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by

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### The stiffness change and the increase in the ultimate capacity for a stiff pile resulting from a cyclic loading

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#### Abstract

Today the knowledge concerning a monopile design under a cyclic loading is uncertain and the current standards might lead to the failure problems or to an uneconomical design. An inappropriate prediction of the stiffness for pile-soil system might lead to vibration problems resulting from the resonance that can occur between the natural frequency of the wind turbine and the frequencies of cyclic loads.

In the paper the experimental results of small-scale tests on a stiff monopile are presented to outline the change in stiffness during the cyclic loading and the change in the ultimate pile capacity. The results confirm the increase of stiffness and the increase in bearing capacity resulting from cyclic loading. Performed analysis provides a better understanding of the problem and reveals some correlations that can be useful in the future design of stiff monopiles.

#### 1 Introduction

Problems concerning energy are well-known and require a continuously increasing focus on renewable energy sources. During last years a huge rise of installed wind capacity has been reported [WWEA, 2013]. The wind turbine concept gives a non-limited resource of the energy, reducing problems of CO<sub>2</sub> production. Situating the wind farms offshore results in a less turbulent, stronger wind, which allows a more effective energy production. However, the costs of offshore wind turbines greatly exceed the costs of onshore structures and therefore, a strong impact should be put on the cost stabilization.

A significant part of the total costs for an offshore wind turbine concerns the foundation

design. Currently, the most often used solution in the offshore area is a monopile concept. As far as the installation process is well-known, the behavior under the cyclic loading from wind and waves is poorly accounted in the literature. An increasing knowledge concerning this aspect might lead to notably changes in costs of offshore wind turbines and the cost decrease will contribute to an increase of competitiveness of the offshore wind energy in the general energy market.

The results of small-scale tests are presented in order to analyze the changes in stiffness for soil-pile system and the ultimate soil-pile resistance influenced by cyclic loading. They are preceded with the description of current standards for laterally loaded piles and also with a short state of art concerning this issue. The analysis demonstrates that both, the soil-pile stiffness and the post-cyclic resistance increase due to the long term cyclic loading conditions. This will affect not only limit state designs, but also the eigenfrequency of the wind turbine.

#### THE STIFFNESS OF WIND TURBINE

The reliability of the foundation design can be attributed to the better understanding of the stiffness for the soil-pile system. The stiffness is important as it contributes to the eigenfrequency of the whole turbine. As the wind turbines are sensitive to vibration that affects the proper operation of a turbine, it is extremely important to avoid the resonance, that can occur between the eigenfrequency of the structure and the frequencies coming from cyclic loads. Moreover, the action of the rotor exerts the aerodynamic loads at frequencies of 1P, which is the rotor frequency and 3P, which is the frequency at which the blades pass the tower. Consequently, the wind turbines are designed in a manner that their frequency does not lie close to these loads frequencies, as softsoft structure, soft-stiff structure and stiff-stiff structure as indicated in Figure 1.1.



**Figure 1.1:** Typical design ranges for offshore wind turbines [Kirkwood & Haigh, 2014]

The most safe solution is the stiff-stiff, though the most expansive. Currently, the soft-stiff solution is chosen by the most of designers. However, the sufficient stiffness of the structure must be provided. As long as the stiffness of the tower and the monopile can be calculated, determination of the stiffness for soil-pile interaction is rather problematic. The most often used analytical approach for laterally loaded piles is the p-y curve method outlined in DNV [2011] and API [2005].

#### THE P-Y METHOD

Current standards use the Winkler approach, where the lateral soil capacity is presented as a system of uncoupled springs. The stiffness of each spring is obtained from the p-y curve, which show the relation between the mobilized resistance of surrounding soil, *p*, and the lateral displacement, *y*. This relation for pile embedded in cohesionless soil is estimated based on the full-scale tests results of slender piles, [Reese *et al.*, 1974], [O'Neill & Murchinson, 1983]. Equation (1) describes this dependency.

$$p = A \cdot p_u \cdot tanh\left(\frac{k \cdot X}{A \cdot p_u} \cdot y\right) \tag{1}$$

Here  $p_u$  stands for the static ultimate resistance, *X* for the depth below soil surface and *k* is the initial modulus of subgrade reaction, that can be determined based on the friction angle. *A* factor for the cyclic loading is set to 0.9 what means, that the ultimate soil resistance is assumed to degrade as an effect of cyclic loads.

