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Model Predictive Control for Flexible Power Consumption of Large-Scale Refrigeration Systems*

Seyed Ehsan Shafiei, Jakob Stoustrup and Henrik Rasmussen

***Abstract*— A model predictive control (MPC) scheme is introduced to directly control the electrical power consumption of large-scale refrigeration systems. Deviation from the baseline of the consumption is corresponded to the storing and delivering of thermal energy. By virtue of such correspondence, the control method can be employed for regulating power services in the smart grid. The proposed scheme contains the control of cooling capacity as well as optimizing the efficiency factor of the system, which is in general a nonconvex optimization problem. By introducing a fictitious manipulated variable, and novel incorporation of the evaporation temperature set-point into optimization problem, the convex optimization problem is formulated within the MPC scheme. The method is applied to a simulation benchmark of large-scale refrigeration systems including several medium and low temperature cold reservoirs.**

I. INTRODUCTION

The structure of power systems, especially in Europe, is changing from a centralized one to a decentralized one due to distributed generation with high penetration of renewable sources. This change leads to several new challenges that can be handled in a smart grid, where both production and consumption of electricity are managed efficiently. To achieve such efficient demand-side management, consumers should be equipped with control systems that can actively respond to the grid requirements.

Demand response (DR) is a component of smart energy demand for managing costumer consumption of electricity. One strategy for DR implementation is real-time pricing [1] in which the load level of a consumer is optimized in response to electricity prices. Another strategy (considered for this study) is to directly manage the energy consumption of consumers. Implementation of such strategy requires at least two levels of design [2]: a higher level to dispatch the energy/power demand to consumers, and a lower level control design specific for each autonomous consumer providing balancing services. The latter is the focus of this paper.

A typology of ancillary services was identified by [3], where different services like continuous regulation, energy imbalance management, instantaneous contingency reserves, replacement reserves, voltage control and black start were investigated. Based on this typology, the present method facilitates the energy imbalance management services for large-scale refrigeration systems. Regarding the power grid balancing services, the potential of corresponding demand response activities was investigated by [4] for heating, ventilation and refrigeration systems. Considering the refrigeration systems, the associated demand response opportunities were reported in [5].

By means of flexible power consumption, the refrigeration system is supposed to consume at the baseline of its power consumption profile during the normal operation, increase the consumption for downward regulation, and decrease it for upward regulation services in favor of the power grid. The thermal capacity of refrigerated goods are employed for storing and delivering of thermal energy. In the present work, it is assumed that a power reference signal is provided by an aggregator to be followed.

One important challenge of control design for multiple evaporator refrigeration systems is coming from the fact that different cooling units have the same evaporation temperature while providing different cooling capacities. The power/energy management is performed by controlling the individual cooling capacities each depends on the same evaporation temperature as others. Finding optimal cooling capacities as well as optimal evaporation temperature is in general a nonconvex optimization problem. This problem is addressed in [6] using nonconvex model predictive control in a real-time pricing market. In [7], the evaporation temperature is controlled in a separated loop while a supervisory MPC is proposed for energy cost optimization.

Another difficulty arises from the existence of nonlinear dynamics — caused by the fluid dynamics inside the evaporator — between the expansion valve and the actual cooling capacity. In the relevant works presented in [6] and [8], the cooling capacity is taken as control variable that simplifies the dynamic model, but the problem is that it cannot be applied as a control signal to the system.

It is shown in the present paper that by virtue of the faster dynamics of the flow change inside the evaporator comparing to the thermal dynamics, it is possible to describe the cooling capacity by static nonlinearity in terms of the valve opening degree and the evaporation temperature. It is simply achieved by choosing an appropriate sampling time for the MPC. At this point, the model would look like a Hammerstein model. Then, by taking the cooling capacity as fictitious manipulated variable, a model predictive control is formulated using a novel incorporation of the evaporation temperature into the optimization problem. It leads to a higher system coefficient of performance (COP). The proposed method is applied to a simulation benchmark of large-scale refrigeration systems including several medium and low temperature cold reservoirs with a booster configuration of two racks of compressors.

