Appendix B

Model for Single Crack Extension in Lightly Reinforced Beams

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APPENDIX B:

MODEL FOR SINGLE CRACK EXTENSION IN LIGHTLY REINFORCED BEAMS

Prepared by F.A. Christensen and R. Brincker

Abstract

In this appendix the failure behaviour of lightly reinforced concrete beams is investigated. A numerical model based on the fictitious crack approach according to Hillerborg [1] is established in order to estimate the load-deflection curve for lightly reinforced concrete beams. The debonding between concrete and reinforcement is taken into account by introducing a debonded zone with constant shear friction stress. Results are presented for material models representing normal strength concrete (two degrees of brittleness) and high strength concrete. The properties of the model are investigated and the results of the model are compared with results from experiments.

Introduction

The main purpose of this work is to formulate a model well suited for estimation of accurate load-deflection curves for lightly reinforced concrete beams. The aim is to be able to investigate which parameters are important for the stability of the tensile failure in load control. If the yield load F_y of the beam is larger than the local peak load F_t corresponding to tensile failure of the concrete, then the tensile failure is normally accepted to be considered stable. Thus, the determination of the local peak load corresponding to concrete tensile failure is essential in the analysis. It is well known, that those kinds of failure problems are governed by fracture mechanical effects, and thus, a fracture mechanical approach must be used in order to establish accurate estimates of the ratio F_t/F_y for different sizes of the structure.

Bosco and Carpinteri [2] formulated a model based on linear elastic fracture mechanics. They showed that, according to this model, the failure mode changes when the beam depth is varied, the reinforcement ratio remaining the same. Only when the reinforcement ratio is inversely proportional to the square root of the beam depth, the mechanical behaviour is reproduced by the model. Baluch et al. [3] have shown how strain softening of the concrete can be taken into account in the model proposed by Bosco and Carpinteri. Due to the limitations of linear elastic fracture mechanics (does not describe the initial stages of cracking in a reinforced structure, and does not describe the weaker size effects introduced by non-linear fracture mechanical models), several other researchers have used an approach where the main crack at the midpoint of the beam is modeled as a fictitious crack [4-6].

In this appendix, a non-linear fracture mechanical approach is adopted for the concrete tensile failure using the fictitious crack model introduced by Hillerborg [1]. The softening relations are assumed to be linear or bi-linear, and different softening parameters are used to represent normal

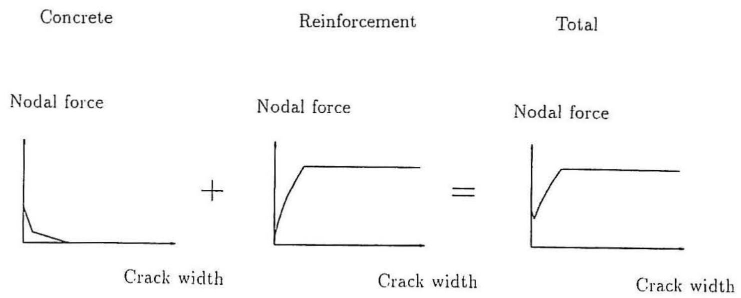

Figure B1. The relation between the nodal force and the crack width for the composite node representing both concrete and the force from the reinforcement.

strength concrete and high strength concrete. When a crack in a reinforced concrete beam starts opening, the reinforcement starts getting loaded taking over the stresses relieved by the tensile failure. For this reason, the modeling of the debonding process between concrete and reinforcement is of great importance. In this work, a simple constant shear friction model is used for the debonding stress.

Model Formulation

The model used in this investigation is based on the sub-structure method introduced by Petersson [7], and reformulated for a beam in three-point bending by Brincker and Dahl [8] in a way that makes it possible to obtain the entire load-deflection curve. Only the basic ideas of the model will be presented here. A beam subjected to three-point bending is considered. It is assumed that a crack starts to extend at the midpoint of the beam and a fracture zone develops in front of the crack tip. According to the fictitious crack model a point on the crack extension path can be in one of three possible states: 1) elastic state 2) fracture state (material is softened by microcracking) and 3) a state of no stress transmission (crack fully developed).

