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Receiver Heterogeneity Helps: Network Coding for Wireless Multi-Layer Multicast

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Abstract—**Heterogeneity amongst devices and desired services are commonly seen as a source of additional challenges for setting up an efficient multi-layer multicast service. In particular, devices requiring only the base layer can become a key bottleneck to the performance for other devices. This paper studies the case of a wireless multi-layer multicast setting and shows that the judicious use of network coding allows devices with different computational capabilities to trade-off processing complexity for an improved quality of service. As a consequence, individual devices can determine their required effort, while bringing significant advantages to the system as a whole. Network coding is used as a key element to reduce signaling in order to deliver the multicast service. More importantly, our proposed approach focuses on creating some structure in the transmitted stream by allowing inter-layer coding, in order to create more opportunities for recovering the base layer promptly. We propose a design and analyze its delay distribution and mean performance under various system conditions and present a first implementation to verify our analysis and demonstrate the applicability of our approach.**

I. INTRODUCTION

Efficient video delivery for devices with heterogeneous requirements and capabilities has posed significant challenges from a network use perspective. Although it is possible to deliver different video qualities to different users by using separate data streams, this solution is highly inefficient as it does not exploit the inherent dependencies of these data streams. Multiple Description Coding (MDC) and Scalable Video Coding (SVC) have provided alternatives to cater to users with different quality demands.

More recently, network coding has shown an interesting potential for enhancing the performance of layered schemes for achieving higher throughput in the network, e.g., [1], [2], [3] or compensating for inherent packet losses in wireless environments, e.g., [4]. In particular, work in [1] studied the case of layered multicast on wireline networks proposing a simple message passing algorithm to solve the demands of multiple receivers and exploiting on demand decoding at intermediate nodes for enhanced performance. [2] provided a generalization to the approach in [1] presenting an algorithm that solves the problem for two layers optimally for certain natural objective functions as well as useful heuristics for the case of three layers. The work in [3] provided heuristics for coding across multiple layers, as [1], but without allowing for decoding at interior nodes and assuming full knowledge of the network's topology. [5] studied the joint design of multi-resolution codes and network coding while [4] provided
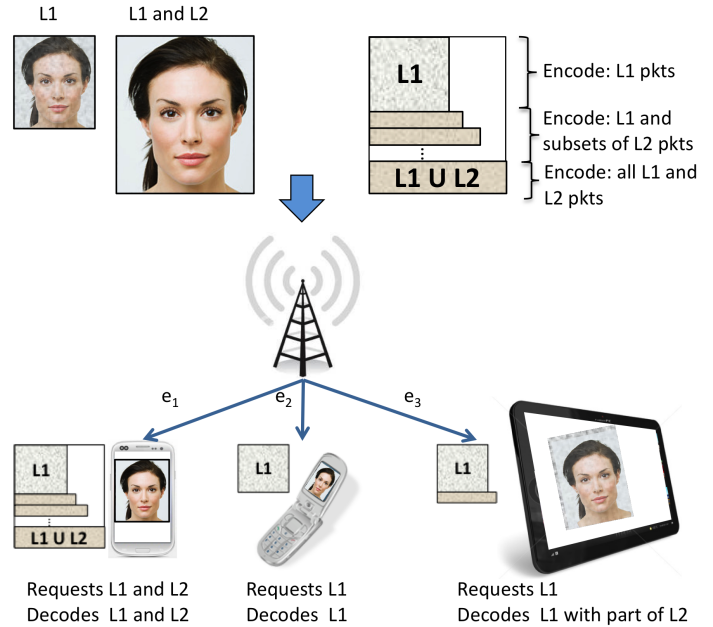

Fig. 1: Basic broadcast topology and encoding decoding matrices

network coding structures for better delay/reliability in the presence of multi-layer codes for video applications. Random linear coding strategies with overlapping and non-overlapping time windows are compared in the multi-layered setting in [6]. Optimization of rateless code schemes for diverse users were studied in [7] and [8].

