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Grunnet, Jacob Deleuran; Soltani, Mohsen; Knudsen, Torben; Kragelund, Martin Nygaard; Bak, Thomas

Published in:
European Wind Energy Conference and Exhibition, EWEC 2010

Publication date:
2010

Document Version
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

Citation for published version (APA):
Grunnet, J. D., Soltani, M., Knudsen, T., Kragelund, M. N., & Bak, T. (2010). Aeolus Toolbox for Dynamics Wind Farm Model, Simulation and Control. In *European Wind Energy Conference and Exhibition, EWEC 2010 : Conference Proceedings* <http://www.ewec2010.info/index.php?id=106>

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Aeolus Toolbox for Dynamics Wind Farm Model, Simulation and Control

Jacob D. Grunnet*, Mohsen Soltani, Torben Knudsen, Martin Kragelund and Thomas Bak

Abstract

This paper presents the wind farm simulation model developed in the EU-FP7 project, AEOLUS. The idea is to provide a publicly available simulation package for researchers developing farm level control solutions. With the software it is possible to auto generate a wind farm simulation model in Matlab/Simulink based on turbine parameters and farm geometry. The input to the farm simulator is power set points of individual turbines. Outputs from the farm simulation are power production, nacelle wind speed and fatigue loads (damage equivalent loads) of each turbine.

1 Introduction

As the penetration of wind energy increases the task of ensuring an efficient, stable, predictable and controllable supply becomes ever more important[1, 2].

By integration of wind turbines in large farms, the control task is extended from single turbine control to wind farm level control. The current farm operators use the most basic methods of controlling wind farms to both fulfill the power demand by the network and distribute the loads in the wind farm. They are usually based on turbine on/off commands which do not provide optimality in neither power production nor turbine loads[3].

The first step toward controller design is to have a dynamical model which represents the real process, here the wind farm, and a suitable simulation environment. There are two main objectives for developing such model.

First, the model should allow the real time simulation of flows in the wind field, e.g. when wind turbines in the upwind row have experienced an extreme gust, the traveling of the gust and its effects on

downwind turbines should be modeled. Wind farm control designers can then evaluate power and loading of the wind farm while the specific controller is developed.

Second, the model should include the main aerodynamic effects of the wind farm in order to let the controller manage the flow resource in the wind field. This in fact gives the wind farm operator the opportunity to optimize their desired objective functions, e.g. a cost function which includes power production and the cost of fatigue loads on the wind farm.

Previous efforts to model wind farms have studied either aerodynamic effects[4, 5] or grid integration issues[6]. Each of those studies had its own objectives: farm feasibility analysis and power integration respectively.

There also exist several simulation packages for wind farms, but they are not freely available and have a very strong focus on aeroelastics, e.g. HAWC2[7].

Therefore a model which is able to fulfill minimum requirements for wind farm control designers is lacking.

One option would be to expand a freely available simulation code such as FAST[8], but again the focus is on aeroelastics, and simulating several turbines at once would be very computationally expensive.

It is also desired that the toolbox is accessible from standard control design tools e.g. Mathworks Simulink, and although there exists a prototype Simulink interface for FAST it is not apparent how this could be expanded to include wake effects.

Based on the above objectives the AEOLUS SimWindFarm (SWF) Simulink toolbox is developed. It consists of four elementary components; wind turbine dynamics, wind field interactions and dynamics, wind farm controller, and electrical network operator. The model is simple enough for engineers to design wind farm controllers, but on the other hand, it still represents the main dynamics of a real wind farm.

The toolbox is flexible such that any farm layout

*All authors are with Aalborg University, Section for Automation and Control, Frederik Bajers Vej 7C, 9220 Aalborg, tlf +45 99408702, fax +45 99401739, {grunnet,sms,tk,mkr,tba}@es.aau.dk

can be used and the custom turbine models can easily be added. The toolbox also includes an ambient wind field generator where mean wind speed, turbulence intensity and grid resolution can be specified.

Furthermore a number of visualisation and post processing tools are included to perform e.g. fatigue analysis and wake animation.

This paper presents the toolbox and the models used as well as a wind farm control example demonstrating the use of the toolbox. The toolbox is available at <http://www.ict-aeolus.eu/SimWindFarm>.

