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*Publication date:*  
1995

*Document Version*  
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Kirkegaard, P. H., Andersen, P., & Brincker, R. (1995). *Identification of an Equivalent Linear Model for a Non-Linear Time-Variant RC-Structure*. Dept. of Building Technology and Structural Engineering, Aalborg University. Fracture and Dynamics Vol. R9533 No. 68

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**FRACTURE & DYNAMICS  
PAPER NO. 68**

**Aalborg Universitetsbibliotek**

530005197898



**Presented at the International Workshop on Structural Damage Assessment  
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# Identification of an Equivalent Linear Model for a Non-Linear Time-Variant RC-Structure

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## ABSTRACT

This paper considers estimation of the maximum softening for a RC-structure subjected to earthquake excitation. The so-called Maximum Softening damage indicator relates the global damage state of the RC-structure to the relative decrease of the fundamental eigenfrequency in an equivalent linear model with slowly varying stiffness properties, displaying the combined damage effects of the maximum displacement ductility of the structure during extreme plastic deformations and the stiffness deterioration in the elastic regime, also called final softening. In order to use the Maximum Softening damage indicator robust estimates of the fundamental eigenfrequency of the equivalent linear structure is required. Among many methods available for system identification application the Recursive Prediction Error Method (RPEM) and the Recursive Least Square (RLS) estimation using ARMAX models are investigated and compared with ARMAX models used on a running window. The techniques are evaluated using simulated data generated by the non-linear finite element program SARCOF modeling a 10-storey 3-bay concrete structure subjected to amplitude modulated Gaussian white noise filtered through a Kanai-Tajimi filter. The results show that an ARMAX model used on a running window seems to give the most robust estimate of the fundamental eigenfrequency.

## 1. INTRODUCTION

During severe dynamic excitation such as major earthquakes the response of reinforced concrete structures is influenced by non-elastic stress-strain behaviour (hysteresis). Any plastic cyclic deformation implies that the structure suffers local or global damages, ranging from harmless cracking of hitherto uncracked cross-sections to bond deterioration at the interface between reinforcement bars and concrete, crushing of the concrete in compression zones, rupture of reinforcement bars and stirrups etc. It has become common practice to instrument important buildings which may be exposed to excessive dynamic excitations in order to control the damage accumulation as measured by the sequential stiffness and strength deterioration. These damages are displayed in the dynamic response of the structure in terms of increased eigenperiods. The so-called Maximum Softening damage indicator is based on this principle, relating the global damage state of the structure to the relative decrease of the fundamental eigenfrequency, Di Pasquale et al. [1], Nielsen et al. [2] and Nielsen et al. [3]. The development of any reliability measure such as the Maximum Softening in future excitations is of paramount interest at the estimation of the residual reliability of the partial damaged structures. Further, prediction of damage is also interesting in order to decide whether or not it is necessary to repair or demolish

a structure. However, in order to use the Maximum Softening damage indicator robust estimates of the fundamental eigenfrequency of the equivalent linear structure are required. Among many techniques proposed for system identification application adaptive techniques such as the recursive prediction error methods seem to be available tools to identify time-variant civil engineering structures, see e.g. Pandit et al. [4], Kozin et al. [5] and Safak [6]. The aim of the present paper is to investigate different system identification techniques with respect to identify an equivalent linear model for a non-linear time-variant RC-structure. System identification techniques can fall into two categories depending on whether they operate on data in time domain, or on the Fourier transform of the data in the frequency domain, see e.g. Jung [7] and Söderström [8]. Frequency domain techniques have been the most popular, mainly due to their simplicity, see Davies et al. [9]. However, such system identification techniques involve averaging temporal information, thus discarding most of their details. For structural systems, whose parameters are expected to degrade with time, this tradeoff of temporal information for frequency information is not always justifiable. However, recently there have been made research concerning structural engineering applications of recursive time-domain techniques capable of tracking time variation of parameters. In this paper the Recursive Prediction Error Method (RPEM) and Recursive Least Square (RLS) estimation using ARMAX models are investigated and compared with ARMAX models used on a running window. The investigations will be based on times series simulated by a non-linear finite element program SARCOF, which has been verified to be able to predict accurately the response of deteriorating RC-structures with well-defined structural parameters (bending stiffness of cracked and uncracked of all beam-elements must be specified) when the structure is exposed to different levels of peak ground excitation. The program estimates the fundamental eigenfrequency of the equivalent linear structure at each time step. The example considered in this paper is a 10-storey 3-bay concrete test model of a RC-structure subjected to earthquakes at different levels of peak ground excitation given as amplitude modulated Gaussian white noise filtered through a Kanai-Tajimi filter.

