



AALBORG UNIVERSITY
DENMARK

Aalborg Universitet

Manual for wave generation and analysis

software in Matlab

Jakobsen, Morten Møller

Publication date:
2015

Document Version
Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

Citation for published version (APA):
Jakobsen, M. M. (2015). Manual for wave generation and analysis: software in Matlab. Aalborg: Department of Civil Engineering, Aalborg University. DCE Technical reports, No. 191

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- ? Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- ? You may not further distribute the material or use it for any profit-making activity or commercial gain
- ? You may freely distribute the URL identifying the publication in the public portal ?

Take down policy

If you believe that this document breaches copyright please contact us at vbn@aub.aau.dk providing details, and we will remove access to the work immediately and investigate your claim.



DEPARTMENT OF CIVIL ENGINEERING
AALBORG UNIVERSITY

Manual for wave generation and analysis software in Matlab

Morten M. Jakobsen

Aalborg University
Department of Civil Engineering
Group Name

DCE Technical Report No. 191

Manual for wave generation and analysis software in Matlab

by

Morten M. Jakobsen

July 2015

© Aalborg University

Scientific Publications at the Department of Civil Engineering

Technical Reports are published for timely dissemination of research results and scientific work carried out at the Department of Civil Engineering (DCE) at Aalborg University. This medium allows publication of more detailed explanations and results than typically allowed in scientific journals.

Technical Memoranda are produced to enable the preliminary dissemination of scientific work by the personnel of the DCE where such release is deemed to be appropriate. Documents of this kind may be incomplete or temporary versions of papers—or part of continuing work. This should be kept in mind when references are given to publications of this kind.

Contract Reports are produced to report scientific work carried out under contract. Publications of this kind contain confidential matter and are reserved for the sponsors and the DCE. Therefore, Contract Reports are generally not available for public circulation.

Lecture Notes contain material produced by the lecturers at the DCE for educational purposes. This may be scientific notes, lecture books, example problems or manuals for laboratory work, or computer programs developed at the DCE.

Theses are monographs or collections of papers published to report the scientific work carried out at the DCE to obtain a degree as either PhD or Doctor of Technology. The thesis is publicly available after the defence of the degree.

Latest News is published to enable rapid communication of information about scientific work carried out at the DCE. This includes the status of research projects, developments in the laboratories, information about collaborative work and recent research results.

Published 2015 by
Aalborg University
Department of Civil Engineering
Sofieldalsvej 9-11
DK-9200 Aalborg SV, Denmark

Printed in Aalborg at Aalborg University

ISSN 1901-726X
DCE Technical Report No. 191

Contents

Preface	vii
1 Manual for wave generation and analysis software in Matlab	1
1.1 Wave generation	1
1.2 Wave analysis	2
2 Implementation of 3D wave analysis	5
2.1 Maximum Likelihood Method	6
2.2 Bayesian Directional Method	7
Bibliography	9

Preface

This Manual is for the included wave generation and analysis software and graphical user interface. The package is made for Matlab and is meant for educational purposes. The code is free to use under the GNU Public License (GPL). It is still in development and should be considered as such.

If you have questions, suggestions, or additions to the code you can contact the author.

Morten Møller Jakobsen
mmj@civil.aau.dk
<http://homes.civil.aau.dk/mmj/>
Sofiendalsvej 11, Room: 11.218
9000 Aalborg

Chapter 1

Manual for wave generation and analysis software in Matlab

The wave generation and analysis software presented here is an open source software package for Matlab. While the code is inspired by the Wavelab by Andersen (2010) and DIWASP by MetOcean Solutions LTD (2002) software it is written independently. The software by this author is for educational purposes only and should not be used commercially. The software has a Graphical User Interface (GUI) facade. The interface calls separate generation and analysis scripts that can run without the GUI.

To start the program browse to the *GUI* folder and open the `GenerationGUI.m` or `AnalysisGUI.m` in Matlab, then click *Run* or press *F5*.

