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OPTIMIZATION OF MEASUREMENTS ON DYNAMICALLY SENSITIVE STRUCTURES USING A RELIABILITY APPROACH

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SUMMARY

Design of a measuring program devoted to parameter identification of structural dynamic systems described by random fields is considered. The design problem is formulated as an optimization problem to minimize the total expected costs due to failure and costs of a measuring program. Design variables are the numbers of measuring points, the locations of these points and the required number of sample records. An example with a simply supported plane, vibrating beam is considered and tentative results are presented.

1. INTRODUCTION

The identification of parameters in dynamic models for civil engineering structures based on measured input-output data has been a topic of research for several researchers in the last decades, see e.g. Kozin et al. [1] and Eykhoff [2].

The value of information achieved by identifying the different model parameters that characterize a dynamic structure depends on the system identification methods and on how the measurements are performed. The information achieved can be used to update the reliability of the structure. Further, and - not less important - an evaluation of the dynamic design model can be performed, i.e. the validity of the mathematical model adopted in the design can be checked and this model can be updated for further investigations.

An important problem which arises with full-scale measurings is how to design the experiment to obtain best knowledge on the random field taking account of the basis of measurings in a finite number of points, so that records obtained from those locations yield dynamic parameter estimates with the least uncertainty. What should be the number of sensors, necessary for obtaining sufficient information on the random fields, where should they be placed and finally how many measurements should be made. The number of sensors is usually limited to minimize the cost of instrumentation.

Though various system identification methods have been developed for identifying the different parameters, few researchers, see e.g. Shah et al. [3], have looked at the question of where to locate sensors in a structure to obtain the best estimates of the partially unknown parameters. The problem of optimal sensor positioning has often found its solution from practical considerations, e.g. location on the antinode (maximize signal-to-noise ratio) or deck level in offshore structures (minimize measuring cost). To improve the signal-to-noise ratio while recording the structural response, the sensors often need to be located as closely as possible to the antinodes. Following this strategy, an instrumentation lay-out can be designed. However, if we have more than one important mode and, more than one sensor, the problem of optimal measuring lay-out becomes very difficult.

To the knowledge of the authors there is not developed mathematical optimization procedures for measuring program designing where the financial cost of the measuring program and benefit of new information is taken into account.

In this paper a method to determine an optimal measuring program to obtain the best estimates of the unknown parameters is proposed based on a cost function introduced to make a trade-off between benefit of the new information and the cost of the measuring program. The method is especially developed for dynamically sensitive structures where the reliability of the structural system is sensitive to the dynamic parameters.

After discussing sources and types of uncertainties which may be reduced through the acquisition of additional information, a reliability based measuring design problem is formulated. Next the proposed optimization procedure is used in a simple example where the parameter in the optimization problem is the minimum required number of independent sample records. The example considered is a vibrating plane beam model subjected to random transverse loading. Finally, some tentative results are presented concerning the question, "Should additional information be obtained" ?.

The reliability calculations in this paper are performed with the computer program PRADSS, see Sørensen [4].

The optimization problems are solved by using the NLPQL algorithm, see Schittkowski [5].

2. RELIABILITY BASED DESIGN OF A MEASURING PROGRAM

First-Order Reliability Methods

A reliability analysis is based on a reliability model of the structural system. The elements in the reliability model are failure elements modelling potential failure modes of the structural system, e.g. fatigue failure. Each failure element is described by a failure function $g(\overline{x},\overline{p}) = 0$ in terms of a realization $\overline{x}$ of a random vector $\overline{X} = (X_1, X_2, .., X_n)$ and deterministic parameters $\overline{p}$. $\overline{X}$ is assumed to contain n stochastic variables describing e.g. load, strength, geometry etc. Realizations $\overline{x}$ of $\overline{X}$ where $g(\overline{x},\overline{p}) \leq 0$ corresponds to failure states in the n-dimensional basic variable space while $g(\overline{x},\overline{p}) > 0$ corresponds to safe states.