The ultimate resistance is obtained from equation (2) for a shallow depth and equation (3) for a deep depth. The depth is considered to be deep, when the value of shallow depth soil resistance becomes bigger belong those two. Coefficients  $C_1$ ,  $C_2$  and  $C_3$  are based on the friction angle.

$$p_{us} = (C_1 \cdot x + C_2 \cdot D) \cdot \gamma' \cdot x \tag{2}$$

$$p_{ud} = C_3 \cdot D \cdot \gamma' \cdot x \tag{3}$$

The equations shows that the response of laterally loaded pile is only dependent on the pile diameter, *D*, but not on the embedded length, *L*, or the eccentricity of the load, *e*.

The stiffness obtained from the p-y curve is

decreased for cyclic loading, A = 0.9. The approach does not account for the magnitude, the direction of loading and for the number of cycles. Another aspect is the usefulness of the method for currently design monopiles. The method was originally designed for a slender piles, that behave flexible under the lateral loads. Today monopiles has a significantly smaller slender ratio and their response on the lateral load is considered to be rigid. The behaviour of stiff monopile under the lateral load is presented in Figure 1.2. The displacement at the pile toe that occur for a rigid pile results in additional shear stress, which increase the lateral resistance. Therefore the use of the method for large diameter piles is doubtful.



**Figure 1.2:** A rigid monopile for offshore wind turbine [LeBlanc *et al.*, 2010]

#### CURRENT PRESENTED CONCEPTS

Recently the behaviour of cyclic laterally loaded monopiles is investigated with a strong interest. The change in stiffness as an effect of lateral cyclic loads has been already analyzed by many researchers. Especially small scale tests and numerical models are used as both are more cost-efficient solutions in comparison with full-scale models.

The change in stiffness effecting from cyclic loading was confirmed in papers of Little & Briaud [1998]. The research based on the results of full-scale slender piles indicates the stiffer response in the second series of cycles as a result of sand densification in the first, preceding series of cycles.

Long & Vanneste [1994] after exploring 34 full-scale tests of both, slender and stiff piles, deduced that the soil resistance is decreased by performed cycles. However, it is stated in limitation that the soil densification might happen for loading ratio below 0 (between one and two-way loading).

Klinkvort *et al.* [2010] by performing a centrifuge small-scale test on a stiff pile has proved that the soil bearing capacity indeed increases as a response of the cyclic loads. The small increase in secant stiffness was observed, even though the data were scattered significantly. All these tests were performed with a small number of cycles, not sufficient for assessing the long-term effects for offshore turbines.

The rig adjusted for performing more cycles was used by LeBlanc *et al.* [2010]. The stiff pile embedded in dry sand of loose and mediumdense state was tested for the accumulated rotation and the change in stiffness as an effect of long-term cycles (N=8 000 - 60 000). The logarithmic dependency between the increase in stiffness and the number of cycles has been found. This trend is also related to the load magnitude and the direction of loading, but independent on the soil density.

The same approach and similar equipment was used by Abadie & Byrne [2014] and Lada *et al.* [2014]. The stiffness increase as a logarithmic function of the number of cycles was found to be dependent on the soil density. In



**Figure 1.3:** The set-up: sketch with dimensions (left picture), the sand container (a), the displacement and force transducers (b) and the cyclic loading side (c)

the small-scale centrifuge test performed by Kirkwood & Haigh [2014] this changes of stiffness are explained by forming of locked in soil stresses around the pile caused by a reversal in loading. Therefore, while designing offshore wind turbines, a strong attention should be paid on the possible migration of the natural frequency of the soil-pile system.

#### 2 Small-scale tests

A series of small-scale tests have been performed at Aalborg University. The rig provided by the Geotechnical Laboratory allows to conduct proper tests, with a number of cycles exceeding 50 000. Therefore, the results can be used to analyze the monopile response for a long-term lateral cyclic loading conditions.

#### EXPERIMENTAL MODEL

The rig used in the research was based on the mechanical equipment used by LeBlanc *et al.* [2010]. Such a setup allows for performing

both static and cyclic tests. Figure 1.3 presents the experimental rig.

The Aalborg University Sand No. 1 of investigated properties given in Table 2.1 is used.

