(D)effecting the Child

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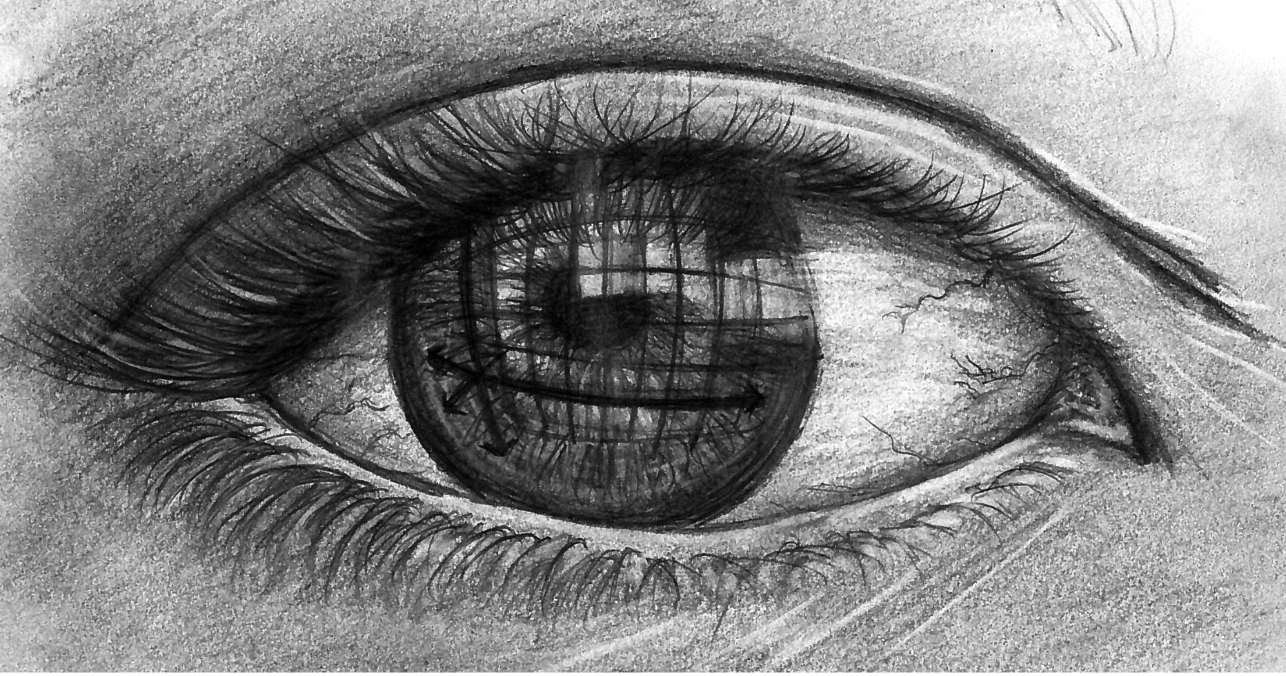

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THE SCIENTIFICATION OF THE SELF THROUGH
SCHOOL MATHEMATICS

BY
MELISSA ANDRADE-MOLINA

DISSERTATION SUBMITTED 2017

AALBORG UNIVERSITY
DENMARK

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ENGLISH SUMMARY

This Ph.D. dissertation explores the socio-political dimension of mathematics education. It problematizes the power effects of school geometry, particularly Euclidean geometry, and of Chilean school mathematics curriculum in the fabrication of modern and cosmopolitan subjects. Based on Foucault's discursive analysis strategies focusing on power/knowledge relationships, this study examines how a subject becomes the desired "scientific self" through technologies of the self, in terms of governmentality. In other words, it maps the scientification of the self through school mathematics, in which the desired citizen is shaped via the learning of elementary geometry.

The mapping is conducted in two moments. First, it uses a discourse analysis of the self-evident truths, within schooling, about the "goodness" of mathematics for the future of the students. These truths have granted mathematical knowledge salvation narratives as a secure way to achieve success, welfare, health, social commitment and economic stability. Second, it uses a historization of the present to map the continuities and discontinuities that make possible to fabricate a desired rational and scientific citizen. It seeks to understand how truths about the need for a scientific child for economic progress are established at a given place in a given time.

As a result of these two analytical strategies, this thesis demonstrates the emergence of what is called the (d)effect of power of a specific scientific subjectivity. In which, school mathematics not only aims at developing valuable skills and to improve the quality of education and the quality of life. It also has a disciplinary and normalizing (homogenizing) effect in the fabrication of the productive and desired subjected body.

As an article based thesis, or a collection of papers, the structure of this dissertation is composed of a framing text and twelve papers. The framing text consists of three parts: a prologue, eight small stories, and an epilogue. This is followed by the twelve articles. The prologue articulates the story of this research from its beginning as *a chronicle of events.* This section narrates the movements taken, from school geometry to art, to physics, to science, to socio-political mathematics education. This section also presents the twelve papers as a progression of diverse emerging problematizations to challenge the 'taken for granted' truths about school mathematics. Truths such as the belief in the need for logical thinking to cope with modern society or that welfare can be achieved through mathematics proficiency. *A chronicle of events* also presents the analytical strategies used in the twelve papers. These analytical strategies have been used to understand the relationship between

power and knowledge in mathematics education, in such relationship. Knowledge and power cannot be separated.

The prologue continues by describing how the articles in this dissertation were constructed. The section—*Rhizome: bending spacetime*—presents how the papers are compiled as a mapping of entangled events and discourses that have been (re)produced in different spatiotemporal configurations. The framing text, and also some of the articles of this dissertation, is intended to build a rhizome—a concept introduced by Deleuze and Guattari. The Rhizome is used as an analytical strategy to problematize what is perceived as self-evident and universal truths. It uses analogies of physics to give a visual image of the rhizomatic structure: a multiverse connected through wormholes. This analogy enables to present discourses to be flexible and mutable formations. The prologue finishes by introducing the following section of the framing text. *Through the Kafkaesque glass.* This section makes a connection between the fictions written by Franz Kafka and Foucault's understanding of power. This connection is used to merge the eight parts of a story that conform the second part of the framing text.

The second part of the framing text—the story—is structured around the twelve articles. Each article should be taken as a wormhole that connects the multiverse of eight parallel universes. These universes are unfolded in eight parts: *the dreamed, the desired, the shaped, the assessed, the rescued, the mutated, the (re) produced, and the (d)effected*. This structure is an attempt to portray a story without an unambiguous causality. The eight parts depict the story of the (d)effects of power in the scientification of the self through school mathematics as a technology of the self. The third part of the framing text, epilogue, unfolds the concept (d)effected, the class/room and the mathematical gaze within the scientification of the self.

The final section of the framing text contains the twelve papers. The papers contend how school mathematics is supposed to fabricate rational and logical subjects, via technologies and dispositives of power. School mathematics not only aims at developing valuable skills and to improve the quality of education and of life. It also has a disciplinary and normative effect. However, in developing countries of the western world this role of mathematics education appears to be 'natural'. The articles of this dissertation deal with techniques and strategies of government in the teaching and learning of mathematics. By building on Foucault's historical perspective on subjectivation, self and subjectivity are constituted in power relations and different forms of governance. These power techniques shape people's wishes, aspirations, interests, and convictions. These govern habits and desires towards the fabrication of the Modern subject as a *qualified citizen* (paper 10), a *scientific self* (paper 2), a *sightless eyed* (paper 1), what is called to be the *Mindniac* (paper 11). The papers map the dominant narratives within the *neoliberal dreams in mathematics education* (paper 8), the *promises of welfare* (paper 9), and *standardized assessment* (paper 6) as a dispositive of power (papers 5 and 7). The papers also present *the effects of power of school geometry* (paper 4) for the shaping of a desired self through the learning of Euclidean geometry that has been historically thought to shape a *scientific thinker* (papers 3 and 12).

This thesis contends that accountability, competition (through national and international standardized assessment) and marketing are not only one of the most important goals of the education, but, at the same time, these are a mean for achieving a better economy and progress through the teaching and learning of school mathematics. Schooling inserts students into school practices where they regulate themselves to become the desired scientific citizen, competent and excited to learn mathematics. The dreams of a rationalized society: Long live to the maths!

DANSK RESUME

Denne ph.d.-afhandling undersøger matematikundervisning fra et sociopolitisk perspektiv. Den problematiser magteffekterne af skolegeometri, specifikt euklidisk geometri, og af den chilenske matematikundervisnings læseplaner i udformning af det moderne og kosmopolitiske subjekt. Med udgangspunkt i Foucaults diskursive analysestrategier og magt-analytiske værktøjer undersøges, hvordan et subjekt bliver forvandlet til et ønsket "videnskabeligt selv" (*scientific self*) igennem styringsteknologier. Afhandlingen udfører med andre ord en kortlægning af, hvordan selvet videnskabeliggøres (*scientification of the self)* igennem matematikundervisning, hvorigennem det ønskede selv fabrikeres igennem indlæringen af elementær geometri.

Kortlægningen finder sted på to måder. Først udføres en diskursanalyse af de sandheder, som er blevet selvindlysende indenfor skolen og undervisningen, og som vedrører den indiskutable "godhed" af matematik, som noget der er nødvendigt og ønskeligt i forhold til elevernes fremtid. Der er opbygget en fortælling om frelse, hvor matematisk viden formodes at skulle være vejen til individets og samfundets succes, velfærd, sundhed, socialt engagement og økonomisk stabilitet. For det andet skrives en historie om nutiden (*a history of the present*) igennem en kortlægning af de kontinuerte og diskontinuerte omstændigheder, som gjorde det muligt at fabrikere en ønsket rationel og videnskabelig borger med dette specifikke indhold og på den specifikke måde. Afhandlingen søger hermed at forstå, hvordan sandheden omkring nødvendigheden af et videnskabeligt barn, som skal sikre økonomisk fremgang, er blevet etableret på et bestemt tidspunkt og bestemte geografiske omstændigheder.

Som et resultat af disse to analysestrategier, påvises hvad der refereres til som en (d)effect af specifikke videnskabelige subjektiviteter: Skolematematik har ikke alene som formål at udvikle værdifulde færdigheder for at optimere uddannelseskvaliteten og livskvaliteten. Skolematematik har også en disciplinerende og normaliserende (homogeniserende) effekt i form af en fabrikering af en produktiv, ønsket og subjektiveret krop.

Afhandlingen er en artikelbaseret afhandling og er sammensat af en rammetekst og tolv artikler. Rammeteksten består af tre dele: en prolog, en historie (med otte små dele), og en epilog. Denne efterfølges af de tolv artikler. Prologen fortæller historien om afhandlingens forskning fra sin begyndelse som *en krønike af begivenheder*. Dette afsnit fortæller om de bevægelser, der blev udtaget, fra skolegeometri, til kunst, til fysik, til videnskab, og til den sociopolitiske matematikuddannelse. Dette afsnit også præsenterer afhandlingens tolv artikler som en progression af forskellige problematiseringer, der udfordrer de sandheder om

skolematematik, der tages for givet i matematikundervisningen. Disse fabrikerede sandheder er blandt andet troen på nødvendigheden af logisk tænkning og matematiske færdigheder for, at mennesker skal kunne klare sig i samfundet efterfølgende og for, at man kan opretholde velfærdsstaten. *En krønike af begivenheder* præsenterer også de analytiske strategier, der blev brugt i disse 12 artikler. Disse strategier er blevet brugt til at forstå forholdene mellem magt og viden og den indflydelse, disse forhold har på matematikundervisningen. Viden og magt kan ikke adskilles i disse forhold.

Prologen fortsætter ved at beskrive, hvordan artiklerne i afhandlingen blev konstrueret. Afsnittet—*Rhizome: bending spacetime*—iscenesætter hvordan artiklerne er sammensat som en kortlægning af sammenviklede begivenheder og diskurser, som er blevet om- og reproduceret i forskellige temporal-rumlige konfigurationer. Rammen, og nogle af artiklerne i afhandlingen bygger et *rhizome*—et begreb introduceret af Deleuze og Guattari. *Rhizomet* bruges som analytisk greb til at problematisere, hvad der er opfattes som selvindlysende og universelle sandheder. Det bruger analogier til fysik til at præsentere et visuelt billede af en rhizomatisk struktur: et multiverst netværk af relationer forbundet via ormehuller. Denne analogi demonstrerer, hvordan diskurser er fleksible og mutable formationer. Prologen afsluttes med at introducere den følgende sektion af rammeteksten: *Through the Kafkaesque glass*. Denne tekst laver en forbindelse mellem fiktioner skrevet af Franz Kafka og Foucaults forståelse af magt. Denne forbindelse bruges til sammenflette de otte dele af historien, der udgør anden del af rammeteksten.

Den anden del af rammen—de historien—udgør afsættet for at forstå forbindelserne mellem afhandlingens tolv artikler. Hver enkelt af artiklerne skal betragtes som et ormehul, der forbinder multiverset af otte parallelle universer. Disse universer udfoldes i otte dele: *the dreamed, the desired, the shaped, the assessed, the rescued, the mutated, the (re)produced, and the (d)effected.* Deres struktur er et forsøg på at lave en fortælling uden en bestemt entydig kausalitet. De otte dele skildrer historien om (d)effekten af magt i den videnskabeliggørelse af selvet, som finder sted igennem skolematematik forstået som en selvteknologi. Den sidste del af rammeteksten, epilogen, udfolder begrebet "(d)effected" og det matematiske blik på et videnskabeliggjort selv.

Resten af afhandlingen udgøres af de tolv artikler. Artiklerne argumenter for, hvordan skolematematik foreskriver skabelsen af rationelle og logiske subjekter igennem magtteknologier og dispositiver. Skolematematik har ikke blot som formål at udvikle værdifulde kompetencer og optimere uddannelseskvalitet og livskvalitet. Den har også en disciplinerende og normaliserende effekt. I den vestlige verdens udviklingslande ser det dog ud til at være helt "naturligt". Artiklerne beskæftiger sig først of fremmest med styringsteknikker og styringsstrategier i matematik undervisning. Med udgangspunkt i Foucaults historiske perspektiv på subjektivering, argumenteres for, at selvet og subjektiviteten konstitueres i magtrelationer konkretiseret i forskellige styreformer. Magtteknikker leder menneskers ønsker, forhåbninger, interesser og overbevisninger. Disse styrer vaner og ønsker henimod fabrikationen af det moderne subject som en *kvalificeret borger* (artikel 10), *et videnskabeligt selv* (artikel 2), et *øje uden syn* (artikel 1), som kaldes *Mindniac* (artikel 11). Artiklerne kortlægger de dominerende narrative indenfor *de neoliberale drømme i matematikundervisningen* (artikel 8), *løfterne om velfærd* (artikel 9) og standardiserede evalueringer (chapter 6) og *som et magt-dispositiv* (artiklerne 5 og 7). Artiklerne præsenterer også *magteffekterne af skolegeometri* (artikel 4) i udformningen af et ønsket selv som en videnskabelig tænker (Artikel 3 og 12).