An overview of the toolbox will be given in the next section, in section 3 the wind, wake and turbine models will be presented, section 5 gives an example of a 10 turbine farm simulation and section 6 concludes the paper.

2 Overview

SWF is designed as a Simulink toolbox which can generate wind farm models based on the users choice of layout and turbine. The simulink model generated has the structure shown in Figure 1.

The two main elements are the turbines block and the wind field block. The turbines block simulates the dynamics of the farms wind turbines based on wind speed and power set point. Each of the turbines produce a standard set of outputs for use when designing the farm controller along with the coefficient of thrust, C_T , of the individual turbines, used to calculate the wake effects. Both these blocks are explained in greater detail in section 3.

Included are also an example farm controller, network controller and grid models, which can be replaced with the whatever models or algorithms the control designer wants.

The example farm controller is a simple proportional distribution algorithm, which distributes the power reference to the turbines based on a simple estimate of its current production capacity and the total power demand from the network operator.

The network operator includes a number of modes such as:

Absolute Explicitly specify farm power output.

Delta Ensure there is always a specific amount of reserve power.

Frequency Help keep the grid frequency constant.

2.1 Scope

As the toolbox focuses on the control aspect of wind farm design a number of limitations are imposed on the farm in order to simplify the simulation models. The major limitations are listed below:

Constant mean wind speed The mean wind speed is constant throughout the simulation.

Constant mean wind direction The wind speed direction is fixed with respect to the farm layout. If another wind direction is wanted, a new simulation model must be generated with a rotated layout.

2D wind field The wind field is only generated in the hub height plane, currently no wind shear or tower shadow effects are considered.

Fixed turbine yaw No yaw dynamics of the turbines are considered as the wind direction does not change.

2.2 Wind Fields

The ambient wind field is the major input to the simulation and SWF defines the field using a 2d grid in the (x, y) plane at hub height, see fig. 2. The wind fields must have a constant average speed and direction, but the grid and field sizes can be freely chosen by the user.

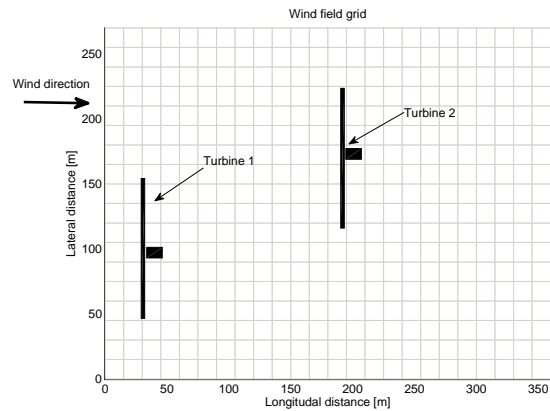


Figure 2: A wind field grid

The wind field generator accepts mean wind speed, turbulence intensity, grid size, length and width of the wind field as input and it uses Veers algorithm[9] to generate a wind field with a spectrum according to IEC 61400-1.