1.1 Wave generation

To generate unimodal waves¹ a Random Phase Method (RPM) is used. The method uses inverse Fourier transformation to calculate the coefficients in the discrete spectrum based on Frigaard and Andersen (2010). For the bimodal spectrum the much slower superposition of regular waves is used.

Running the wave generation software the *GenerationGUI* opens as seen in Fig. 1.1. The output formats implemented is a generic type based on Hawkes et al. (1997) “*Comparative Analyses of Multidirectional Wave Basin Data*” and another format readable by Wavelab.

Clicking the *Load Array* button opens the *WGArray* window. After selecting the nodes to be used for the array and the wave parameters the target spectra can be examined using the *Plot* option as seen in Fig. 1.2. The text based outputs from the software is put into structures for a clean workspace. The two structures *geninput*

¹single peak spectrum, i.e. only swell waves or wind waves

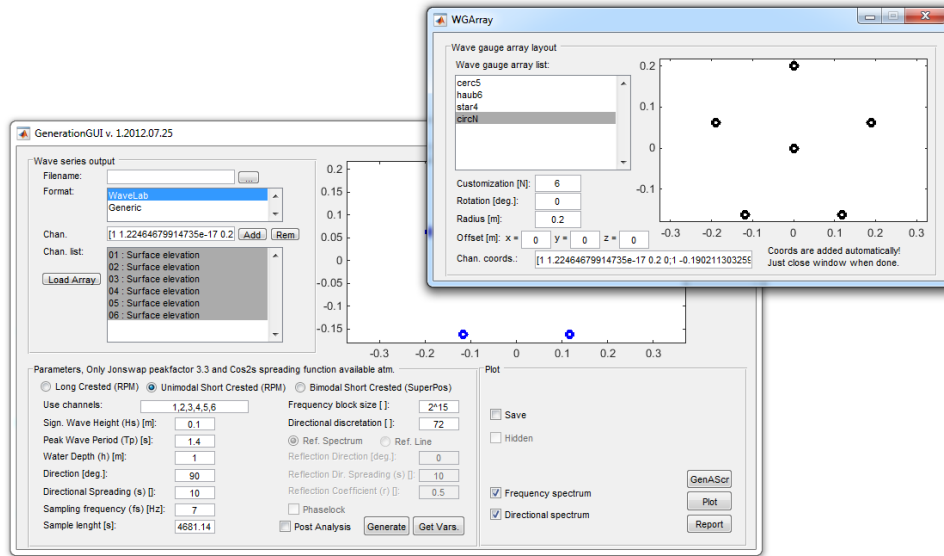


Figure 1.1: Wave Generation GUI.

and *generation* contain the inputs to the generation scripts and the output respectively. To run the wave analysis on the generated wave series the *Post Analysis* checkbox can be ticked. Alternately the saved file can be analyzed manually later.

1.2 Wave analysis

The 3D wave analysis methods referred to as Bayesian Directional Method (BDM) and Maximum Likelihood Method (MLM) has been implemented as seen in Fig. 1.3. The methods and implementations are explained in Section 2.

To do the analysis the water depth and the sampling frequency used in the samples are needed. The selection of incident and reflected wave directions are only used to process reflection coefficients and the calculation of separate incident and reflected significant wave heights.

The output from the analysis software can be plotted similar to the wave generation in Fig. 1.2. The two structures *anainput* and *analysis* contain the inputs to the model and the output respectively.

