I = \frac{(d_1 - D_1/2)^2}{4D_C} (erf^{-1}(\frac{C_{cr} - C_0}{C_i - C_0}))^{-2} \tag{8}$$

where C_i is the initial chloride concentration, C_{cr} is the critical chloride concentration at which corrosion starts, and $d_1 - D_1/2$ is the concrete cover. For plain concrete of moderate strength ($f_{cu} \approx 30$ N/mm²) reported values of D_C are in the range between $1 \cdot 10^{-8}$ and $5 \cdot 10^{-8}$ cm²/sec.

When corrosion has started then the diameter $D_I(t)$ of the reinforcement bars at time t is modelled by

$$D_I(t) = D_1 - C_{Corr} i_{corr} t \tag{9}$$

where D_1 is the initial diameter, C_{corr} is a corrosion coefficient, and i_{corr} is the rate of corrosion. The area of a reinforcement bar is then modelled using the following formulation

3

$$A(t) = \begin{cases} nD_i^2 \frac{\pi}{4} & \text{for } t \le T_I \\ n(D(t))^2 \frac{\pi}{4} & \text{for } T_I \le t \le T_I + D_i / (0.0203 \cdot i_{corr}) \\ 0 & \text{for } t > T_I + D_i / (0.0203 \cdot i_{corr}) \end{cases}$$

(10)

$$T_I = \frac{x_d^2}{4D_C (erf^{-1}(\frac{C_{cr}(x_d,t) - C_0}{C_i - C_0}))^2} \quad , \quad D(t) = D_i - 0.0203 \cdot (t - T_I) \cdot i_{corr}$$

$A(t)$ is the area of reinforcement bars $[mm^2]$ at the time t years, n is the number of reinforcement bars, D_i is the diameter of a single bar $[mm^2]$ and T_I is the corrosion initiation time in years. The value "0.0203" in the estimation of $D(t)$ will vary depending on the circumstances.

The initiation time of corrosion is determined based on values of $C_0, C_i, D_C, x_d, C_{cr}$. After the deterioration is started the corrosion rate is modelled by the corrosion current i_{corr} only.

The model for $A(t)$ (and the value of i_{corr} used) relates to an average deterioration of the reinforcement in the concrete. An important aspect of corrosion in addition to the average corrosion is the maximum penetration (pitting of reinforcement). Pitting of reinforcement may have more influence on the reliability than the average deterioration due to localised much higher weakening of the reinforcement. The ratio R between the maximum penetration PC_{max} and the average penetration PC_{av} has been estimated by a number of authors to be between 4-10, see e.g. [7]. Pitting corrosion is not included in this investigation.

The stochastic variables used in the deterioration modelling are: initial chloride concentration on surface, initial chloride concentration in concrete, diffusion coefficient for the concrete, cover to reinforcement, critical chloride concentration, and rate of corrosion.

4. Example

This following example is used to illustrate the proposed methodology. The example is based on an existing UK bridge, but some limitations and simplifications are made. The bridge was built in 1975.
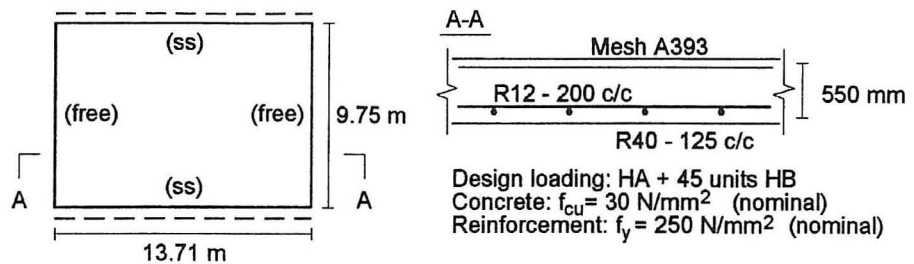

Figure 1. Bridge data.