Denne afhandling hævder, at ansvarlighed (accountability), konkurrence (igennem nationale og internationale tests) og marketing ikke alene anses som nogle af de vigtigste mål i uddannelse. De anses også som nogle af vigtigste midler til at opnå en bedre økonomi and fremskridt gennem deres indlejring i matematikundervisningen. Skoling indsætter elever i skolepraksisser, hvor de lærer at regulere dem selv som ønskede videnskabelige borger, kompetente og ivrige efter at lære matematik. Drømmene om det gennemrationaliserede samfund: Matematikken længe leve!

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Open your mouth only if what you are about to say is more beautiful than silence.
Unknown.

Sometimes words are redundant or just not enough for expressing gratitude. Making a list of the people who have accompanied me in this journey is not necessarily a demonstration of appreciation. I was not alone! I would never have accomplished this Ph.D. without those who inspired me, who challenged me, who supported me, who talked to me, who read me, who listened to me, who advise me, who laughed at me, who understood me, who believed in me, who rejected me, who accepted me, who waited for me, who funded me[1], and without the one that left this Earth before seeing each other again… someday we will meet again.

I would like to say thank you to all of you.

[1] This research was funded by the National Commission for Scientific and Technological Research, CONICYT PAI/INDUSTRIA 79090016, in Chile

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Things should be made as simple as possible, but not simpler

Albert Einstein

PROLOGUE

This story began not with a question, but with a hopeless insistence on a matter: the presence of Euclidean geometry in school mathematics curriculum. More than a research query it was an intriguing curiosity to understand the interplay between the intuitive and perceptual encounter of school geometry problems—asserted to be from "real life" situations—and the axiomatization of space prompted by school mathematics curriculum. This dissertation became more an experiment, from a socio-political positioning, that expanded to a wide variety of aspects concerning this interplay. What started as a problematization of the power effects of school geometry as a technology of government that fabricates the modern subject, ended up merging with issues that connect mathematics and school mathematics with large-scale assessment, progress and the economic growth of nations, and the scientification of the self. Euclidean geometry opened the path to a broader aspect, often neglected, of mathematics education: school mathematics as a consumer's good for an ever-growing economy.

A CHRONICLE OF EVENTS

Since the beginning, this research was thought as an exploratory study of the connections between the optically perceived world (Crary, 1992)—what the eye is able to see—, and two and three-dimensional school geometry—or the space perceived in school. The first approximation started by looking at how "epistemological forms" shape the way in which humans understand the world. Taking artists as a first approximation, the study of paintings (as images) helped in noticing, for instance, the influence of religious beliefs in normalizing painters' thoughts and perception of reality, and therefore, their representations of the world. As an example, in theocentricism, Gothic paintings were about how reality appears to be to God (Kvasz, 1998). This led to depicting the optically perceived world, not as a person might see it, but as God perceived it: to God objects are not influenced by perspective given his omnipresence (figure 1).

This starting point enabled to examine how two approximations of "the visual", from theoretical physics—optics and its applications—and from phenomenological judgments are contrasted with Euclidean metrics[2]. In this movement, the work of Daston and Galison (2007) served as inspiration to problematize "the visual" as a historical configuration embedded in practices of objectivity for the scientification of knowledge. In their book, *Objectivity*, the study of images in scientific atlases

[2] In the section "The objectivation of space", in the paper *The sightless eyes of reason*, and in the section "The training of the untrained eye", in *The effects of school geometry in the shaping of a desired child.*

shed light on how the practices of knowing have effects both in the forms of knowing and in the subject—subjectivation.

Figure 1: Gothic painting[3]

They present a historical configuration of objectivity as a scientific practice that aspires to a form of knowledge that bears no trace of the knower (Daston & Galison, 2007), a way to break the mental world of individual subjectivity, a suppression of the self by the self. Reasoning with Daston and Galison's work, geometry was not a matter of seeing nature as it was, but as objective "epistemic virtues" to be studied. In other words, scientific practices led to the suppression of the visual as a process of seeing, through the eyes, for the achievement of an axiomatization of the visual through optics—physics—and through the study of the anatomy of the eye: A scientific gaze to approach the visual. Or as Foucault would express, a medical gaze able to dehumanizing the eyes:

> The access of the medical gaze into the sick body was not the continuation of a movement of approach that had been developing in a more or less regular fashion since the day when the first doctor cast his somewhat unskilled gaze from afar on the body of the first patient; it was the result of a recasting at the level of epistemic knowledge (savoir) itself, and not at the level of accumulated, refined, deepened, adjusted knowledge (connaissances). (Foucault, 1975, p. 137)

Nonetheless, approaching optics by looking at the physics of light (Newton (n.d.) in figure 2) was not a promising end for this analysis. And even though I enjoy physics as much as mathematics, the understanding of how Maxwell's equations set the theoretical foundations to establish a connection between optics and electromagnetism or their consistency with quantum optics, was not very fruitful to

---

[3] The Miracle of the child attacked and rescued by Augustine Novello. Painted by Simone Martini around 1328 (Retrieved from: https://commons.wikimedia.org/wiki/File:Simone_Martini_071.jpg)

grasp the initial research conundrum. [*If I may add an anecdote here: I still remember when I was at my desk following a demonstration of optics by using Fourier transforms, and my supervisor looked at my screen and said, almost surprised, "Melissa… are you reading mathematics?"*]. However, the reading of Euclid and Newton's treatises on optics led to problematize, how space has been objectified as an annulation of the self to escape from subjective interpretations. Such annulation granted space, in natural sciences, the privilege of being reached only by mathematics[4]. The discussion was not about the axiomatization of the visual—the mathematization of the optically perceived world—and its connection with school geometry; rather it was on the axiomatization of space. Building on Daston and Galison (2007), the suppression of the self led to perceive space and geometrical knowledge as decontextualized, and as universal and timeless: as objective scientifization of space.

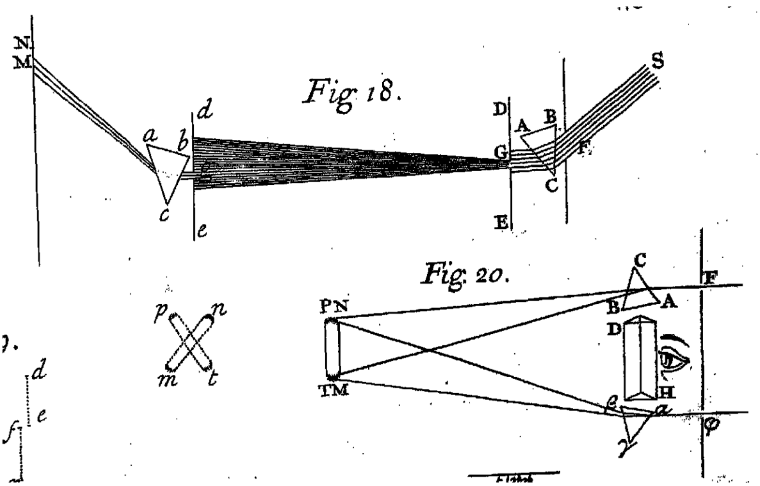

Figure 2: Newton's studies on the refraction of light

At that time I was puzzled about how in the school mathematics curriculum Euclid's axioms and postulates seem to prompt to an abstract understanding of the world. While in physics these help in understanding the Universe: "in special relativity, two particles on parallel trajectories would remain parallel forever, just as in Euclidean geometry" (Hamilton, 2017, p.51). Moreover, Einstein's special theory of relativity has approached the curved space by using Euclid's metrics.

> The behaviour of measuring-rods and clocks is influenced by gravitational fields, *i.e.* by the distribution of matter. This in itself is

[4] This exploration is also part of the section "The objectivation of space", in the paper *The sightless eyes of reason*.

> sufficient to exclude the possibility of the exact validity of Euclidean geometry in our universe. But it is conceivable that our universe differs only slightly from a Euclidean one, and this notion seems all the more probable, since calculations show that the metrics of surrounding space is influenced only to an exceedingly small extent by masses even of the magnitude of our sun. We might imagine that, as regards geometry, our universe behaves analogously to a surface which is irregularly curved in its individual parts, but which nowhere departs appreciably from a plane: something like the rippled surface of a lake. Such a universe might fittingly be called a quasi-Euclidean universe. (Einstein, 2001, pp. 115–116)

Within the special theory of relativity, Euclidean geometry enables to interpret "locally inertial frames" inscribed in a topological space that resembles—is homeomorphic to—a Euclidean 4D space ($\mathbb{R}^4$) (Hamilton (2017, p. 143), in Figure 3). The four-dimensional space-time continuum "in its most essential formal properties, shows a pronounced relationship to the three-dimensional continuum of Euclidean geometrical space" (Einstein, 2001, p.58).

Figure 7.4 shows an embedding diagram of the spatial Schwarzschild geometry at a fixed instant of Schwarzschild time t. To the polar coordinates r, θ, ϕ of the 3D Schwarzschild spatial geometry, adjoin a fourth spatial coordinate w. The metric of 4D Euclidean space in the coordinates w, r, θ, ϕ, is

$$dl^2 = dw^2 + dr^2 + r^2 do^2 \ . \tag{7.57}$$

The spatial Schwarzschild geometry is represented by a 3D surface embedded in the 4D Euclidean geometry,

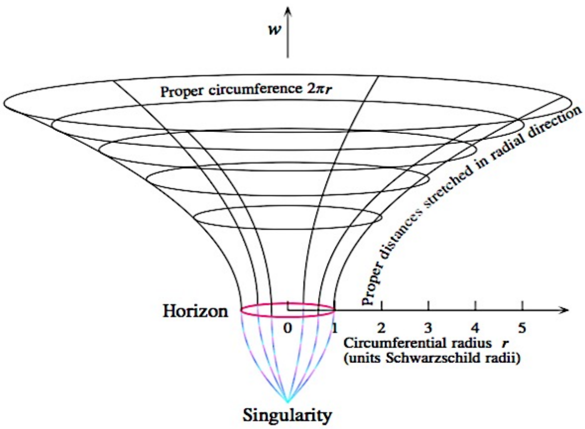

Figure 3: Embedding diagram)

This issue has been approached in the first paper: *The sightless eyes of reason: Scientific objectivism and school geometry.* Here it was the first attempt to problematize the discourse on school geometry. The journey started acknowledging the toolbox of Foucault's work as an introductory analytical strategy. The notion of "taken-for-granted" assumptions that circulate within social discourses helped in locating the dominant narratives of mathematics education towards the fabrication of, what it was called, *a desired child*: a platonic ideal product of schooling. It also helped in setting the ground to understand discourses neither as an imposition by a superior force nor as isolated assemblies, but as produced by the interaction of diverse spheres of social life (Foucault, 1972; 1982). When brought to mathematics education research, from a socio-political perspective, these circulating truths become naturalized and unproblematized understandings of mathematics education practices. This paper aimed at problematizing the dominant narratives about school geometry and evidencing elements of power effects of school mathematics in the fabrication of students' subjectivities. On the one side, there was a discourse within research in science and mathematics education, in documents by the National Research Council and from the Chilean Ministry of Education about the importance of developing visualization and spatial thinking. On the other side, there were curricular guidelines that present "everyday life problems" to be solved by using an axiomatization of space. Foucault's work allowed interpreting this mismatch as an evidence of the fabrication of the "sightless eyes of reason". In which school geometry, as a technology of the self (Foucault, 1982), pursues shaping the scientific minds of the future by cutting the links with the body—in this case, the visual.

The exploration conducted in this paper set the discussion around the fabrication of the sightless eyes of reason. Part of the research in science education analyzed in this first paper raised the importance of visualization and spatial reasoning for STEM domains. This relevance has been expressed through the implication of spatial techniques in the breakthrough of significant scientific discoveries (see e.g. Newcombe, 2010; Kozhevnikov et al., 2007). The need of incorporating "spatiality" in the curriculum has been a discourse circulating in research in mathematics education for many years. However, its justification in connection with locating talented youth with potential in STEM domains (Wai et al., 2009), or in fuelling school readiness to perform long-term gains in STEM domains—from early years—(Verdine et al., 2014), is quite recent. The aim of schooling became the locating of specific children needed for STEM domains and the encouragement of children to pursue science. This circulating discourse changed the focus of the research; the desired child was not expressed only regarding their potential abilities, skills, techniques, etc.—the ultimate development of reason and logic. It was also expressed in terms of "what the desired child is ought to become": the desired child for a desired job.

This exploration on the circulating discourses on science education research shifted the aim of the research to trouble the apparent connection between geometry and the development of science. It was conducted in two parts. Firstly, a discourse analysis, as an analytical strategy built again in Foucault's work, was useful in examining the (re)production of a naturalized truth about Euclidean geometry. In the paper *Shaping a scientific self: a circulating truth within social discourse*, discourses outside the fields of mathematics and mathematics education were taken to explore how the learning of Euclidean geometry, considered as a proper way of doing science, was ought to shape scientific thinking. Within this, it was believed that the understanding of Euclid's axioms and postulates enabled the access to all forms of human knowledge. Such dominant discourse was entangled in the forms of reasoning in areas such as architecture, theology, political science, literary education, and others. The *Elements* proved to be, to other fields of inquiry, a consistent, axiomatic, and rigorous paradigm, taken as a scientific model (Hartshorne, 2000). The concern for the production of potential scientists—the productive citizens—was explained by the learning of the method deployed in the *Elements*: A deductive system rooted in proofs and demonstrations. Secondly, as an exploration of how the method deployed in Euclid's books was entangled in Western notions of science. In the paper *The Euclidean tradition as a paradigm for scientific thinking*, statements regarding the constitution of a proper scientific approach were analyzed from three fields of knowledge: architecture, political science, and theology. Understanding the *Elements* as a rigorous paradigm inspired by Plato's Academy and Aristotle's organization of science, enable to explain the dominance of Euclidean geometry as a path to think "scientifically," for example, the theological argument in which the learning of elementary geometry was thought to develop reason (Frank, 2007). In this light, school geometry is not contributing only in shaping students forms of thinking about space and visualizing the world, but in conducting students to follow the path of what schooling perceives a real scientist is in order to fabricate productive citizens able to solve problems by using reason and logic.