In first-order reliability methods (FORM), see e.g. Madsen et al. [6], a transformation $\overline{T}$ of the generally correlated and non-normally distributed variables $\overline{X}$ into standardized normally distributed variables $\overline{U} = (U_1, U_2, .., U_n)$ is defined. Let $\overline{U} = \overline{T}^{-1}(\overline{X},\overline{p})$. In the $\overline{u}$-space the element reliability index β is defined as

$$\beta = \min_{g(\overline{T}(\overline{u}),\overline{p})=0} (\overline{u}^T\overline{u})^{\frac{1}{2}} \tag{1}$$

If the failure function is not too non-linear the probability of failure P_F can with good approximation be determined from

$$P_F \approx \Phi(-\beta) \tag{2}$$

where $\Phi(\cdot)$ is the standard normal distribution function.

Sources and Types of Uncertainty

In general, the additional information from field tests would not eliminate all the uncertainties in a reliability problem. In assessing reliability of a structure different sources of uncertainty are pertinent. In Der Kiureghian [7] the evaluation of the reliability index under the following four sources of uncertainty are discussed: Inherent variability, estimation error, model imperfection and human error. Inherent variability, often called randomness, may exist in the characteristics of the structure itself or in the environment to which the structure is exposed.

Estimation error arises from the incompleteness of statistical data and our inability to accurately estimate the parameters of the probability models that describe the inherent variabilities. Model imperfection arises from our use of idealized mathematical models to describe complex phenomena. Finally, the human error uncertainty arises from errors made by engineers or operators in the design, construction or operation phases of the structure.

Inherent variability is essentially a state of nature and the resulting uncertainty may not be controlled or reduced, i.e. the uncertainty associated with inherent variability is something we have to live with. The uncertainty associated with estimation error, model imperfection and human error may be reduced through the acquisition of additional data, the use of more accurate models and implementing rigorous quality control measures in the design, construction and operation phases of a structure.

The available statistical information, objective and subjective, on relevant variables and the set of mechanical and probabilistic models and their associated error estimates constitute the state of knowledge in a reliability problem. The state of knowledge is said to be perfect when complete statistical information and perfect models are available; otherwise, the state of knowledge is said to be imperfect. Real engineering problems invariably deal with imperfect states of knowledge.

The acquisition of additional information, such as performing a full-scale measuring of a structure, of course, will require the time, energy, and financial resources. The added cost for this new information should be included or reflected in design of a measuring program. The added cost may be justified if it eliminates a significant part of the uncertainty, thus leading to a lower expected probability of failure of the structure.

Optimization Problem

In order to design an optimal measuring program it is suggested, based on the various aspects discussed above, to minimize the costs due to failure and the costs connected with the measuring program. It is assumed that dynamic system parameter estimates in the identification problem are based on m sample records, at each of the N measuring points, achieved by performing a full-scale measuring program. An optimization problem of an optimal measuring program can now be formulated as

$$
\begin{aligned}
min \quad & C(N, m, \overline{z}) = C_F P_F(N, m, \overline{z}) + C_M(N, m, \overline{z}) && (3)\\
s.t \quad & N > 0 && (4)\\
& m \geq 0 && (5)\\
& z_i^l \leq z_i \leq z_i^u \qquad i = 1, 2, ..., N && (6)
\end{aligned}
$$

where N, m and $\overline{z}$ are the optimization variables. C_F is cost of failure and C_M is cost of N measurings at geometrical locations $\overline{z}$ where m sample records are taken in each point. The expected cost C is the objective function . P_F is probability of failure after measurings have been performed. As constraints upper and lower limits on the measurings locations, lower limits on the sample records and number of measuring points are given. The optimization problem may also be based on the system reliability. Then, the system reliability index is used instead of the element reliability index in (2).

The probability of failure P_F in (3) is calculated using the first-order reliability methods. The failure function, related to e.g. fatigue, is written

$$g(N, m, \overline{z}, \overline{p}, \overline{x}, \overline{s}) = 0 \tag{7}$$

where the realizations $\overline{x}$ of the random vector $\overline{X}$ describe uncertainties, such as those arising from inherent variabilities, which are irreducible by measuring the structural response. Uncertainties which are reducible, such as those from estimation error, are described by realizations $\overline{s}$ of the random vector $\overline{S}$. The distribution of $\overline{S}$ depends on m. I.e. it is assumed that uncertainty of the statistical information may be described by the number of sample records. This problem is discussed in the following example.