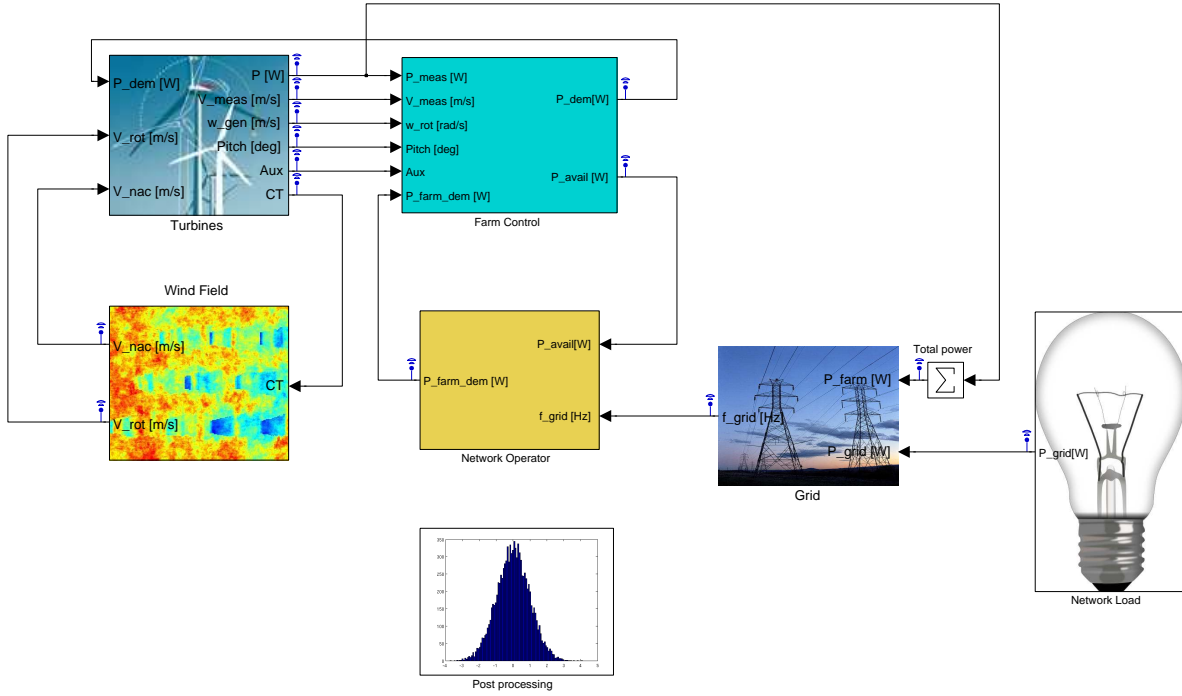


Figure 1: A screen shot of a SWF generated farm model showing the top-level structure of the simulation.

2.3 Turbines

The other major component of the wind farm is the turbines, and the basic SWF package includes a simple model of the NREL 5MW baseline turbine[10]. However it is very easy to add new turbines by simply adding a simulink model of the desired turbine to the turbines library. It will thus automatically be possible to create new wind farm simulations with that turbine or any mix of available turbines.

In order for a turbine model to be usable by the toolbox it must adhere to a set of interface specifications. The full details can be found on the toolbox website¹, but the basic requirements will be listed here. Note that the NREL 5MW model is completely transparent including scripts to easily alter the parameters, and it is therefore recommended to copy the NREL model and change the necessary parameters instead of implementing a new turbine model if at all possible.

The main interface requirements are that the inputs and outputs of the new model correspond to the expectations of the toolbox, as shown in Figure 3 and listed in table 1.

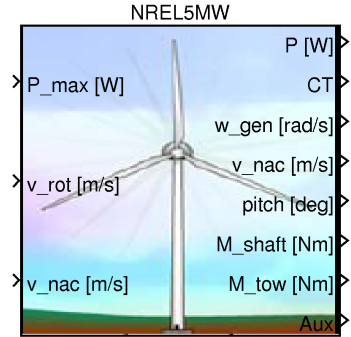


Figure 3: The NREL 5MW Simulink turbine block

3 of the outputs are absolutely necessary for the simulation to run.

- Produced power
- Coefficient of thrust
- Measured nacelle wind speed

The rest of the outputs are only used for post-processing and can be replaced by zero signals.

The Auxiliary output is not used in the NREL 5MW model, but is added such that extra sensor outputs that might be available from a specific turbine model

¹<http://ict.aeolus.eu/SimWindFarm>

Inputs	Unit	Description
Power demand	W	Power reference from the farm controller
Effective wind speed	$\frac{m}{s}$	Averaged over rotor wind speed
Nacelle wind speed	$\frac{m}{s}$	The wind speed at the nacelle grid point
Outputs		
Produced power	W	Power produced by the turbine
Coefficient of thrust	none	Current turbine thrust coefficient
Generator speed	$\frac{rad}{s}$	Generator shaft speed
Measured nacelle wind speed	$\frac{m}{s}$	Nacelle wind speed as measured by the anemometer
Pitch angel	deg	Pitch angle sensor measurement
Main shaft torsion moment	Nm	The torsion moment of the rotor shaft (only for post proc.)
Tower bending moment	Nm	The tower root bending moment (only for post proc.)
Auxiliary outputs	none	User defined outputs

Table 1: Input and outputs of the SWF turbine block

can be added e.g. tower acceleration or LIDAR measurements.