## 2. SYSTEM IDENTIFICATION TECHNIQUES

In this section the different system identification techniques adopted in this paper for identification of an equivalent linear model for a deteriorating RC-Structures under stochastic dynamic loading is described.

### 2.1 Estimation of Dynamic Parameters using ARMAX Models

Linear dynamic systems are generally described by continuous time domain ordinary or partially differential equations. Modern recording systems however are all digital and give measurements in discrete form. One commonly used approach to convert equations from continuous time domain to discrete time domain is the covariance equivalent approximation.

This means if an ARMA( $2n, 2n-1$ ) model is used for a linear  $n$ -degree of freedom system it can be shown that the covariance of the response due to the ARMA model and that of the assumed white noise excited structure will be identical, see e.g. Kozin [5]. Given a measured response  $y(t)$  the ARMA( $n_a, n_c$ ) model is defined as

$$y(t) + a_1 y(t-1) + \dots + a_{n_a} y(t-n_a) = e(t) + c_1 e(t-1) + \dots + c_{n_c} e(t-n_c) \quad (1)$$

where  $y(t)$  is obtained by filtering the Gaussian white noise  $e(t)$  through the filter described by the Auto Regressive polynomial, consisting of  $n_a$  AR-parameters  $a$ , and the Moving Average polynomial, consisting of  $n_c$  MA-parameters  $c$ . Using (3) implies that a structure can be identified without knowing the excitation which can be assumed to be a stationary Gaussian white noise. However, if the excitation  $x(t)$  of a  $n$ -degree of freedom linear system is measured an ARMAX model can be used, see e.g. Safak [6]

$$\begin{aligned} y(t) &+ a_1 y(t-1) + \dots + a_{n_a} y(t-n_a) \\ &= b_1 x(t-1) + \dots + b_{n_b} x(t-n_b) + e(t) + c_1 e(t-1) + \dots + c_{n_c} e(t-n_c) \end{aligned} \quad (2)$$

By introducing the following polynomials in the backward shift operator  $q^{-1}$ , defined as  $q^{-j} y(t) = y(t-j)$

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \dots + a_{n_a} q^{-n_a} \\ B(q^{-1}) &= b_1 q^{-1} + \dots + b_{n_b} q^{-n_b} \\ C(q^{-1}) &= 1 + c_1 q^{-1} + \dots + c_{n_c} q^{-n_c} \end{aligned} \quad (3)$$

eq. (4) can be written in a more compact form as

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})} x(t) + \frac{C(q^{-1})}{A(q^{-1})} e(t) \quad (6)$$

The roots of  $A(q^{-1})$  are the poles of the model whereas the roots of  $B(q^{-1})$  are the zeroes. Assuming that the model is stable the poles are in complex conjugated pairs. The relationship between the poles  $p_j$  and the modal parameters is given by

$$p_j = e^{2\pi f_j T(-\zeta_j + i\sqrt{1-\zeta_j^2})} \quad j = 1 \dots n_a \quad (7)$$

where  $f_j$  and  $\zeta_j$  are the natural eigenfrequency and damping ratio of the  $j$ th mode.  $T$  is the sampling period. It is seen that each complex conjugated pair of poles corresponds to a simple-damped oscillator, see Safak [6]

$$f_j = \frac{|\lambda_j|}{2\pi T}, \quad \zeta_j = \frac{-\text{Re}(\lambda_j)}{|\lambda_j|} \quad (8)$$

Setting  $\lambda_j = \ln(p_j)$  the modal parameters are obtained from the following equations where  $||$  denotes the modulus and  $\text{Re}$  the real part of the complex number  $\lambda_j$ .