**Table 2.1:** The soil properties for Aalborg UniversitySand No.1

Property	Value
Specific grain density, d <sub>s</sub> [g/cm <sup>3</sup> ]	2.64
Maximum void ratio, e <sub>max</sub> [-]	0.858
Minimum void ratio, e <sub>min</sub> [-]	0.549
Particle size, d <sub>50</sub> [mm]	0.14
Uniformity coefficient, $\frac{d_{50}}{d_{100}}$ [-]	1.78

Table 2.2: The characteristic properties of pile

Property	Value
Pile diameter, D [mm]	100
Embedded pile length, L [mm]	500
Wall thickness, <i>t</i> [mm]	5
Load eccentricity, e [mm]	605
Slenderness ratio, L/D [-]	5

Soil is prepared before each test in the same manner, so the comparison between tests is reliable. The preparation provides a saturated, dense sand state, with the relative density,  $I_D$  aimed for 80-90 %. Such a soil state is considered to be an adequate simulation of offshore conditions.

An open-ended aluminum monopile is used for all tests. The pile properties are given in Table 2.2. The soil stiffness is predicted to be around 4 MPa, so according to the most often used criterion proposed by Poulos and Hull (1989), the rigid response of the pile due to lateral load is expected [Sørensen *et al.*, 2012]. The more thorough description of the soil preparation and the equipment is given in the paper [Lada *et al.*, 2014].

#### EXPERIMENTAL PROGRAM

The static tests are performed in order to determine the ultimate pile capacity,  $M_R$ . The results of cyclic tests are used for investigation of the accumulated rotation and the change in stiffness due to the increasing number of cycles. After each cyclic test, the post-cyclic static test is conducted and the results are used for investigation of the change in the ultimate resistance. For both loading conditions, static and cyclic, the drained behaviour of the sand is assumed.

The effects of cyclic tests are investigated by using the approach presented by LeBlanc *et al.* [2010]. Tests are described by the loading characteristics,  $\zeta_b$  and  $\zeta_c$ , which are determined by equation (4) and (5).

$$\zeta_b = \frac{M_{max}}{M_R} \tag{4}$$

$$\zeta_c = \frac{M_{min}}{M_{max}} \tag{5}$$

Here, the maximum and the minimum moment of cyclic loading is denoted as  $M_{max}$  and  $M_{min}$ .

In order to provide realistic conditions the target  $\zeta_b$  for the most of tests is set to be close to 0.3. 30 % of the ultimate capacity and around 10<sup>7</sup> cycles is used to assess the Fatigue Limit State, FLS. However, such a number of cycles would be too time consuming, therefore a number of  $5 \cdot 10^4$  cycles is assumed to be relevant.

Different values of  $\zeta_b$  are prescribed for tests, when the dependency of the results on this parameter is assessed.

#### 3 Results and discussion

The increase of the bearing capacity as a result of cyclic loading was already proved in others papers, [Klinkvort *et al.*, 2010] and [Lada *et al.*, 2014]. However, any dependency for this increase has been presented. By analyzing the results of small-scale tests some relationships between the change in the ultimate capacity and the loading characteristics are observed.

The post-cyclic soil resistance is normalized by the static ultimate capacity and plotted versus  $\zeta_b$  in Figure 3.1. A linear dependency is observed, where almost all of the chosen data fit to the function. When the loading amplitude is increased, the bigger increase in the resistance is obtained. The test with  $\zeta_b = 0.514$  was stopped after 8 000 cycles due too excessive tilting of the lever in the equipment. Nevertheless, it is already seen that the value of post-cyclic resistance increases significantly.

When plotting the results in relation to  $\zeta_c$  also a specific trend is observed. The results are presented in Figure 3.2. The selected tests are described by  $\zeta_b \approx 0.3$ . A small increase in the soil resistance is obtained after the tests between one-way and two-way cyclic loading, whereas a significant change in bearing capacity is found after the tests characterized by  $\zeta_c$  close to -1 or to 0.



**Figure 3.1:** Normalized post-cyclic soil resistance dependency on  $\zeta_b$ 



**Figure 3.2:** Normalized post-cyclic soil resistance dependency on  $\zeta_c$ 

The change in the ultimate capacity might be interpreted as an increase in the stiffness of the soil. The change in stiffness for 50 % strength in the post-cyclic tests was investigated, but no correlation with the load characteristics was obtained. Also the influence of the load characteristics on the value of cyclic unloading stiffness was assessed. The results of cyclic stiffness after around 50000 cycles are presented in Table 3.1. From overall tests the minimum increase of stiffness was assessed for 34 % for a one-way cyclic test. The maximum increase was found for a two-way loading test with  $\zeta_c = -0.915$ . The increase of stiffness for the tests between one-way and two-way loading lie between those two values. The results of the increase in the cyclic stiffness for  $\zeta_b = 0.3$ are plotted in Figure 3.3.