The bridge was designed for 45 units HB load, see [8]. The bridge has a span of 9.755 m, the width is 2×13.71 m, and the slab thickness is 550 mm (see figure 1).

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Based on the corrosion data shown in table 1 the expected area of the reinforcement as a function of time can be calculated, see figure 2.
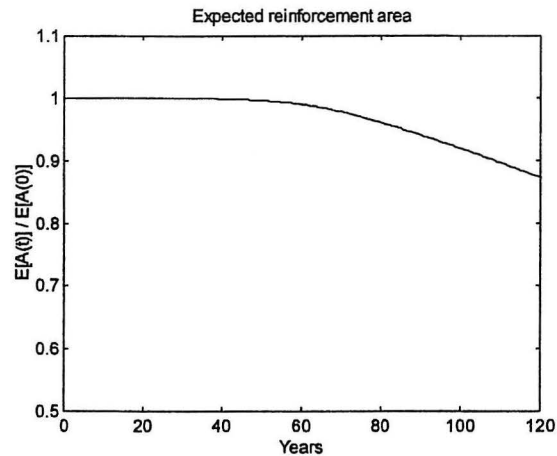

Figure 2. Reinforcement area A(t) as a function of time.

Reliability profiles for the two limit states discussed in section 2 are calculated on the basis of the stochastic modelling shown in tables 1 and 2.

Stochastic variables: Yield line limit state				
No	Type	Par. 1	Par. 2	Description
1	Normal	550.0	10.0	Thickness of slab [mm]
2	LogNormal	30.0	6.0	Cube strength of concrete [MPa]
3	Normal	23.6	0.4	Density of concrete [kN/m3]
4	LogNormal	289.0	25.0	Yield strength: longitudinal reinforcement [MPa]
5	Normal	60.0	8.0	Cover on longitudinal reinforcement [mm]
6	LogNormal	289.0	25.0	Yield strength: transverse reinforcement [Mpa]
7	Normal	86.0	8.0	Cover on transverse reinforcement [mm]
8	Fixed	10053.0	-	Longitudinal reinforcement area (initial) [mm2]
9	Fixed	565.0	-	Transverse reinforcement area (initial) [mm2]
10	Gumbel	0.352	0.026	Static load factor [-]
11	Normal	1.27	0.20	Dynamic load factor [-]
12	Normal	1.08	0.072	Chloride concentration on surface [%]
13	Fixed	0.0	-	Initial chloride concentration [%]
14	Normal	35.0	2.5	Diffusion Coefficient [cm2/sec]
15	Normal	0.4	0.05	Critical Chloride concentration [%]
16	Uniform	2.5	0.29	Corrosion parameter [-]
17	Normal	1.0	0.05	Model uncertainty variable [-]

Table 1. Stochastic modelling used for the ULS.

Stochastic variables: Crack width limit state				
No	Type	Par. 1	Par. 2	Description
1	Normal	60.0	9.0	Concrete cover [mm]
2	Normal	125.0	12.5	Distance between reinforcement bars [mm]
3	Normal	40.0	1.2	Diameter of reinforcement bar [mm]
4	Normal	550.0	27.0	Thickness of slab [mm]
5	Normal	200.0E3	6.0E3	Young's modulus [N/mm2]
6	Normal	3.4	0.68	Tensile strength [N/mm2]
7	Gumbel	1.0	0.10	Model uncertainty [-]
8	Gumbel	0.352	0.026	Static load factor [-]
9	Normal	1.27	0.20	Dynamic load factor [-]
10	Normal	1.08	0.072	Chloride concentration on surface [%]
11	Fixed	0.0	-	Initial chloride concentration [%]
12	Normal	35.0	2.5	Diffusion Coefficient [cm2/sec]
13	Normal	0.4	0.05	Critical Chloride concentration [%]
14	Uniform	2.5	0.29	Corrosion parameters [-]

Table 2. Stochastic modelling used for the SLS.