The parameters  $\theta$  of the ARMAX model are estimated by minimizing a quadratic error function,



$V(\theta)$ , defined as

$$V(\theta) = \frac{1}{2} \frac{1}{N - t_s + 1} \sum_{t=t_s}^N \varepsilon(t, \theta)^2 = \frac{1}{2} \frac{1}{N - t_s + 1} \sum_{t=t_s}^N (y(t) - \hat{y}(t, \theta))^2 \quad (9)$$

$\hat{y}(t, \theta)$  is the predicted response and  $t_s = \max(n_a, n_c) + 1$ . To calculate the parameter estimates a numerical minimization method must be chosen, see e.g. Ljung [7].

In order to deal with the order of the ARMAX model Akaike, see Akaike [10], suggested an Information Theoretic Criterion (AIC) of the type

$$AIC = \log\left[\left(1 + \frac{2n}{N}\right) \cdot V(\theta)\right] \quad (10)$$

where  $N$  is the number of data in the time series,  $n$  is the number of parameters. The *AIC* criterion penalizes using too high model orders, i.e. their value may increase with increasing model order. The model structure giving the smallest value of the criterion is selected. An important characteristic of ARMAX-models is that it is possible to get unbiased estimates of the AR-parameters, see e.g. Pandit [4], where estimates of the variances of the estimated parameters can be estimated by the Cramer-Rao lower bound, see e.g. Kirkegaard [11]. Model validation is the final stage of the system identification procedure. In fact model validation overlaps with model structure selection. Since the system identification is an iterative process various stages will not be separated: models are estimated and the validation results will lead to new models etc. Model selection involves the selection of the form and the order of the ARMAX model, and constitutes the most important part of the system identification. Model validation is to confirm that the model estimated is a realistic approximation of the actual system. A throughout description of the problem of model selection and validation is given in Ljung [7]. One of the dilemmas in the model validation is that there are many different ways to determine and compare the quality of the estimated models. First of all, the subjective judgement in the model validation should be stressed. It is the user that makes the decision based on numerical indicators. The variance of the parameter estimates can be such an indicator. It is also important to check whether the model is a good fit for the data recording to which it was estimated. Simulation of the system with the actual input and comparing the measured output with the simulated model output can also be used for model validation. One can also compare the estimated transfer function with one estimated by FFT. Statistical tests of the prediction errors are also typically used numerical indicators in model validation.

## 2.2 ARMAX Model used on a Running Window

In order to identify time-varying systems an ARMAX model is estimated on a running window which size is chosen as  $2.4 T_0$ . The parameter estimates are located at the centre of the window. An ARMA model used on a running window in stead of using the ARMAX model can also be used. This means that it is assumed the measured excitation  $x(t)$  is a white noise sequence similar to  $e(t)$ . This implies that the parameters  $b_i$  and  $c_i$  are combined into a single parameter.

### 2.3 Recursive ARMAX Model

In stead of using ARMAX and ARMA models on a running window (off-line) a recursive form (on-line) can be used where a model is adjusted at each time step. Such an approach has two main advantages: 1) it requires much less memory in the computer since the calculations are done sequentially using only the latest segment of data, and 2) it can detect time varying characteristics at each time step, see Ljung [7]

The principle of recursive identification is given by

$$\theta(t) = f(\theta(t-1), x(t), y(t)) \quad (11)$$

where the parameter estimates at time step  $t$  are given as a function  $f(\cdot)$  of the parameter estimates at time step  $(t-1)$ , the excitation  $x(t)$  and response  $y(t)$  at time step  $t$ , respectively.

The parameters of the ARMAX model for a given time series pair  $y(t)$  and  $x(t)$  are estimated on-line by using stochastic approximation, Ljung [7]. This implies that a method for parameter estimation known as the Recursive Prediction Error Method (RPEM) can be obtained. The RPEM algorithm is given as, Ljung [7]

$$\begin{aligned} \theta(t) &= \theta(t-1) + R^{-1}(t) \psi(t) \epsilon(t, \theta(t-1)) \\ R(t) &= \lambda R(t-1) + \psi(t) \psi^T(t) \end{aligned} \quad (12)$$

where  $R(t)$  is the Hessian matrix given as the second derivative of  $V(\theta)$  with respect to the parameter vector

$$\theta = (a_1, \dots, a_{n_a}, b_1, \dots, b_{n_b}, c_1, \dots, c_{n_c}). \quad (13)$$