**Table 3.1:** The results of cyclic unloading stiffness,  $\zeta_b \approx 0.3$ 

ζ <sub>c</sub>	k <sub>N,unload</sub> [ Nm <sup>o</sup> ]	$\frac{k_{N,unload}}{k_{1,unload}}$ [ - ]
0.03	1 061.70	1.34
-0.107	783.63	1.48
-0.152	1 122.70	1.80
-0.268	951.03	2.64
-0.329	1 065.51	2.03
-0.383	1 070.33	2.15
-0.554	958.79	2.50
-0.831	414.94	1.53
-0.915	833.50	3.32
-0.929	918.47	2.71



Figure 3.3: The total increase in the unloading, cyclic stiffness

The chosen trend is not perfectly fitted to the data, however it can be concluded, that when approaching  $\zeta_c = -1$  the change in the cyclic unloading stiffness become bigger for given

tests. No dependency for the change in stiffness on  $\zeta_b$  is found. Also no correlation for the final value of the cyclic stiffness and the load characteristics was found. It can be concluded that apart of the stiffness, there might be some others parameters that influence the post cyclic resistance.

The analysis shows that due to the cyclic loading the final stiffness is always bigger than the initial one. However, in some tests the increasing tendency in stiffness was changed into the falling trend. As a result, the biggest increase of the stiffness do not always correspond to the biggest number of cycles, but to the number of cycles between. This shows that during a life-time of a turbine the cyclic loading changes significantly the stiffness and therefore, some fluctuations in the soil-pile system stiffness should be included in the design.

The dependency between the number of cycles and the post-cyclic resistance is also investigated. The test group considered was loaded with the same weights, aiming in one-way cyclic loading with the magnitude of around 30 % of ULS. The results are presented in Figure 3.4.



**Figure 3.4:** The increase of ultimate resistance due to the number of cycles

Clearly, it can be concluded that there is a relationship between the change in the ultimate bearing capacity and the number of cycles. It seems like the cycles strengthen the soil, and when the number of cycles increase, the increase of the ultimate bearing capacity is observed. A logarithmic function of N was adjusted to the data and the reasonable fit is obtained.

The result of post-cyclic resistance after a test with 10 000 cycles differs from the logarithmic function. However it should be underlined that many external and internal factor can influence the test results and some of them can differ from the expectation.

As far as the rest of results are in a good fit with the function, following statement are concluded:

- The bearing capacity of soil-pile system increase logarithmically with the number of cycles,
- The logarithmic dependency should be still investigated and confirmed with more test results.

The same tests are used for the investigation of the stiffness changes with the increasing number of cycles. The results of the rotational stiffness from the post-cyclic tests are plotted in Figure 3.5. The tangential stiffness for 50 % of ULS is taken as the representative value. The rotational stiffness from the static tests is also plotted for the number of cycle N=1.

In the same manner, the cyclic unloading stiffness changes in tests are plotted against the increasing number of cycles, and also a logarithmic dependency is obtained, Figure 3.6.

The change in the ultimate capacity due to the cyclic loading is clearly related to the increasing stiffness of the soil-pile system. The increase of the stiffness can be clarified with the fact that the plastic strains that occur during the cycles might expand the yielding surface. As the soil become stronger, it possesses a greater stiffness.



Figure 3.5: The change in stiffness for post cyclic tests



**Figure 3.6:** The increase of the rotational stiffness due to the number of cycles

Nevertheless, those increasing dependency with the number of cyclces should be still investigated in order to define, whether the logarithmic function of cycles can be used in predicting the post-cyclic resistance and the change in stiffness for the design of offshore wind turbines.

#### 4 Conclusion

Nowadays the use of offshore wind turbines is increasing, as it gives an unlimited source of energy that can be produced more effectively in comparison to the onshore wind farms. Therefore, the design of the foundation must be developed in order to limit the total costs of turbines and also to provide their operation to be undisturb and more efficient.

