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Ejersbo, Lisser Rye; Misfeldt, Morten

Published in:
CERME10

Publication date:
2017

Document Version
Accepted author manuscript, peer reviewed version

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Lisser Rye Ejersbo[1] and Morten Misfeldt[2]

[1] Aarhus University, DPU, Copenhagen, Denmark; lre@edu.au.dk

[2] Aalborg University, Copenhagen, Denmark; Misfeldt@learning.aau.dk

This article is a description and discussion of a design research project in which we introduced a research idea about the influence of language on number concepts development into praxis on a school in grades K-3. Danish children have difficulties remembering the Danish number names because the Danish language resembles a primitive number concept in mathematical thinking. In the project, we renamed the numbers between 11- 99 after the base-10 system. Our hypothesis was that this system would help Danish students to get a more secure concept about the base-10 system. The project lasted for three years ending in spring 2016. Our results were so convincing that the school decided to continue using the mathematical number names, and other schools that heard about the project seem to be interested in using the system as well. In other words, the project goes from being a top-down project to a bottom-up project.

Keywords: Design research, base-10 system, grades K-2, top-down and bottom-up project.

Introduction

In this article, we describe a design research project, which took place in a public school in the primary grades K-3. We started with a hypothesis, and then it moved on to a design and then contact with a public school. We ran the project for three years and then worked with the school on its continuation. The main concern of our paper is that of existing and anchoring projects that have proven successful and engaged (part of) the organization with which we have collaborated.

Our research idea was provoked by the fact that the Danish number names (Ejersbo & Misfeldt 2011; 2015) have an etymology and wording that are both peculiar and impractical. For example, in Danish, the number 73 is *treoghalvfjerds* (three and half-four), the number 32 is *toogtredive* (two and thirty), and the number 16 is *seksten* (sixteen). These old roots are unknown to most students. Furthermore, the number names are abbreviated. In Danish, the number 70 or *halvfjerds* (half-four) was once named half-four-times-twenty (in Danish halvfjerdsindstyve), but the *times-twenty* (in Danish sindstyve) has been lost in the counting numbers but retained in the ordinal numbers (for further explanation, see Ejersbo and Misfeldt, 2011). Another concern is the irregularity of the number names between 10 and 20, where 11 and 12 have unique names while 13 to 19 each end with a ten. Also, the two-digit numbers from 13 to 99 have an inversion property (the ones are said before the tens), and the tens have names inspired by a 20-base system.

Even though Danish number names are particularly irregular, most European languages break away from the clear regularity of the base-10 place value system. So understanding the effects of such irregularities on mathematics teaching and learning is interesting. With that motivation, we developed a hypothesis of how speaking about numbers with specific regular words that resemble the base-10 system would be beneficial for learning about the numbers in an easier way. This

hypothesis has been tested with a three-year long intervention at a school in the Copenhagen area. The project has ended, and the intervention was to a large extent successful. The data about student learning confirms that the classes where the new words for numbers system was successfully implemented have very strong performances in the areas related to number sense and arithmetic. Therefore, some of the teachers and the school organization would like to continue the work. We consequently discuss the question of how to exit intervention projects while anchoring relevant practices from the intervention into the organization.

In the paper we explore how this initial hypothesis has been activated as a way of using language as a didactical tool in a design research project. We will describe the state of the art that allowed us to develop the project idea, and how the project proceeded. Then we will briefly state our results in terms of a more well developed and tested set of hypotheses. Finally, we will describe our *exit strategy* for leaving a better practice at the school and reaching out to other schools and teachers.

Number concepts, base-10 and number names

We know from the research literature that there are major differences in the kind of system and regularities a language uses to describe numbers. Most European countries have an irregular naming system for numbers between 11 and 20; both the German and Dutch systems feature an inversion property of the numbers between 13 and 99, similar to the Danish one. These inversion effects were studied by Moeller, Pixner, Zuber, Kaufman, and Nuerk (2011) for two-digit numbers, showing how inversion-related difficulties predict later arithmetic performance (for an overview, see Ejersbo and Misfeldt, 2011, 2015).