$\psi(t)$  is the gradient of  $y(t)$  with respect to  $\theta$  and  $\lambda$  is a forgetting factor normally. The intermediate steps of the derivation of (12), as well as the equation for calculating  $\psi(t)$  is given in Ljung [7]. In order to start the recursion initial values of the parameters need to be specified. It can be shown that for stable systems the effect of initial values diminishes very rapidly with time, thus they can be assumed zero. The RPEM represents a general family of recursive system identification methods, Ljung [7]. There are several other methods, such as e.g. the recursive pseudolinear regression, maximum likelihood estimation, and the recursive least square method, that all can be considered as the special forms of the RPEM method for particular forms of the ARMAX model, Ljung [7]. One particular model structure, which will be shown to be appropriate for ground motions, is the ARX model, also known as the equation error method. The ARX model corresponds to  $C(q^{-1}) = 1$  in equation (14). That is

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})} x(t) + \frac{1}{A(q^{-1})} e(t) \quad (14)$$

This implies that the parameter vector becomes

$$\theta = (a_1, \dots, a_{n_a}, b_1, \dots, b_{n_b}). \quad (15)$$

and

$$\varphi(t) = [-y(t-1), \dots, -y(t-n_a), x(t-1), \dots, x(t-n_b)]^T \quad (16)$$

which implies that for an ARX model it can be shown that the best estimate of the output at time  $t$  is

$$y(t) = \theta^T \varphi(t) \quad (17)$$

Equation (17) describes a linear regression equation where  $\varphi(t)$  is the regression vector. Since the ARX model is a special form of the ARMAX model, the general equations (12) is also applicable to ARX models where  $\varphi(t)$  is used in stead of  $\psi(t)$  in (12). This special form of the RPEM method is also denoted recursive least square (RLS).

### 3. EXAMPLE: IDENTIFICATION OF AN EQUIVALENT LINEAR MODEL

In this example the system identification techniques described in section 2 will be investigated in a simulation study. The investigations will be based on times series simulated by a non-linear finite element program SARCOF, Mørk [12], which has been verified to be able to predict accurately the response of deteriorating RC-structures with well-defined structural parameters (bending stiffnesses of cracked and uncracked of all beam-elements must be specified). The program estimates the fundamental eigenfrequency of the equivalent linear structure at each time step. The computer program SARCOF is a non-linear finite element program which is able to handle severe inherent material non-linearities and it is able to handle the following items:

- Unsymmetric cross-sections with different yield capacities at positive and negative bending.
- Interaction of bending moments and axial forces.
- Stiffness and strength degradation during plastic deformation.
- Pinching effect of moment-curvature relation due to shear loading.
- Finite extensions of plastic zones at the end of the beams.



The program is based on a full non-linear description of the internal degrees of freedom, which controls the hysteresis. In order to save computer time, the external degrees of freedom, i.e. the global displacements are described by truncated expansion in the eigenmodes of the undamaged structure, see Mørk [12].

### 3.1 Test Structure

The computer model, see figure 4, models a test structure, see Cecen [13], which consists of two 10-storey, 3-bay frames working in parallel with storey weights, uniformly distributed, attached in between.

The total height of the structure is 2.29 m and all storey heights are uniformly distributed. The columns and beams in the structure are 0.038 m wide, 0.051 m and 0.038 m deep for the columns and beams, respectively. Furthermore, all columns and beams are symmetrically reinforced. The following values are used for the density  $\rho = 2500 \text{ kg/m}^3$ , the stiffness  $E = 2.0 \cdot 10^{10} \text{ N/m}^2$  and the damping ratio  $\zeta = 0.035$ . The stiffness and strength deterioration are modelled using a Clough-Johnston hysteretic model. In this deteriorating model the limit value  $e_0$  is taken as 26 and the decay parameter  $e_1$  is taken as 12. The first eigenfrequency of the structure is 3 Hz. The test model is shown in figure 1. The  $M_i$ 's refer to different material properties, see Cecen [13].