The initial interest of this research was to move towards issues regarding the body as a "body without senses" for school mathematics practices[5]. However, the main comment I received from the first paper—*The sightless eyes of reason*—was about the taking of school geometry as a technology of government. The training of the self, as the conduct of conduct, through school geometry for the fabrication of the desired child was not self-evident, it needed clarification. Therefore, in the paper *The effects of school geometry in the shaping of a desired child* the original research interest shifted. Chilean curricular guidelines were analyzed to locate expressions of the desired product of schooling through geometry. Examining these materials with Foucaultian lenses enabled to map the shaping of the sightless eyes as a training process in which students regulate their own conduct[6]. This movement opened the path to addressing the effects of power and governmentality in school mathematics,

[5] But again, the initial intention was to set the discussion around the axiomatization of the visual.

[6] In the section "The training of the self", in the paper The effects of school geometry in the shaping of a desired child.

in which humans become subjects through the objectifying effects of scientific knowledge (Foucault, 1982). In this light, mathematics education converts into an important space of Modern government where subjectivity is fabricated. Building on Foucault's notion of governmentality as the "conduct of conduct" (Foucault, 2008), subjects self-regulate through technologies that are systematized, regulated, and reflected modes of power (Foucault, 1997). The section "Shaping the child through school geometry" addresses how geometry in schools insert students in mathematic practices in which they should accept to neglect their senses in order to navigate space according to certain desirable attitudes. The space deployed by schooling as a Cartesian plane inscribed in Euclidean metrics is contrasted with Lefebvre's (1991) notion of space as a product of concrete practices[7] to examine "school space" as a vector model in which students should navigate in terms of a coordinate system. *The effects of school geometry in the shaping of the desired child* reflects on how these technologies conduct individuals to change, structure, and constitute themselves as subjects, by enunciating the practices of the self—"models that [the subject] finds in his culture and are proposed, suggested, imposed upon him by his culture, his society, and his social group" (Foucault, 1984a, p. 291)—that pursue the fabrication of the desired child. In a Foucaultian sense, power enables the subject to embrace these models and act with them productively.

> [It] permit individuals to effect by their own means or with the help of others a certain number of operations on their own bodies and souls, thoughts, conduct, and way of being, so as to transform themselves in order to attain a certain state of happiness, purity, wisdom, perfection, or immortality. (Foucault, 1988, p. 18)

Until this point, school geometry as a technology of power for the shaping of a scientific self was the main focus of research. But, while analyzing the discourses circulating in Chilean school guidelines, an example from the Pisa sample test was used to portray the expressions of the desired child. This movement led to look at large-scale assessment in mathematics, or standardized assessment, as a dispositif of power: where power becomes concrete. To Foucault (1980, p. 194), the *dispositif*, or apparatus, is a network between diverse elements: "discourses, institutions, architectural forms, regulatory decisions, laws, administrative measures, scientific statements, philosophical, moral and philanthropic propositions." In his explanation, he contends three meanings of the *dispositif*: as a system of relations established by the above elements, as the nature of the connection between these elements, and as a formation that responds to historical-given "urgent needs." In this regard, standardized assessment is not only an instrument but also becomes a "thoroughly heterogeneous ensemble" that has a "dominant strategic function" (Foucault, 1980).

[7] In the section "Space through trained eyes", in the paper The effects of school geometry in the shaping of a desired child.

> [A] particular discourse can figure at one time as the programme of an institution, and at another it can function as a means of justifying or masking a practice which itself remains silent (Op. cit., p, 194).

The paper *Not another typical story, yet not a new critique: A journey to utopia across standardized assessment* looks at the continuities and discontinuities of the discourses around large-scale assessment from national and international agencies. Such discourses have been promoting the "goodness" of these types of evaluation for the development and progress of nations, or economies on OECD's terminology. The analysis of these discourses enables to explore the historical making of subjects in Chile and to problematize the "governing by numbers" (see Rose, 1991) in the (re)production of two statements: *higher scores means better quality* and *competitiveness and accountability lead to higher performance, raising incomes, social mobility and welfare*[8] This paper, by using a "not academically structured" style of writing, aimed at portraying standardized assessment as a *dispositif* that effects power in terms of governmentality. Governing people becomes "a versatile equilibrium, with complementarity and conflicts between techniques which assure coercion and processes through which the self is constructed or modified by himself" (Foucault, 1993, p. 204).

The gaze then was turned towards assessment in school mathematics as an important part of the dispositif. The exploration began with an analysis of the curricular guidelines for Chilean teachers in order to assess visualization. The paper *Assessing visualization: An analysis of Chilean teachers' guidelines* analyses the discourses about what is ought to be privileged in the classroom, in terms of assessment, by looking at the recommendations for teachers in curricular guidelines. It is contended that assessing visualization in schools, being an ability asserted to be important by policy makers and international agencies, has no recommendations on how it should be assessed. There is evidence of the effects of power of large-scale assessment in the conduction of teachers' conducts towards a training tradition. In this move the Ministry of Education advises teachers to use similar problems and style of questions as the standardized national test. By homogenizing both, classroom assessment and standardized tests, it is aimed to certify students' competencies on solving problems through the axiomatization of space, and through the mathematical gaze of the sightless eyes.

The paper *Tales of evaluation in school mathematics* is a story regarding the power effects of testing in the shaping of the desired child in connection with those of the fabrication of the sightless eyes of reason. This tale aims at depicting the entanglement between circulating truths about standardized assessments, the need for learning mathematics, and the axiomatization of space. It begins by raising the

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[8] In the section "This is…", in the paper Not another typical story, yet not a new critique: A journey to utopia across standardized assessment.

importance given to mathematics in school curriculum as a proper language for the sciences. In this configuration, mathematics was considered a useful tool for the improvement of science teaching (Deschamps et al., 1970). The analysis takes discourses around large-scale assessment and curricular guidelines to portray the role granted to assessment for the normalization of mathematics education intertwined with neoliberalism. Within these circulating discourses, assessment systems were taken as the path to achieving quality as a guiding system to help in decision-making processes. But large-scale assessment merged those processes with economic concerns, taking a whole new meaning. The need for higher proficiency in mathematics is explored through historical and cultural strategies as a fixation of raising scores to achieve the illusion of producing the desired scientific self.

School mathematics is not only a technology to shape the desirable productive scientific minds of the future able to see an axiomatized world. It is also a means to efficiently allocate resources, as a constant process of judging current policies to increase quality by competition of achieving rewards, and also as a system for guiding the decision-making process for economic growth and social welfare. This reading led to problematize circulating discourses naturalizing that better results in mathematics have to be taken as a synonymous of a good economy. Such belief grants school mathematics the status of an investment, a consumer's good for progress. Such contention is explored in the paper *Incepted neoliberal dreams in school mathematics and the "Chilean experience."* Here, school mathematics is analyzed in its configuration in a neoliberal form of economic steering. By using a series of dreams, the paper portrays how historical configurations—political, economical, and educational—are entangled with the teaching and learning of mathematics for the shaping of a desired and productive citizen. Mathematics is not only a technology of government for the shaping of the Modern subject. Under consumerism and competitiveness, mathematics is taken as the ideal product of the marketable education. Standardized tests results in mathematics are being correlated with social welfare and being used to predict students' future income. A marketable education leads to parents paying more tuition fee to obtain a better product. From this reading, mathematics becomes the merchandise of a system that needs to have consumers feeling addicted to it, by recognizing in it the key to a successful future[9].

As a safe bet for maintaining a marketable education and for shaping the productive citizens needed for progress, mathematics has been acknowledged as a redeemer of the economy. After the economic crises of 2008, OECD recommended investing in education. The paper *Be the best version of yourself! OECD's promises of welfare through school mathematics* analyses the naturalized truths in OECD's discourse on a state of welfare. Numeracy proficiency is taken as one of the key aspects to ensure a greater economic progress. It is raised the need of having "well-equipped-citizens"

[9] In the section "Incepted neoliberalism", in the paper Incepted neoliberal dreams and the "Chilean experience".

for people to reach their full potential. This is achieved by (re)producing a discourse in which higher mathematics attainment is translated into better health, longer life, higher employment rates, higher income, and many other advantages[10]. This narrative embodies the desired child as a mathematically skilled entrepreneur willing to participate fully in society. And, at the same time, it embodies double gestures of who needs to be saved, that produces processes of exclusion with those of inclusion—abjection (Popkewitz, 2008). The one in need of salvation becomes at the same time the included and the excluded: the "abjected." As Popkewitz (2011, p. 42) says: "The different child is to be rescued and saved from his or her unliveable spaces. The space of the all children is the space of a difference and abjection that cases the Other into unliveable spaces"[11].

The dominant narratives about the "well-equipped-citizens" inspired to analyze how discourses about the need of "qualified citizens" have been circulating as a historical configuration. The paper *The fabrication of qualified citizens: from the "expert-hand worker" to the "scientific minded"* explores the narratives on mathematics education as the key to assuring economic progress and growth by fabricating qualified people. For example, scientific and rational citizens were thought to be necessary to overcome the state of misery and poverty produced by colonization in Chile (Encina, 1981). Diverse measurements were taken to guarantee and enhance the proper teaching of science in schools: Fetching foreign experts—teachers and policy makers—, introducing European and American methods of teaching, and modifying the school mathematics curriculum to fit European standards. Science was thought to shape the "expert-hand worker" needed for increasing the production of agriculture and the industries: "Making science popular is the only path to improve […] the earth does not produce what it should without expert hands to grow it" (Amunategui, 1856, in Labarca, 1939, p. 144). Within this discourse, elementary geometry and drawing were considered powerful tools to "educate the senses"[12], and to develop skills that required applied knowledge—for example, in the study of the functioning of a machine[13]. Geometry and mathematics were considered useful tools for the sciences until the beginning of the 20th century in Chile when German teachers raised the concern about the status given to mathematics. Mathematics in school stopped being operational for workers. Instead, by following European

[10] In the section "OECD's promises of welfare", in the paper Be the best version of yourself! OECD's promises of welfare through school mathematics.

[11] In the section "From the hoped to the feared", in the paper Be the best version of yourself! OECD's promises of welfare through school mathematics.

[12]In the section "From the fabrication of expert-hand workers…", in the paper The fabrication of qualified citizens: from the "expert-hand worker" to the "scientific minded"

[13]In the section "…To the "technical worker…", in the paper The fabrication of qualified citizens: from the "expert-hand worker" to the "scientific minded"

standards, it became a necessary knowledge for securing the entrance to University. The new qualified citizen was shifted to the "highly educated scientific minded"[14] .

The "trusting in science" for assuring progress led to entangle economic interests, techniques of fabrication of citizens, neoliberal ideals of a marketable education, and the desire for the men of science. Such intertwine enabled to produce the comic book: *"Mindniac" the reasonable citizen of schooling (Chilean edition)*. In this paper, the desired of a scientifically skilled citizen emerged as a circulating and dominant discourse and the main purpose of schooling in Chile. Such need of a particular type of person has been fabricated through diverse techniques of power. And, although it would be easy to view the emergence of these types of persons as a timeline when arranged chronologically—the technical, the reasonable, the competent, and the well equipped—it is not. The fabrication of the Mindniac is an entanglement amongst all four movements. Each dominant narrative does not end when the other starts. Such discourses are even reproduced today as expressions of the need for highly qualified, scientific minded, self-entrepreneurs and consumers of knowledge, and life-long learners. This reading enables to understand each movement as (re)producing a specific feature of the Mindniac to overcome a state of underdevelopment of nations.

Finally, this need of a scientific child raised by the dominant discourses led to problematize the *Shaping of the scientific self* through Euclid's *Elements* and its historical configuration as necessary knowledge to be learned in schools. In the paper *The amalgam of faith and reason: Euclid's Elements and the scientific thinker*, two taken-for-granted statements were problematized. Christianity, in the Middle Age, pursued its expansion by articulating a discourse of faith with Aristotelian logic[15]. The *Elements* were taken as a perfect example of the axiomatization of science and a model for achieving certainty of faith. Scholastics studied and used Euclidean geometry as a deductive system rooted in proofs[16]. Christianity, through Neoplatonism, gave a sacred role to mathematics as a vehicle to approach God, the divine. And, therefore, the *Elements* were not only thought to teach scholastics to use reason and logic. They were also thought to teach the architecture of divinity, the language of God: geometry[17]. In medieval schools, geometry and arithmetic were part of the *quadrivium*, along with astronomy and

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[14] In the section "To the fabrication of the scientific minded", in the paper The fabrication of qualified citizens: from the "expert-hand worker" to the "scientific minded"

[15] In the section "The amalgamation of faith and science", in the paper The amalgam of faith and reason: Euclid's Elements and the scientific thinker.

[16]Iin the section "When scholastics met Euclid", in the paper The amalgam of faith and reason: Euclid's Elements and the scientific thinker.

[17] In the section "Sacred mathematics and the path to God", in the paper The amalgam of faith and reason: Euclid's Elements and the scientific thinker.

music. The Church played an important role in the structuring of school. A role that is possible to see even today in modern education (see Tröhler, 2011). Christianity helped giving mathematics a privileged position in schools for the fabrication of a scientific citizen.

THE RHIZOME: BENDING SPACETIME

The chronicle above is a short story about a series of events. One of the possible paths to narrate the movements followed in this dissertation. Until this point, the story portrays a straightforward timeline. It is a depiction of a chronological mapping of how the research moved from its initial conundrum to diverse apparently disconnected interests. The papers are connected not as a sequence of events, but as continuities and discontinuities. And although certain shifts occurred throughout this exploration, each paper contributes to the mapping of the scientification of the self through school mathematics, in which the shaping of the desired citizen via the learning of elementary geometry has been entangled. It is difficult to show the movements and its connections "linearly" as a structured continuous writing. This story would be better told as a hypertext, as a vibrating and flexible discourse (re)produced throughout space and time: A sort of multiverse connected through wormholes, a rhizomatic assemblage.