In a wireless multi-layered multicast setting, receivers with different computational power and demands make use of different types of encoded packets. We present a scheme that splits higher layers into sublayers and sends inter and intra-layer packets with different probabilities. The advantage of this flexibility is that it can increase the coding advantage of users with low-demand by extracting information from inter-layer packets. To the best of our knowledge, the effect of layer sizes on this coding advantage has not been investigated. The scheme takes the synthesis of users into account and determines its parameters, sublayer sizes and probabilities, based on user preferences.

The rest of the paper is organized as follows. Section II describes the wireless multicast problem, and Section III presents

the proposed solution with formulas for the performance of different user types. Performance comparison is discussed in Section IV and numerical results are presented in Section V.

II. PROBLEM FORMULATION AND CONTRIBUTIONS

Network coding requires an increased computational complexity from user devices, which may have limitations on its applicability. Increasing the number of packets encoded together also increases the complexity of the decoding phase. Network coding algorithms for multi-layer content typically distinguish two types of coded packets, according to the number of layers the packet contains data from:

- **Intra-layer** packets contain data from one single layer only, in our case the base layer.
- **Inter-layer** packets may contain encoded packets from several layers, and they require higher computational capacity.

Users may have different preferences on the type of the encoded packets. Figure 1 shows an example with two layers and presents three user types with different demands and computation abilities. User 1 requests two layers because of its screen with high resolution and its computational capability to decode inter-layer packets. Thus, it exploits inter-layer packets mixing the two layers for recovering both available layers. User 2 has a lower computation power and only requests the first layer, so it only exploits intra-layer packets containing the base layer only. Finally, User 3 also requests the first layer only due to its screen limitation. However, since it is willing to invest additional computational effort to get a better service, it will also extract information from inter-layer packets. In our example, User 3 only exploits a part of the inter-layer packets.

We introduce a scheme for wireless multi-layer multicast which takes heterogeneity of users into account. It addresses the problem of finding the trade-off between sending intra-layer packets of the base layer, and inter-layer packets mixing the base layer and one refinement layer. In addition to the concept of mixing inter- and intra-layer packets, we divide higher layers into sublayers. An inter-layer packet contains encoded packets from the base layer and some encoded packets from one specific sublayer. The reason for the concept of sublayers is that it decreases computational complexity and increases useful information extracted from inter-layer packets for User 3. The nature of the analysis and implementation using sub-layers allows us to easily map scenarios with more than two layers into our overall solution.

The overall goals of our work are the following:

- Reduce (and make more deterministic) the time to get the base layer for all receivers as well as reducing their time to recover all desired layers.
- Exploit the inherent, heterogeneous computing capabilities of different devices to improve their overall performance.
- Provide a single encoding structure that allows heterogeneous receivers to improve their service quality. Since we assume the different data packets have the potential to

be received at each destination, these destinations should have the ability to use them if needed.

- Provide an explicit trade–off in performance between different types of receivers.

III. PROPOSED SCHEME

We consider a source S transmitting coded packets. The data is split in n layers, namely, Layer 1 (L_1), Layer 2 (L_2), ..., Layer n (L_n) where l_i packets compose layer i. We say that L_i is higher than L_j if $i > j$. Correspondingly, if $i < j$, then Layer i is lower. In order to use L_i, a receiver needs to also decode all the data packets corresponding to lower layers. The source creates linear combinations of only L_1 packets with probability p_1, while with probability p_i it will generate coded packets involving all L_1 packets and some or all of the L_i packets. The latter contains several cases, where we divide L_i in K_i sublayers of size d_i packets, each sublayer with probability $1/K_i$ is to be chosen. The reason for this code structure is that there are different L_1 receivers. For example, L_1 receivers with limited computing capabilities will only use L_1 packets. However, L_1 receivers with more computational resources, e.g., a mobile device with a fast processor but a small screen, can exploit some of the combinations of L_1 and L_i packets. Our goal is in part to characterize the appropriate p_i and K_i to improve performance of the different receiver types. Note that a larger K_i will benefit L_1 receivers with additional computing capabilities, because they will be able to decode Layer 1 without getting all degrees of freedom to decode both L_1 and L_i. However, a larger K_i makes for a less efficient code, i.e., requiring more coded packets to decode both L_1 and L_i.