Modelling of the Cost Function

One of the main difficulties with the above optimization problem is knowledge of C_F and C_M.

When a structure fails one needs to pay various costs such as repair costs, reconstruction costs, clean up costs, loss of income, costs due to loss of social prestige and possible deaths. The total cost of failure C_F may range from e.g. 2 to 5 times the initial costs for an offshore structure, see e.g. Marshall [8]

The costs of obtaining the new information C_M is to cover not only the sample records but also the costs of statistical analysis of the information and an appropriate share of costs of planning. A simple and useful function for the cost of a measuring program is $C_M = C_0 + C_1 m + C_2 N$. C_0 may be interpreted as representing the costs of the instrumentation and planning. C_1 may be interpreted as an additional cost per sample record and C_2 is the cost of an additional sensor. In some cases a more complicated cost function can be used, e.g. when a learning effect is introduced in the statistical analysis.

3. EXAMPLE WITH A RANDOMLY VIBRATING BEAM

In this example, the proposed optimization procedure is used in an example where a simply supported vibrating plane, continuous elastic steel beam model subjected to random transverse loading is considered.

In this example a measuring program which only consists of one sensor, N=1, is considered. I.e m is the only optimization variable. The measuring point is placed at $L/2$ where L is the length of the beam. Optimization of the number of sensors and sensor locations is not taken into account because it is assumed that a solution of the full optimization problem (3)-(6) will result in unacceptable long computation time. Therefore an implementation of the optimization problem for solving the problem of optimal sensor lay-out is disregarded. If this simplified approach should be used to estimate the optimal number of sensors, the optimization problem in (3)-(6) can be solved sequentially for varying N where the optimal locations of the N measuring points are estimated by using a simple method for optimal sensor location, see e.g. Shah et al. [3].

Further, it should be emphasized, that the errors considered in this example are only the statistical sampling errors inherent in analysis of finite amounts of data. Beyond the errors of finite samples are a multitude of other potential errors that may have accumulated during the data acquisition. It is assumed that such errors have been controlled and that the time history records received for analysis accurately represent the physical phenomenon of interest.

Structural Model of Vibrating Beam

The beam considered is subjected to a distributed tranverse loading represented by $P(z,t)$ per unit length of the simply supported beam. We assume that its equation of motion is given by

$$EI\frac{\partial^4 y(z,t)}{\partial z^4} + C_d\frac{\partial y(z,t)}{\partial t} + M\frac{\partial^2 y(z,t)}{\partial t^2} = P(z,t) \tag{8}$$

where $y(z,t)$ is the deflection of the beam at time t and distance z from its end. M is its mass per unit length, C_d is the viscous damping coefficient per unit length and EI is the bending stiffness of the beam. We also assume that $P(z,t)$ is a zero-mean stationary Gaussian stochastic process with a covariance function given by

$$E[P(z_1,t_1),P(z_2,t_2)] = \psi^2\delta(z_1-z_2)\delta(t_1-t_2) \tag{9}$$

where δ is Dirac's delta function and ψ is the intensity of random fluctuations of the loading. What we have assumed is that the stochastic load is white noise with respect to both the time and the space parameter.