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II. SYSTEM DESCRIPTION AND PROBLEM STATEMENT

In this section, configuration and model of a typical supermarket refrigeration system is described.

A. CO_2 Booster Refrigeration System

A basic layout of a typical refrigeration system including several cooling units with two racks of compressors in a booster configuration is shown in Fig. 1. Starting from the receiver (REC), two-phase refrigerant (mix of liquid and vapor) at point '8' is split out into saturated liquid ('1') and saturated gas ('1b'). The latter is bypassed by a bypass valve (BPV), and the former flows into expansion valves where the refrigerant pressure drops to medium ('2') and low ('2′') pressures. The electronic expansion valves EV_MT and EV_LT are responsible for regulating the air temperature inside the medium temperature (MT) and the low temperature (LT) cooling units, respectively, by controlling the entering mass flows into the evaporators. Flowing through medium and low temperature evaporators (EVAP_MT and EVAP_LT), the refrigerant absorbs heat from the cold reservoir. The pressure of low temperature units (LT) is increased by the low stage compressor rack (COMP_LO). All mass flows from COMP_LO, EVAP_MT and BPV outlets are collected by a suction manifold at point '5' where the pressure is increased again by high stage compressors (COMP_HI). Afterward, the gas phase refrigerant enters the condenser to deliver the absorbed heat from cold reservoirs to the surrounding. The receiver pressure is regulated by the high pressure valve CP_HP. The detailed thermodynamic analysis of such systems is described in [9].

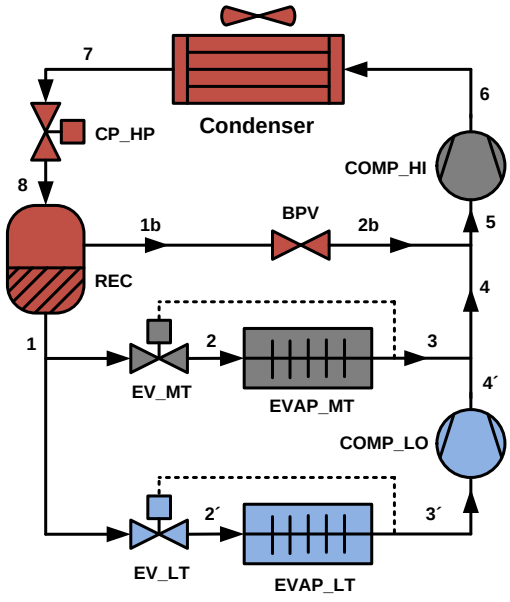

Fig. 1. Basic layout of a typical supermarket refrigeration system with booster configuration.

B. Cooling unit dynamics

In the cooling units, heat is transfered from foodstuffs to cooled air, $\dot{Q}_{foods/air}$, and then from cooled air to circulated refrigerant, $\dot{Q}_e$, which the latter is also known as cooling capacity. There is however heat load from supermarket indoor, $\dot{Q}_{load}$, formulated as a variable disturbance. Here, we consider the measured air temperature entering the evaporator area as the cold unit temperature, T_{air}. Using lumped modeling approach [10], the following dynamical equations are derived based on energy balances for the mentioned heat transfers.

$$MCp_{foods}\frac{dT_{foods}}{dt} = -\dot{Q}_{foods/air} \tag{1}$$

$$MCp_{air}\frac{dT_{air}}{dt} = \dot{Q}_{load} + \dot{Q}_{foods/air} - \dot{Q}_e \tag{2}$$

where MCp denotes the corresponding mass multiplied by the heat capacity. The energy flows are