The method is implemented by dividing the midpoint section of the beam into a number of nodes. For each node a relation between the nodal force and the crack width is used as input to a numerical solving scheme. In order to model a reinforced concrete beam, the node located at the place of the reinforcement has to represent both the reinforcement and the concrete. Since the bond-slip between the reinforcing bars and the concrete substantially influences the response of the beam, it is necessary to take this effect into account. This is done by assuming a constant shear stress at the debonded interface between the rebars and the concrete. To simplify the problem the force of the reinforcement bar is assumed to act directly on the faces of the concrete tensile crack. The relation between nodal force and the crack width is found by superposition of the contributions from the concrete and from the reinforcement as shown in Figure B1.

Using a model like this, the complete relation between the load and the deflection at the midpoint of the beam can be estimated. An example of a load-deflection curve is shown in Figure B2 where a bi-linear softening relation has been used to represent the concrete tension failure. Stress distri-

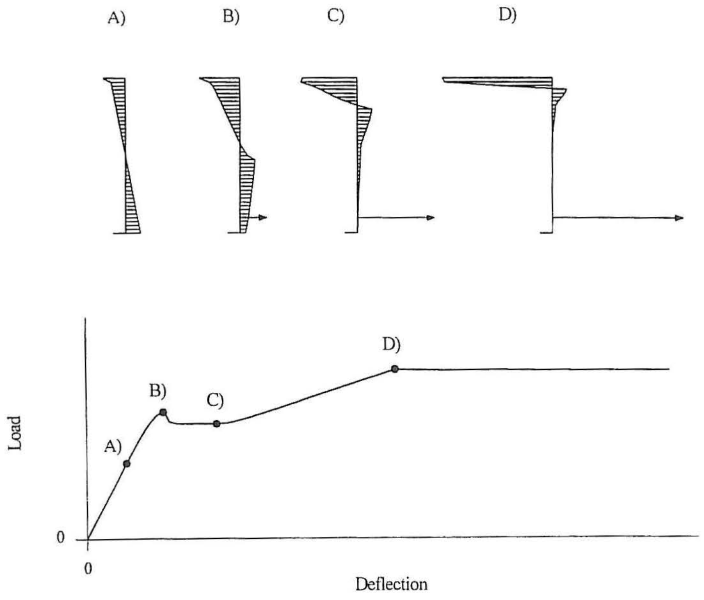

Figure B2. Bottom: The load-deflection curve divided into different parts according to the stress distribution. Top: Stress distributions, A) The tensile stress is reached at the bottom of the beam, B) peak load at first crack, C) The critical crack width is reached at the bottom of the beam, D) The yield stress is reached in the reinforcement.

butions corresponding to certain points on the load-deflection curve are also shown in Figure B2. Until point A) the beam behaviour is purely elastic and point A) represents the point where the tensile stress is reached at the tensile side of the beam. The crack starts to extend, and while the crack width is small, a zone of approximately constant failure stress will be present in the tensile side of the beam. This plastic-like stress distribution causes the load to increase until the peak load at first cracking is reached, point B). At point C) the critical crack width is reached at the bottom of the beam. When the reinforcement contribution becomes higher as the crack starts to open, the load starts to increase in the load-deflection diagram. Hereafter the load increases rapidly until the yield stress is reached in the reinforcement, point D).

The drop in load between the peak load at first cracking and the yield plateau, is characteristic for the failure response of lightly reinforced beams. At any stage of the failure process, the load carrying capacity of a lightly reinforced concrete beam can be divided into contributions from the concrete and the reinforcement respectively. For the model, these two contributions together with the total load capacity are shown in Figure B3. At small deflections the load is carried by the concrete, while at larger deflections (when the concrete is cracked) the load is carried by the reinforcement.

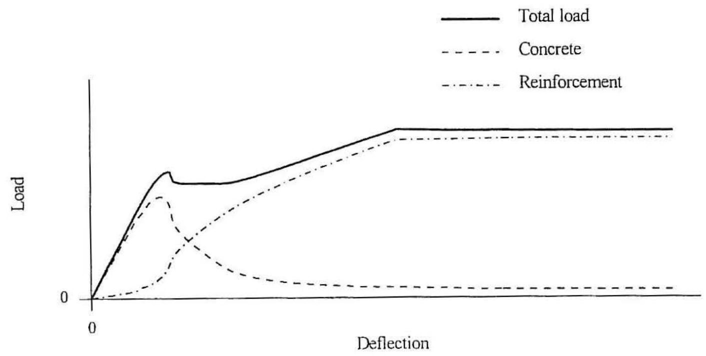

Figure B3. The load deflection curve divided into contributions from reinforcement and concrete, respectively.