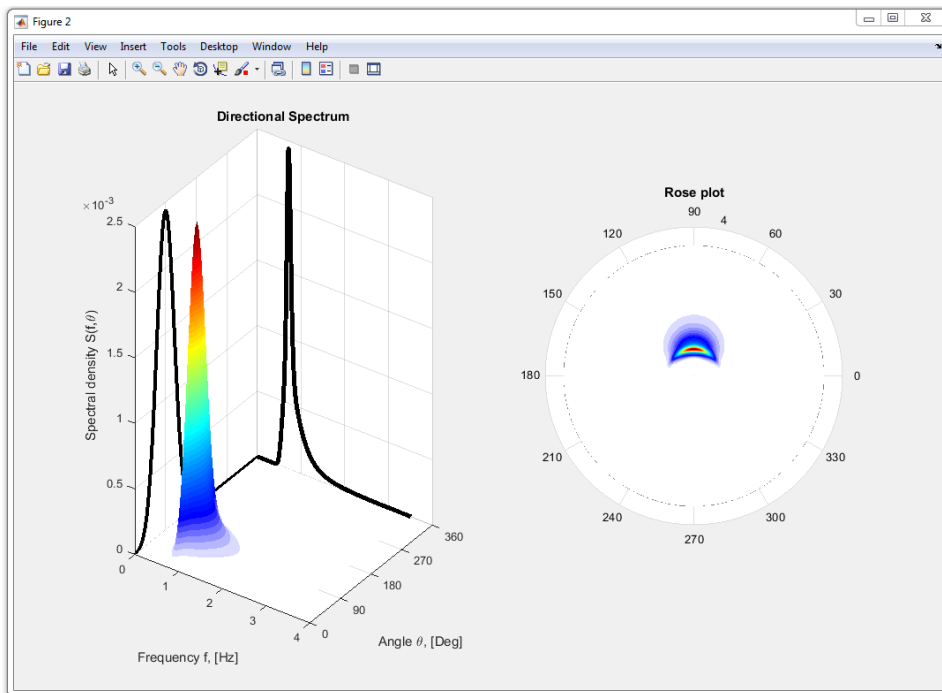


Figure 1.2: Plots of two common representations of the directional spectrum.

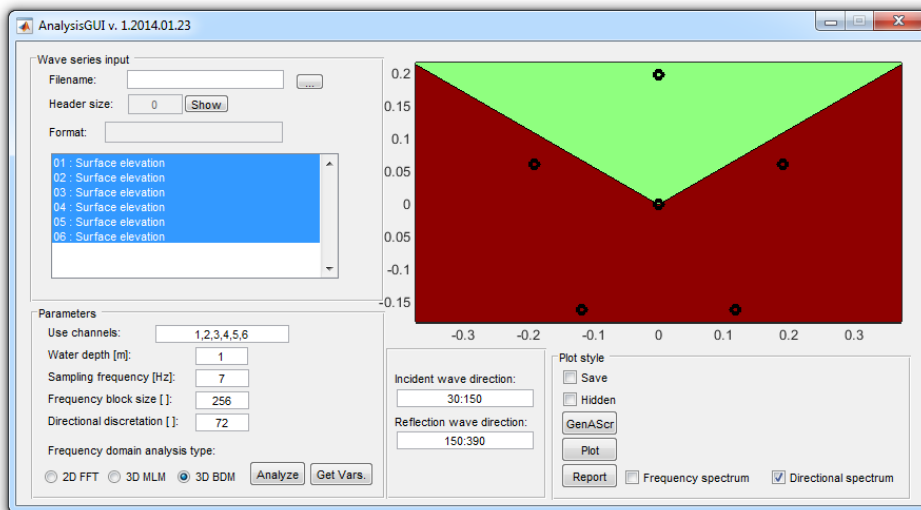


Figure 1.3: Wave analysis user interface.

Chapter 2

Implementation of 3D wave analysis

It is recommended to read the literature which outlines and explains the most commonly used methods to estimate the directional spectrum i.e. Hashimoto et al. (1987); Isobe et al. (1984); Davis and Regier (1977); Sand (1979); Benoit et al. (1997); Hawkes et al. (1997). This section will be an abridged version of the underlying theory and an explanation of how these methods are derived from the probabilistic methods.