The choice of $1/K_i$ as the probability to choose sub-layer i is optimal for cases where all receivers have the same channel loss probability. This choice can be modified in the event of channel asymmetries or if some sub-layers are known to be discarded by all devices interested in L_1. However, this optimization is out of the scope of our current work.

A. Preliminaries

For our analysis, we make the following assumptions

- **Large Finite Fields:** Arithmetic operations are performed in a finite field with a large number of elements. Thus, a coded packet of a specific sub-layer will provide an independent linear combination if the rank at the receiver can be increased with any coded packet of the given sub-layer.
- **Minimal Feedback from Receivers:** Receivers provide only minimalistic feedback indicating that the receiver has successfully decoded its intended layer(s). This allows the system to manage a large number of receivers with limited signaling.
- **Communication Channel:** Transmissions are broadcasted to different nodes in the network. Unless stated otherwise, we focus on the case of a wireless, single hop broadcast network as in Figure 1. Packet losses are assumed to be independent.

B. Encoding, Recoding and Decoding Approaches

The following descriptions are based on our implementation of the algorithms in the Kodo [9] network coding library. The implementation and simulations used in this paper can be downloaded as a standalone package from [10].

- **Encoder:** In order to implement the layered encoding we used a simple scheme requiring only three minimal changes to an existing RLNC encoder. 1) Before encoding a symbol randomly select a coding layer L_m according to the layer probabilities p_m, where $0 \leq m \leq n$. 2) Generate only non-zero coding coefficients up until the size d_m of the chosen coding layer. 3) Include the layer index into the encoded symbol allowing the decoder to easily identify which layer was used for the encoding.

- **Decoder:** In order to implement the proposed scheme we needed to construct a decoder capable of decoding a specific layer L_i while utilizing j out of a total n layers, where $i \leq j \leq n$. As with the encoder this goal was achieved in three stages (see Fig. 2). 1) Extract the layer index of the incoming symbol. If the layer index is larger than j discard the symbol. 2) Otherwise pass the symbol to the *elimination decoder*. The purpose of the elimination decoder is to remove the L_j contribution in the incoming symbols so that it becomes useful for decoding layer L_i. 3) If the elimination decoder successfully removed the L_j contribution from the incoming symbol it can be passed to the L_i decoder for actual decoding. With this structure we are able to deal with all choices of L_i, L_j and L_n.

- **Recoder:** Recoding at intermediate nodes without altering the coding structure requires the system to control which sub-layers can be combined for generating a coded packet of a given sub-layer. A simple approach lies in creating a random linear combinations of all coded packets of that sub-layer and sub-layers that have less data packets. This exploits the structure of the source's stream to preserve such structure. Clearly, this recoding procedure benefits higher sub-layers. A more advanced and computationally demanding approach is to perform partial decoding of higher (sub-)layers in order to exploit packets from these in the recoding of lower (sub-)layers.

Remark 1. *Although we study the case of two layers, the management of multiple layers is straightforward in terms of the encoding, recoding, and decoding schemes. The reason is that we are inherently defining sub-layers for layer L_2. Some of these sub-layers can also be full layers in future settings. Clearly, changes in the probabilities of sending each sub-layer will change to provide the desired service.*