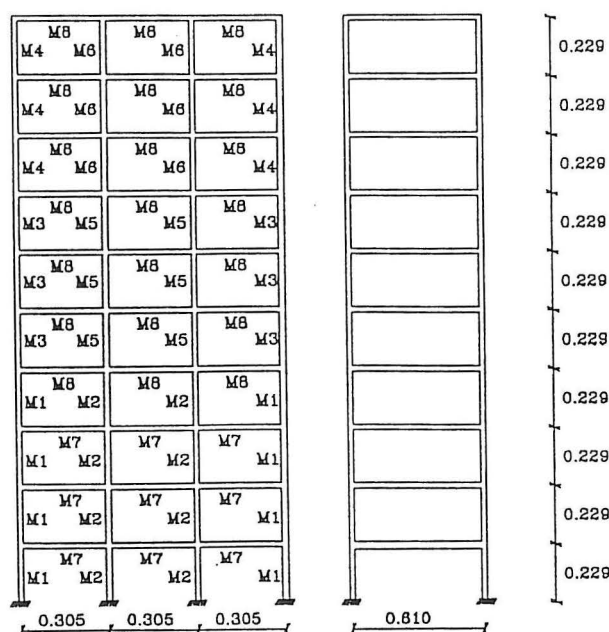


Figure 1: Computer model for the 10-storey 3-bay Reinforced Concrete Frame.

The excitation applied to the test structure was simulated earthquake runs for the basement motions that were patterned after the North-South component of the acceleration history measured at El Centro during the Imperial Valley Earthquake of 1940. The acceleration process at the ground surface is determined as the response process of an intensity modulated Gaussian white noise filtered through a Kanai-Tajimi filter. The excitation process  $F(t)$  is then obtained from the stochastic differential equations, see Tajimi [14]

$$F(t) = 2\zeta_s \omega_s \dot{U}(t) + \omega_s^2 U(t) \quad (18)$$

$$\ddot{U}(t) + 2\zeta_s \omega_s \dot{U}(t) + \omega_s^2 U(t) = -\beta(t)W(t) \quad (19)$$

$F(t)$  can then be interpreted as the negative part of the ground surface acceleration, and  $\zeta_s$  and  $\omega_s$  are the damping ratio and the circular eigenfrequency of a single degree-of-freedom shear model of the underlying subsoil.  $W(t)$  is a unit intensity white noise with the auto-spectral density function  $0.5\pi$  generated by the broken line process model of Ruiz and Penzien, see Clough et al. [15]. The deterministic modulation function used is given by Jennings et al [16]

$$\beta(t) = \beta_0 \begin{cases} \left(\frac{t}{t_1}\right)^2, & 0 \leq t \leq t_1 \\ 1, & t_1 < t < t_0 + t_1 \\ \exp(-c(t - t_0 - t_1)), & t_0 + t_1 \leq t \end{cases} \quad (20)$$

The following parameters are used for the Kanai-Tajimi filter and the modulation function. The damping ratio in the Kanai-Tajimi filter is chosen as 0.65 and the circular frequency is chosen as  $19.8.s^{-1}$ . In the modulation function the decay parameter  $c$  is 0.2. The excitation has maximum acceleration at  $t_1 = 3$  sec. and duration of the strong motion is  $t_0 = 3$  sec.

The integrated dynamic system is in SARCOF solved by a 4th Runge-Kutta Scheme. The time step is selected as 0.004 sec., where it has been proven that no drift occurs in the simulated signal.

### 3.2 Maximum Softening

The changes in stiffness are normally very high and the stiffness changes are very fluctuating during an earthquake. Figures 2-3 present an example of how the 1st. eigenperiod  $T(t)$  of the equivalent linear structure and therefore also the stiffness is slowly varying of the 10 storey, 3 bay reinforced concrete structure. The reason for this fluctuating behaviour is simply due to the fact that the structure changes rapidly from being in the elastic to the plastic regime. As illustrated in figures 2-3 a measured change in the eigenperiod is of course not suitable for prediction of the

damage at a given time  $t_1$  during the quake. It is therefore necessary to perform a smoothing of the measured eigenperiod which corresponds to time-averaging the structural degradation. A time-averaging method of the instantaneous period has been proposed by Rodriquez-Gomez [17] and is based on the principle of a moving averaging window in the following way. The smoothed value  $\langle T(t_1) \rangle$  at the time  $t_1$  is evaluated as