2.4 Post processing

As part of the toolbox a number of post processing tools are available to guide the control designer. Foremost is the fatigue tool which is based on MCrunch[11]. It can compute rainflow counts and damage equivalent loads for tower bending and main shaft torsion, such that the fatigue load of different farm control strategies can be evaluated.

Secondly there are two visualisation tools; one shows wake meandering and merging at a turbine to visualise the effect of multiple upstream wakes reaching the turbine rotor. The other provides an animation of the wind field including wake effects, which can be used to judge interactions between the ambient wind field and the wake effects in the farm.

3 Models

This sections describes the models employed by SWF, but focuses on the wind and turbines blocks as they are the most significant contribution of the simulator package.

3.1 Wind field modelling

The wind simulation and modelling can be divided into an ambient field model, which describes the wind field if no turbines where present and a wake model which describes the turbines effect on the ambient wind field.

In SWF both models assume Taylor's frozen turbulence hypothesis for inviscid flow[12] to be true. This greatly simplifies the effort of generating an ambient wind field, and provides relatively simple equations for the wake effect models.

3.1.1 Ambient wind field

The ambient wind in a wind farm is usually described by spectral matrices describing the wind speed variation at a number of points in the wind farm and their relation using the method described in [9].

The wind model assumes a constant mean wind speed and a zero lateral mean wind speed, i.e. the mean wind direction is constant in the longitudinal direction.

It would therefore be natural to implement a wind field simulation as a number of stochastic processes, one for each point. However due to the frozen turbulence assumption it is only necessary to simulate stochastic processes in the first line in the lateral direction in the park. These velocities are then assumed to travel with the average wind speed in the longitudinal direction. In this way the wind speeds at downwind grid points will eventually be generated.

The wind field is generated according to the recommendations in IEC 61400-3 concerning offshore turbines, which state that for non-site specific wind conditions the parameter values in IEC 61400-1 (2005) can be used.

The spectrum used is the Kaimal spectrum

$$S_k(f) = \sigma_k^2 \frac{4 \frac{L_k}{U}}{(1 + 6f \frac{L_k}{U})^{\frac{5}{3}}} \quad (1)$$

where L_k is velocity component integral scale parameter, f is the frequency in Hertz, k denotes the velocity component, U is the hub height mean wind speed and σ_k is the variance determined by the turbulence intensity, T_i , given by

$$\sigma_x = T_i \left(\frac{3}{4} U + 5.6 \right) \quad (2)$$

$$\sigma_y = 0.8 \sigma_x \quad (3)$$

The coherence between two channels separated laterally by distance l is given as

$$C_k(f) = e^{-c_k f \frac{l}{U}} \quad (4)$$

where c_k is the coherence parameter for each wind direction.

As the hub height is assumed to be above 60m then $L_x = 340.2$ and $L_y = 113.4$ and according to [13] c_x and c_y can be set to 7.1 and 4.2 respectively.

Notice that the simple wind field model using the frozen turbulence assumption has a coherence of 1 in the longitudinal direction, which does not conform to IEC 61400-1, and care should thus be taken not to design controllers that rely on this fact.

3.1.2 Wake effects

Due to the frozen turbulence assumption and the constant mean wind direction it is possible to rank turbines relative to each other as being either downwind or upwind.

For wake effect calculations at a given turbine it is only necessary to consider upwind turbines, and as this relationship is fixed it considerably simplifies the calculations.

SWF considers three wake effects; deficit, expansion and center, where wake deficit is a measure of the decrease in downwind wind speed, wake expansion describes the size of the downwind area affected by the wake and wake center defines the lateral position (meandering) of the wake area, see Fig. 4 and [5].