C. Delay Performance of Different Receiver Types

In this subsection, we give exact values for the expected number of packages users need to receive in order to be able to decode the demanded layer(s). We present formulas for all the three types of users presented in Section II. Calculations
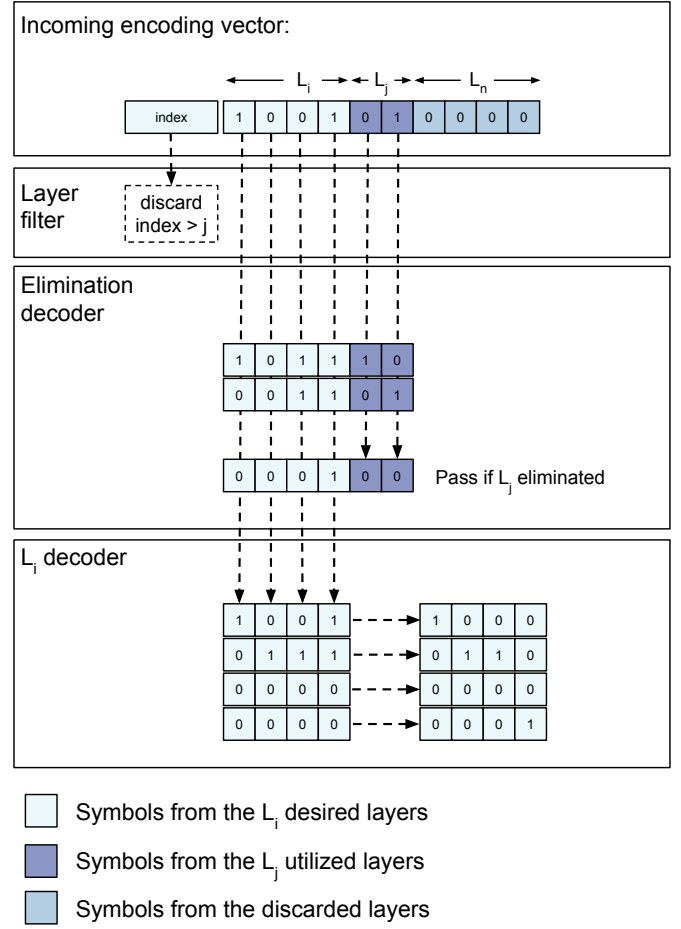

Fig. 2: Decoding procedure for a L_i decoder with $d_i = 4$, $d_j = 6$ and $d_n = 10$. Symbols are passed through three stages where the first two only conditionally forward the symbol. Decoding is complete when the L_i decoder reaches full rank.

can be extended for the general case of n layers applying similar techniques.

Definition 1. *A packet is called a **1-packet, 2-packet** and **1-2-packet** if it is an encoded packet from original packets of L_1, L_2, and both layers, respectively.*

Let x_1 denote the total number of coded packets received when L_1 becomes decodable for a receiver using only 1-packets. Then, since the last packet received must be a 1-packet and the number of previously received packets has a binomial distribution,

$$\Pr(x_1 = n) = \binom{n-1}{l-1} p^{l_1} (1-p)^{n-l_1}$$

The expected value of x_1 can be expressed with the following formula:

$$E(x_1) = \sum_{n=l_1}^{\infty} n \binom{n-1}{l-1} p^{l_1} (1-p)^{n-l_1}$$

Definition 2. *For a 1-2-packet m let m_1 and m_2 denote the 1-packet and 2-packet reduced from m by taking the coefficient vectors of only L_1 and L_2, respectively. Similarly, we define M_1 and M_2 for a set M of 1-2-packets. For such a set M, let $SP(M) := (|M| - rank(M_2))^+$ be called the **surplus** of M.*

Let $\Pr_{SP}(N, K, d, b)$ denote the probability that a random set of 1-2-packets has surplus exactly b. Note that the surplus of such a set is the sum of surpluses of K disjoint subsets containing packets from a certain division. Then, $\Pr_{SP}(N, K, d, b)$ can be calculated recursively for $K > 1, N > 0$:

$$\Pr_{SP}(N, K, d, b) =$$
$$\sum_{n=0}^{d+b} \binom{N}{n} \frac{1}{K^n} \left(1 - \frac{1}{K}\right)^{N-n} \Pr_{SP}(N - n, K - 1, d, b'),$$

where $b' = b - (n - d)^+$.