$$\langle T(t_1) \rangle = \frac{1}{T_a} \int_{t_1 - \frac{T_a}{2}}^{t_1 + \frac{T_a}{2}} T(t) dt \quad (21)$$

where  $T_a$  is the length of the averaging window, which should be sufficiently large, so that the local peaks are removed. On the other hand,  $T_a$  should not be selected so large that intervals of increased plastic deformation are not displayed in  $\langle T(t_1) \rangle$ . The value  $T_a = 2.4 T_0$  is recommended as a reasonable compromise, Rodriquez-Gomez [17], where  $T_0$  is the 1st eigenperiod of the equivalent linear structure.

Based on  $\langle T(t_1) \rangle$ , the instantaneous softening,  $\delta(t)$ , of a structure is defined as Çakmak et al.[1]

$$\delta(t) = 1 - \frac{T_0}{\langle T(t) \rangle} \quad (22)$$

The maximum softening damage indicator,  $\delta_m$ , is the maximum of  $\delta(t)$  during the seismic excitation. Obviously, the damage indicator  $\delta(t)$  is non-decreasing with time and attains values in the range  $[0;1]$ , where  $\delta(t) = 0$  corresponds to an undamaged structure.

In figure 2 and 3, respectively the instantaneous damage  $\delta(t)$  is shown as a function of time for two earthquakes at level 0.3 g and 1.0 g. The damage is estimated by using different lengths of the running window.



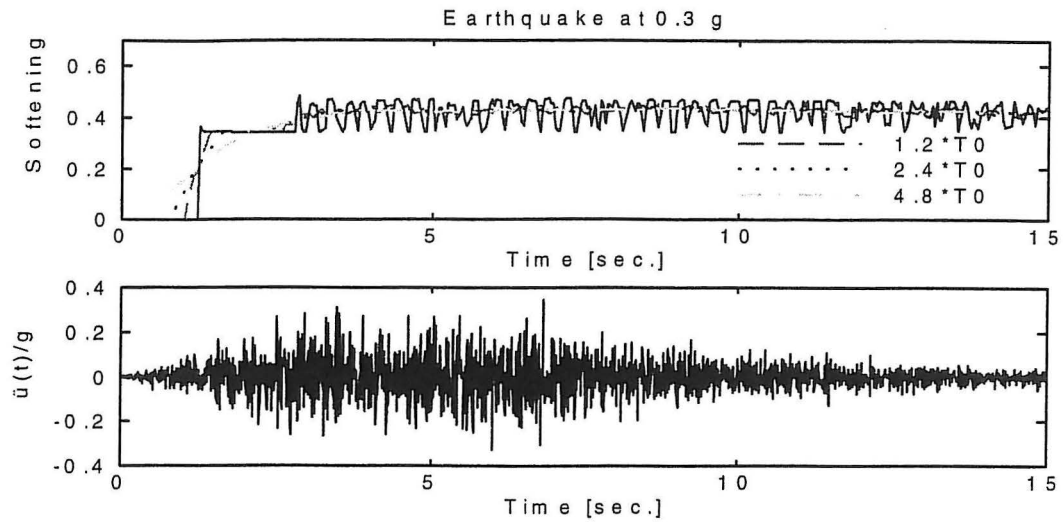


Figure 2: The damage indicator  $\delta(t)$  as a function of time for different choices of the length of the running window. Earthquake = 0.3 g.