> [T]here are very diverse map-tracing, rhizomeroot assemblages, with variable coefficients of deterritorialization. There exist tree or root structures in rhizomes; conversely, a tree branch or root division may begin to burgeon into a rhizome. The coordinates are determined not by theoretical analyses implying universals but by a pragmatics composing multiplicities or aggregates of intensities. A new rhizome may form in the heart of a tree, the hollow of a root, the crook of a branch. Or else it is a microscopic element of the root-tree, a radicle, that gets rhizome production going. Accounting and bureaucracy proceed by tracings: they can begin to burgeon nonetheless, throwing out rhizome stems, as in a Kafka novel. (Deleuze and Guattari, 1987, p. 15)

A rhizome allows a non-hierarchical multiplicity of entryways, no beginning or end, as neural networks (Figure 4). According to Deleuze and Guattari (1987, p. 12), the rhizome "is altogether different, a map and not a tracing [...] The map is open and connectable in all of its dimensions; it is detachable, reversible, susceptible to constant modification". At first glance, it might seem that the papers are random movements that occur independently, without a connection between them. Throughout the papers, there are visible differences, and at the same time there are similarities and intertwines. All events in the papers are entangled in different times, and in different places. At times, it seems to encounter redundancies of sections or arguments that give the impression that the same path was taken twice: Starting at the same point of departure and suddenly diverging in separate paths or converging from two different entryways into the same. For example, the fabrication of the scientific child through the learning of elementary (Euclidean) geometry is entangled in most of the papers. The starting point of school geometry as a technology of the self by analysing curricular guides—teacher guidelines, map of progress, learning standards, school textbooks for teachers and students, and so on—ended up intertwining with the axiomatization of space in the pursue of suppressing the self on the practices of knowledge (*The sightless eyes of reason*), with pedagogical techniques in which students train themselves to see with a mathematical gaze (*The effects of school geometry*), with the effects of power that teachers' guidelines have in students subjectivities (*Assessing visualization),* with the scientific thinker shaped by the Christian quest to amalgamate faith and reason (*The amalgam of faith and reason*), and so on. Such movements give the illusion that two or more papers are similar. However, the same path not always leads to the same scene: it is a rhizomatic web. And it has been a challenge to find a way to capture how this thesis moves throughout the rhizome as a series of entangled events with an apparent disconnection between them.

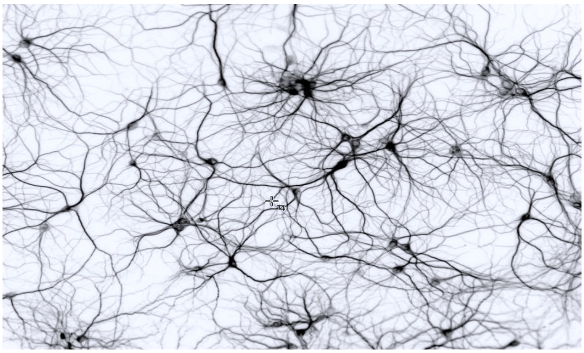

Figure 4: Neural Network

Discourses are not fixed, they move throughout spatiotemporal conditions. At times they mutate, they shift, and they oscillate. At times they remain the same. At times "the specific power of a concept is lost in its excessive and unquestioned circulation" (Saar, 2002, p. 231). Discourses are constantly being produced and reproduced, emerging and being perpetuated. They are flexible, four-dimensional (XYZT). They circulate at unison; there is no conception of causality or chronology. They are all together, entangled. And to express this in words is complex. When a discourse analysis is conducted, it is usual to look at statements as encapsulations of a certain time and place: The understandings of a particular era to grasp the current expressions and the order of things. This enables to capture the flows by historicizing the present"[18], following techniques of the new cartographer (Deleuze,

[18] Such rhizomatic historicizing configurations can be seen in Not a typical story, not a new critique, in Incepted neoliberal dreams and the 'Chilean experiment, and in Mindniac

2006). The rhizomatic construction in the development of this research was thought as a parallelism of discourses functioning as a multiverse. There is no linearity of events in alignment with Foucault's rejection of causality.

> We consider the understanding of the way one event succeeds another as a specifically historical issue, and yet we do not consider as an historical issue one which in fact equally so: understanding how two events can be contemporaneous. I would like to point out, moreover, that history is quite frequently considered as the privileged site of causality: all historical approaches should aim at highlighting relations of cause and effect […] As soon as relations of logical type, like implications, exclusion, transformations, are introduced In historical analysis, it is obvious that causality disappears. But we have to rid ourselves of the prejudice that history without causality would no longer be history. (Foucault, 1999, p. 92)

Before introducing the connections between the papers, a visual representation of the rhizomatic movement followed is necessary. And for the sake of having a mental image, a physics understanding of the expanding universe will be used. Early representations of the Solar System were portrayed as spinning flatly, orbiting around the Sun (Vita-Finzi, 2016, p. 2, in Figure 5). "Laplace imagined that the planets began life as an extended cloud of gas slowly rotating about the Sun. Over time, the gas cloud cooled and contracted, rotating more rapidly as it did so and flattening into a disk" (Chambers & Mitton, 2014. p. 45).

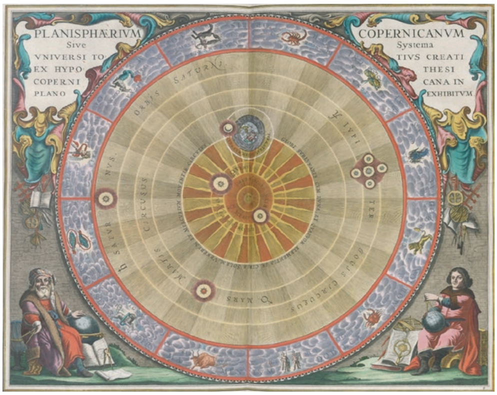

Fig. 1.1 Copernicus' Sun-centered system. Note the assumption that the entire universe is embraced by the planisphere. From Andreas Cellarius' *Planisphaerium Universi Totius Creati ex Hypothesi Copernicana in Plano*, 1660, From http://www.staff.science.uu.nl

Figure 5: Early representations of the Solar System

Other studies have shown that the Universe keeps expanding, and, therefore, objects keep moving. "Poincaré laid many of the foundations for what we now call chaos theory. In reality, it is impossible to determine the motion of the planets far into the past or the future with certainty" (Chambers & Mitton, 2014. p.46). There is no sense of homeostasis. The understanding of the accelerating expansion of the Universe, by beholding the observable Universe, led to search how fast the Universe is expanding and also, and most interesting for this visual representation, how objects move within this expansion and their trajectories. For example, the Solar System is not moving flatly, as earlier pictures depicted. Recent studies are proposing the helical motion in which the Solar System is embedded (see Keshava Bhat, 2008).

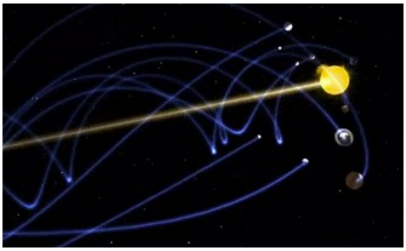

Figure 6: Motion of the Solar System through the space[19]

Cosmological studies in physics help in elucidating how the Universe was created. There exist many theories, one of them is the interplay between the Higgs boson in the Big Bang theory. This theory enables to explain how the universe was produced, how it behaves, and also, how it might be (its future). For example, the particle Higgs Boson allows explaining the homogeneity of the Universe as known today.

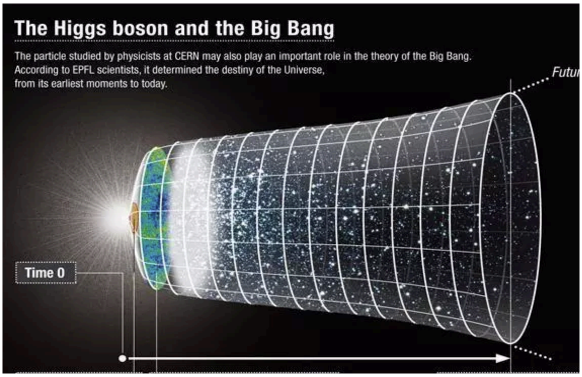

Figure 7: The Higgs boson and the Big Bang [20]

[19] Retrieved from: https://www.universetoday.com/wp-content/uploads/2013/12/tumblr_mj0vvcqnZx1qdlh1io1_400.gif

[20] Retrieved from: https://phys.org/news/2011-09-higgs-boson-size-universe.html

This analogy helps in explaining that even though discourse analysis enables a historical understanding of power and governmentality in the fabrication of subjectivities (Foucault, 1991), discourses keep fluctuating in an expanding Universe, as the helical motion of the Solar System. Discourses are not fixed or isolated, as the early depictions of the Milky Way. The emergence of discourses does not happen spontaneously. Discourses are progressively configured by human interactions. Connecting with the Big Bang Theory, according to physicists the Universe began with an extreme amount of heat accumulated in a dense point (the size of an atom); the explosion created matter—fundamental particles as quarks and photons. Afterward, the Universe commenced to expand and, therefore, to cool down. The cooling of such fundamental particles formed matter as we know today—such as stars and planets. All of this is to explain that words, even enunciated beautifully, do not mean taken-for-granted statements. Words are fundamental particles. Social interactions need to expand to create a "cooling effect" to form naturalized truths. And, as the Higgs boson enables to homogenize the Universe, power, through discourse, operates as a homogenizing force, by conducting individuals forms of acting and being in the word according to tempo-spatial conditions. If we imagine the (re)production of discourses as an expanding Universe, taken-for-granted statements collide, change, and evolve under certain circumstances. They live within many other naturalized truths. They are pulled or attracted by the "gravitational force" of "social spheres," and power is effected everywhere. Discourses, as the Milky Way, keep moving in this expanding Universe.

Like a rhizome, the Universe has no beginning or end, and it is impossible to study completely: only the observable Universe is reachable, while the rest remains a mystery. Events become a specific time and place, collations, entanglements. This is the visual rhizome: a complex expanding Universe, a spacetime continuum, where events take place within the observable or unobservable, probably light years away. The techniques of the new cartographer help in mapping the effects of power, the dispositives of power, but not all in an exhaustive manner. It requires many movements, a variety of strategies: Discursive and non-discursive analysis, historizations of the present, genealogies, mapping, tracing.... And also a wide range of analytical tools: concepts (statements, enunciations, truths, subjectivity, poker/knowledge relations, governmentality, *dispositif*, technologies of the self, scientific objectivism, abjection, space as a social practice…), structures (rhizome, quasi-self-similar fractals, post-structuralism…), diverse entryways (discourses, images, policies, history, fields of knowledge…), diverse paths (tales, dreams, fictions, historizations…), diverse gazes (education, geometry, mathematics, physics, optics, theology, architecture, political sciences, phenomenology, art, philosophy, science…), diverse methods (archeology, genealogy, locating, clustering, reading, historicizing…), diverse materials (treatises on diverse fields of knowledge, history of Chile, history of mathematics, OECD publications, curricular guides, textbooks, PISA, Chilean national tests, history of education, theories of education…), diverse directions (economy, progress, mathematics and science as the salvation, the sightless eyes, the scientific thinker, the Mindniac…), and so much more. The observable Universe keeps expanding, where all discourses are connected, nothing is isolated. Not even songs, tales or films. As Foucault asserted, power comes from everywhere!

> The omnipresence of power: not because it has the privilege of consolidating everything under its invincible unity, but because it is produced from one moment to the next, at every point, or rather in every relation from one point to another. Power is everywhere; not because it embraces everything, but because it comes from everywhere. And "Power," insofar as it is permanent, repetitious, inert, and self-reproducing, is simply the over-all effect that emerges from all these mobilities, the concatenation that rests on each of them and seeks in turn to arrest their movement. (Foucault, 1990, p. 93)

THROUGH THE KAFKAESQUE GLASS

From the papers and materials analyzed for this thesis, there exist at least seven circulating discourses pursuing the scientifization of the self through school mathematics, in which the shaping of the desired citizen via the learning of elementary geometry is entangled. The following chapters are thought as multiverses of taken-for-grated truths connected by the wormholes, produced by each paper, to map the rhizome. The circulating truths of the observable Universe will be told as a Kafkaesque narrative to power. There is an undeniable link between Kafka's fictions and Foucault's understanding of power (see Mellamphy & Biswas Mellamphy, 2005).

> Some studies show the relevance of Foucault's concepts of power and 'disciplinary society' to Kafka's work. Others refer to Foucault's work to illustrate Kafka's depiction (and parody) of bureaucracy's unrelenting control and the nature of totalitarian regimes. Yet another approach connects Kafka's depiction of space to Foucault's theory of the *panopticon* developed in his study of the prison *Discipline and Punish. The Birth of the Prison.* Finally, some studies focus on aspects of subjectivity and regard K. as a subordinated hero in a context pervaded by power. (Bogaerts, 2011, p. 100)

Many authors have made a Foucaultian reading to Kafka's writing (Bogaerts, 2011; Curtis, 2010). And what attracts of these fictions is that they are able to depict Foucault's understanding of power and its effects on the fabrication of subjectivities within a disciplinary society, for example in *The Trial*, *Metamorphosis*, and *The Castle*. In *Poseidon* (Kafka, 1971), Kafka narrates the story not of the God describe by mythology, but the story of a bureaucrat far from being the divine master of the seven seas. "All Poseidon's attempts to find a new position are doomed to failure, since as a god of the oceans he is destined to assume this managerial post over the waters of the world" (Gray, Gross, Goebel, & Clayton, 2005, p. 221). What Kafka is illustrating with *Poseidon* is that no one is able to secape form the bureaucracy of the Modern world, not even a God.