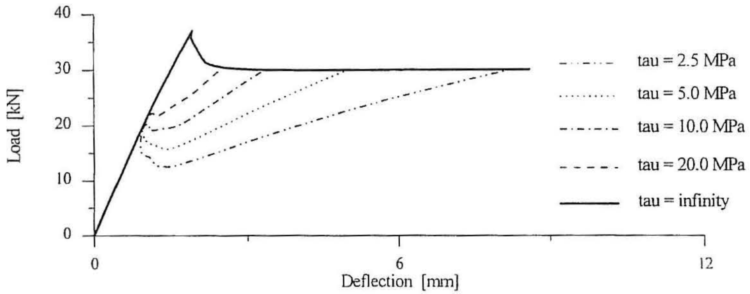

Figure B4. Load-deflection curves for the 200×400×4800 mm beam of normal strength concrete. The reinforcement ratio is 0.25%. The friction stresses τ are varied from 2.5 MPa to infinity.

For a given softening relation for the concrete, the pull-out of the reinforcement (the debonding between concrete and reinforcement) strongly controls the failure behaviour of the beam after the peak load. In the model a constant shear friction stress τ_f is assumed to act on the debonded interface between concrete and reinforcement. In Figure B4, load-deflection curves are shown for a $200 \times 400 \times 2400$ mm beam where the friction stresses are varied from 2.5 MPa to infinity. As it appears, the value of the friction stress significantly influences the behaviour of the beam between the peak load at first cracking and the yield plateau. In the literature values of the friction stress are typically reported in the range 3-8 MPa. for instance, Planas [6], found about 5 MPa for ribbed reinforcement. In this interval the peak load is not influenced much, whereas the drop in load after the peak is more sensitive to changes in the value of τ_f. Note that the limit $\tau_f \to \infty$ defines a "master curve" in the sense that all failure responses for finite values of τ_f might be considered as deviations from this curve around the peak point of the master curve.

Table B1: Softening properties of normal strength and high strength concrete

Material property	Normal strength concrete (bi-linear softening)	Normal strength concrete (linear softening)	High strength concrete
Tensile strength f_t $[MPa]$	3.0	3.0	5.0
Youngs modulus E $[MPa]$	40,000	40,000	40,000
Fracture energy G_F $[N/mm]$	0.120	0.240	0.120

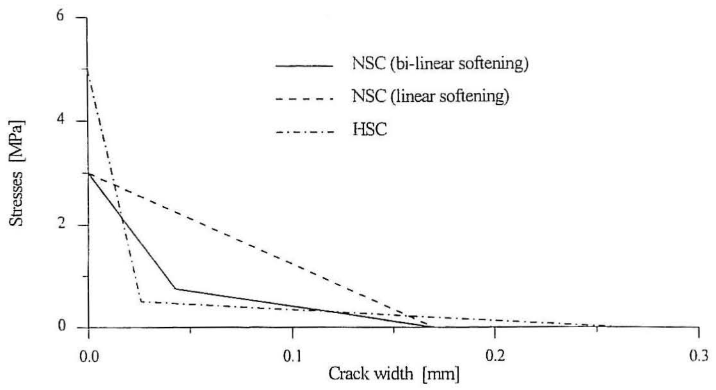

Figure B5. Softening relations for the three different materials used in the modelling.

Properties of the Model

In the following section some typical results from the model are given for a normal strength concrete with a linear and a bi-linear softening relation and a high strength concrete with a bi-linear softening relation. Material parameters representing these materials are given in Table 1 and the softening relations are shown in Figure B5. The softening parameters for the bi-linear softening relations are representative for the two types of concrete used in the experimental investigation (Appendix A). Failure responses are shown for four beam depths of 100, 200, 400 and 800 mm. The b/h-ratio is equal to 0.5 and the l/h-ratio is equal to 12. For all the simulations the debonding friction stress is 5.0 MPa and the yield stress of the reinforcement is 500 MPa.