Similarly, for $b > 0$ let $\Pr_{SP}^*(N, K, d, b)$ denote the probability that a set of N random 1-2-packets has surplus exactly b and the last packet m increases the surplus, that is $SP(M) > SP(M - m)$. Then, we have

$$\Pr_{SP}^*(N, K, d, b) = K \sum_{n=d+1}^{d+b} \binom{N-1}{n-1} \frac{(K-1)^{N-n}}{K^N}$$
$$\times \Pr_{SP}(N - n, K - 1, d, b - (n - d)).$$

Now we are ready to express the expected number of packets a receiver needs for decoding, if both 1-packets and 1-2-packets are used. Let $x_{1|2}$ denote the number if packets received when L_1 becomes decodable. Note that $x_{1|2} \leq l_1 + l_2$. Then, according to whether the last packet is a 1-packet or a 1-2-packet we can distinguish between two cases.

$$\Pr(x_{1|2} = N, \text{last packet is a 1-packet}) =$$
$$\sum_{n=1}^{l_1} \binom{N-1}{n-1} p^n (1-p)^{N-l_1} \Pr_{SP}(N - n, K, d, l_1 - n)$$

$$\Pr(x_{1|2} = N, \text{last packet is a 1-2-packet}) =$$
$$\sum_{n=0}^{l_1} \binom{N-1}{n} p^{l_1} (1-p)^{N-l_1} \Pr_{SP}^*(N - n, K, d, l_1 - n)$$

$$E(x_{1|2}) = \sum_{N=l_1}^{l_1+l_2} N \Big(\Pr(x_{1|2} = N, \text{last packet is a 1-packet})$$
$$+ \Pr(x_{1|2} = N, \text{last packet is a 1-2-packet}) \Big)$$

Let $\Pr_{SP2}(N, K, d, b)$ denote the probability that a set of N random 1-2-packets has surplus at least b and all the divisions are decodable.

$$\Pr_{SP2}(N, K, d, b) = \sum_{n=d}^{N-(K-1)d} \binom{N}{n} \frac{1}{K^n} \left(1 - \frac{1}{K}\right)^{N-n}$$
$$\times \Pr_{SP2}(N - n, K - 1, d, (b - (n - d))^+)$$

Let $\Pr_{SP2}^*(N, K, d, b)$ denote the probability that a random set of N 1-2-packets has surplus at least b, all the divisions are decodable and the last message completes a division.

$$\Pr_{SP2}^*(N, K, d, b) = K \binom{N-1}{d-1} \frac{1}{K^d} \left(1 - \frac{1}{K}\right)^{N-d}$$
$$\times \Pr_{SP2}(N - d, K - 1, d, b)$$

Let $\Pr_{exSP2}(N, K, d, b) = $ denote the probability that a random set of N 1-2-packets has surplus exactly b and all divisions decodable.

$$\Pr_{exSP2}(N, K, d, b) = \sum_{n=d}^{d+b} \binom{N}{n} \frac{1}{K^n} \left(1 - \frac{1}{K}\right)^{N-n}$$
$$\times \Pr_{exSP2}(N - n, K - 1, d, b - (n - d))$$

Let x_{12} denote the number of packets needed to decode both layers. According to the type of the last packet there are three cases:

1) last packet is a 1-packet
2) last packet is a 1-2-packet and L_1 is completed with this packet
3) last packet is a 1-2-packet and a division in L_2 is completed with this packet

Note that cases (1) and (2) imply that $x_{12} = l_1 + l_2$. Hence, $x_{12} > l_1 + l_2$ implies case (3).

$$\Pr(x_{12} = l_1 + l_2 \text{ and case i,}) =$$
$$\sum_{n=1}^{l_1} \binom{l_1 + l_2 - 1}{n - 1} p^n (1-p)^{l_1 + l_2 - n}$$
$$\times \Pr_{exSP2}(l_1 + l_2 - n, K, d, l_1 - n)$$

$$\Pr(x_{12} = l_1 + l_2 \text{ and not case i,}) =$$
$$\sum_{n=0}^{l_1} \binom{l_1 + l_2 - 1}{n} p^n (1-p)^{l_1 + l_2 - n}$$
$$\times \Pr_{exSP2}(l_1 + l_2 - n, K, d, l_1 - n)$$

$$\Pr(x_{12} = N > l_1 + l_2) = \sum_{n=0}^{N-l_2} \binom{N-1}{n} p^n (1-p)^{N-n}$$
$$\times \Pr_{SP2}^* \big(N, K, d, (l_1 - n)^+\big)$$