Different studies (Miura & Okamoto, 1989; Miura, Okamoto, Chungsoon, Steere, & Fayol, 1993; Miura, Okamoto, Vlahovic-Stetic, Kim, & Han, 1999) compared Japanese, Chinese, Korean, and English-speaking American first graders' (6–7 years old on average) cognitive representations and understanding of place value. The findings confirmed that the Asian-language speakers showed a preference for using base-10 representations to construct numbers, whereas English speakers favored using a collection of units. Note that a significant difference between American and Asian number names appears between the numbers 11 and 19, exactly when the base-10 system starts to use two digits. In Miura and Okamoto's (1989) study, children were asked to construct the numbers 11, 13, 28, 30, and 42 from sets of ten and unit wooden blocks. The results showed that 91% of the American first graders used unit blocks to represent the numbers on their first attempt. In contrast, about 80% of the Asian children used sets of ten blocks when representing the numbers on their initial attempt. These differences in cognitive representation were mainly ascribed to language (Miura et al., 1993).

Learning to count and understand the base-10 system are cognitive challenges involving many small steps. We have chosen to focus on oral counting, the cardinal principle of combining a name with a cardinal value, and the combination of words for a number, its cardinal value, and the digit sign.

Children typically learn the names of numbers as a long list of words and demonstrate knowledge of the stable order principle by almost always saying number words in a constant order while

emphasizing the last number (Goswami, 2008). The names are developed as sounds connected to the number of objects in the sets.

The developmental shift to understanding the number name as a cardinal value requires a qualitative shift in children's representation of numbers. The cardinal principle requires comprehension of the logic behind counting (Goswami, 2008) and the ability to judge the size of a set. It relies on a representation of quantitative information in which the coding of smaller quantities is different from that of larger quantities (Goswami, 2008). Children's conceptual understanding of numeration depends on their ability to make a connection between a number name and its cardinal value, which they learn to do by grouping and quantifying sets of objects (Thomas, Mulligan, & Goldin, 2002).

Learning how to connect the number name, its cardinal value, and the digit sign is another challenge. As discussed, two different systems must be combined with different representations. Becoming an expert at combining these two systems means developing rapid access to an automatic use of written numbers and simultaneously being able to multitask to solve other problems in parallel. If the two systems are iconic and support each other, the child encounters less difficulty in learning this skill, as is the case for Japanese-speaking children. If the two systems are irregular and therefore conflict with each other, it is more problematic for the child to understand and remember the connection among the name, the cardinal value, and the sign. Duval (2006) described this situation as a conversion between registers and observed that the congruent conversions seem the easiest for students, meaning that the representation in the starting register is transparent to the target register. One obvious solution is, therefore, to use a fully regular approach to saying the names of the numbers, which means saying "one-ten and four" instead of fourteen and so on. It is possible and easy to *create* such a logical system for naming the numbers in Danish, and thus this became our main project idea.

This reasoning helped to form the project idea of using such logical number names as a didactical tool.

The design of a research idea

Occupied by these issues of why and how different languages influence number concepts and perhaps even the ability to learn simple arithmetic, we designed a three-year project to take place at a Danish public school in the suburbs of Copenhagen.

Using design research (Cobb & Gravemeijer, 2008), we formulated our hypotheses for empirical investigation. Design research is defined as a family of methodological approaches in which instructional design and research are interdepended. It is continuous process between design and analysis as a method to develop the teaching processes through cycles. In our case we consist designing the plan, discussing materials, conduction the iterative runs, and evaluating the different cycles. The hypotheses were grounded in our initial understanding of the difficulties that Danish children experience with the Danish number names. The research builds on the following two hypotheses:

1. Number names function as cognitive artifacts; hence, a concordance between spoken and written language is sensible.

2. Language constitutes concepts, which is why clear terminology seems effective in developing lucid concepts.

Project intervention

To address the question of the influence of number names on number concepts, we contacted a Danish public school that could be interested to run the project together with us. We already knew the school, which made the access easier. We were invited to a meeting with the leading team of the school, including the headmaster, together with a small group of teachers from the school. We presented our project, and the participants accepted it for one year as a start. An evaluation would decide if it should continue additionally for two more years. We decided to involve all 10 classes—three grade 2, three grade 1, four kindergarten classes—and 9 teachers in the primary section of the school. The project combined the renaming of numbers with supporting the teachers in instructing the students. In each class, 20–35% was children who had migrated from other countries, but all the children spoke Danish, and all the teaching was in Danish. The entire research project was planned to last for three years. The data consists of students' performance in classroom observations, a number understanding test, teachers' portfolios, interviews with teachers and students, and notes from collaboration with teachers. At the end of the projects, we used the national test for evaluating the students' competencies in Algebra and Numbers.