Relation between the directional spectrum $S(f, \theta)$, the frequency spectrum $S(f)$ and the spreading function $D(\theta|f)$ is shown in Eq. 2.1.

$$S(f, \theta) = S(f) \cdot D(\theta|f) \quad (2.1)$$

The spreading function is subject to the constraint in Eq. 2.2.

$$\int_{-\pi}^{\pi} D(\theta|f) d\theta = 1 \quad (2.2)$$

To do the 3D analysis it is necessary to estimate the directional wave spectrum $S(f, \theta)$. A relation between the directional wave number-spectrum $S(k, \sigma)$ and the spectral matrix $\Phi(f)$ is outline through Fourier Transformation in Eq. 2.3 Isobe et al. (1984).

$$\Phi_{mn}(\sigma) = \int_k H_m(k, \sigma) H_n^*(k, \sigma) e^{-ik(\mathbf{x}_{nm})} S(k, \sigma) dk \quad (2.3)$$

Where k is the wave number can be determined from the dispersion equation using either Chebyshev(used here) or Newton-Raphson convergence theorem. H is a transfer function which depends of the type of sensor data used, $H = 1$ is used if the time series are from surface elevation measurement by Hashimoto et al. (1987); Benoit et al. (1997). \mathbf{x}_{nm} represents the distance between each gauge pair, which can be described by Cartesian or Polar coordinates in Eq. 2.4 and 2.5 respectively.

Eq. 2.1 is used to isolate the spreading function rather than the directional wave spectrum, cf. Sand (1979).

$$\frac{\Phi_{mn}(f)}{S(f)} = \int_{-\pi}^{\pi} H_m(f, \theta) H_n^*(f, \theta) e^{-ik((x_m - x_n) \cdot \cos(\theta) + (y_m - y_n) \cdot \sin(\theta))} D(\theta|f) d\theta \quad (2.4)$$

or

$$\frac{\Phi_{mn}(f)}{S(f)} = \int_{-\pi}^{\pi} H_m(f, \theta) H_n^*(f, \theta) e^{-ik r_{mn} \cos(\theta - \beta_{mn})} D(\theta|f) d\theta \quad (2.5)$$

The solution to Eqs. 2.3, 2.4 and 2.5 is the subject to a wide range of analyzing procedures. Those used in the software are explained in the following.

2.1 Maximum Likelihood Method

The first implementation of the Maximum Likelihood Method was introduced in the field of seismic research but is useful within marine research using wave gauge arrays as well, cf. Capon et al. (1967); Capon (1969).

$$\hat{P}(\gamma, k) = \left[\sum_{m,n} \Phi_{mn}(\gamma)^{-1} e^{ik\mathbf{x}_{mn}} \right]^{-1} \quad (2.6)$$

where \hat{P} is the estimate of the power output subject to the Maximum Likelihood filter. γ is the normalized frequency $\gamma = \omega T$. \hat{P} is proportional to the directional wave spectrum, so under the constraint of Eq. 2.2 the expression can be rewritten to Eq. 2.7.

$$\hat{S}(k, \sigma) = \alpha \cdot \left[\sum_{m,n} \Phi_{mn}(\sigma)^{-1} e^{ik\mathbf{x}_{mn}} \right]^{-1} \quad (2.7)$$

$\Phi_{mn}(\sigma)^{-1}$ being the inverse spectral density matrix and α being a proportionality constant. To include other types of sensory data an extension was made to the MLM, which included the transfer function H as suggested by Isobe et al. (1984).

$$\hat{S}(k, \sigma) = \alpha \cdot \left[\sum_{m,n} \Phi_{mn}(\sigma)^{-1} H_m^*(k, \sigma) H_n(k, \sigma) e^{ik\mathbf{x}_{mn}} \right]^{-1} \quad (2.8)$$

The implementation in the analysis code uses the spreading function as suggested earlier together with the constraint in Eq. 2.2 to determine α . Then knowing the spreading of the waves the directional wave spectrum is determined by Eq. 2.1. One of the issues with the method is the inverted matrix that makes the method vulnerable to truncation errors in the spectrum where there is little energy.