In the two top plots in Figure B6 load-deflection curves are shown for different beam sizes of normal strength concrete with a bi-linear softening relation and a linear softening, respectively. In both cases the reinforcement ratio is equal to 0.25%. Note the strong size effect on the results and the more ductile behaviour of the beam with the linear softening relation (doubled fracture energy). The bottom plot in Figure B6 shows similar results for high strength concrete. Note the more brittle behaviour of these beams. For all three cases, keeping the reinforcement ratio constant, the ratio F_t/F_y is increasing when the beam size is decreased. This indicates, that in order to reproduce the same F_t/F_y ratio when the beam depth is increased, the reinforcement ratio must be decreased. More results from this kind of investigation are shown in the main part

of the chapter.

By comparing the load-deflection curves for the normal strength concrete with bi-linear and linear softening (doubling the value of G_F) it can be observed that the reinforcement ratio should be increased when the fracture energy is increased to maintain the same F_t/F_y ratio. Further, when the tensile strength is increased the reinforcement ratio should also be increased to maintain the F_t/F_y ratio.

Some of these effects are taken into account by the brittleness number for reinforced concrete proposed by Bosco and Carpinteri [2]

$$N_P = \frac{f_y\sqrt{h}}{K_{Ic}}\varphi \qquad (1)$$

where K_{Ic} is the fracture toughness of the concrete. It is assumed that the fracture toughness can be approximated by $K_{Ic} = \sqrt{EG_F}$. Varying the reinforcement ratio, the failure response can now be estimated for different values of the brittleness number N_P.

In Figure B7 the value of N_P is varied ranging from 0.15 to 0.30 for the three materials (for the 200 $\times$ 400 $\times$ 2400 mm beam). The results show that the peak load at the first cracking F_t is equal to the yield load F_y for values of the brittleness number N_P at about 0.20 for both types of normal strength concrete. However, for the high strength concrete, the corresponding value is about 0.30. This indicates, that the brittleness number does not account for an isolated increase of the strength. This is to be expected since the brittleness number is a linear fracture mechanical parameter that does not include any description of the shape of the softening curve.

Further, It should be noticed that, although the value of the brittleness number is kept constant, the load-deflection curves for the high strength concrete beams show a much more brittle behaviour than the load-deflection curves for the normal strength beams, bottom plot, Figure B7. Again, this is due to the fact that the brittleness number does not include description of the shape of the softening relation.

Figure B8 shows how the failure response changes when the brittleness number is kept constant and the beam depth is varied. For all the three different materials it can be stated that the failure behaviour changes significantly with the beam depth. These conclusions are believed to be generally valid for the case of no initial crack. Using a non-linear approach like the fictitious crack model, the case of no initial crack can be analysed as it has been shown here. However, using linear fracture mechanics, an initial crack must be present, and thus, the basic assumptions for using the brittleness number are in principle not satisfied. A similar investigation can be carried out assuming the presence of an initial crack, and for this case, the ratio F_t/F_y becomes more stable in the case of constant brittleness number. The main results of this investigation are shown in the main part of the chapter.

Results shown in Figures B6-B8 are normalized by the yield force on the load scale and by the deformation corresponding to point A) in Figure B2 on the deformation scale.