$$\dot{Q}_{foods/air} = UA_{foods/air}(T_{foods} - T_{air}), \tag{3}$$

$$\dot{Q}_{load} = UA_{load}(T_{indoor} - T_{air}), \tag{4}$$

and

$$\dot{Q}_e = UA_e(T_{air} - T_e) \tag{5}$$

where UA is the overall heat transfer coefficient, T_e is the evaporation temperature, and T_{indoor} is the supermarket indoor temperature. The heat transfer coefficient between the refrigerant and the display case temperature, UA_e, is described as a linear function of the mass of the liquefied refrigerant in the evaporator [11],

$$UA_e = k_m M_r, \tag{6}$$

where k_m is a constant parameter. The refrigerant mass, $0 \leq M_r \leq M_{r,max}$, is subject to the following dynamic [12],

$$\frac{dM_r}{dt} = \dot{m}_{r,in} - \dot{m}_{r,out}, \tag{7}$$

where $\dot{m}_{r,in}$ and $\dot{m}_{r,out}$ are the mass flow rate of refrigerant into and out of the evaporator, respectively. The entering mass flow is determined by the opening degree of the expansion valve and is described by the following equation:

$$\dot{m}_{r,in} = OD\ KvA\sqrt{\rho_{suc}(P_{rec} - P_e)} \tag{8}$$

where OD is the opening degree of the valve with a value between 0 (closed) to 1 (fully opened), P_{rec} and P_e are receiver and suction manifold (evaporating) pressures, ρ_{suc} is the density of the circulating refrigerant, and KvA denotes a constant characterizing the valve. The leaving mass flow is given by

$$\dot{m}_{r,out} = \frac{\dot{Q}_e}{\Delta h_{lg}} \tag{9}$$

where Δh_{lg} is the specific latent heat of the refrigerant in the evaporator, which is a nonlinear function of the suction pressure (or equivalently evaporation temperature). When the mass of refrigerant in the evaporator reaches its maximum value ($M_{r,max}$), the entering mass flow is equal to the leaving one.

C. Compressor Power and System COP

The electrical power consumption of each compressor bank is calculated by

$$Pow_c = \frac{1}{\eta_{me}} \dot{m}_{ref}(h_{o,c} - h_{i,c}), \tag{10}$$

where $\dot{m}_{ref}$ is the total mass flows into the compressors, and $h_{o,c}$ and $h_{i,c}$ are the enthalpies at the outlet and inlet of the compressor bank and are nonlinear functions of the refrigerant pressure and temperature at the calculation points. The constant η_{me} indicates overall mechanical/electrical efficiency considering mechanical friction losses and electrical motor inefficiencies [13]. The outlet enthalpy is computed by

$$h_{o,c} = h_{i,c} + \frac{1}{\eta_{is}}(h_{is} - h_{i,c}), \tag{11}$$

in which h_{is} is the outlet enthalpy when the compression process is isentropic, and η_{is} is the related isentropic efficiency given by [14] (neglecting higher order terms).

$$\eta_{is} = c_0 + c_1(f_c/100) + c_2(P_{c,o}/P_{suc}) \tag{12}$$

Where f_c is the virtual compressor frequency (total capacity) of the compressor rack in percentage, $P_{c,o}$ is pressure at the compressor outlet, and c_i are constant coefficients.

The total coefficient of performance is defined as ratio of the total cooling capacity over the total power consumption of the compressors.

$$COP = \frac{\dot{Q}_{e,tot}}{Pow_{c,tot}} \tag{13}$$

The COP is calculated by

$$COP = \frac{x_{MT}\Delta h_{lg,MT} + x_{LT}\Delta h_{lg,LT}}{\frac{1}{\eta_{MT}}(h_{oc,MT} - h_{ic,MT}) + \frac{x_{LT}}{\eta_{LT}}(h_{oc,LT} - h_{ic,LT})}, \tag{14}$$

where indices MT and LT relate the calculated values to the medium and low temperature sections, respectively. Parameters x_{MT} and x_{LT} are ratio of the refrigerant mass flow of MT and LT evaporators to the total flow rate, and $\eta_{MT} = \eta_{me,MT}\eta_{is,MT}$ and $\eta_{LT} = \eta_{me,LT}\eta_{is,LT}$. The enthalpy terms are nonlinear function of the evaporation temperature (T_e) and/or the condensation pressure (P_c) as $\Delta h_{lg}(T_e)$, $h_{oc}(P_c)$, and $h_{ic}(T_e)$ as well as the corresponding refrigerant temperatures.