The first year

The cooperation with the teachers and the classes were only possible because of the positive attitude from the headmaster of the school. She left it to us, the researchers and the teachers, to run the project. But she and the leading team was helpful and showed interest the whole time. The parents were informed officially by the headmaster about the project.

An in-service course for the involved teachers was the first thing to arrange and run. At this course many questions came up and were discussed. Should the teachers always rename both the names of the numbers? How should the fractions be named? Would the student get to know the normal Danish number names? We made a lot of decisions that day and agreed that the teachers should write a log with further questions that we as researchers should answer, either by discussion or by recording answers in the log.

The participants were now ready to start the next school year with the mathematical numbers.

Kindergarten: The Kindergarten (K) teachers were used to cooperating with each other and continued this work with the mathematical numbers. We observed the classrooms regularly and had follow-up meetings. The K teachers used both the mathematical number names and the normal number names when they named a number, or the students read a number. They also arranged joint counting for all using mathematical numbers, and they made materials for student use that helped the students to be aware of the base-10 position system, and the students became very familiar with the mathematical numbers. The parents were further informed at a meeting with the kindergarten teachers only; all in all, they implemented the mathematical numbers very easily.

Grade 1: The project proceeded differently in the first-grade classrooms. All these classes had new teachers, which is normal for students in the first grade in Denmark. We were in a real-world

situation with all the mess that exists there. The three first-grade classes had three different teachers who did not work together very often, and none of them continued the work done by the K class teachers. So the routines disappeared. The big difference in practice between first grade and the K class caused some chaos during the first two months. Furthermore, some of the first-grade teachers left the school or their classes during that period. But new teachers came, and during November and December the classes also worked regularly with the mathematical numbers. We were lucky that one of the newcomers believed in the project idea and was very involved with it. He became a teacher in two of the three first-grade classes, and his presence was a great benefit for the project.

Grade 2: We never observed any of these classes, but met with the teachers and discussed how they could implement the mathematical numbers in the best way. These students had already been in school for two years, and we decided that while we could not expect that they would naturally use the mathematical numbers, they should know them.

Evaluation: In the first year we were very busy collecting data, observing the seven classes, and trying to find the best ways to implement the mathematical numbers. The teachers' log idea never caught on, so we solved any problems during our monthly meetings with the teachers. At the end of the year, we tested all the students. The outcome of the test showed us that the student used both names for the numbers quite naturally. There was a slight tendency that the students were more secure from the spoken mathematical numbers to written numbers than from the spoken normal numbers to written numbers. In the K classes, we noticed that the students were much more secure in correctly recognizing and naming numbers between 10 and 20 than was normal for these classes.

During the year, we used the design research method as a way to evaluate the actual lesson related to how the whole project was running. We exchanged good ideas and noticed the progress and difficulties. We solved the difficulties in different ways and changed some of our means; but not our goal. We made a report of the first year with our results and data. It was positively received by the leading team, who decided to let the project continue for the next two years.

In our plan for the second year we decided not to observe in the K classes because it was new students, but more or less the same practice. Still, we meet with the K teachers once a month. We would do a brief orientation for the new mathematics teachers in the four new first grades, and we would follow the second grades more intensively.

The second year

For each year, we expanded our research with new K classes and with that also thirds grates. The K teachers could more or less develop and repeat with their new students what they had done first year. The newcomers in the K teacher group were taught by especially one of the K teachers taking the major responsibility for informing the new teachers. As we discussed in our meetings with the teachers, the work in the K classes went very smoothly.

The four new first grades had new teachers, luckily only two teachers with two classes each. They did not know anything about the project before they chose to teach first-grade mathematics, but cooperated from the start. We met with them and introduced them to the project, and visited their classrooms several times during the year.

The three second grades were the most interesting, because the students were in their second year of the project. Their teachers were very engaged and consistent in the use of the mathematical numbers. Each time we observed the classroom we talked with the students and asked how they felt using two different names for one number. The answers were surprising, because we had expected that the students might be irritated that they should remember to use the mathematical numbers:

Student 1: It is fun, and we like to use the mathematical number names, because then we always are able to remember the names.