The design of the monopiles for offshore turbines is normally based on p-y curve approach, where the ultimate soil-pile capacity has a big impact on the relation between the soil resistance and the corresponding lateral deformation of the pile. Contrary to the recommended practice, where the ultimate capacity is degraded, the results of small scale tests show that the soil become stronger due to the cyclic loading. The relation between the increase in the ultimate soil resistance and the load characteristics,  $\zeta_b$  and  $\zeta_c$ , is found. As an effect of cycles, the soil capacity increases more when bigger load magnitude is applied. Also it is concluded that the most of increase happens for a one-way and two-way loading tests, whereas the tests between show a smaller growth. Additionally, the logarithmic relation between the increase in the ultimate capacity and the number of cycles was found.

The small scale tests results reveal also the change in the rotational stiffness. The logarithmic function was also obtained as a good fit for the relationship between the number of cycles and both, the total change in the cyclic unloading stiffness and the post cyclic stiffness of 50 % strength. The dependency between the changes in stiffness and the load characteristics was difficult to obtained. Only a clear increase of the total change in cyclic stiffness was found when  $\zeta_c$  approaches -1. This indicates a stronger sand densification for two-way loading in comparison to one-way

loading, where the smallest increase of the stiffness is found. However, it contradicts to the findings concerning the post-cyclic resistance, as the maximum increase is observed not only for the two-way loading but also for the one-way loading.

The accurate prediction of the soil-pile system is extremely important for the design of the whole structure. Most often the design predicts the eigenfrequency of the turbine to lie between 1P and 3P. The change in stiffness increases the eigenfrequency, which lead to problems related to the resonance occurring between the loading frequencies and the natural frequency of the turbine.

#### References

- Abadie, C.N., & Byrne, B.W. 2014. Cyclic loading response of monopile foundations in cohesionless soil. *Physical Modeling in Geotechnics - Gaudin & White*, **ISBN 978-1-138-00152-**7, 779–784.
- API, American Petroleum Insitute. 2005. *Recommended practice for planning, designing and constructing fixed offshore platform*. Downloaded: 10-10-2013.
- DNV, Det Norske Veritas. 2011. *Design of Offshore Wind Turbine Structures*. Downloaded: 10-10-2013.
- Kirkwood, P.B., & Haigh, S.K. 2014. Centrifuge testing of monopiles subject to cyclic lateral loading. *Physical Modeling in Geotechnics* - *Gaudin & White*, ISBN 978-1-138-00152-7, 827–831.

Klinkvort, R.T., Leth, C.T., & Hededal, O. 2010.

Centrifuge modeling of a laterally cyclic loaded pile. *Physical Modelling in Geotechnics Ű Springman, Laue & Seward*, **ISBN 978-0-415-59288-8**, 959–964.

- Lada, A., Gres, S., Nicolai, G., & Ibsen, L. Bo. 2014. Response of a stiff monopile for a longterm cyclic loading. *Department of Civil Engineering, Aalborg University*, part of a Master thesis.
- LeBlanc, C., Houlsby, G.T., & Byrne, B.W. 2010. Response of stiff piles in sand to long-term cyclic lateral loading. *Geotechnique 60*, **No.2**, 79–90.
- Little, Robert L., & Briaud, Jean-Louis. 1998. Full scale cyclic lateral load tests on six single piles in sand. US Army Engineering District, Final Raport.
- Long, J.H., & Vanneste, G. 1994. Effects of Cyclilc Lateral Loads on Piles in Sand. *Journal of Geotechnical Engineering*, Vol.120, No.1, 225–244.
- O'Neill, M.W., & Murchinson, J.M. 1983. An Evaluation of p-y Relationships in Sand. *a report to the American Petroleum Institute*.
- Reese, L.C., Cox, W.R., & Koop, F.D. 1974. Analysis of Laterally Loaded Piles in Sand. Proceeding of the Sixth Annual Offshore Technology Conference, Houston, Texas, paper no. OTC 2080, 473–484.
- Sørensen, S.P.H., Brødbæk, K.T., Møller, M., & Augustesen, A.H. 2012. Review of laterally loaded monopiles employed as the foundation for offshore wind turbines. DCE Technical Reports, Department of Civil Engineering, Aalborg University, Aalborg, Denmark, No.137.
- WWEA, World Wind Energy Association. 2013. Small Wind World Raport. URL: http://www. wwindea.org. Downloaded: 25-03-2014.

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