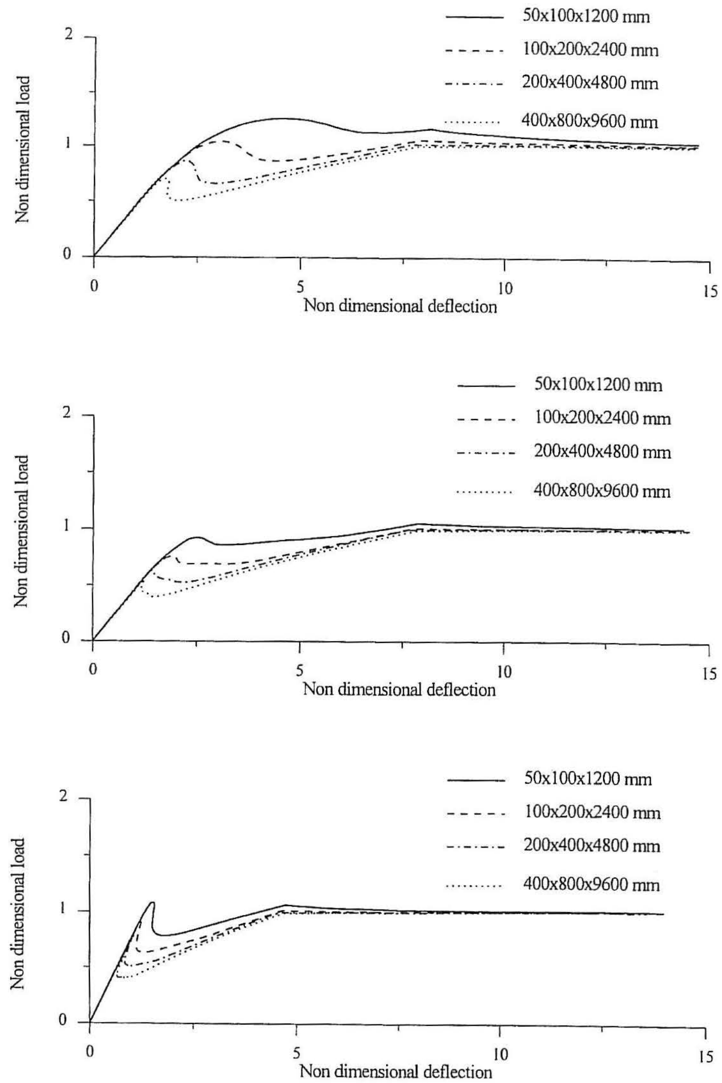

Figure B6. Size effects on the failure response for constant reinforcement ratio $\varphi = 0.25\%$. Top: Normal strength concrete with linear softening. Middle: Normal strength concrete with bi-linear softening. Bottom: High strength concrete with bi-linear softening.

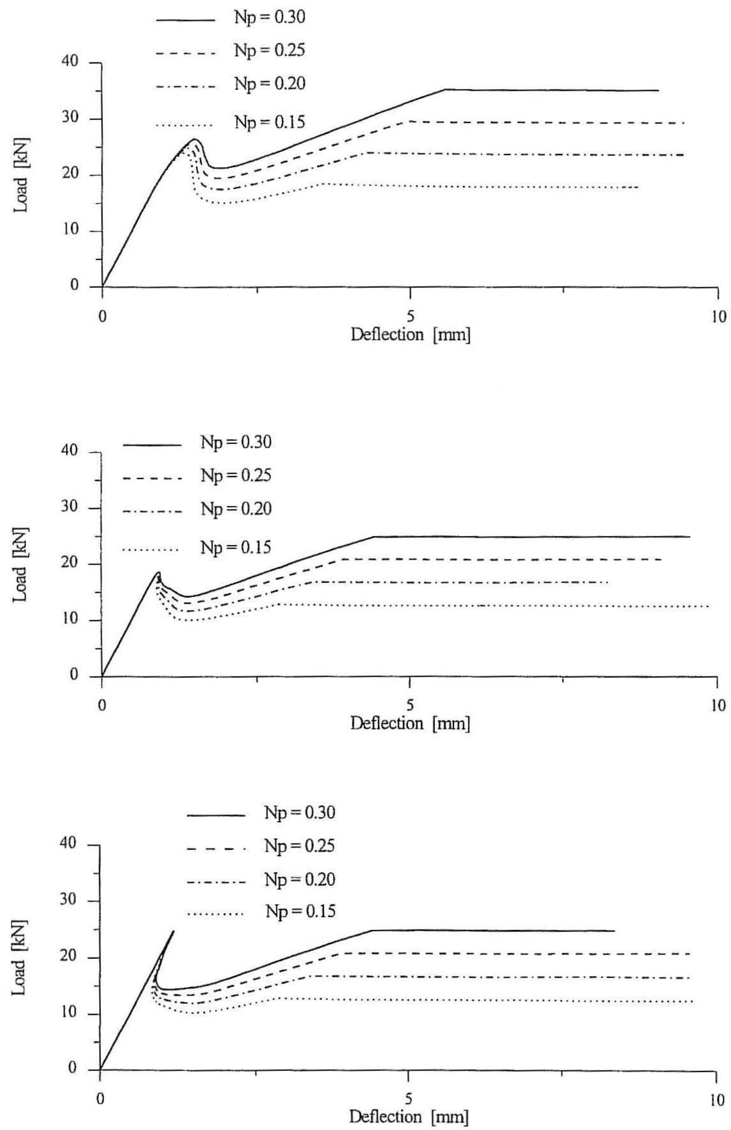

Figure B7. Variation of the failure response with brittleness number N_P for the $200 \times 400 \times 4800$ *mm* beam. Top: Normal strength concrete with linear softening. Middle: Normal strength concrete with bi-linear softening. Bottom: High strength concrete with bi-linear softening.