Modelling of the Failure Function

The failure function in (7) is modelled as a fatigue failure element by using the Palmgren-Miner rule in combination with SN- curves. Then the accumulated fatigue damage D can be written, see Wirsching[9]

$$D = \frac{T_L}{T_0 K}(2\sqrt{2})^k\Gamma(1+\frac{k}{2})\sigma_s^k \tag{10}$$

where T_L is the expected lifetime. Here we use T_L=25 years. σ_s is the standard deviation of the stress process and T_0 is mean period of a stress cycle. The stress process is assumed to be zero-mean Gaussian narrow-banded. $\Gamma(\cdot)$ is the gamma function. k and K are parameters in the SN-curves. Here k is modelled as a constant, k=3, and K is modelled as a random variable. Stress concentration is neglected. Now the fatigue failure function can be written for a given location

$$\begin{aligned} g(N,m,\overline{z},\overline{p},\overline{x},\overline{s}) = -ln(D) &= -ln(T_L) + ln(T_0) + ln(K) \\ &- kln(2\sqrt{2}) - ln(\Gamma(1+\frac{k}{2})) - kln(\sigma_s) \end{aligned} \tag{11}$$

As σ_s and T_0 shall correspond to the measured response at the N locations the two quantities are functions of $N, m, \overline{z}, \overline{p}, \overline{x}$ and $\overline{s}$

Now, we define an identified auto-spectral density of the stress process $S_{ss}^{Iden}(\omega)$ at $L/2$ where L is the length of the beam. I.e. we consider the fatigue failure at $L/2$. The superscript $Iden$ indicates an indentified spectrum which corresponds to information achieved from a measuring program. Once the spectrum of the stress process has been estimated, variance and mean period of a stress cycle can easily be calculated by

$$\sigma_s^2 = \int_{-\infty}^{\infty} S_{ss}^{Iden}(\omega)d\omega \qquad T_0 \approx 2\pi\frac{\sigma_s}{\sqrt{\int_{-\infty}^{\infty}\omega^2 S_{ss}^{Iden}(\omega)d\omega}} \tag{12}$$

Modelling of Measured Spectral Estimates

Now, we assume that we have measured the structural response at $L/2$. The connection between the identified auto-spectral density $S_{ss}^{Iden}(\omega)$ of the stress process and the assumed measured spectral estimates of the response process is established in the following way.

First, we identify the structure from the measured spectral densities by fitting a model to the assumed measured spectral densities. Secondly, by using the fitted model, we estimate the measured stress spectrum which we have defined above.

We define the measured auto-spectral density of the response process by

$$S_{yy}^{Mea}(\overline{p}, \overline{x}', s_l, \omega_l) = S_{yy}^{Mod}(\overline{p}, \overline{x}', \omega_l) s_l \tag{13}$$

where s_l is a realization of a random variable S_l modelling the statistical uncertainty of the spectral estimates at the $l'th$ angular frequency ω_l. $\overline{x}'$ is realizations of a random vector $\overline{X}'$ containing parameters describing the system, e.g. damping, mass etc. The random vector $\overline{X}'$ describes the inherent uncertainty. The elements in $\overline{X}'$ and the variable K in the fatigue model form the random vector $\overline{X}$. The superscript Mod indicates that it is a response spectrum based on a model.

Using the modal spectral analysis method, see e.g. Lin [10], the spectral density of the response process $S_{yy}^{Mod}(\overline{p}, \overline{x}', \omega_l)$, can be estimated. Because of the statistical uncertainty there is an error ϵ between measured and the theoretical spectral densities estimates. I.e

$$S_{yy}^{Mea}(\overline{p}, \overline{x}', s_l, \omega_l) = S_{yy}^{Mod}(\overline{p}, \overline{\theta}, \omega_l) + \epsilon(\omega_l) \tag{14}$$

where $\overline{\theta}$ is a vector containing the identified model parameters. It is noticed that it is the same theoretical model which is used for calculation of $S_{yy}^{Mod}(\overline{p}, \overline{\theta}, \omega)$ and $S_{yy}^{Mod}(\overline{p}, \overline{x}', \omega)$. The spectrum $S_{ss}^{Iden}(\omega)$ is then found as $S_{ss}^{Iden}(\omega) = S_{yy}^{Mod}(\overline{p}, \overline{\theta}, \omega_l)T$ where T is a constant. It is assumed that a best estimate of the parameters $\overline{\theta}$ can be obtained by minimizing the total sum of squared errors Err^2, given by

$$Err^2 = \sum_{k=1}^{N_R} (\epsilon(\omega_k))^2 \tag{15}$$

where N_R are the number of spectral estimates. An equation for the total squared error is obtained by introducing the error, defined by re-arranging (14), into equation (15).