D. Problem Statement

Here the problem is to designing a control algorithm enabling large-scale refrigeration systems to follow the assigned power reference by an aggregator while optimizing the coefficient of performance.

It can be seen from (14) and the corresponding dependences of enthalpies that the COP is a function of mass flows coming from evaporators, condensation pressure, and evaporation temperature. The mass flows are dictated by operating conditions of the display cases and controlled by the corresponding expansion devices. We assumed the condenser fan speed is at the maximum level, so the condensation pressure is changed by changing the outdoor temperature. The only remained manipulated variable to change the COP is the evaporation temperature. Therefore, the maximum COP can be achieved by maximizing the evaporation temperature.

III. MPC FORMULATION

A model predictive control scheme that can address the above problem is formulated in this section. The objective function for power following is defined as:

$$J_{Pow} = \sum_{k=1}^{N} \|Pow_c[k] - Pow_{ref}[k]\|_2^2 \tag{15}$$

where Pow_{ref} is the power reference, k denotes the current time instant, and N is the prediction horizon in terms of number of time steps (samples). Manipulated variables are the opening degrees of the expansion valves (OD) and the evaporation temperature set-point ($\hat{T}$).

Looking into system dynamics, it turns out that the power consumption (Pow_c) is the nonlinear function of the evaporation temperature (T_e); and the cooling capacity ($\dot{Q}_e$) is also a nonlinear function of both the evaporation temperature and opening degree of expansion valves (OD). In the following it is shown that how a convex optimization problem can be formulated by (i) introducing a fictitious manipulated variable; (ii) novel incorporation of T_e into the MPC scheme; and (iii) choosing appropriate sampling time and prediction horizon.

A. Problem Convexification using Synthetic Input

Considering $\dot{Q}_e$ as fictitious manipulated variable, the indoor temperature (T_{indoor}) as measurable disturbance and the cold reservoir temperatures (T_{air}, T_{food}) as state variables, we can formulate the discrete-time linear dynamics for each cooling unit as follows.

$$x[k+1] = Ax[k] + Bu[k] + B_d d[k] \tag{16}$$

where $x = \begin{bmatrix} T_{foods} & T_{air} \end{bmatrix}^T$, $u = \dot{Q}_e$, and $d = T_{indoor}$. The parameters are

$$A = \begin{bmatrix} -\frac{UA_{foods/air}}{MCp_{foods}} & \frac{UA_{foods/air}}{MCp_{foods}} \\ \frac{UA_{foods/air}}{MCp_{air}} & -\frac{UA_{foods/air}+UA_{load}}{MCp_{air}} \end{bmatrix}, \tag{17}$$

and

$$B_1 = \begin{bmatrix} 0 \\ \frac{-1}{MCp_{air}} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ \frac{UA_{load}}{MCp_{air}} \end{bmatrix}. \tag{18}$$

The first state variable is subject to the following constraint due to food safety promise:

$$x_{1,min} \le x_1 \le x_{1,max} \tag{19}$$

where $x_{1,min} = T_{foods,min}$ and $x_{1,max} = T_{foods,max}$ are the limitations on the food temperature. The input constraint is given by

$$0 \le u \le u_{max} \tag{20}$$

with $u_{max} = UA_{e,max}(T_{air} - T_e)$ where $UA_{e,max} = k_m M_{r,max}$.