It has been shown in [5] that a good approximation of the meandering is to consider the wake center as a passive tracer which moves downwind with the mean wind speed. This model corresponds well with the frozen turbulence assumption, such that the wake center, expansion and deficit at a given point p downwind from a turbine at time t_1 is defined by the wind field at the turbine and its coefficient of thrust at the time of tracer release, t_0 ,

$$t_0 = t_1 - \frac{d}{U} \quad (5)$$

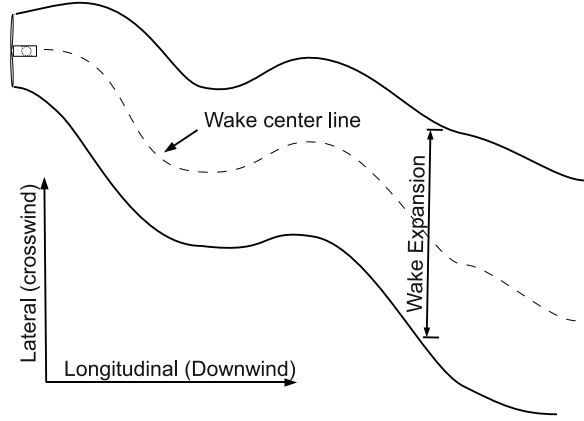


Figure 4: Wake meandering of a turbine, defined by its wake center and expansion.

where d is the longitudinal distance between the upwind turbine and p .

The deficit from turbine i at distance d can according to [14] be approximated as

$$a_i(d) \approx 1 - \frac{1}{2} C_{T,i}(t_0) \left(1 + \frac{d}{4R} \right)^{-1} \quad (6)$$

where $C_{T,i}(t_0)$ is the coefficient of thrust of turbine i at time t_0 and R is the radius of the rotor.

The wind speed at p distance d downstream from turbine i can then be calculated as $v_x(p) = a_i(d)u_x(p, t)$, where u_x is the ambient wind speed in the longitudinal direction i.e. the theoretical wind speed if no turbines were present.

Notice that the deficit depends on the coefficient of thrust, C_T , and as C_T decreases when less energy is extracted from the wind it is possible to decrease the deficit by de-rating a turbine, thus enabling a farm control strategy to control the deficits in the farm.

However the wake deficit is only applied for p in the wake area which can be computed from the wake center and expansion as shown in Fig. 4.

Expressions for wake center and expansion was developed in [15, 14], and in SWF in particular we use (7) from [14] to calculate the wake expansion radius, which is a simplified model independent of C_T .

$$e_i(d) = 2R \sqrt{1 + \frac{d}{4R}} = \sqrt{4R^2 + dR} \quad (7)$$

The wake center is computed as a passive tracer, such that the center at time t_k is a function of the center at time t_{k-1} and the average lateral wind

Table 2: Main properties for NREL 5-MW turbine

Rated power	5MW
Configuration	Upwind, 3 Blades
Control	Variable speed, collective pitch
Wind	
Cut-in	$3 \frac{m}{s}$
Rated	$11.4 \frac{m}{s}$
Cut-Out	$25 \frac{m}{s}$
Rotor	
Cut-in	6.9rpm
Rated	12.1rpm
diameter	126m
Hub	
Diameter	3m
Height	90m

speed, over the wake area,

$$W_i(d, t_{k-1}) = \{(x, y) | x = d \wedge y \in W_y\}$$

$$W_y = [w_i(d, t_{k-1}) - e_i(d); w_i(d, t_{k-1}) + e_i(d)] \quad (8)$$

$$w_i(d, t_k) = w_i(d, t_{k-1}) + \bar{v}_{y,i}(d, W_i(d, t_{k-1})) \quad (9)$$

where $w_i(d, t_k)$ is the wake center at time k , $W_i(d, t_{k-1})$ is the wake area distance d from the turbine at time t_{k-1} and $\bar{v}_{y,i}(d, W_i(d, t_{k-1}))$ is the average lateral wind speed over the wake area.

Using equations (7), (6) and (9) it is possible to calculate the wake contributions for each turbine and this enables us to calculate the actual wind speed at any point in the farm as

$$v_x(x, y) = u_x(x, y) \prod_{i \in I} a_i(d_i) \quad (10)$$

where I contains the indices of all turbines where the point (x, y) is in its wake area, $y \in W_i(x - P_{x,i})$, with $P_{x,i}$ being the longitudinal position of the i 'th turbine.