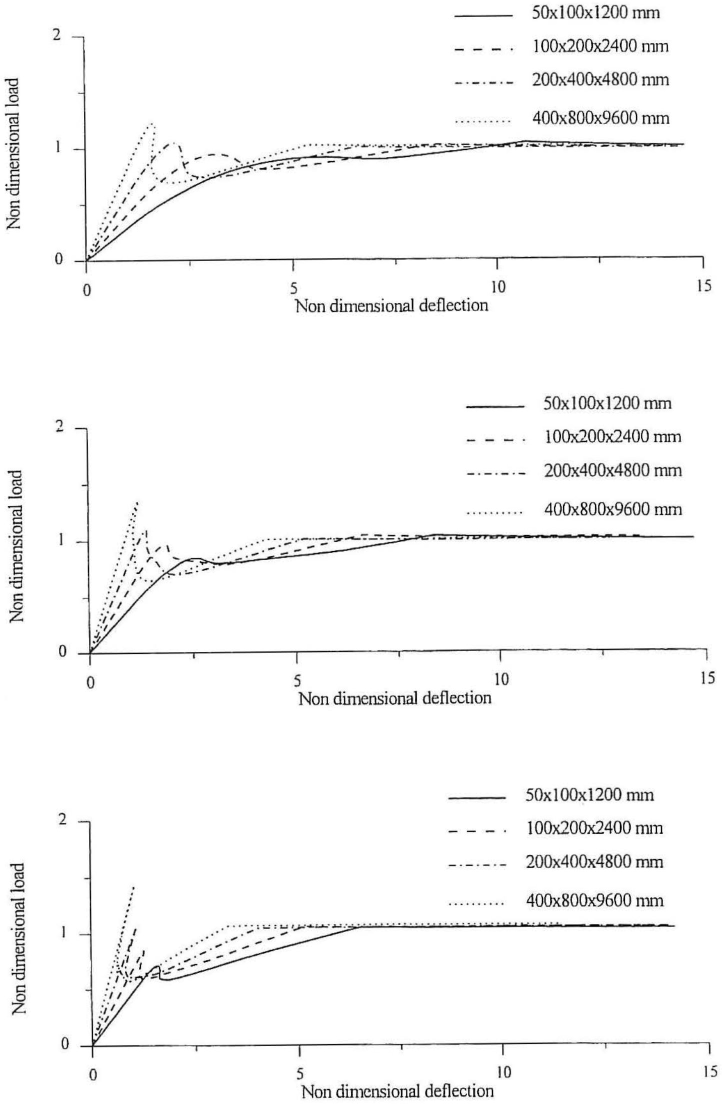

Figure B8. Size effects on the failure response for constant Brittleness number N_P. Top: $N_P = 0.20$ and normal strength concrete with linear softening. Middle: $N_P = 0.20$ and normal strength concrete with bi-linear softening. Bottom: $N_P = 0.30$ and high strength concrete with bi-linear softening.