$$E(x_{12}) = (l_1 + l_2) \Pr(x_{12} = l_1 + l_2)) +$$
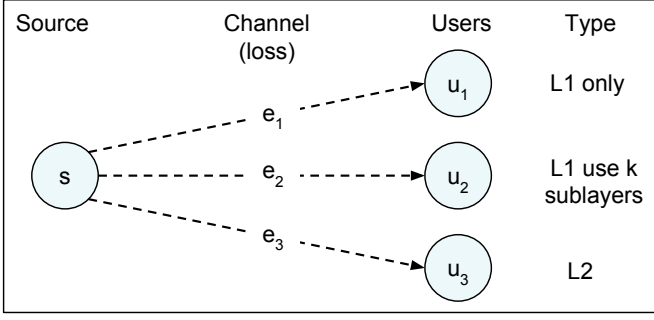$$\sum_{N=l_1+l_2+1}^{\infty} N \, \Pr(x_{12} = N > l_1 + l_2)$$

Fig. 3: Simulator setup with a single source s broadcasting to three users u_1, u_2 and u_3 with three different decoding requirements. During transmission packets are lost with independent erasure probabilities e_1, e_2 and e_3.

D. Optimization Criteria

The system's optimization criteria can depend on the requirements of the wireless system. For example, if the goal is to minimize the time of the reception of a video frame (with different available layers), the goal is to make all receivers decode at the same time their respective data. On the other hand, if the goal is to optimize energy consumption of the system, then the use of different sub-layers for decoding will affect the computational effort (and processing energy) of the individual users and of the system as a whole.

The key of our approach is that not only p_1 can be used as the variable to tune performance, but rather one of a large group including the p_i choices for the different sub-layers as well as the number and size of each sub-layer.

Our goal is not to provide a comprehensive discussion of the different optimization options, but rather to show that our procedure opens the door to more flexible and practical optimizations.

IV. PERFORMANCE COMPARISON

In this section, we simulate the performance of the three different user types introduced in Section II and compare it to the analytical results obtained in the previous sections.

The basic setup of the simulator is shown in Fig. 3. As shown a single source is broadcasting to the three users, each packet sent is lost with independent loss probability e_1, e_2 and e_3. One receiver uses only L_1 coded packets, one uses k_{use} sub-layers to help in decoding L_1, and the latter gathers all coded packets to recover L_1 and L_2. Although in our analysis we focused on the case with $k_{use} = K$, we shall explore in more details these options for receivers interested in L_1.

In our numerical results, we use as key performance metrics the mean number of received packets by each receiver type in order to decode its intended layer(s) and also the mean total number of transmissions to satisfy all three receivers.

V. NUMERICAL RESULTS

This section provides numerical results using the implementation described in Section IV and the analysis from Sec-
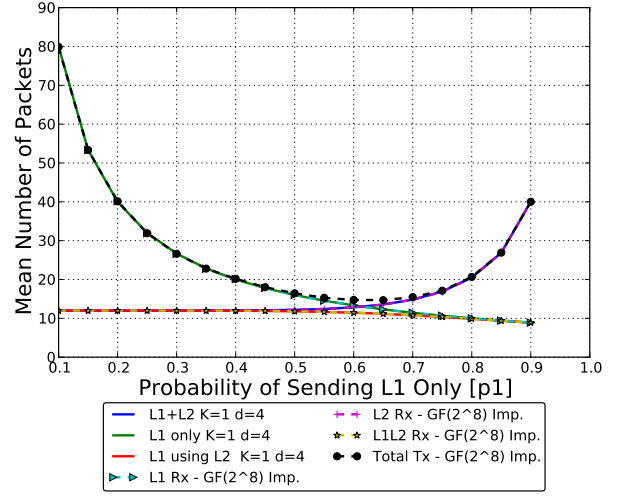

Fig. 4: Comparing theory and implementation for the number of received packets per receiver when L_2 has 4 packets and L_1 has 8 packets and $K = 1$.

tion III-C to both confirm our analytical results and illustrate the potential of the proposed mechanism.