3.2 NREL Turbine Model

The NREL 5MW simulation model used in SimWind-Farm is a simplified aeroelastic model based on lookup tables for the aerodynamics (C_P/C_T), a simple 3rd order drive train model, a 1st order generator model, 1st order pitch actuator, and a 2nd order tower dynamics. The main properties of the turbine can be seen in table 2.

The turbines are controlled using the control strategy from [10] which includes a simplified start up procedure and pitch control for full load operation.

Figure 5 illustrates the structure of the NREL 5MW with variables explained in table 3. The inputs

required by the turbine model are the wind speed at the nacelle, the average wind speed over the rotor area (effective wind speed), and the power reference supplied by the farm controller. The outputs available to the farm controller are produced power, measured generator speed, nacelle wind speed, and blade pitch angle.

Table 3: Turbine variables

Ω	rotor speed
ω	generator speed
β	pitch angle
M_{shaft}	main shaft torque
M_{gen}	generator torque
F_{tow}	tower thrust
P_{dem}	power demand
P_{ref}	generator power reference
β_{ref}	pitch angle reference
V_{nac}	nacelle wind speed
V_{rot}	average wind speed over the rotor
V_E	effective wind speed
V_{meas}	measured wind speed
β_{meas}	measured pitch angle
P_{meas}	measured produced power
ω_{meas}	measured generator speed

The aerodynamics of the turbine can be described using two static relationships (11) and (12).

$$M_{shaft} = \frac{1}{2} V_{rot}^3 \rho A C_P(\lambda, \beta) \Omega^{-1} \quad (11)$$

$$F_{tow} = \frac{1}{2} V_{rot}^2 \rho A C_T(\lambda, \beta) \quad (12)$$

where C_P and C_T are two look-up tables derived from the geometry of the blades with inputs tip speed ratio ($\lambda = \frac{R\omega}{V_{rot}}$) and pitch angle. The parameters are air density, ρ , and rotor disc area, A .

The 3rd order drive train model is based on two rotating shafts connected through a gearbox with torsion spring constant K_{shaft} , viscous friction B_{shaft} , and gear ratio N .

$$\dot{\Omega} = \frac{1}{I_{rot}} (M_{shaft} - \phi K_{shaft} - \dot{\phi} B_{shaft}) \quad (13)$$

$$\dot{\omega} = \frac{1}{I_{gen}} (-M_{gen} + \frac{1}{N} (\phi K_{shaft} + \dot{\phi} B_{shaft})) \quad (14)$$

$$\dot{\phi} = \Omega - \frac{1}{N} \omega \quad (15)$$

where ϕ is the shaft torsion angle and I_{gen}/I_{rot} are the inertias of the generator and rotor respectively.

In the baseline NREL 5MW turbine there is no generator model, but a simple 1st order model is included in this benchmark, with input P_{ref} and time

4 Farm Controller

The example farm controller distributes the power demands to the turbines proportionally to the available power of each turbine

$$P_{a,i} = \frac{1}{2} \rho A v_{meas,i}^3 C_{p,max} \quad (22)$$

$$P_a = \sum P_{a,i} \quad (23)$$

where $P_{a,i}$ is the available power at turbine i , $v_{meas,i}$ is the measured nacelle wind speed at turbine i and $C_{p,max}$ is the maximum coefficient of thrust of the turbine.

The power can be distributed according to the farm power demand, P_d , as

$$P_{d,i} = P_d \frac{P_{a,i}}{P_a} \quad (24)$$

with $P_{d,i}$ being the power demand sent to the i 'th turbine.

5 Example

In order to demonstrate the capabilities of SWF a 10 NREL 5MW turbine farm has been created with the layout as shown in Figure 7. The mean wind speed is $9 \frac{m}{s}$ with a turbulence intensity of 10%, which is well below the NREL 5MW rated wind speed of $11.4 \frac{m}{s}$

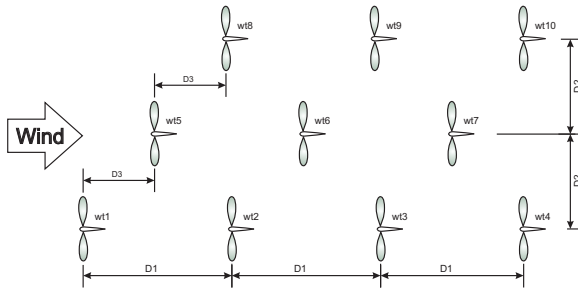


Figure 7: The configuration of the example wind farm. The mean wind direction is marked with an arrow. D1 is 600m, D2 is 500m and D3 is 300m

The network operator is in frequency control mode, trying to keep the frequency constant at 50Hz, despite a changing grid load.