> *POSEIDON sat at his desk, going over the accounts. The administration of all the waters gave him endless work. He could have had as many assistants as he wanted, and indeed he had quite a number, but since he took his job very seriously he insisted on going through all the accounts again himself […]. It cannot be said that he enjoyed the work; he carried it out simply because it was assigned to him; indeed he had frequently applied for what he called more cheerful work, but whenever various suggestions were put to him it turned out that nothing suited him […] the great Poseidon could hold only a superior position […]. As a result he had hardly seen the oceans, save fleetingly during his hasty ascent to Olympus, and had never really sailed upon them. He used to say that he was postponing this until the end of the world, for then there might come a quiet moment when, just before the end and having gone through the last account, he could still make a quick little tour. (Kafka, 1971, pp. 478-479).*

Here I would like to provide a new reading of the power effects of school mathematics as a technology of government by following a Kafkaesque style of writing. Kafka's book *The Trial* (Kafka, 1956)*,* considered to articulate power, examination, and surveillance, has inspired the story of the *(d)effecting of the child*. In *The Trial* Kafka narrates the story of Josef K's arrest on the morning of his thirty-first birthday. K knew he had done nothing wrong, but he was not told why he was arrested. He went through a complicated process of bureaucratic procedures leading to his trial.

> The Trial represent a simultaneity in which disparate historical, disciplinarily discourses collide a moment of rupture, exchange, and confrontation, only possible in that precise historical envelope from which the author spoke. In other words, it represents a singular, historical moment at the threshold of legal and disciplinary transformation, a moment that fluctuated between the archaic modalities of linguistic based discipline and modern surveillance. The major point of this paper is that Foucault challenges the idea that power is wielded by people or groups by way of episodic or sovereign acts of domination or coercion, seeing it instead as dispersed and pervasive. (Yari, et al., 2013, p. 160).

Each chapter tells a different story, each of which is inspired by the disciplinary techniques portrayed in *The Trial.* It does not have a chronological order. All chapters are to be taken as a multiverse entangled through wormholes, the papers. The papers are going to appear throughout the chapters as:

The sightless eyes of reason: Scientific objectivism and school geometry	Sightless eyes Sightless eyed Sightless eyes of reason
Shaping a scientific self: a circulating truth within social discourse	Shaping a scientific self A scientific self
Euclidean tradition as a paradigm for scientific thinking	Euclidean tradition As a paradigm for scientific thinking Euclidean tradition as a paradigm for scientific thinking
The effects of school geometry in the shaping of the desired child	Effect The effects of school geometry The effects of school geometry in the shaping of the desired child
Not another typical story, yet not a new critique: A journey to utopia across standardized assessment	Stories A journey to utopia across standardized assessment
Assessing visualization: An analysis of Chilean teachers' guidelines	Assessing Assessing visualization Assessing of visualization
Tales of evaluation in school mathematics	Tales Tales of evaluation
Incepted neoliberal dreams in school mathematics and the "Chilean experience"	Incepted neoliberal dreams
Be the best version of yourself! OECD's promises of welfare through school mathematics	The best version of Be the best version of yourself Promises of welfare Promises of welfare through school mathematics
The fabrication of qualified citizens: from the "expert-hand worker" to the "scientific minded"	Fabricating qualified citizens The fabrication of qualified citizens
"Mindniac" the reasonable citizen of schooling (Chilean edition)	Mindniac Mindniacally
The amalgam of faith and reason: Euclid's Elements and the scientific thinker	Amalgam of faith and reason Scientific thinker

…Si tu n'es pas comme tout le monde, c'est que tu es anormal, si tu es anormal, c'est que tu es malade. Ces trois catégories: n'être pas comme tout le monde, n'être pas normal et être malade, sont tout de même très différentes et se sont trouvées assimilées les unes aux autres.

(Foucault, 2004, p. 95)

PART ONE: THE DREAMED

Tonight I'm gonna have myself a real good time
I feel alive and the world it's turning inside out. Yeah!
I'm floating around in ecstasy. So don't stop me now, don't stop me
'Cause I'm having a good time, having a good time
I'm a shooting star leaping through the skies
Like a tiger defying the laws of gravity
I'm a racing car passing by like Lady Godiva
I'm gonna go, go, go. There's no stopping me
I'm burning through the sky. Yeah! Two hundred degrees
That's why they call me Mister Fahrenheit
I'm travelling at the speed of light
I wanna make a supersonic man out of you

Don’t stop me now by Queen

One morning a mother asked her little child what does he want to become when he grows up. The child, as naïve as children can be, answered, “I want to become an astronaut.” “Why an astronaut?” His mother asked with stunning surprise, “I thought you wanted to become a rock-star, you are always singing and pretending to play the guitar.” Silence… a speechless little kid, astonished by her comment, thought and thought why her mother seemed to be so surprised. Was not he good enough to become an astronaut? Was not he smart enough? But his mother had always pictured him as someone more creative. “Well,” the mother continues, “if you want it, you have to study very, very hard.” “If you succeed in school, then it would be so easy for you to become whatever you want,” she added.

When the father came home later that night—because he worked many hours a day to overcome the family economic crisis—the mother told him about the child’s dream of becoming an astronaut. “What a silly idea!” the father said. “He needs to find a good job to provide for this family! He wants to become something impossible; he needs to go to school, go to college and then to work in a high paid job, not like me,” he added, “we are saving for his studies not for his dreams.” “If he likes space so much,” he paused, outraged, “he can become a physicist, an astrophysicist, a scientist, not that unreachable and unreal illusion. He wants to be floating in outer space. He needs to stop dreaming and star wanting something practical with his life. He needs to think about his future.”

The child, already in bed, heard the whole conversation. Was his dream so out of place? His father wanted him to become someone productive for the system, someone earning a higher income. A high-paid worker who does what he is asked for. Someone who goes to work for a certain amount of hours every day, and then is able to enjoy his passions, outside labor hours: Just another employee of the bureaucratic system. He knew his dream was not impossible. After all, many people have achieved their goals of becoming astronauts. And he also knew those people did not accomplish their goals by praying to God, or by winning a contest. "They studied!" He said out loud, submerged in the darkness of his room, "schools are the place where people get closer to their dreams." However, he was very afraid of his father to voice his desires again. "This will be a secret," he said before falling asleep.

A few days after this conversation, the child entered to school. He knew he needed to be a good student, with the best grades he could obtain to become an astronaut. In the first years, he learned about animals, about nature, about the solar system, about mathematics. "Mathematics, why mathematics?" He questioned. All he did was memorizing… why was it helpful? Teachers always answered, "You need it for the test." School mathematics considered useful only for testing, only to pass exams. He talked about wanting to become an astronaut with one of his teachers. "Astronauts use mathematics every day," the teacher said. Uncertain about the answered the child agreed to engage more deeply with mathematics. He knew that if he wanted to go to a good university, his parents wouldn't be able to afford it. He and his parents needed to invest. He needed a student loan or a grant. And for that, he ought to have good grades.

His father, knowing the economic situation of the family, started saving for his son's education since the child was a little boy. Going to University was not a possibility the child was able to reject. "There is a certain chain of training, any good job would ask for the applicant qualifications, University gives an advantage, also degrees. They will select the best option for them, the best applicant," the father said to the mother while deciding to start saving for his son's tuition fees. The father was clear about his son's future and how higher education gives an advantage in any situation. That was not negotiable for the father. "Education is an investment!" the father constantly said to the child, "and you have to acknowledge it and ***be the best version of yourself.***"

The father knew very well, due to his own experience, that education was not simply a social right. For him, education was not only compulsory for making people wiser or for developing certain skills needed to perform better in University or in life. For many years, going to school was an investment. It was a consumers' good with the essence of equity. For a long time, free school choice faded in economic policies leading to an era of consumerism of education: voucher and rewards system, excessive tuitions, expensive quality. "They disguised it by talking about welfare, but is all about a marketable system," the father said to himself, "What is welfare, anyways? What is welfare but a fancy word to obsess us about the future of our children." Competitiveness was the result of a group of people charmed by an ideal of optimizing economic benefit: the ***incepted neoliberal dreams.*** People charmed by the gains produced by citizens willing to consume education for a personal benefit.

"The more you pay, the better educated," the father said to the child. "We are paying for your school, it is not for free, you know it very well." he paused for a moment trying to find the right words, "we are paying for your school for you to have a better life than your parents. Out there, it is all about how much money you make." The "promising jobs" were always advertised in the newspapers when students had to decide what to study in college, and advertisements for astronauts were not on the list. The highest paid jobs in the market were always the most desirable, however, not always achievable. They were left for highly qualified, trained people not for anyone, and the father knew it very well. Although, achieving the desired "quality" level to be considered a proper applicant for those jobs was only obtained by a University diploma, not by innate skills. And of course… it was a game. Universities were ranked, and the entering gate was a test. The better the school, the better possibilities to enter those highly ranked Universities… the better the school, the more expensive it gets. "At the end, it is all about money," the father explained to his son, "The more we pay for your education, the better probability of you having a good result in the test, do you understand? We are investing in your future. You have to secure a job to assure your future, a future that should be much better than ours." The child, overwhelmed by his father's words, terrified and speechless, agreed to think more about his future and less about impossible dreams. He did not understand the importance that his father gave to money, but he was only a child yet.

He was taught from the very beginning that education was a safe bet for a prosperous life. Everybody he knew, every place he went to, they all seemed to agree that college opens the gates to a better future. He knew that was not the only option, but almost everyone agreed that it was the safest. Going to University and being highly educated were, apparently, synonyms of intelligence. "Sometimes it does not make sense," the child said to himself, "going to college is so natural nowadays, it is not even a right, it is not mandatory, but… if one decides not to go if one fails to enter, then what? I wonder if someone without a fancy degree could engage in political discussions and be taken seriously by mainstream media. Or would that someone be taken as a joke, as just one more on a sea of average people? But if that someone had a diploma in political sciences… people would perceive him vastly different, the opposite. That someone would have a voice." Education started to take a new attribute for him. And suddenly, proper education, higher education opens a world of possibilities beyond imagination. That was the dream.

PART TWO: THE DESIRED

Come as you are, as you were, as I want you to be
As a friend, as a friend, as and old enemy
Take your time, hurry up, the choice is yours, don't be late
Take a rest as a friend as an old memoria
Memoria, memoria, memoria, memoria
Come doused in mud, soaked in bleach, as I want you to be
As a trend, as a friend as an old memoria, memoria
And I swear that I don't have a gun

Come as you are by Nirvana

"Why is going to school compulsory? Why is education needed? Do only educated people reach their goals?" He thought to himself on his way to class. He woke up that morning feeling empty, having plenty of questions and no answers to satisfy his restless mind. He knew he had to be at school every day, in his best behavior, if he wanted to succeed. He did not understand the purpose of each school subject and their respective syllabus, but he did not feel in a position to question it. Mathematics was always intriguing for him. All his mathematics teachers always said to the class "mathematics is a universal language." "Universal language? If it is so universal, why is it not possible to travel the globe without learning other languages? Do all people speak the language of axioms and numbers?" he wondered. The more he read, the more he began questioning everything around him. Science and mathematics in schools were not complementary subjects. Mathematics was a form of language useful for science. After all, the need to secure the economic development of nations increased the necessity of scientific and rational citizens. "So, are we supposed to become scientists?" he said astonished by the realization that a scientific citizen was the desired goal. "Is that why I want to be an astronaut? Is that why my father said I have to study physics or astronomy? Why did he not say arts or literature? Is that why my mother associated creativity with music? Is it because reasoning is a synonym of doing science?"

The functioning of the school and its pedagogical devices were intriguing for him. Apparently, these did not make sense to him. With the movement of Modern Mathematics, mathematics in school was thought as an effective tool with valuable implications for the improvement of science teaching based on logical thinking. "Of course I will need mathematics if I want to perform well in science," he said convinced of this statement. But the connection between the usefulness and necessity of mathematics for his future performance as an astronaut was still blurry to him. All documents he browsed referred to mathematics applied to a particular subject: magnetism in physics, acid and bases in chemistry, bacteria growth in biology, and so much more. But when it came to mathematics, pure school mathematics, mathematics seemed to be so distant of being applicable: calculating areas and volumes of ideal figures and shapes by using a coordinate system, counting squares to measure the length of objects, calculating probabilities by flipping coins. School mathematics seemed so abstract and obscure for him without a field to interpret it, as if schools were forcing reality to become part of the practices of certainty.

Geometry was no exception. Schools guidelines aimed at the teaching of spatial abilities with a space restricted to a particular form of seeing and thinking about reality. "They want me to solve problems with theorems I would never apply in life," he thought to himself while reading his school textbook as if the system would have failed him to provide useful mathematics. But this did not imply misimplementations by mathematics school textbooks' editors and authors, or inadequate readings of the syllabus by teachers. School mathematics helped in fabricating the scientific minds of the future: a ***Mindniac***. And so, ***shaping a scientific self*** required suppressing the self to perceive space and geometrical knowledge as decontextualized, as universal and timeless. Geometrical rules that could be applied to any time and place, regardless of mutable factors: Platonic and untouchable knowledge. Demonstrations, proofs, axioms, theorems appeared repeatedly in his mathematics lessons. "Why do I need to make demonstrations? Why do I have to learn so many theorems?" he seemed to collapse by no finding a proper answer to his questions. At the beginning, geometry seemed very entertaining to him. Playing with tangible figures, objects that he was able to manipulate, to produce, to reproduce, to depict, to touch. As he grew up, everything started to change around him. The joy started to vanish, and school space became more and more objective and axiomatic. Mathematical tasks in the classroom and in assessment became the epitome of abstraction. Reality seemed a very distant image to him, a blurry memory. "Why should I do it like this?" he thought to himself while solving one test. "How did it end up like this?" he added. And even though he did not understand the place of those tasks in school, he knew it was part of the game. A game he simply was not able to escape. Was he at all allowed to escape it? Of course he could refuse, of course, he could resist, could he?

He knew he had to play the game and he knew its rules, however, this did not stop him from questioning everything to understand why he had to play it. He was intrigued by the positioning of mathematics in schools and by its apparent usefulness for science. "The more abstract, the more quantifiable, the more structured… the better science?" he wondered. For many years, before science was granted with its name, Euclid's *Elements* entered the world of Western thinkers as a structured guide for doing science. The ***Euclidean tradition as a paradigm for scientific thinking*** set the grounds to believe that geometrical knowledge was the access key to all forms of knowledge. Incredulous of such idea he began questioning the entanglement of the *Elements* as a scientific model: "Is it Euclidean geometry supposed to shape scientists?" He doubted. He tried to understand how geometry, as a form of mathematical language, was useful for the economic development of nations by fabricating scientific and rational citizens. Somehow, reason appeared as fundamental in the shaping of a logical thinker through mathematics. However, its entanglement with the *Elements* was not straightforward for him. Geometry in schools was not shaped as the *Elements*. "Where is all of this coming from?" This question puzzled him for several of his school years.