The optimization problem in (15) is solved using the NLPQL algorithm which requires the gradients of the objective function. Here the gradients of the objective function are numerically calculated . The optimization variables $\overline{\theta}$, which describe the theoretical model, are mass per unit length M, the intensity of random fluctuations ψ of the loading and the modal damping ratio ζ_1 of the first mode . For simplicity we only take one mode into account.

Modelling of the Uncertain Quantities

In this example we have 10 random variables. No model uncertainty is considered.

The fatigue strength random variable K is modelled as $LN(6400MPa, 1024MPa)$ where LN signifies a log-normal distribution.

The statistical uncertainty of the theoretical response spectrum, see (13), is modelled by six random variables, i.e. that the $\overline{S}$-vector contains six independent random variables. The statistical error of the spectral estimates, from finite Fourier transforms, consists of a bias and a random error. Here, for simplicity, we only consider the random error. The random error is reduced by computing an ensemble of estimates from m different subrecords and averaging the results to obtain a final smooth estimate of the spectrum. Spectral estimates of a stationary Gaussian white noise process are χ^2_{2m} distributed with $2m$ degrees of freedom. The coefficient of variation ϵ of a estimate is $\epsilon = 1/\sqrt{m}$,i.e. the six random variable in the $\overline{S}$-vector are modelled using a mean value equal to one and a coefficient of variation ϵ. In Bendat et al. [11] the properties of statistical uncertainty of spectral estimates are discussed in details .

The uncertainties of the parameters describing the system are modelled by three independent random variables M, ψ and ζ_1 whose statistical characteristics are shown in Fig. 1. $\epsilon_{apriori}$ is a coefficient of variation which describes the uncertainty of the apriori information before the measuring program is performed. $\epsilon_{inherent}$ is a coefficient of variation which describes the inherent uncertainty in the random variables based on our judgement. The response spectrum in (13) is calculated by using this coefficient of variation. We assume that the expected values are the same before and after the measuring program has been performed. The sensitivity of the measuring program design to variation of the assumed mean values after performing the measuring program will be considered in the example.

Variable	Distribution	Mean Value	$\epsilon_{apriori}$	$\epsilon_{inherent}$
M	N	1.0	0.10	0.05
ψ	N	1.0	0.20	0.15
ζ_1	LN	1.0	0.30	0.15

Fig. 1: Statistical characteristics in SI units. (N: normal , LN: lognormal).

In the calculations the mean values (1.0) are multiplied by values corresponding to a lightly damped system, the modal damping ratio of first mode is 0.04, and the mass, length and bending stiffness are modelled so the frequency of the first mode is 0.4 Hz .

Gradients of the Cost Function

As mentioned above the NLPQL algorithm is used to solve the optimization problem (3)-(6) for N=1. The NLPQL algorithm requires the gradients of the cost function (3). The derivative with respect to the optimization variable m is

$$\frac{\partial C}{\partial m} \approx \varphi(-\beta)\frac{\partial(-\beta)}{\partial m}C_F + C_1 = \varphi(-\beta)\sum_{i=1}^{n_s}\frac{\partial(-\beta)}{\partial \sigma_i}\frac{\partial \sigma_i}{\partial m}C_F + C_1 \qquad (16)$$

where φ is the standard normal density function. As the standard deviation of S_i is $\sigma_i = \frac{1}{\sqrt{m}}$ the derivative $\frac{\partial(\sigma_i)}{\partial m} = -\frac{1}{2}(m)^{-1\frac{1}{2}}$. n_s is the number of random variables in $\overline{S}$

The derivative $\frac{\partial(-\beta)}{\partial \sigma_i}$ are estimated from , see Madsen et al. [6]

$$\frac{\partial \beta}{\partial \sigma_i} = \frac{1}{\beta}\sum_{l=1}^{n_s+n} u_l^* \frac{\partial\{T_l^{-1}(N, m, \overline{z}, \overline{p}, \overline{x}^*, \overline{s}^*)\}}{\partial \sigma_i} \qquad (17)$$

where the $*$ indicates values at the design point .