The simulation was run for 1000 seconds with a loads ranging from 5.4MW to 15MW. The resulting grid frequency is presented in Figure 8 which illustrates that the controller regulates the frequency to 50Hz with a steady state error of 0.15Hz due to only

proportional gain control being used in the simple network operator implementation. Adding an integral term to the controller should remove the steady state error, but as the frequency control is not the focus of this paper, this is left as an exercise to the toolbox user.

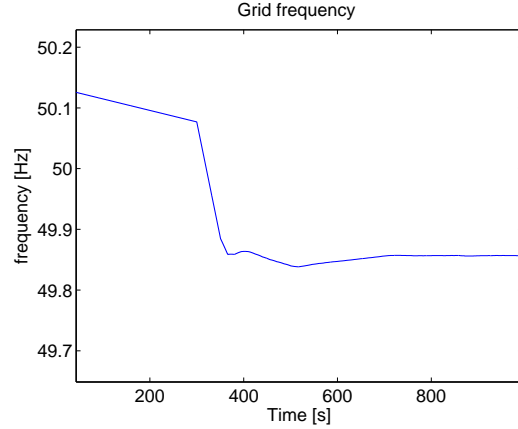


Figure 8: Grid frequency during the simulation

As the steady state error is constant the power produced by the farm must necessarily match the grid load, and as can be seen from Figure 9 produced power follows the grid load through the step and eventually matches it as the frequency stabilises.

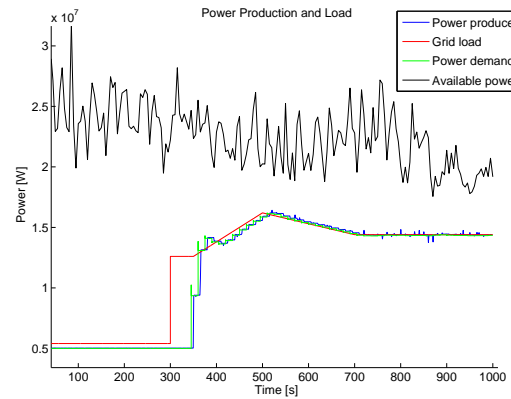


Figure 9: Power and load during the simulation. The available power is the farm controller estimate, the power demand is the network operator's desired farm production

Perhaps more interesting for the main purpose of the toolbox is the wake effects of the turbine. Figure

10 shows the longitudinal wind speeds in the wind farm at 4 different time instances approximately 11 seconds apart.

The frozen turbulence assumption can clearly be seen in the four figures as the ambient wind field moves downwind and the effect of the turbines on the wind field is clear as the wakes of the turbines are very distinct. Notice how the dynamic power reference to the turbines produce “gaps” in the wake.

The deficit caused by the wind turbine wake is also aptly illustrated by Figure 11, which shows the average wind speeds at the nacelles for the four turbines in the bottom most row. Note that in order to illustrate the deficit without disturbance from the grid frequency controller this illustration is done with the network operator in “absolute” mode.

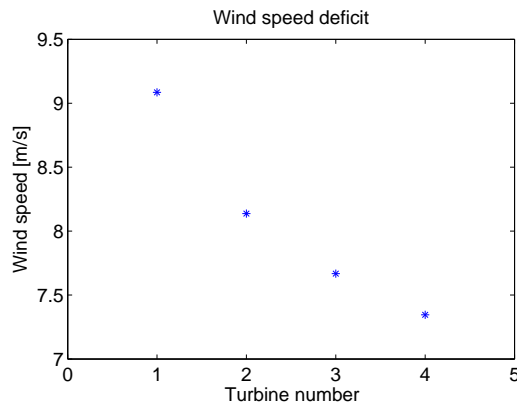


Figure 11: Average wind speed at turbines 1 to 4.