The general traffic highway load model in the Eurocode 1, Part 3 (ENV 1991-3:1995) for lane and axle load is applied. The load effects produced by the Eurocode model (lane and axle load) are multiplied by a static load factor (extreme type 1) and a dynamic load factor (normal). See e.g. stochastic variables 10 and 11 used for the yield line limit state. Several load cases are considered in the project. In this paper only the load case with 2 marked lanes loaded is included.

The normalised reliability profile for the yield line ULS (full width failure) and the corresponding probability of failure profile are shown in figure 3. The reliability index at time t=0 is β_0=11.5. Due to the size of the concrete cover (mean value 60 mm) the deterioration does not have any effect until year 70.
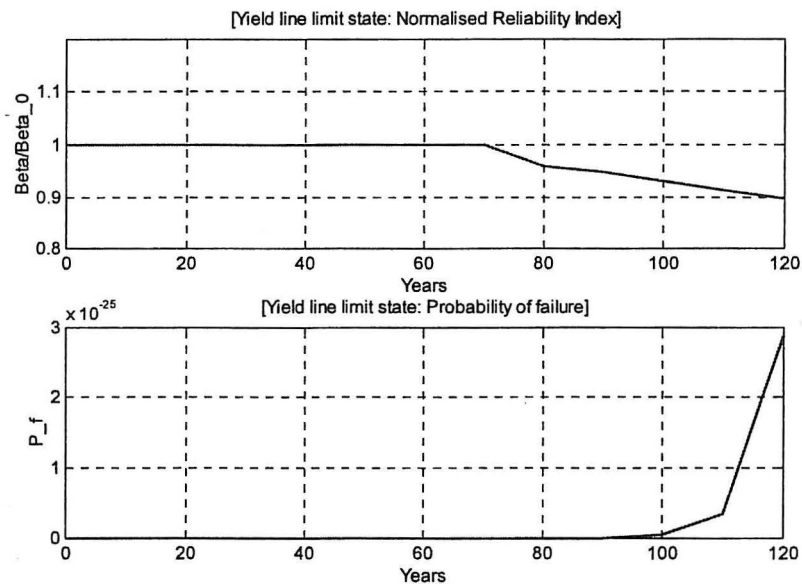

Figure 3 : Reliability profiles using a yield line limit state.

The results from the sensitivity analysis with regard to the mean values are shown for t=0 years and t=120 years in figure 4. The most important variables are, as expected, the thickness of the slab, the yield strength of the reinforcement, and the model uncertainty. Observe that the magnitude of sensitivity with regard to the cover changes from negative at time t=0 to positive at time t=120 due to the corrosion.
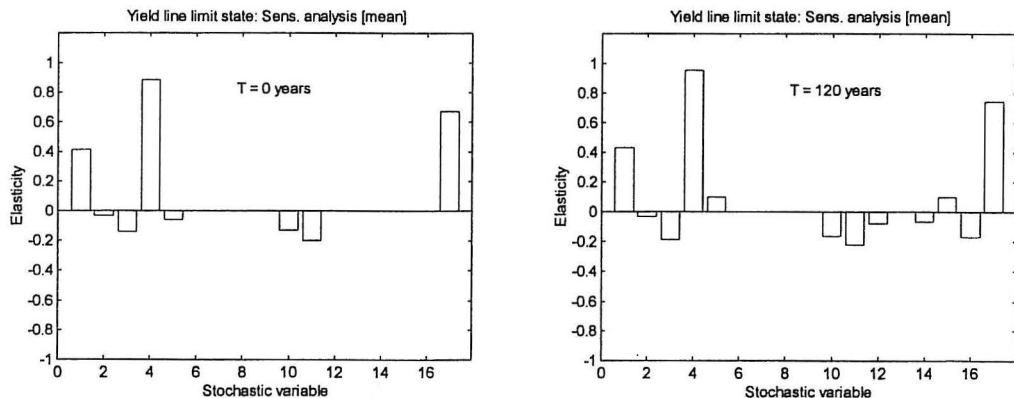

Figure 4 : Sensitivity analysis for yield line limit state at t = 0 years and at t = 120 years.