2.2 Bayesian Directional Method

The Bayesian Directional Method is the method suggested by default as it has shown to be more accurate and reliable by Hashimoto et al. (1987); Hashimoto and Kobune (1988). This method only makes assumption about local smoothness of the spectrum. By introducing the sample data functions x and y and implicitly assuming that the expression is subject to a priori information Bayes' Theorem is expressed in Eq. 2.9.

$$p(y|x) = \frac{p(y)p(x|y)}{p(x)} \quad (2.9)$$

Considering x as the measured data function the model distributions $p(x|y)$ is interpreted as the likelihood function $L(x, \sigma^2)$. $p(y)$ is the prior information redefined as $p(x|u^2, \sigma^2)$ and $p(y|x)$ is the posterior distribution, $p_{post}(x|u^2, \sigma^2)$. This leads to the proportionality in 2.10.

$$p_{post}(x|u^2, \sigma^2) \propto L(x, \sigma^2)p(x|u^2, \sigma^2) \quad (2.10)$$

Where the prior distribution is given by 2.11 and the likelihood function by 2.12.

$$p(x|u^2, \sigma^2) = \left(\frac{u}{\sqrt{2\pi}\sigma} \right)^K \exp \left(-\frac{u^2}{2\sigma^2} \sum_{k=1}^K (x_k - 2x_{k-1} + x_{k-2})^2 \right) \quad (2.11)$$

$$L(x, \sigma^2) = \frac{1}{(2\pi\sigma^2)^N} \exp \left(-\frac{1}{2\sigma^2} \sum_{i=1}^{2N} \left(\phi_i - \sum_{k=1}^K \alpha_{i,k} \exp(x_k) \right)^2 \right) \quad (2.12)$$

where

$$M = N(N + 1)/2 \quad (2.13)$$

$$\phi_i(f) = \Re \left(\frac{\Phi_{mn}(f)}{S(f)W_{mn}(f)} \right), \quad i = 1, \dots, M \quad (2.14)$$

$$\phi_i(f) = \Im \left(\frac{\Phi_{mn}(f)}{S(f)W_{mn}(f)} \right), \quad i = M + 1, \dots, 2M \quad (2.15)$$

$$\alpha_{i,k}(f) = \Re \left(\frac{\Delta\theta H_m(f, \theta_k) H_n^*(f, \theta_k) e^{ik\mathbf{x}_{nm}}}{W_{mn}(f)} \right), \quad i = 1, \dots, M \quad (2.16)$$

$$\alpha_{i,k}(f) = \Im \left(\frac{\Delta\theta H_m(f, \theta_k) H_n^*(f, \theta_k) e^{ik\mathbf{x}_{nm}}}{W_{mn}(f)} \right), \quad i = M + 1, \dots, 2M \quad (2.17)$$

Where N is the number of wave guages, W is a weighting function, u is a hyperparameter and σ is the standard deviation. These are chosen through Akaike (1980)'s Bayesian Information Criterion (ABIC), by minimizing Eq. 2.18.

$$ABIC = -2\ln \int L(x, \sigma^2)p(x|u^2, \sigma^2)da \quad (2.18)$$

Which should be maximized to obtain the estimate of x . In the implementation x is discrete, which is distinguished by x_k , where $k = 1, \dots, K$. The directional spreading function is then obtained from Eq. 2.19.

$$\hat{D}(\theta|f) = \sum_{k=1}^K e^{x_k(f)} I_k(\theta) \quad (2.19)$$

$$I_k(\theta) = \begin{cases} 1 & : (k-1)\Delta\theta \leq \theta < k\Delta\theta \\ 0 & : \text{otherwise} \end{cases} \quad (2.20)$$

It is significant to note that the method to calculate W of the spectral density functions was greatly improved by Hashimoto (1997). The newer method is implemented here which is defined in Eq. 2.21.

$$W_{mn}(f) = \sqrt{(\Phi_{mm}(f) \cdot \Phi_{nn}(f) \pm C_{mn}(f)^2 \pm Q_{mn}(f)^2)/2N_a} \quad (2.21)$$

Where N_a is the number of ensemble averages. When calculating the weighting function Eq. 2.16 uses positive coincident spectral density and negative quadrature while 2.17 uses negative coincident spectral density and positive quadrature. It should be noticed that this new method can result in $W_{mn}(f) = 0$, and in this rare case it is revert back to the old method in the implementation.