Results

The optimal solution of m for a measuring program with one sensor placed at $L/2$ is estimated. It is assumed that the cost function can be modelled as

$$C_0 = 10^6 \ DKK., \quad C_1 = 500 \ DKK., \quad C_2 = 10^5 \ DKK., \quad C_F = 10^9 \ DKK. \quad (19)$$

In Fig. 2 the objective function C and the number of sample records m are shown as functions of number of iterations. As starting point $m = 10$ is used.

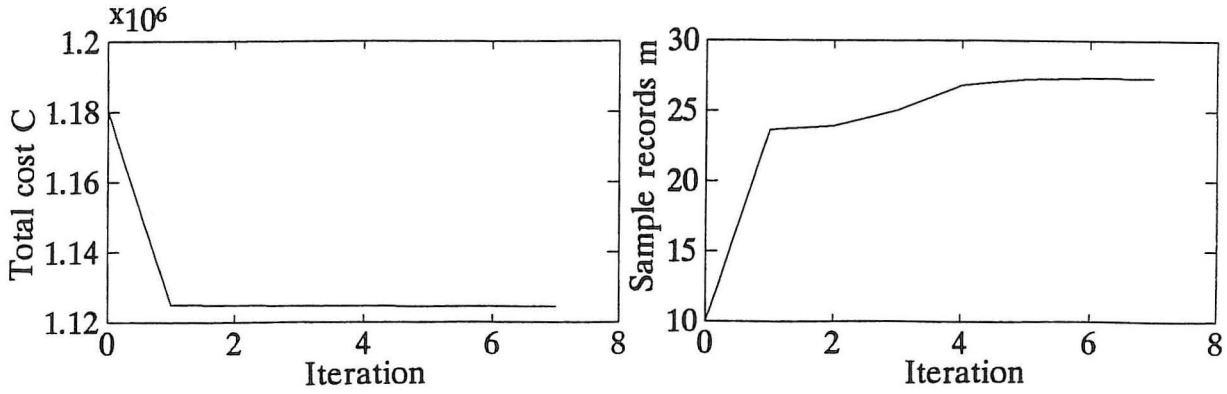

Fig. 2: Iteration history. (Optimal values $m = 27.37$, $C = 1124597$ DKK.)

It is seen that convergence is obtained after 7 iterations. The computer time used on a VAX-8700 was 1098 sec.

The optimal solution of m for various values of cost of failure C_F and estimates of the inherent uncertainty is shown in Fig. 3 and Fig.4, respectively. In Fig. 4 optimal solutions of m to different estimates of the expected value of the modal damping μ_{ζ_1} are considered.

C_F (DKK.)	P_F	C (DKK.)	C_M (DKK.)	m
10^5	$5.65\,10^{-3}$	1101123	1100555	1.11
10^6	$9.03\,10^{-4}$	1102624	1101720	3.44
10^7	$1.64\,10^{-4}$	1105097	1103455	6.91
10^8	$3.71\,10^{-5}$	1110538	1106825	13.65
10^9	$1.09\,10^{-5}$	1124597	1113685	27.37

Fig. 3: The optimal solution of m for various values of cost of failure C_F.

μ_{ζ_1}	P_F	C (DKK.)	C_M (DKK.)	m
0.030	$1.11\,10^{-2}$	2255887	1145095	90.19
0.035	$4.66\,10^{-4}$	1165738	1119130	38.26
0.040	$3.71\,10^{-5}$	1110538	1106825	13.65
0.045	$1.12\,10^{-5}$	1105082	1103955	7.91
0.050	$6.15\,10^{-6}$	1103416	1102800	5.60

Fig. 4: The optimal solution of m for various values of μ_{ζ_1}, ($C_F = 10^8$ DKK.).

It is seen, as expected, that m increases when C_F increases, which means that acquisition of more information is of course more relevant when cost of failure increases. Further, it is seen

from Fig. 4 that an optimal solution of m actually depends not only on the actual values which are presumably known (here deterministic values) but also on the value of the parameter μ_{ζ_1} which is to be identified.