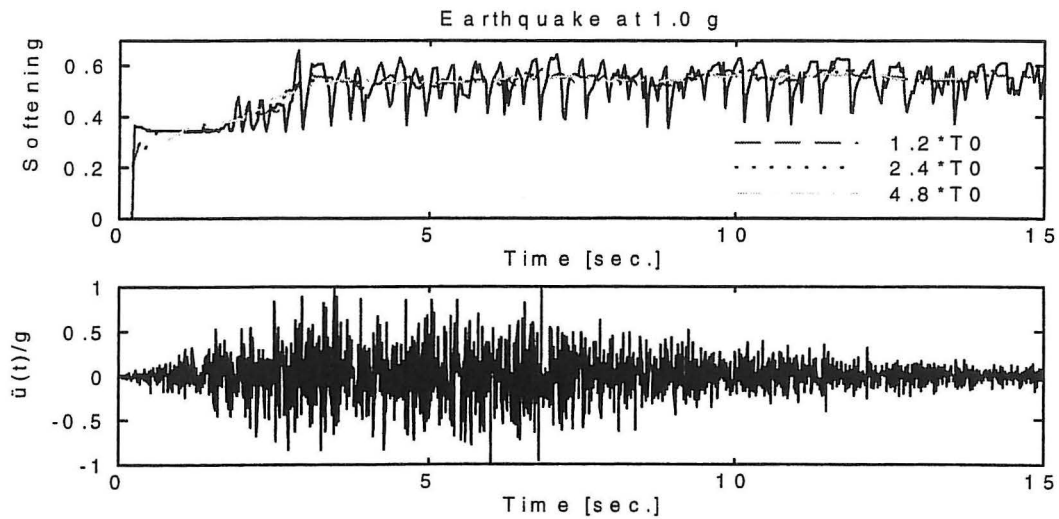


Figure 3: The damage indicator  $\delta(t)$  as a function of time for different choices of the length of the running window. Earthquake = 1.0 g.

From figure 2 and 3, respectively it is seen that the choice of the length of the running window proposed in Rodriques et al. [17] at  $2.4 T_0$  seems to be a reasonable compromise. This means that in the following that size of window is used.

### 3.2 Results

In this section the earthquake responses considered in section 3.1 are analysed using following 4 system identification approaches:

- ARMAX + running window
- ARMA + running window
- RPEM (RARMAX)
- RLS (RARX)

The second approach where an ARMA model instead of an ARMAX model is used assumes that the measured excitation  $x(t)$  is an white noise sequence similar to  $e(t)$ . This implies that the parameters  $b_i$  and  $c_i$  are combined into a single parameter. As described in section 2.1 the first step in applying the system identification techniques outlined above to a practical problem is to select and validate the model for the system. Here, the model validation and selection part of the identification process will not be described in details. A computer program based on MATLAB [18] was developed to carry out the calculations.

Figures 4 and 5 show the time variation of the softening estimated by the four system identification techniques for an earthquake at 0.3 g and 1.0 g, respectively. The ARMAX and ARMA models were used on a window with a length at  $2.4 T_0$ . The forgetting factor  $\lambda$  used in the RPEM and RLS algorithm was chosen as  $\lambda = 0.99$ . It was found that the estimated results by the RPEM and RLS seem to be very sensitive to the choice of the forgetting factor and the initial parameter estimates. The results shown in the figures 4 and 5 indicates that the ARMAX model used on a running window and the RARX model give the robust estimates of the fundamental frequency and with that the softening. Although encouraging results have been obtained in other fields with the RPEM, see e.g. Ljung [7], it seems that it cannot be recommended for earthquake-engineering applications.