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The normalised reliability profile for the crack SLS and the corresponding probability of failure profile are shown in figure 5. The reliability index at time t=0 is β_0=7.1. Due to the size of the concrete cover (mean value 60 mm) the deterioration does not have any effect until year 90.
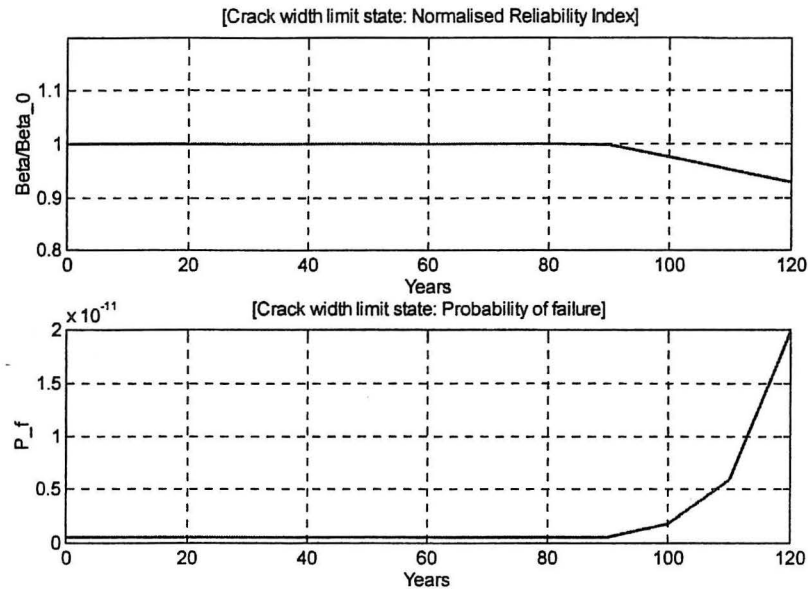

Figure 5 : Reliability profiles using a crack width limit state

The results from the sensitivity analysis with regard to the mean values are shown for t=0 years and t=120 years in figure 6. The most important variables are as expected the concrete cover, the diameter of the reinforcement, the thickness of the slab, and Young's modulus. Observe that the magnitude of the sensitivity with regard to the cover is decreasing from time t=0 to time t=120 due to the corrosion.
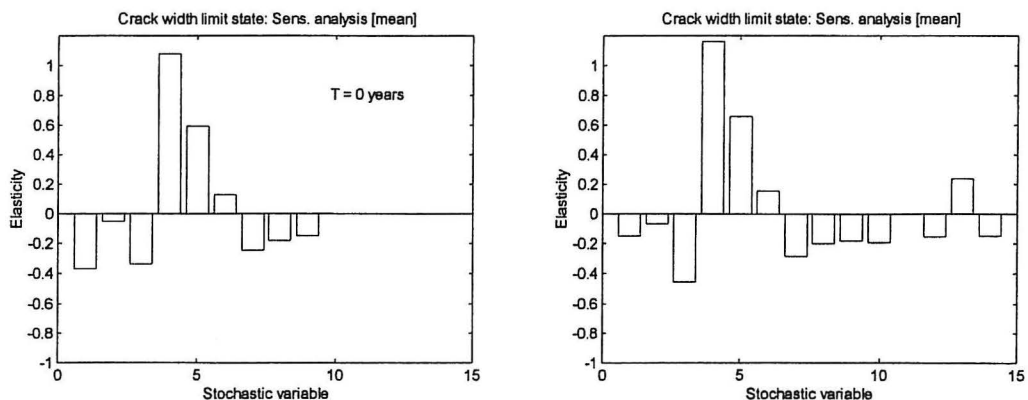

Figure 6 : Sensitivity analysis for crack width limit state at t = 0 years and at t = 120 years.

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