In Fig. 5 we have shown an example of the objective function C and the reliability index β for various values of m. In this case $\mu_{\zeta_1} = 0.04$ and $C_F = 10^8$ DKK. It is seen that the apriori information, $\beta^{apriori} = 2.93$, corresponds approximately to two sample records and that the value of $m = 13.65$ found by (3) seems to correspond to the minimum of C. Further, it is seen that the value of additional information which can be achieved by the measuring program is bounded by the reliability index corresponding to perfect information, $\beta^{perfect} = 4.47$ for $m \to \infty$.

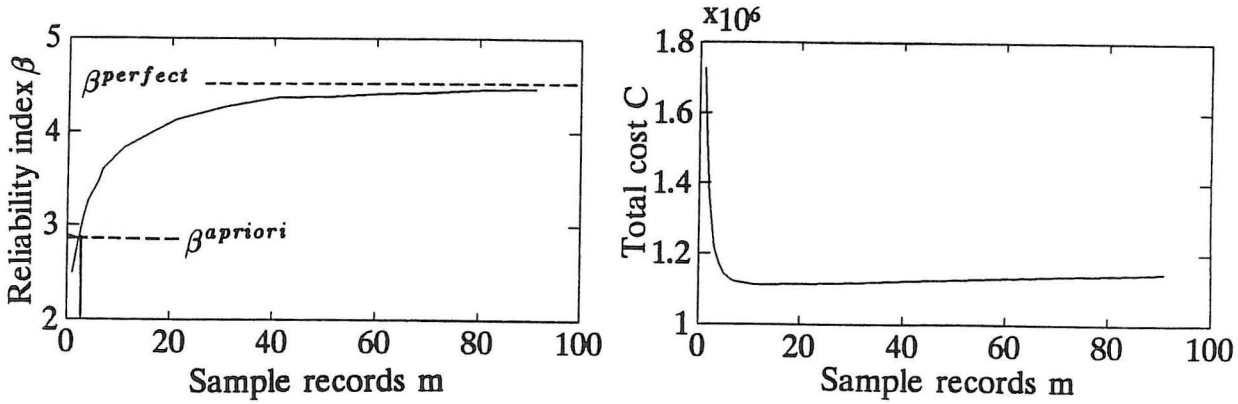

Fig. 5: Values of C and β to variations of m.

Now, the value of perfect information is defined as

$$(VI)_{perfect} = (\Phi(-\beta^{apriori}) - \Phi(-\beta^{perfect}))C_F \tag{20}$$

$(VI)_{perfect}$ is simply the difference between the total expected cost of failure corresponding to apriori information and perfect information, respectively. This value of information represents the maximum cost that may be allowed for acquisition of additional information. According to (20), the value of information from performing a measuring program is

$$VI = (\Phi(-\beta^{apriori}) - \Phi(-\beta))C_F \tag{21}$$

where the reliability index β corresponds to the optimal measuring program. If VI exceeds the cost of the measuring program C_M it should be performed.

Here, e.g. for $C_F = 10^8$ DKK., the value of information $VI = 165790$ DKK. which is less than the cost of the optimal measuring program $C_M = 1106825$ DKK., see Fig. 3. I.e. no measurings should be performed.

It is noticed that the expected cost of failure $C_F P_F$ corresponding to apriori information is 169500 DKK. I.e. by considering the reliability level of the structure it is also possible to get a conditional answer to the question, "Should additional information be obtained"?.

Additional information is also uninteresting if the number of sample records corresponding to apriori information exeeds the number of sample records m estimated from the optimization problem.

4. CONCLUSIONS

Design of an optimal measuring program is formulated as an optimization problem to minimize the total expected costs due to failure costs and costs of the measuring program. All the calculations are based on the apriori knowledge of the data properties and engineering judgement. Tentative results from an example with a simply supported plane vibrating beam indicate that the method works. However, to prove the pratical value of this approach, more complex examples should be investigated. Such examples are currently under the authors' consideration.

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