Substituting (13) into (15) and treating $\dot{Q}_{e,tot}$ as the sum of fictitious control inputs give

$$J_{Pow} = \sum_{k=1}^{N} \left\| \frac{\sum_{i=1}^{m} u_i[k]}{COP} - Pow_{ref}[k] \right\|_2^2, \tag{21}$$

where *COP* is calculated using (14) at time instant k and kept constant all over the horizon. In order to avoid the oscillation of the control signal, the following cost function is introduced that is a standard approach in MPC formulations.

$$J_{\Delta u} = \sum_{k=1}^{N} \|u[k] - u[k-1]\|_2^2 \tag{22}$$

For now, the optimization problem ca be defined as:

$$\begin{aligned} \min_{\mathbf{u}} \quad & J_{pow} + W_{\mathbf{u}} J_{\Delta \mathbf{u}} \\ \text{subject to} \quad & \mathbf{x}[k+1] = \mathbf{A}\mathbf{x}[k] + \mathbf{B}\mathbf{u}[k] + \mathbf{B_d}\mathbf{d}[k] \\ & \mathbf{x}_{1,min} \leq \mathbf{x}_1[k] \leq \mathbf{x}_{1,max} \\ & 0 \leq \mathbf{u}[k] \leq \mathbf{u}_{max} \end{aligned} \tag{23}$$

where the vector and matrix notations are used to show all cooling dynamics as a large-scale multivariable system, and $W_{\mathbf{u}}$ is a weighting factor.

Note that the solution of the above optimization problem, $\mathbf{u}$, cannot directly be applied as a control input to the system. This is more elaborated in the sequel.

B. Novel Incorporation of T_e into MPC Scheme

Note that the $\hat{T}_e$ of MT section is different from of LT section, and remind the fact that several cooling units at each section have the same corresponding evaporation temperature. The COP can be kept at the highest point by keeping T_e as high as possible up to the point that enough cooling capacity is provided to cold reservoirs to preserve the required temperatures. This can be achieved by adding the following cost to the objective function.

$$J_{T_e} = \sum_{k=1}^{N} \|\hat{T}_e[k] - T_{e,max}\|_2^2 \tag{24}$$

where $T_{e,max}$ is the maximum value that T_e is allowed to reach. Thus the MPC pushes the evaporation temperature up to the highest value. It should also be constrained as $\hat{T}_e \leq T_{e,max}$ The Moreover, in order to make sure that the resulted cooling capacity from the optimization problem is coincide with the evaporation temperature, the upper limit of the input constraint (20) is modified as

$$u_{max} = UA_{e,max}(x_2 - \hat{T}_e) \tag{25}$$

with $x_2 = T_{air}$. Now, the MPC algorithm can be formulated using the following optimization problem with novel incorporation of $\hat{T}_e$.

$$\begin{aligned} \min_{\mathbf{u},\hat{T}_e} \quad & J_{pow} + W_{\mathbf{u}} J_{\Delta \mathbf{u}} + W_e J_{T_e} \\ \text{subject to} \quad & \mathbf{x}[k+1] = \mathbf{A}\mathbf{x}[k] + \mathbf{B}\mathbf{u}[k] + \mathbf{B_d}\mathbf{d}[k] \\ & \mathbf{x}_{1,min} \leq \mathbf{x}_1[k] \leq \mathbf{x}_{1,max} \\ & 0 \leq \mathbf{u}[k] \leq \mathbf{U}\mathbf{A}_{e,max}(\mathbf{x}_2 - \hat{T}_e) \\ & \hat{T}_e \leq T_{e,max} \end{aligned} \tag{26}$$

where W_e is the weighting factor for compromising between the evaporation temperature and the other terms in the objective function.

Remark 1: Choosing a small value for W_e may result in a lower T_e (larger distance to $T_{e,max}$) which is equivalent to a smaller COP value. Choosing a large value for W_e, on the other hand, leads to a higher $\hat{T}_e$ and consequently a better COP, but a smaller constraint set for the decision variable (cooling capacity). So it curtails the flexibility in controlling the power consumption. Thus, depending on the DR services that the refrigeration system would provide, W_e compromises between the flexibility of power consumption control and optimality of COP.