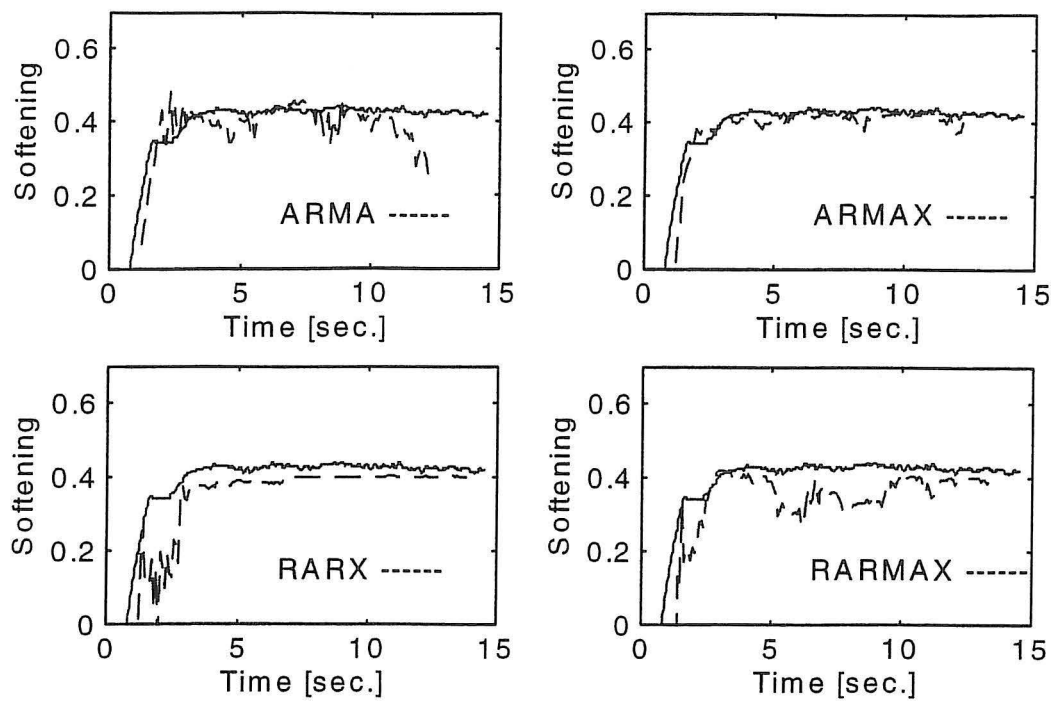


Figure 4: The estimated softening as a function of time. Earthquake = 0.3 g.

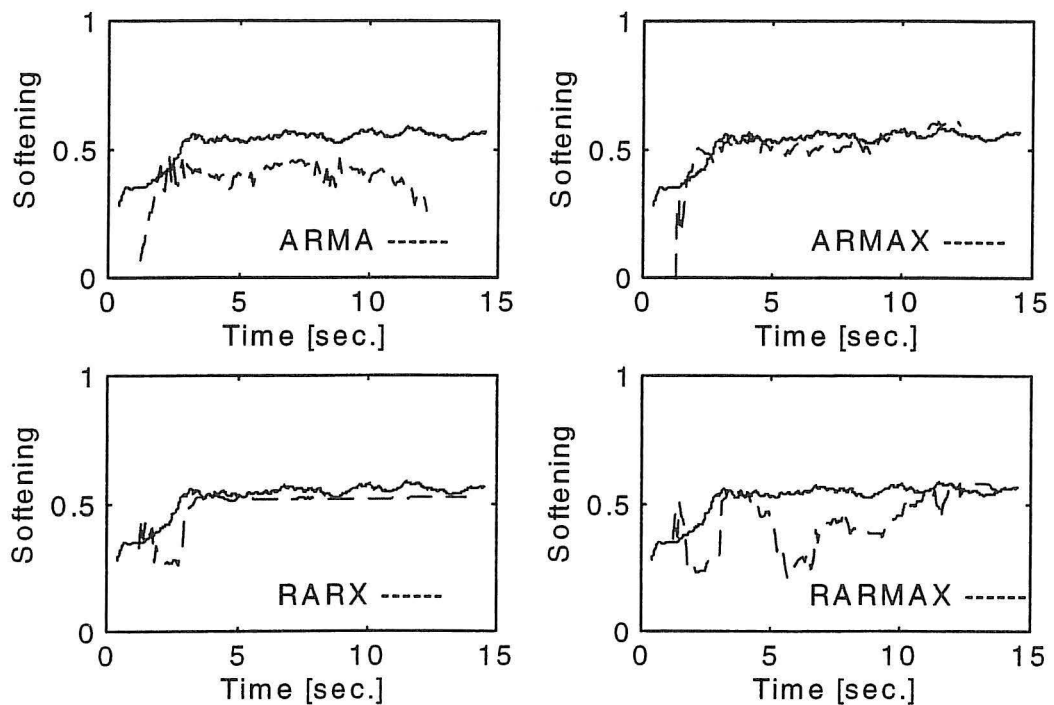


Figure 5: The estimated softening as a function of time. Earthquake = 1.0 g.



## 4. CONCLUSIONS

The results presented in this paper show:

- how an equivalent linear model for a time-variant RC-structure can be estimated using the Recursive Prediction Error Method (RPEM) and the Recursive Least Square (RLS) and ARMAX models used on a running window.
- that an the RLS and the ARMAX model used on a running window seems to give the most robust estimate of 1st eigenfrequency

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