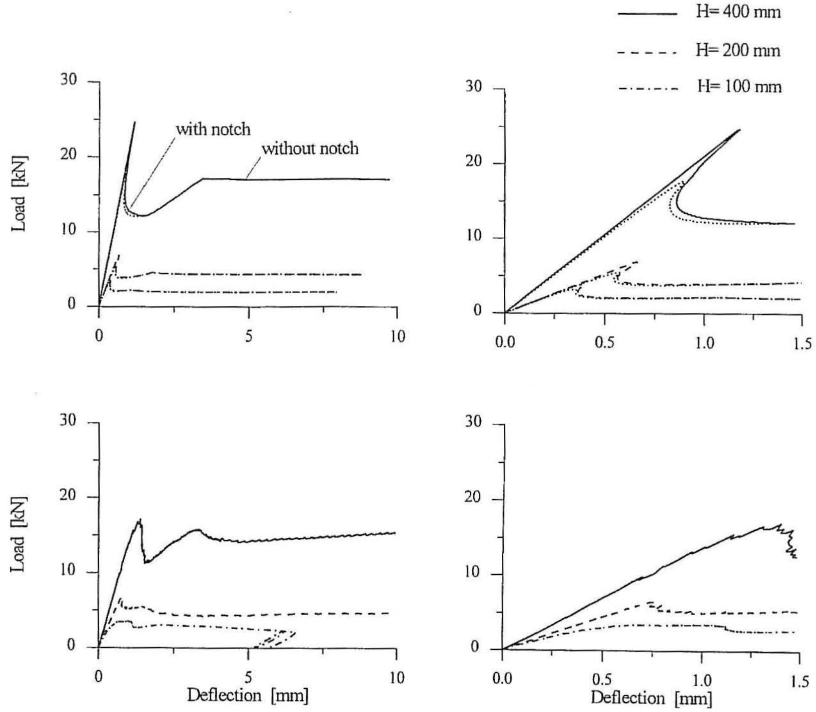

Figure B9. Failure response comparison between model and experimental results for the reinforcement ratio of 0.14 % and high strength beams. Top: Model results. Bottom: Experimental results.

Comparison with Experimental Results.

In Figure B9 model results and typical results obtained by experiments are compared for the high strength concrete and a reinforcement ratio of 0.14 %. The failure response is simulated for the case of no initial crack and for the case of an initial crack with an initial depth of 7.5 % of the beam depth.

Evaluating the model it should be noticed, that no tuning of the model has been performed. All material parameters in the model have either been taken from the experimental investigation described in Appendix A or as typical values reported in the literature.

As it appears, the model clearly overestimates the peak value in the case of no initial crack, whereas the simulated failure response shows a better agreement with experimental results if an initial crack is present. Similar conclusions can be drawn for the normal strength concrete and for other reinforcement ratios. The results indicate that an initial crack should be assumed in all experimental cases of a size of about the chosen value or may be slightly larger. The "valley" after the concrete tension failure peak seems to be well estimated by the model, indicating that it seems reasonable to assume a constant frictional shear stress in the debonded zones at each side of the crack.

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APPENDIX C:

MULTIPLE CRACKING AND ROTATIONAL CAPACITY OF LIGHTLY REINFORCED BEAMS

Prepared by F.A. Christensen, M.S. Henriksen and R. Brincker

Abstract

In this appendix a model is formulated for the rotational capacity of reinforced concrete beams assuming rebar tension failure. The model is based on a classical approach and establishes the load-deflection curve of a reinforced concrete beam. The rotational capacity is then obtained as the area under the load-deflection curve divided by the yield moment of the beam. In calculating the load deflection curve, the cracking process of the concrete is ignored. By assuming that all cracks are fully opened, the energy dissipated during cracking of the concrete is taken into account by simply adding the total tensile fracture energy to the total plastic work obtained by the classical analysis.

Model Formulation

Before cracking of the concrete both the concrete and the reinforcement are assumed to behave elastically, and no slip is assumed between concrete and reinforcement. Assuming a linear variation of the normal beam strain over the cross-section, the stress distribution is obtained by classical beam theory.

When the tensile strength is reached at the tensile side of the beam, the concrete is assumed to crack. Further, cracks are assumed to be formed during constant bending moment (no decrease of the bending moment) and are allowed to extend until the level of the neutral axis. The tension force from the reinforcement is balanced by compression stresses in the concrete. The size of the compression zone is obtained by assuming a uniform distribution of the compression stresses and using an equilibrium equation. At the cracked section the tensile force in the reinforcement is transferred to the surrounding concrete by assuming a formation of two debonded zones around the crack with constant shear friction τ_f. In Figure C1 the stress distribution at a cracked section of the beam is shown immediately before and after formation of a crack.

Calculation Procedure

The crack development is initiated when the tensile strength is reached at the tensile side of the beam in the cross section with maximum bending moment. This corresponds to the situation shown in Figure C1. As the load increases, cracks might form in neighbour sections. If the bending moment is equal to the cracking moment at section II a new crack will be formed at section II. If the bending moment is less than the cracking moment, the load is increased causing the debonded zones to extend. By repeating this procedure cracks are formed one by one until tensile