Figure 4 shows the performance of the three type of receivers when $K = 1$ and compares the analysis results with the implementation with [10] when using $GF(2^8)$ for its finite field operations. On the one hand, this figure shows that the theoretical and practical results match. On the other hand, it shows that receivers with high computational power can recover L_1 significantly faster than receivers using only L_1 for all values of p_1. If the system attempts to minimize the overall completion time, a $p_1 \approx 0.6$ will be chosen to strike a balance between L_1 and L_2 receivers. However, an L_1 receiver to decode 30% faster if it exploits inter-layer packets for the same setting.

Figure 5 shows that splitting into $K = 4$ sub-layers but letting the receivers decide how many of them (k_{use}) to use to decode L_1 allows for reducing the number of received coded packets before decoding. As shown in the figure, even the use of a single sub-layer, i.e., $k_{use} = 1$, improves significantly the performance of receivers attempting to recover L_1 without requiring a large increase in the processing complexity. In this case, these receivers would need to decode 20 data packets instead of 16 data packets.

More importantly, Figures 4 and 5 show that our novel encoding structure allows for the system to have more predictable and controllable behavior for L_1 receivers. This also means that the system is less sensitive to the choice of p_1 to determine overall performance. Furthermore, our structure provides more degrees of freedom for the receivers and the transmitter to optimize the overall system performance while exploiting each device's heterogeneous capabilities. In this way, more powerful receivers with adverse channel conditions can exploit additional processing for coping with their current channel and attain a better service quality. A key aspect is that
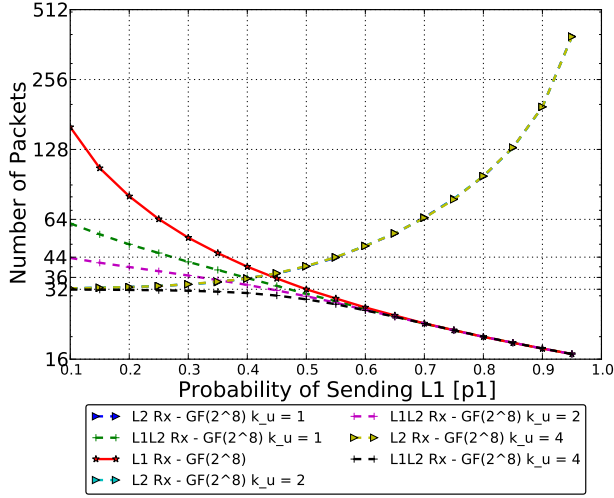
Fig. 5: Number of received packets before decoding from the three receiver types, when each layer has 16 packets, and dividing L_2 into four sub-layers each of size four packets. k_{use} indicates how many of the sub-layers are being used by the receiver interested in L_1 but exploiting part of L_2.

the receiver could make this choice independently from other devices' policies.

VI. PERSPECTIVES

Although we analyzed the case of no feedback or minimalistic feedback without altering our policy, it is worthwhile to study more dynamic policies. Namely, the probability p_1 (or of any sublayer of Layer 2) could be changed after a group of receivers has finished. For example, p_1 could be made zero if all computationally limited receivers have been satisfied, thus allowing for a more efficient code structure for the remaining receivers. Clearly, this requires that the system knows about which users are actively receiving the data.

VII. CONCLUSIONS

This paper presents a novel code structure and approach to manage multi-layer, multicast streams with high potential in practical wireless systems. The underlying idea was to create a code structure that allowed some receivers to focus on receiving only coded packets containing the base layer, others receiving all packet types, and finally enable receivers interested only in the base layer but willing to commit additional resources to use packets that contained packets from the refinement layer in order to decode the base layer swiftly. Although the previous two receiver types had been considered in the past, our take on permitting on-demand, partial decoding of the refinement layer to obtain the base layer is unique. A byproduct of our design is the fact that the system can now have a wider array of tuning parameters, which should enable more interesting optimization options and efficient solutions. Our implementation results show that the novel coding structure has a disruptive effect and benefit over the entire system, reducing the time to complete the transmission.

Future work shall focus more deeply on developing efficient policies to exploit our approach both in the current context or as a way to enable intermediate nodes in networks to extract lower layers to improve multicast throughput, as proposed in [1].

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