Remark 2: The direct physical relationship between $\dot{Q}_e$ and T_e is not included in the optimization problem. In lieu, to make sure that the resulted $\dot{Q}_e$ is feasible to achieve at the concluded T_e, its constraint set is manipulated by $\hat{T}_e$ as another decision variable in (26).

C. Control Inputs

As pointed out in Section III-A, the cooling capacity ($u = \dot{Q}_e$) resulted from the optimization problem is not an actual control signal. It is however function of the manipulated variables, *i.e.,*the evaporation temperature and the opening degree of expansion valve. The former is directly given by the MPC algorithm, but the latter needs more elaboration.

After applying a new *OD*, the latent mass dynamic (7) reaches the steady-state after around 4 minutes which results:

$$\text{steady-state} \;\Rightarrow\; \dot{M}_r \simeq 0 \;\Rightarrow\; \dot{m}_{r,in} \simeq \dot{m}_{r,out}. \tag{27}$$

Using (27), (8), and (9), the opening degree is calculated as

$$OD \simeq \frac{\dot{Q}_e}{\Delta h_{lg} K v A \sqrt{\rho_{suc}(P_{rec} - P_e)}}\ , \tag{28}$$

where P_{rec} is assumed constant [10], and Δh_{lg}, ρ_{suc} and P_e all are functions of T_e which is regulated to $\hat{T}_e$. All in all, for MPC implementation, at each sampling time, u and $\hat{T}_e$ are the solutions for (26) based upon which the opening degree is calculated as

$$OD = K_e(\hat{T}_e)u, \tag{29}$$

where $K_e(\hat{T}_e) = \left[\Delta h_{lg} K v A \sqrt{\rho_{suc}(P_{rec} - P_e)}\right]^{-1}$ is updated at each sample time k. The proposed MPC scheme is summarized in Algorithm 1.

Remark 3: There are local stable superheat controllers operating on the expansion valves to make sure the refrigerant is completely vaporized (superheated) at the outlet of the valves. This is for compressors safety. The superheat control loop is much faster than the MPC and is in the steady-state at each MPC step. In this work, we impose a certain value of superheat degree in our simulation model to take its effect into account.

D. Sampling Time and Prediction Horizon

In order to choose an appropriate sampling time, T_s, and prediction horizon, N, the limits of each should be investigated. For energy balancing services, the consumer should respond in around 10 minutes [3], and for the proposed power following approach even a faster sampling period, $T_s < 10$ min, would be more favorable. In accordance with the discussion made in Section III-C, the sampling time should also be $T_s > 4$ min.

Algorithm 1 MPC implementation

Prediction

Load

Pow_{ref} from higher level aggregator

Compute

COP and keep it fixed all over the horizon

Solve

$$\min_{\mathbf{u},\hat{T}_e,\varepsilon} \quad J_{pow} + W_{\mathbf{u}} J_{\Delta\mathbf{u}} + W_e J_{T_e} + W_r \|\varepsilon\|_2^2$$

subject to

$$\mathbf{x}[k+1] = \mathbf{A}\mathbf{x}[k] + \mathbf{B}\mathbf{u}[k] + \mathbf{B_d}\mathbf{d}[k]$$
$$\mathbf{x}_{1,min} - \varepsilon \leq \mathbf{x}_1[k] \leq \mathbf{x}_{1,max} + \varepsilon$$
$$\varepsilon \geq 0$$
$$0 \leq \mathbf{u}[k] \leq \mathbf{U}\mathbf{A}_{e,max}(\mathbf{x}_2 - \hat{T}_e)$$
$$\hat{T}_e \leq T_{e,max}$$