For a long time, the *Elements* were thought to be the vehicle to access the world of ideas through the path of proofs and demonstrations. The structure of the *Elements* was taken as a secure model to achieve certainty and truth. Suddenly, Euclidean geometry was believed to unfold a deductive and axiomatic way of thinking. It was thought to prompt to the enlargement of the mind and to increase the capacity for reasoning. A historical dominance of the *Elements,* as ***the amalgam of faith and reason*** for the constitution of a particular subject, enabled having students following the path of what schools perceived a real scientist is. And even though he was never instructed in geometry by using the *Elements* in the classroom, he was able to see some connections taking place: the uses of proofs, the axioms, the theorems, the structured procedures of how to make a proper demonstration. It was all there, the whole time in front of him. It was about ***the fabrication of qualified citizens.*** However, the target was not in creating suitable people to become scientists, but to become logical thinkers, problem-solvers, and productive citizens encouraged to use reason and to see the world with **s*ightless eyes***. And there was no way to escape from it, was it? Which options did he have? He knew he had to learn mathematics because he needed the best grades he could obtain. "It is better to let some things take their natural course," he said agreeing to participate in these practices of scientifization. He thought that the best option was letting things follow their natural flow. Perhaps resisting was detrimental for his future endeavors. Perhaps he needed to start questioning less about all he could not control and start worrying a bit more about himself. After all, he felt he could adjust to fit in what was desired.

PART THREE: THE SHAPED

Oh no love! You’re not alone
You're watching yourself but you're too unfair
You got your head all tangled up but if I could only make you care
Oh no love! You’re not alone
No matter what or who you've been
No matter when or where you've seen
All the knives seem to lacerate your brain
I've had my share, I'll help you with the pain. You're not alone
Just turn on with me and you're not alone
Let's turn on with me and you're not alone (wonderful)
Let's turn on and be not alone (wonderful)
Gimme your hands cause you're wonderful (wonderful)
Gimme your hands cause you're wonderful (wonderful)
Oh gimme your hands.

Rock ‘n’ roll suicide by David Bowie

One day the child woke up. He was old enough to go to school; he already was six years old. Getting up of the bed was not easy; he had to be at school at 8 o’clock in the morning. He had always struggled with socializing. He was very afraid to leave his parents that day. The school bell rang; it was time to meet his classmates and his teacher. Slowly he started to walk towards the classroom, he sat in the back hoping not to be noticed. His classmates were friendly. They all spoke to each other as if there were a connection he simply could not feel with others. He waited alone for the teacher to arrive. In the first period, the first assignment the teacher asked the students was to draw what they want to be in the future. He glimpsed his classmates: firemen, ballet dancers, singers, soccer players, he did not know what he wanted to be. The ones closer to him looked very confident on their future expectations. All of them knew so fast what they wanted to be. Why didn’t he? He thought about his father’s work, twelve hours a day on a job he did not seem to enjoy; he was always complaining. His mother worked fewer hours than his father, but she had the same dissatisfaction. “Is that what employment does?” he thought to himself while trying to find some proper answer to the task.

And then, he remembered about a cartoon he watched when he was younger. A cartoon on the stories of an astronaut that discovers other forms of life besides humans, the tales of an adventurous man able to see the wonders of the observable universe. The child was always intrigued not by the adventures alone but also by the living of an astronaut in outer space. He wanted to know everything about the Universe. He felt excited and started drawing himself as an astronaut floating in outer space next to the rocket, as if he was enjoying zero gravity and the amazing view not depicted in the picture, probably the Earth, probably the Moon, or beyond. He was the only one who wanted to become a scientist, if astronauts can be considered as such. The teacher told the whole class they have to fight for their dreams. Everybody gave precise justifications to their drawings, except for him. He did not want other people to know he did not have any aspirations prior the teacher's drawing invitation. He did not want the rest of the class making fun of him; he only said, "Because I like space." The teacher kept the drawings, and a few days later she called all parents to separate meetings. His mother met the teacher, after the meeting she asked him what he wanted to become when he grows up. His parents' reactions were unexpected. He thought they would be somehow proud of him not wanting to have a low paid job. Apparently, that was one of the most important features of a job according to his father. Astronauts are well paid, but sadly for him, it was considered just as a childish dream.

"It cannot be as impossible as they say," he thought, "I have to fight for my dreams as my teacher said." Throughout his life, he always heard his teachers, parents, and mostly all people he knew that knowledge is power. And, therefore, education becomes a mean to empower people. "Highly educated people are constantly perceived as the most successful," he added, "Education is compulsory for a reason." He decided to give the best of him, as his father said: "***be the best version of yourself.***" At first, he felt a joy with mathematics. Geometry was about manipulating figures and drawing their representations, which he liked. It felt much more like a game to him, playing with dough to emulate shapes, playing with building blocks or wooden sticks to construct structures, and so much more. Everything seemed so useful, so practical, even the learning to recognize the elements of geometrical figures. Mathematics appeared to be so applicable in his daily life. But, to do what? He realized that cutting cubes along its diagonals was a practice that he would, probably, never do. Counting squares to measure figures, with what purpose? He could easily use a measuring tape. "Helpful, might be… necessary, that is questionable," he thought. Counting squares, an innocent school mathematics practice, just a little novelty amongst many others that seemed not to have any explanation for the child, was, indeed, not so innocent. He wanted to know the purpose. He was very curious. After all, he had to engage in those practices. Suddenly, concrete objects began to vanish, to become only distant memories. The "real" three-dimensional space promised was progressively mutating into an axiomatized, objectified, immutable truth. Geometry became an abstract image only reachable by reason and logic, only seen through the ***sightless eyes of reason***.

Gradually, what started as a gameish and motivational manner of learning geometry in the classroom transformed into school practices far from being entertaining to the child. What once were "real" situations changed into a complex and unnecessary understanding of the world. ***Assessing visualization*** was solely a whimsical promise. "I do not understand why!" he said to himself trying to solve a problem for an upcoming test. Solving problems of concrete and contextualized situations was thought to be crucial in schools. Even international tests measured children's abilities to sort them out. But not all answers had the same scores. A credit system created to grade each answer according to guidelines to rate them individually. Better answers than others. And, of course, the best answer with the highest score was desirable. Here, another game started to take shape. Not the old game with the use of the hands or the eyes, but with the use of the mind. Unraveling the right way of solving a quest became a contest, developing the skills needed to it became an obsession, and ***shaping a scientific self*** became a compulsion.

It was not clear for the child why he needed such elaborate steps to solve problems that did not require it. The use of the Cartesian coordinate system to redistribute an apartment by giving coordinates to the furniture was upsetting for him. "Why does this problem should be solved in this way? Why do I need coordinates to relocate the door or the sofa? Will I ever furnish my future home like this?" he thought while solving the task. "Wait… is this what architects and designers do?" he added. With practice, the tasks became less frustrating and more normal. It felt like a routine. He accustomed himself to the practices of the school. Suddenly, the exasperation of older days disappeared to give rise to an acceptance of the trustworthiness of mathematics. ***The effects of school geometry in the shaping of the desired child*** enabled a training of the self in which the child regulates himself to think and act… ***Mindniacally***. And so, relying only on visual judgments to solve problems in the classroom is undesired, sometimes a problematic behavior. With time, studying possibilities of scoring a goal in a soccer match by using vectors in a Cartesianized field was no longer a bizarre idea. "The coordinate system is very useful to calculate the trajectory of the ball with precision. I wonder if soccer players use these techniques to design strategies," he thought while solving the task.

PART FOUR: THE ASSESSED

New blood joins this earth, and quickly he's subdued.
Through constant pained disgrace the young boy learns their rules.
With time the child draws in. This whipping boy done wrong.
Deprived of all his thoughts the young man struggles on and on he's known
...They dedicate their lives to running all of his.
He tries to please them all. This bitter man he is.
Throughout his life the same. He's battled constantly.
This fight he cannot win. A tired man they see no longer cares.
...Never free. Never me. So I dub thee unforgiven.
You labeled me, I'll label you. So I dub thee unforgiven.

Unforgiven by Metallica

School days were a routine. Going to school, sitting still, looking at the teacher, at the whiteboard, at the textbooks, writing everything down, taking important notes, leaving school, doing homework, and going to sleep until the next day to do the same. “Your job is to study, that is your only responsibility,” his father used to tell him, probably to encourage him to continue with his duties. “That is my job, to be a student. I have to be good at it,” the child constantly repeated these words to himself. Exams were not a problem for the child. The teacher often made them work in groups to solve problems, to answer questions. For him, there were unnecessary interactions if he wanted to learn. “I don’t understand why should I cooperate with them. They only want to chat, to waste time, they don’t want to learn,” he thought most of the time. Although, sometimes he felt exams were all they were preparing for. Was he learning? He could not tell the difference. In holidays he was always forgetting what he had learned during the school year. For him, this was a clear sign that he was not really learning. “What is it, then?” he wondered at the beginning of every school year when he was asked to take a diagnostic test for each school subject. “I have to learn it all again!” He felt he was indeed learning but not mathematical content, not axioms, theorems, formulas, or calculations. He was learning how to behave, how to act, and even though he experienced temporary memory loss regarding mathematical knowledge, he never forgot how he should be solving problems or the rules in the classroom, or how to answer a test.

But one day… everything changed. The school was not the same anymore, at least not for him. The teacher was not alone. The principal stood next to her with some papers on their hands. “What’ s going on?” he asked the classmate next to him.

Everybody seemed perplexed. A national test in which they were going to take part... The principal explained them the rules of the test and emphasized that it was only a trial. A trial to tell how good they were and if they needed to improve. The test appeared to be something serious, but the principal made it seemed very casual. “Is he deceiving us?” he thought to himself, puzzled by the fact that the principal took the time to go to the classroom exclusively to explain them the test. They took the test. Somehow the questions seemed familiar… the structure, the choices, the format, everything. Were they secretly being trained to take this national test?

The teacher brought back the results, and these were not good. Labels were put. Everyone started comparing their results as if they were somehow competing with each other. He felt the school was training them to be better, to feel the need to improve, to obtain a higher score, to desire more of themselves: ***to be the best version of*** themselves. It was not about learning anymore. More trials… until the date of the test was fixed. The day started approaching. It appeared in the news as a national phenomenon. Suddenly the child understood that it was not only a regular and casual test… it was a nationally standardized system to rank all schools according to their students’ average performance. It was not an innocent game to know if he and his classmates needed to improve. It seemed so much more than that. However, he could not decipher what it was.

They said the test did not measure knowledge and procedures. At first, his exams, in the classroom, were completely the opposite. It was only about knowledge. But as he was getting older and as the national tests were on the horizon… exams commenced resembling the trial test. And so, the ***tales of evaluation*** began. Solving problems became the epitome of the classroom, and of assessment. “They help us to think logically, all problems need the use of reason,” he began problematizing the questions in the tests, “as if that kind of thinking can be boosted, developed, can it?” “If so, how?” he added. The sole idea of being part of something bigger than his dream of becoming an astronaut was overwhelming. The results of the actual national test were not for him to enjoy. The results were given to the school as an average. It wasn’t his performance alone... It was the performance of the school. It was the success of the teacher. It was the decisions made by the administrators and the principal. Students were not the ones of scrutiny, but the school. Was he wasting his whole school years just to make the school be labeled as “successful”? What about him?

He started questioning how mathematics was always advertised as a practical and useful knowledge to overcome daily life challenges. At least, this was in the introduction to every school mathematics textbooks, he owned throughout his school years. But… Was he learning? He knew he learned how to take exams, how to answer questions, how to optimize his time to find the right solution, but… was he learning mathematics? “Mathematics is a universal language,” “Mathematics is important,” his teachers said. “Am I learning mathematics or just learning enough mathematics to succeed in the test?" Which choices did he have? He knew he needed the expertise in testing to do well in the final test to apply to the University. He knew he needed the skills to optimize the time frame given to each question. He knew his teacher needed them to perform at least above average to keep her job. He knew the school needed the score to stay open, to attract more students. He knew the school ranking was important also for his future. He knew that attending in a school positioned as successful gave a higher social status to people. His parents were not able to pay a more expensive school just for him to feel he was learning. Results were all he would be. "Numbers, only numbers," he said, "In ***a journey to utopia across standardized assessment,*** there is nothing more than numbers!" Indeed, the tests were measuring valuable skills for policy makers and teachers: the use of concepts, the ability of reasoning, or the capability of solving problems. But, in the end, the resulting score was not interpreted as those skills. The score was taken as the level of mathematics proficiency, and he knew it very well. A score able to talk about his entire future… A score able to predict his future income… A number that was hunting him, that was tricking him to see himself only as a number. A number that was making him desire to study more and to participate in mock tests. He felt the pressure; they all felt it.

Words announced eloquently are always attractive to innocent ears, to lost souls. It was usual to hear people judging others not according to their own grades or to their problem-solving skills, the two things that are the most valued features inside the classroom. But people judged others in relation to their school place in the ranking. "Madness, this is complete madness," he said after overhearing a conversation between two girls. "I have the best grades in my class, I am the first place," one of the girls said to the other, probably bragging about her merits, the child could not tell. "Well, I don't know why you are so proud of it. You go to a bad school, so you are the best of the worst. That is not something to be proud of," the other girl replied. He was surprised. How could the second girl know if the first girl would perform even better in a "higher-ranked" school?

He felt part of a process that he could not understand and he could not control. For him, not even the tests grasped what schooling was supposed to do or what students should be learning. The ***assessing of visualization*** always upset him while taking these national tests. "Visualizing three-dimensional configurations was not important according to the textbooks? But here, only calculations! Where are the promised real-life situations?" he thought to himself when he was studying for the upcoming national standardized test. He knew he needed the grades. He knew he needed the current system to be considered a "qualified element" to continue in the same current system, to go to a good University, to find a good job, to keep being considered as productive. He knew he needed to see with the ***sightless eyes of reason***.