Lastly the rainflow graph from the tower of turbine #2 is shown in Figure 12.

6 Conclusion

In this paper we presented a freely available toolbox for simulating wind farms with focus on the farm control task. The toolbox is highly flexible as it is easy to add or alter turbine models, generate wind fields and specify new farm layouts.

An example of the toolbox capabilities were presented which showed the simulation of a 10 turbine farm with the objective of keeping the grid frequency constant.

The example shows that frequency control is possible with the simulator and it illustrates the wake effects induced by the turbines.

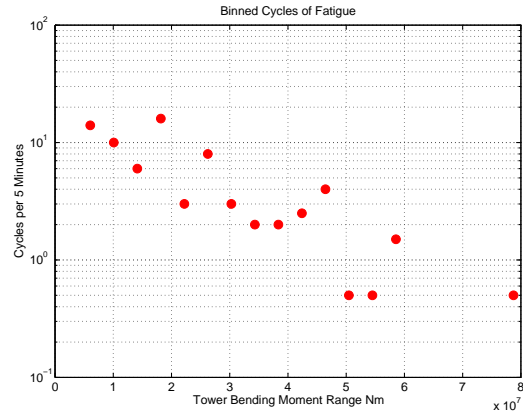


Figure 12: Rainflow count of the tower fore-aft load. The damage equivalent load is $7.8 \cdot 10^6 Nm$.

7 Acknowledgements

This work was sponsored by the EU, through the ICT project Aeolus which is part of the 7th frame program.

Thanks goes to all of the Aeolus project partners, for their input and contributions to the toolbox.

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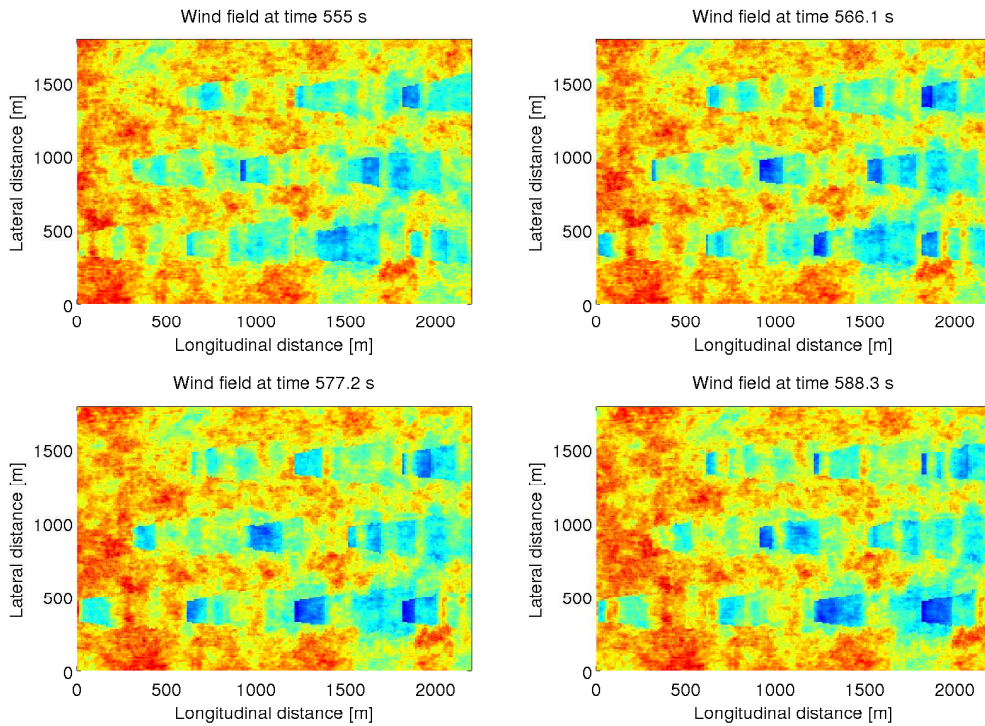


Figure 10: Evolution of the wind field including wakes.

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