Update

$\mathbf{u}[k] =$ first move in obtained $\mathbf{u}$

$\hat{T}_e[k] =$ first move in obtained $\hat{T}_e$

$\mathbf{OD}[k] = \mathbf{K}_e(\hat{T}_e)\mathbf{u}[k]$

Control inputs

$\mathbf{OD}[k],\ \hat{T}_e[k]$

A very short prediction horizon may jeopardize stability of the control system [15]. It is too difficult — if not impossible — to determine the lowest possible prediction horizon analytically. In the objective function (21), COP is kept constant all over the horizon. The longer the horizon, the more bias in COP variations due to the variation of the outdoor temperature — the latter affect the condensation pressure and accordingly the COP. Therefor, the prediction horizon should be long enough to ensure the stability, and, on the other hand, not be so long to fulfill the prediction performance in terms of the COP assumption.

IV. Simulation Results

In this section, the proposed MPC scheme is applied to a high-fidelity simulation benchmark developed based on the model explained in [10]. The model is validated against real data obtained from a supermarket refrigeration system including 7 MT and 4 LT fridge and freezer display cases and a cold room, and two stages of compressor racks.

Based upon the discussion made in Section III-D, the sampling time and prediction horizon are chosen as $T_s = 5$ min and $N = 12$, respectively. In order to have a feasible solution for the optimization problem, slack variables, ε, are employed to soften the state constraints as explained in [16] and also shown in Algorithm 1.

A. COP Optimization

A simple power reference contains the baseline of the power consumption profile during a day is applied to investigate the COP improvement made by the novel $\hat{T}_e$ control method. For this purpose, the proposed method is compared to the case where a fixed $\hat{T}_e$ in the middle of its possible range is applied. The MPC design of the latter is the same as of Algorithm 1, but using a fixed set-point for evaporation temperature.

Fig. 2 shows that how the MPC using the $\hat{T}_e$ control can track a very low baseline while the fixed $\hat{T}_e$ failed to follow, because, otherwise, it would violate the temperature constraints. The reason of rising the baseline after around 9 AM is that the load increases due to increase of the outdoor temperature. The COPs are compared in Fig. 3 where an improvement with the average of 22% is achieved by the COP optimization. The lower baseline the consumer can follow, the lower energy cost it should pay.

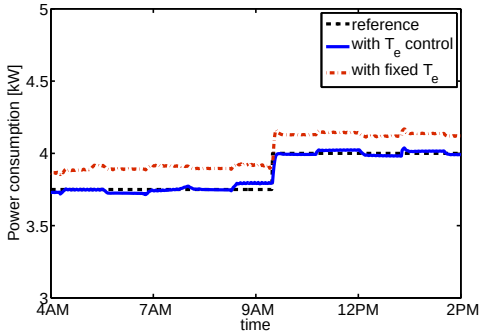

Fig. 2. Following a low baseline profile of a power reference. The MPC scheme with $\hat{T}_e$ optimization shows a satisfactory performance while it fails with the constant $\hat{T}_e$.

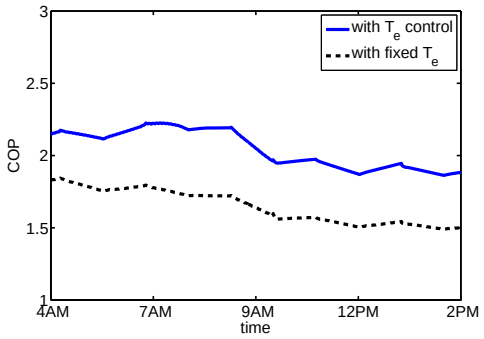

Fig. 3. A higher COP is achieved by the $\hat{T}_e$ control method.

B. Energy Balancing Service

The purpose of the following simulation experiment is to show the ability of the proposed control algorithm in case of significant change in the power reference for upward and downward regulation services. The power reference is increased 75% at 12 PM up to 13 PM for downward regulation services. For upward regulation, the refrigeration system needs to store thermal energy sometime ahead of the service start time. Consequently, at 15 PM the power reference is increased 12.5% for energy storage, and then is dropped significantly (87.5%) at 18 PM up to 19 PM.