Student 2: We also know the cardinal at once.

Student 3: It is a help to remember the names of the normal number as well.

It was a very positive response and we encouraged the students to tell the same to their parents. We believed that it would a good way for the students to inform their parents about their math lessons.

At the end of the year, we used the same test we had used in the first year but only with the first and second grade. We noticed again that the students' understanding of numbers was very good, both for the use of normal number names and for the mathematical number names

Due to the time we had to do the project, and how it developed in the school, we decided to minimize the observation to only the three third-grade classes, but we still met with the teachers from second grade. We did not observe the first grades or meet with their teachers. We continued with the K teachers' meetings.

The third year

We mostly concentrated our research on grade three and the kindergarten classes. The use of the mathematical number names seemed very natural for the students together with the normal number names, and they were bilingual in the numbers from 10–99. In third grade, the students were so familiar with the base-10 value system that they could transfer the knowledge to the decimal numbers, which meant that they easily answered questions correctly when asked to compare numbers like 0.4 and 0.25. In May 2016, the three classes had the national test in mathematics for third grade. Compared to the average of all the third graders in Denmark, one of the third-grade classes—the one that was observed most frequently—had an average score in Numbers and Algebra that was far above average. The other third-grade classes also showed a better result in Numbers and Algebra than the average third-graders.

The conclusion we draw was that the students using the new system showed a better understanding of the base-10 system. We saw these competencies, and met our goals, in all the classes which used the mathematical numbers. And because the national test investigates additional competencies in Numbers and Algebra, we dared to conclude that the students gained from using two names for the numbers.

During the following year, the school had a new headmaster and a new leading board, but because of the results, she decided to continue with the mathematical number names even though the research project stopped.

From Top-town to Bottom-up

With the decision that the school wanted to continue using the mathematical number names after our exit, we needed to design a plan for how it could be possible. Inspired by the research (Jarvis, 1999; Nielsen, 2001) we suggested the following plan:

1. All the teachers at the school should know that the project stopped as a research project, but that the project would continue as an intervention project with the teachers as the drivers.

2. One K teacher should be responsible for introducing the methods in the K classes for incoming teacher and for ideas to be exchanged among the K teachers.

3. A mathematics teacher should be responsible for orientation of the mathematics teachers in first grade each year and arrange a course at the beginning of the new school year, which everybody could join in.

4. There will still be access to the researchers for questions and other things; we are interested in the continuing process.

This plan was first discussed with the involved teachers who agreed to the work they should do, and then it was presented to the headmaster. She also agreed and was willing to find time for the teacher support.

We started the process with a course for mathematics teachers in the K-3 classes in August 2016. There were about 15 people at the course, which was organized and run by a K teacher, a mathematics teacher, and one of the researchers. At this course, we made a quick run through the ideas behind the project and how it had run in the previous three years. The teachers who had previously taught classes and been involved in the project exchanged ideas and views of the learning processes with the mathematical numbers. The K teacher told how she was at an in-service course for K teachers in the Copenhagen region and told about the project and how she and the math supervisor at the school videotaped how she used the mathematical number names in the K classes. The other participants at the in-service course showed a big interest in the project. A similar course will be held again, and the math supervisor has told us that there is already a big interest in this in-service course.

Discussion

This article is less concerned with the actual results of the investigations that we conducted in the school and more concerned with the transition from an intervention driven by research curiosity to an ongoing project driven by the school itself.

In spite of a low budget and few researchers, or maybe because of these limitations, we saw some teachers taking over the project in an especially engaged way. One teacher, in particular, took a lot of responsibility and during a period when other teachers were out sick, he taught all the three classes in third grade. Without him, we are not sure the project would have had the success it had. It

would neither be possible to run such a project without the support and interest we had from the leading team including the head master, who played an important role.

Perspectives

As it looks now, we hope that the use of mathematics number names will spread to other schools and continue to develop. Even though we officially stopped the project at a meeting with all the teachers at the school, we will continue with some kind of support if necessary. We will also stay in touch for our own sake.

Acknowledgment

We would like to thank Husum Skole, Copenhagen, for financial support together with the teachers, who worked for the project.

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