In an attempt to get even better results a *relaxation* of the new estimates are performed when no new solution was found. For stability and to avoid long computational time, the number of iterations are limited which dramatically reduces the computational time at the cost of some accuracy. The expressions in Eq. 2.22 and 2.23 are used to reduce the computational time need for convergence further suggested by Hashimoto (1997).

$$u = ab^m \quad , a = 1.0, b = 0.5, m = 1, 2, \dots \quad (2.22)$$

$$x_0(i) = \ln(1/2\pi) \quad , i = 1, \dots, K \quad (2.23)$$

Bibliography

- Akaike, H. (1980). Likelihood and the Bayes procedure. *Bayesian statistics, Valencia: University Press*, 31(1):143–166.
- Andersen, T. L. (2010). *WaveLab*. Version 3.0 and 3.34. Download: 03-05-2010.
- Benoit, M., Frigaard, P., and Schäffer, H. (1997). Analysing multidirectional wave spectra: A tentative classification of available methods. In *IAHR Seminar Multidirectional Waves and their Interaction with Structures*, 27th IAHR Congress. The International Association for Hydro-Environment Engineering and Research, The National Research Council of Canada.
- Capon, J. (1969). High-Resolution Frequency-Wavenumber Spectrum Analysis. *Proceedings of the IEEE*, 57(8):1408–1418.
- Capon, J., Greenfield, R. J., and Kolker, R. J. (1967). Multidimensional Maximum-Likelihood Processing of a Large Aperture Seismic Array. In *Proceedings of the IEEE*, volume 55, pages 192–211. IEEE.
- Davis, R. E. and Regier, L. A. (1977). Methods for estimating directional wave spectra from multi-element arrays. *Journal of marine research*, 35(3):453–477.
- Frigaard, P. and Andersen, T. L. (2010). Technical Background Material for the Wave Generation Software AwaSys 5. Technical Report 64.
- Hashimoto, N. (1997). *Analysis of the directional wave spectrum from field data*, volume 3 of *Advances in coastal and ocean engineering*, pages 103–143. World Scientific Publishing Co. Pte. Ltd.
- Hashimoto, N. and Kobune, K. (1988). Directional Spectrum Estimation from a Bayesian Approach. *Coastal Engineering*, 1:62–77.
- Hashimoto, N., Kobune, K., and Kameyama, Y. (1987). Estimation of Directional Spectrum using the Bayesian Approach, and its Application to Field Data Analysis. 26(5):57–100.
- Hawkes, P., Ewing, J., Harford, C., Klopman, G., Stansberg, C., Benoit, M., Briggs, M., Frigaard, P., Hiraishi, T., Miles, M., Santas, J., and Schäffer, H. (1997).

- Comparative Analyses of Multidirectional Wave Basin Data. In *IAHR Seminar Multidirectional Waves and their Interaction with Structures*, 27th IAHR Congress. The International Association for Hydro-Environment Engineering and Research, The National Research Council of Canada.
- Isobe, M., Kondo, K., and Horikawa, K. (1984). Extension of MLM for estimating directional wave spectrum. In *Symposium on Description and Modelling of Directional Seas*, volume A6, pages 1–15. DHI & MMI.
- MetOcean Solutions LTD (2002). *DIWASP, a directional wave spectra toolbox for MATLAB*. Centre for Water Research, University of Western Australia, Perth.
- Sand, S. E. (1979). *Three-Dimensional Deterministic Structure of Ocean Waves*. PhD thesis, Technical University of Denmark, Lyngby, Denmark.

Recent publications in the DCE Technical Report Series