The tracking result is represented in Fig. 4. A high performance for the power regulation is obtained by the MPC algorithm. The first step in the power reference after 9 AM is due to the baseline profile. The evaporation temperatures of MT and LT units are shown in Fig. 5. The two big caves in the figures come about when the MPC needs to decrease the evaporation temperature to be able to apply the required

large cooling capacity during the significant increase of the power reference.

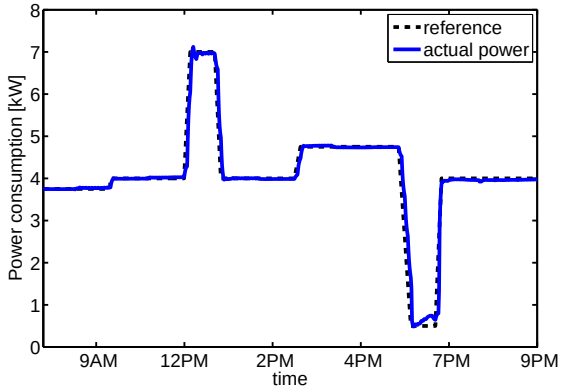

Fig. 4. Power reference following for energy balancing services.

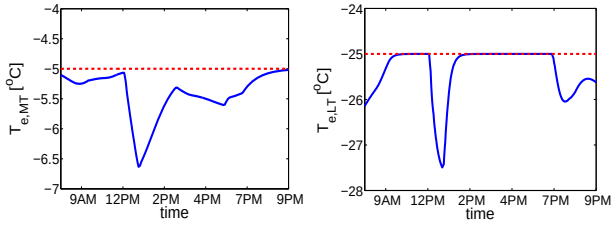

Fig. 5. Evaporation temperatures for both the MT and LT sections.

Fig. 6 shows the opening degrees of the expansion valves. There is a visible correlation between the *OD* variations and variations of the power consumption. The food temperatures and constraints are provided in Fig. 7. The food temperatures increase/decrease by decreasing/increasing the power consumption which shows the correspondence of the electrical power regulations with storing and delivering of the thermal energy.

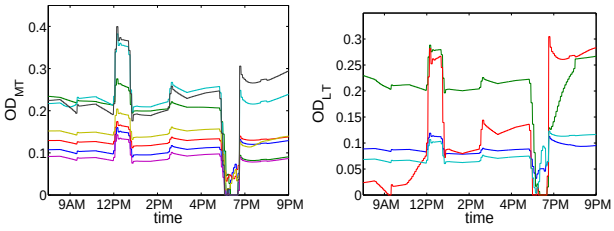

Fig. 6. Opening degree of the electronic expansion valves.

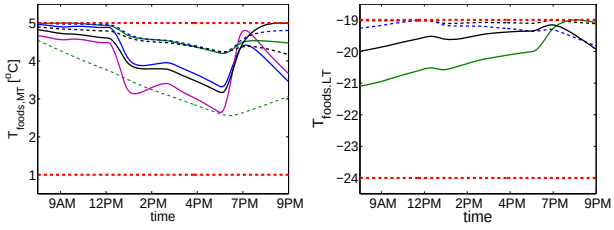

Fig. 7. Food temperatures of the different cooling sites belong to the MT and LT sections. All the temperature constraints are respected.

V. CONCLUSIONS

A model predictive control scheme was proposed for flexible power consumption of refrigeration systems. The proposed control strategy facilitates the demand response required for energy imbalance management services. By introducing a fictitious manipulated variable, a convex optimization problem was formulated within the MPC scheme. A novel incorporation of the evaporation temperature set-point into MPC formulation was presented for COP optimization. The COP improvement with the average of 22% and, consequently, the lower baseline of the power consumption were achieved by the COP optimization method. Simulation experiments showed that the proposed MPC algorithm is able to regulate the power references with significant magnitude changes of at least 75% from the baseline.

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