PART FIVE: THE RESCUED

When you feel all alone
And the world has turned its back on you
Give me a moment please to tame your wild wild heart
I know you feel like the walls are closing in on you
It's hard to find relief and people can be so cold
When darkness is upon your door and you feel like you can't take anymore
...When you feel all alone
And a loyal friend is hard to find
You're caught in a one way street
With the monsters in your head
When hopes and dreams are far away and
You feel like you can't face the day
Let me be the one you call
If you jump I'll break your fall
Lift you up and fly away with you into the night

Crash and burn by Savage Garden

"Why is mathematics so important?" he always asked to himself. It seemed to be a riddle without a solution. In school, mathematics appeared to be as any other school subject, except for the number of hours dedicated to its learning, except for the sudden intellect granted to the student that is "good in math," as if it was a superpower, a mutation to uncover prodigies and geniuses. Higher proficiency and higher grades in all school subjects were desirable, but when it came to mathematics… mathematics gave the impression of being a tool able to save the world. As if mathematics could do wonders for everybody. "Is it because they say it is a universal language?" he continued. He realized people that were labeled as good in mathematics were considered to have higher IQ than the rest of the population, probably the highest. Even though there was no evidence to support these types of claims. "Are all mathematicians above average?" he wondered. Even though popular beliefs say that Einstein was not considered to be bright enough because he failed in school mathematics. "Wasn't it because he was bored of the useless, simplistic and unrealistic tasks?" he thought to himself. A myth inscribed, masking most of the nonsense correlations between mathematics and intelligence.

Indeed, mathematics was taken as a tool to validate the intellect of people, a fair judge of brightness. And, according to the child, with scientists, it is not the same. He began questioning, troubling what he had always heard on the radio, the television, what he had read in the newspapers: when it comes to science… it depends on the scientific field of knowledge. Social sciences did not have the same social appreciation than the natural sciences. Sociological studies did not have the same social value than studies in physics or neuroscience. Quantitative analyses were overpowered by quantifications. "Is it all about numbers?" he wondered. The learning of mathematics in schools was not focused solely on the set of axiomatic tools displayed to learners, but also on the development of a scientific, rational and logical, thinking. The need for ***the fabrication of qualified citizen*** positioned mathematics at the core of education. "There is a mystery, a charming effect of mathematics," he thought. The child began to think about all he had read and heard over the years, not solely on the media, but about all his experiences. "Mathematics was not created only by mathematicians, was it? Was not it made by philosophers, by physicists, by doctors, by economists, by engineers, by all scientists using mathematical models? Am I wrong?" he added. The apparent universality of mathematics as the language of science started to be visible for him. Mathematics seemed to be exceptional, a tool useful for everything, an essential wisdom that everyone needed, a powerful knowledge. It appeared so seductive, a way to access not only the world of ideas but to gracefully achieving scientific certainties, an ***amalgam of faith and reason***, a linkage between the mundane and the divine.

In schools, mathematics as a subject was so significant that it was thought to secure a better future. Numeracy proficiency was the ultimate goal. Mathematics was not useful only to judge intellects, but also to secure people's future, to save them from their doomed. The ***promises of welfare through school mathematics*** assured the achievement of the best opportunities in life. Higher employment rates, higher income, higher health, higher social and political engagement, and so much more: the conquering of the best possible version of each human being. Over the years, the child tried to be reasonable about the assertions regarding the advantages of mathematics. He tried to keep his distance in blindly believing every statement circulating on its "goodness." But this, this made much more sense to him. "I know I will not be able to enter to University if I perform poorly on the test," he thought. "I know almost all high paid jobs require a diploma certifying higher skills. And being good in mathematics increases those probabilities," he added with doubt in his own words. He knew mathematics had a power that enables to give a high-rated label to anyone good in it: a sort of indicator of people's intellect and pedigree. Mathematics was thought to develop problem-solving skills and logical thinking, abilities very beneficial for a potential employee and for a future citizen.

Mathematics was treated as the perfect shaper of qualified workers. It was supposed to develop desirable skills among potential candidates for boosting economy, for wealth. "I do need to learn mathematics. I do not know if I would need it in my job. But if it helps in rating skills and how smart a person is, then, it is a safe way of securing a high paid job. Doesn't it? Although, does it really increase the chances of being employed?" he asked to himself. He knew he needed to develop those skills if he was going to pursue the high paid jobs, if he wanted to secure his future, if he aspired to prosper in life—as his father constantly told him. He knew mathematic proficiency was necessary, but not because he required this knowledge for solving problems in life, he was starting to recognize that he would not use axioms, or theorems, not even the Cartesian coordinate system that his teachers seem to love. He knew he needed the label, as a sort of "good student" tag, and he also needed the grades. Otherwise, he would not be considered successful, but a failure. If mathematics proficiency was truly a key for his future success and financial stability, he had to be good at it. There was no other option for him. Probably there were more ways, plenty unknown possibilities for achieving a better future, but mathematics was the safest bet.

"If it was that simple," he said to himself while preparing for his last participation in the national standardized test of his school life. He was already sixteen years old. He remembered his first test. He was so young, so naïve. Now he realized this test was also a key label he had to carry in pursuing his wishes to go to college. The ***incepted neoliberal dreams*** helped in fabricating a system in which the school became another label for him. High grades in a good school that is at the top ten of the ranking is not the same than the same grades in a bad school ranked at the bottom. He was neither in a good nor a bad school, he was in one positioned a little below the average, but it was the best one his parents could afford. He knew this meant that he had to prepare twice as hard for the final exam to enter to college. This test, the last gate he had to open, not only to become a qualified future employee. It was also his only opportunity to apply for financial aid. It was all about the grades and the scores of the tests. "They all talk about the quality of education, and that only means that quality of life will decrease," he thought to himself. He was exhausted, devastated. In school, he had other subjects to study, plenty of tasks and homework he needed to complete. However, he knew he needed to study for the test. It was his future in the hands of educational policies. It was not a game. "Mathematics, a powerful knowledge, they say," he continued, "mathematics…" He had to become the ***Mindniac***. After all, Mathematics was considered to have the effect of enlightening people, enabling personal fulfillment, facilitating employment and full participation in society. It is supposed to make everything better.

PART SIX: THE MUTATED

Master of puppets I'm pulling your strings
Twisting your mind and smashing your dreams
Blinded by me, you can't see a thing
Just call my name, 'cause I'll hear you scream. Master, master
Just call my name, 'cause I'll hear you scream. Master, master
Master, master, where's the dreams that I've been after?
Master, master, you promised only lies
Laughter, laughter, all I hear or see is laughter
Laughter, laughter, laughing at my cries
Fix me

Master of puppets by Metallica

"Do we even need to be good at mathematics?" he said to himself. He felt in denial. It was a waste of time to start questioning everything. "I'm sure they know what they are doing, I'm sure they know what is the best option," he tried to agree. All his life he was hesitant about the functioning of schooling. He always tried to think that education was not a capitalist interest. He often struggled with the apparent lucrative business behind schools. Now, now he was less doubtful. After all these years, after all the pain and suffering, after all the sleepless nights, he finally saw the benefit of education. The school was a place where individuals get prepared for their adult life. "We learn how to talk, how to read, how to write. We learn about our culture, about history, about the cosmos, about nature. We learn other languages, about other cultures. We learn so many useful skills that separate us from the uneducated ones," he thought to himself after completing his last test. "We have more opportunities, better opportunities," he continued. The ***incepted neoliberal dreams*** were now a necessary evil. "Why I did not realize before? You have to invest if you want a better life. If it were for free, then everybody would achieve the highest positions in the workplace, but someone needs to be the employee. The *Brave new world* of Aldus Huxley makes much more sense," he said somehow convinced.

Schools had been taken, since the beginning, as the place where people can develop the skills needed for their own life, as the place where people can be free to choose whatever path they would want to take in life. It had been thought to possess a pastoral faculty able to rescue people from their destiny. And, within this, mathematics has always had a special place. It has always been granted with the ability of ***fabricating qualified citizens.*** The Christian believer was the first mutation: A person who would develop his intellect and scientific curiosity through the study of Euclid's books. The logical citizen needed for recognizing in the *Bible* the proof of God's existence, the proofs of Christian certainties, the logical and deductive citizen who was able to rationalize the mysteries of faith. Within which, Euclidean geometry had been taken as the ***amalgam of faith and reason.*** However, the shaping of the scientific thinker for Christianity, as a man of faith, was not enough for the progress of nations.

The conquering of wealth went hand in hand with educating people with the necessary knowledge to work the ground: the expert hand worker, a person with a higher understanding of how to grow the ground. The "proper instructed" citizen needed in the starting point for the development of nations, a citizen able to know, by applying scientific knowledge, when and how to grow seeds to optimize the production of aliment. Geometry was taken as a powerful mean to educate the senses. This was the second mutation. Life keeps evolving. The world became industrialized. Expert-hand workers knew everything about agriculture, not gained by experience, but by proper education. Unfortunately, they did not know how to operate and repair the new machinery entering to the land. Another type of citizen was required. Highly qualified engineers and technicians were needed to cope with the modernization of the world. The old agriculture techniques were slow and demanding, compared to the rapid and effortless modern methods. Geometry was not thought as a mean to educate the senses. It was changed to educate abilities. The new mutation required applicable knowledge for the everyday practices in the workplace. Mathematics was understood as a useful science to educate practical and rational skills. It was not enough. The scientific citizen needed should also develop the mind. To learn how to operate machines, how to build them, how to repair them was insufficient to go along with the changes of the world. Astronomers, physicists, chemists, biologists, highly trained engineers were needed: The scientific minded. Mathematics was taken as a corpus of knowledge for educating the mind. More advanced and more abstract mathematics was assumed to shape a greater scientific thinking, to enrich the mind. It was not enough to achieve wealth. Life keeps changing, evolving. The one needed, kept mutating. From a competent entrepreneur to a knowledge consumer to a well-equipped, healthy and successful life-long learner with cosmopolitan desires. It was never enough and it will never be. As for mathematics, it became a core value of modernity; it became the means to master ***the best version of*** the citizen. The logical and rational child useful for the sciences is a dominant narrative that keeps circulating: the ***tales.*** All these mutations were necessary to overcome the complex path to progress and wealth, to overcome wars and economic crises, to take off the label of being underdeveloped, to reach the level of the first world countries, and the status of the world leading economies, to be at the top of the international rankings.

"Learn how to answer the tests and you'll have the world at your feet, eh?" a boy said to him while coming out of the room where the test was being taken. This boy smiled at him, full of confidence. "Is it true?" the child thought. He dedicated all his life to pursue his dream of becoming an astronaut, his whole life he thought grades were necessary, his entire life he committed to studying as hard as he could to secure his place in college. "Was all my sacrifice necessary?" he continued. The confident boy, he probably did not take education as seriously as him. "Is it possible to learn how to solve tests?" he could not believe this boy's opinion, "He will probably have a low score in the test. It is impossible to trick the system," he continued, "Education is necessary for a reason." This confident boy somehow managed to fool the system. He was going to be labeled as a "good student," and he solely learned the techniques necessary to perform efficiently in the tests. He realized this confident boy learned how to be efficient and how to solve problems logically; he learned how to use reason to be more productive. He probably learned the minimum possible to obtain the maximum outcome. But him, the child, he did what he was asked for. He learned everything, even if it was not included in the test. After all, studying was his job. He realized he learned how to be obedient and how to deliver the "good student" profile. "Who is the desired one? Is he? Is I?" he asked himself.

He was confused. "Would standardized tests be able to measure skills or abilities?" he thought. Teachers were prompt to make formative and summative evaluations. Formative tests enable teachers to assess individual progress not only content related. Students would not be reduced only to a number. Although certain capabilities were not included in the guides for teachers, the ***assessing of visualization*** was no exception. Elementary geometry was taken, in the beginning, as a secure way of educating the senses of expert hand workers. Now, the senses were left outside the mathematics classroom. Mathematics was approached with ***sightless eyes***, a perpetual motion in which geometry became an axiomatization of space, a Cartesian world governed by vectors. He knew that this understanding of space was necessary for physics. He knew he needed this mathematical gaze. Probably the confident boy did not want to pursue a scientific career. And even if we wanted, it was highly improbable for him to get accepted; he did not have the mathematical gaze needed for science. He learnt how to optimize time, but he did not follow the path to become a future scientist created by school. The child realized that school had the purpose of shaping people in the right way. "There are certain steps you have to follow," his father used to tell him. And he finally understood those words. Otherwise, he would not be able to apply to a good University. He mutated into everything he did not agree with over the years, everything he challenged, everything he thought was meaningless. He became a ***Mindniac,*** the desired citizen for progress, economic stability, and his own and social welfare.

PART SEVEN: THE (RE)PRODUCED

"Getting angry doesn't solve anything."
How can I help it? How can I help it? How can I help what you think?
Hello my baby. Hello my baby. Putting my life on the brink.
Why don't you like me? Why don't you like me? Why don't you like yourself?
Should I bend over? Should I look older just to be put on your shelf?
I tried to be like Grace Kelly. But all her looks were too sad.
So I try a little Freddie. I've gone identity mad!
I could be brown. I could be blue. I could be violet sky
I could be hurtful. I could be purple. I could be anything you like
Gotta be green. Gotta be mean. Gotta be everything more
Why don't you like me? Why don't you like me? Why don't you walk out the door!
...Say what you want to satisfy yourself.
But you only want what everybody else says you should want

Grace Kelly by Mika

It was summer after his eighth year of school. It was a very challenging year for him. He had participated in the national standardized test. His school did not perform very well according to the national average. It was not the worst performance they had had, but, still, it was below average. This was not enough for his future dreams. “At least, it was one of the best schools in the area,” he said to his mother trying to be positive about the critical situation. Even though he thought he performed brilliantly, the average of the school was the only one considered for measuring the quality of their education. There was a certain stigma given to most of the schools after the publication of the test outcomes. People feel free to judge without acknowledging the work teachers, students, and principals did to obtain a better result each year. “Pressure, nothing more than pressure,” he thought. If he only had more money to pay for a better school, if only his classmates would study a little more, then he would not have to be label as a poor performer. If things could be different, if only there were no ***incepted neoliberal dreams*** about school.

He had been preparing for the tests during the whole school year. He tried to study everyday. “Mathematics is important for every day life,” he said convinced of this statement. Mathematics was treated almost as a sacred knowledge, an ***amalgam of faith and*** reason. It was thought that through its learning people would be able to develop logical, deductive, and rational skills necessary to cope with challenges outside the classroom. Mathematics was advertised as a necessary tool, as a valuable product. It began to take a status of a supra knowledge. It was a Platonic construction in the realm of ideas, of virtues. It was not human, but divine! And everyone who was able to understand it was thought to possess the necessarily trained mind to conquer the world. Mathematics was believed to open the gates of wisdom and opportunities. And so, he knew he needed mathematics. He needed to study. Not only for the test, he needed to become a life-long learner. He needed it because he wanted to be successful. And success was a synonym of wealth. “Knowledge is power, they say,” he thought.

Mathematics was believed to develop scientific reasoning, to uncover certainties. The universal language, the proofs, the demonstrations, the way of thinking, the logic, the vectors, the variables, the formulas, the graphs, the tables. Mathematics was considered the base of all sciences. And geometry, geometry had its special place. Without the Elements, probably science would not be the way we know it today. Geometry was believed to enlarge the mind, to think with precision. Mathematics and geometry held a key role in the shaping of the ***Mindniac***. The *Elements* of Euclid were taken for many years as a powerful model for achieving certainty through axiomatic and a proper method of doing inquiry, an incredible powerful belief that led to taking the *Elements* ***as a paradigm for scientific thinking.*** The road to think scientifically, for ***shaping a scientific self***, for the ***fabrication of qualified citizens***, and to ***effect*** the child through school mathematic practices.

There was an unquestionable necessity of mathematics for citizenship, for society, for the economy. And he knew it. He knew he had to be ***the best version of*** himself. He knew that, despite all introductions in his textbooks, visualization was important for science, not for school. They said that scientific and rational citizens should and would increase economical development of nations. It was not only a belief, for them. It was a fact! Mathematics practices in school aimed at axiomatized space, and to see with a mathematical gaze, with ***sightless eyes of reason***. Somehow, reason and logic were everything they sought for, the ***assessing of visualization*** became unnecessary, it became optional, it became obsolete. He understood that the world outside of school is not interpreted as a Coordinate system, it could be, but it was not something people relied on while living their lives. He did not struggle with the space schools offers, the one with axes and vectors, the one that can be manipulated at will, the one that is exact. He felt in advantage compared to his classmates. The vast majority of them still struggled with basic operations. He could manage to do complex calculations to solve problems, a capacity he was very proud of. “Sometimes I feel superior because mathematics has that power,” he said to himself while trying to give meaning to the sentence: knowledge is power.

“Is my school not allowed to have power because it apparently lacks knowledge?” He continued. Knowledge enables people to empower themselves and give them the illusion of freedom. “If anything,” he followed, “Money is a powerful thing.” He began thinking how his knowledge was limited to the economic situation of his parents. “People with higher income would have the possibility to attend to better schools, the ones at the top. They would have access to other knowledge that is out of my reach," he argued. He had a feeling of injustice. National tests, for him, were the epitome of moral decay. There were people with the freedom to loosely judge the performance of an entire school based on detrimental numbers without any meaning more than the one gave by society. "What if students do not like mathematics and that is what they fail? What if they had personal issues, psychological problems? What if they simply did not care about being tested? What if they did not believe in that system of measurement?" he asked to himself trying to seek for an answer in his own memories, "but, higher education contributes to progress, they say," he added, "The higher education, the better job, and the better life."

That number, that position of his school would hunt him until the next test. He felt he could redeem himself. He thought he could encourage his classmates to study and to set the school in a higher position. He was enthusiastic, only not out of the goodness of his heart; he needed the school to perform better to increase his chances to enter to college. The test was supposed to identify schools in need to provide a higher quality of education and equalitarian opportunities to all. However, those numbers became a doomed hope for a brighter future. There was nothing more that just ***a journey to utopia across assessment***; nothing more than just numbers. Though, necessary numbers. "How else could we know which are the best schools? Education is the key to the future and mathematics is the most valuable and necessary tool for everyday life," he said, "mathematics develops reasoning, which separates us from animals."

The ***tales*** were impregnated in his own beliefs, his own being. They became part of him. It was difficult to escape from the discourses that surrounded him, and even more when those were constructed, created and perpetuated socially. They were everywhere, in school, in media, in his house, among his classmates, teachers, principal, and everybody else. "The system works. It brings order and structure to society," he said. "Mathematics opens the path to a successful life. And if other people do not want to pursue success, they should not follow the same path. Would they have to develop reason, anyways they will not need it," he began to doubt about the advantages of mathematics. "Everybody needs to count, but they have phones with calculators. Who uses their minds to perform daily life computations anymore? And who uses equations and vectors? Scientists do, every day!" he added.

PART EIGHT: THE (D)EFFECTED

I'm an animal. I'm a victim. I'm the answer to your prayers.
I'm a witness on the witchhunt. I'm the monster up the stairs
I'm ghost that's in the mirror. I'm everything that you fear
I'm the riptide. I'm the soul-shock. I'm the voice that's in your head.
...I'm the hunted. I'm the predator. I'm the answer to the riddle.
...I'm the player. I'm the naïve. I'm the one who's not addicted.
...I've seen it all. Still can't taste it.
Smashed to the wall that brought me to my knees.
...I'm just a child with the tears in its eyes
I am holding this gift that is broken.
What do I have left now?

Seen it all by Korn

The child grew up. He was confused. If he was supposed to reason according to school expectations, he was able to realize the impracticability of mathematics to promote problem-solving strategies for future employment: tasks that did not require the context they provided; promised skills that seemed not to be developed, and so on. But, schooling also required him to be quiet. He could not phrase any discomfort. Was he obliged to obey? No, but he knew the system does not work otherwise. “If you raise your voice, you might get fired. If you argue with the teacher, you might get a lower grade,” he thought. During his entire school life, he tried to be and do what he was asked for. The homework, the attitude, the study, the grades, the enthusiasm, the desired to learn, the participation… He tried to be a good student, the best student he could possibly be. He took education as his full-time job like his father used to say to him. After these twelve years of schooling, he began wondering about all he had done. He felt he learned crucial knowledge for pursuing his career. He did not want to be an astronaut anymore. He has got realistic. He was almost an adult. He was going to apply to study physics: the secure option that would enable him to study the Universe and its mysteries. Hopefully, one day, he would be able to go to NASA or Space X. “Dreams do not feed you or clothe you, they do not put a roof over your head, they are just dreams,” he said while deciding what to do with his life, while calculating the exact probability of becoming an astronaut. He was still waiting for his test's outcome, the one needed for applying to University.

“What has mathematics turned me into? What has mathematics done with me? I'm only a nerd to my classmates. A person whose only preoccupation is learning, obtaining the best grades I possibly can. And what has this made me? A weirdo. An isolated human! I feel I lack social skills. I cannot relate to people and sustained conversations outside school topics, they mock me, and they make fun of me as a student. Bullied! "The teacher's pet", they say. But I'm only caring about my future. The rational part of me should care for myself. Was I even a good student? Did I learn how to think by myself, how to solve problems as they expect me to do? Am I qualified to find a job? Are those skills even prompt by the school? Probably not," he wondered. "The image school tries to promote is the one of Spock, only logical, a Vulcan! But he was half human. And we, we are humans. Mathematics has always been presented so accurate, so reliable, so truthful, and so certain. But that works only in school. Of course, I can see a Cartesian world. It is compulsory for school mathematics practices. As if I had special lenses to look vectors and locate points even in the oddest circumstances. For sure I can rotate any figure on a coordinate system to do laborious calculations that make no sense at all outside of school. I have learned how to play the game! I am considered to be productive. Everyone says and thinks I am going to be successful because I performed well in all school subjects, especially in mathematics. But I wondered if my grades are only because I decided to stop concerning about the practicability of school mathematics or because I learned how to play by the rules?" he could not make his mind.

School mathematics was supposed to develop rational, deductive, and logical skills in students. It was believed to conduct habits and desires towards the fabrication of a cosmopolitan subject. It was acknowledged as a key part of the technologies of government to ***fabricate qualified citizens***, ***a scientific self***, the ***Mindniac***, the desired "well-equipped citizen" able to achieve their full potential as a productive and socially engaged person for economic growth. It was not only ***stories*** and ***tales***. It was the ***incepted neoliberal dreams***, the ***promises of welfare***, and the ***assessing***. It was the pursuing of normalization, of homogeneity, the need to "cure" the poor skilled and save him from its limits on the conquering of success. It was the chase for the reasonable and normal: the ***scientific thinker***, the ***sightless eye***d. It was the hopes of the ***Euclidean tradition*** and the dreams of the power ***effects of school geometry*** in the scientification of the self. Indeed school mathematics had effects of power in shaping the subjectivity of this child, but also enabled him to recognize that school practices only take place in school. It was not necessary for him to use the mathematical gaze of the sightless eyes of reason outside the classroom. He knew that school mathematics required him to disconnect from his body. Perception and intuition were the enemies when it came to the logically, rigid and fixed space inscribed in school. He had to detach from his senses to perform accordingly to what was desired. A set of mind that help him discriminate when, where, and how he should use his mathematical tools provided by schooling.

School mathematics and geometry had effects of power in terms of conducting the conduct of the child, by himself. But at the same time, he saw the game being played. He understood it was all part of an illusionary and naïve system. The measuring of productiveness through standardized tests only implied people training solely for the test to obtained a label they probably would not deserve.

People competing against each other during all their school years to obtain poor quality jobs that would probably have them sitting on a desk every day wondering what else could they do with their lives if money did not rule the world. Mathematics does not necessarily mean a better future. "Better for the economy, it depends on the market. Better for social mobility, there is no evidence to support that claim, people move to other countries even without having a degree. What is success and for whom?" he thought. Somehow he became both the desired and the undesired: the person who is able to recognize in mathematics a useful tool for the enlargement of the mind. After all, he felt superior and empowered by this knowledge. And, at the same time, the person who sees in mathematics an unnecessary topic without any particular proved meaning. The scientifization of the self (d)effected him. He became so deductive, logical, and rational that he sought a proper explanation of every aspect of daily life, he began questioning everything again. Was he going to pursue his career? Was he going to become an entrepreneur, a life–long learner, or a cosmopolitan Modern subject?

He was undoubtedly a brilliant student. He probably was the closest a child has ever been of becoming the desired child of schooling. But all he had become was a child, perfectly subjectified to be a scientific thinker, but doomed to a bureaucratic system he did not support. "Real life is messy, it is much more complex. As Batman said: You either die a hero or live long enough to see yourself become the villain," he continued…

People know what they do; frequently they know why they do what they do; but what they don't know is what they do does

Foucault (1984b, p. 95)

EPILOGUE

The disciplinary institutions secreted a machinery of control that functioned like a microscope of conduct; the fine, analytical divisions that they created formed around men an apparatus of observation, recording and training.

(Foucault, 1979, p. 173)

The story of the (d)effects of power in the scientification of the self aims at depicting how subjects, embedded in practices of schooling as an institution, (re)shape and modify themselves, in order to conduct their owns modes of acting and being in the world through the practices of school mathematics and geometry. The story, with a Kafkaesque inspiration, progresses from the child's first approximation to school and his early confrontation with a desired future, until its last test and the waiting to know his results. The multiverse produced by the story enabled to depict a non-linear story.

Hopefully, at this point, it has become clearer how Euclidean geometry, particularly the *Elements*, has historically operated as a fundamental part of the technologies of government toward the shaping of scientific-minded citizens since the Middle Ages. Also, it has become visible the power effects of mathematics education in the fabrication of the sightless eyes of reason as a rationalized mathematical gaze towards the scientification of the self. Historically mathematics has had an important impact in the configuration on Western education, and has held a privileged position not only regarding the status that brings to people but also as an approach to understand the divine: Geometry as the language of God (untouchable); mathematics "ascends from the world of impermanence to thus higher, heavenly plane" (Cohen, 2007, p. 19).

This dissertation contributes to the understanding of how schooling becomes a social institution in a particular spacetime, within concrete historical configurations, and the desire to strive for the fabrication of rational thinkers, and enlightened subjects. In such grid, school mathematics makes part of the technologies of power/knowledge for the making of Modern cosmopolitan subjectivities. This thesis has exposed the "result" of neoliberal techniques of government through school mathematics, and it has also raised the concern regarding economic factors embedded in the teaching and learning of school mathematics in Chile: marketable school mathematics. The classroom is not only the place where students live their lives as students. It is also the place where they students labeled. These labels not only consider abilities, skills, mathematic proficiency or students' grades, but the student's economic class or social and economic standing. Such transformation mutates the group of students in the classroom to a segregation of students: the class/room.

Chile succeeded in creating a system in which economic stability can be translated into better opportunities in life, through the engagement with school mathematic practices. Having the option to pay more for a higher-ranked school is not a possibility for all citizens, and cheaper schools are often ranked lower in national tests. Here, the desired child comes into an unreachable illusion inside a system where everything is already taken to be a failure. All students are already failing, even if they are “good students” (with the highest grades). This happens within a regime of truth in which all possible approximations of becoming what is desired are unfruitful. The desired child remains in the realm of the virtual, of immanence: A platonic desired citizen. Students will never be good enough to become the desired child, not because they are unable to learn mathematics but becomes from the outset they are disabled by the impossibility to reach a Platonic desire. It is this double gesture of power in the cultural and social configuration of mathematics education what allows the double instantiation of the effecting the scientific self and deffecting it at the same time: the (d)effecting. For example, the desired scientific child should conduct herself, through technologies of the self unfolded by school geometry, to fabricate a mathematical gaze (figure 8). However, not all students are able to detach from their bodies as a suppression of the self to reach the realm of truly objective knowledge. The desired child, as a problem solver, productive, qualified, competent and rational child, not only understands that in school mathematics practices she is prompt to use her mathematical gaze, but also realize, that this gaze becomes unproductive outside the classroom. And therefore, in school, she unlinks her senses to engaged in the platonic envisioned that is mathematics.

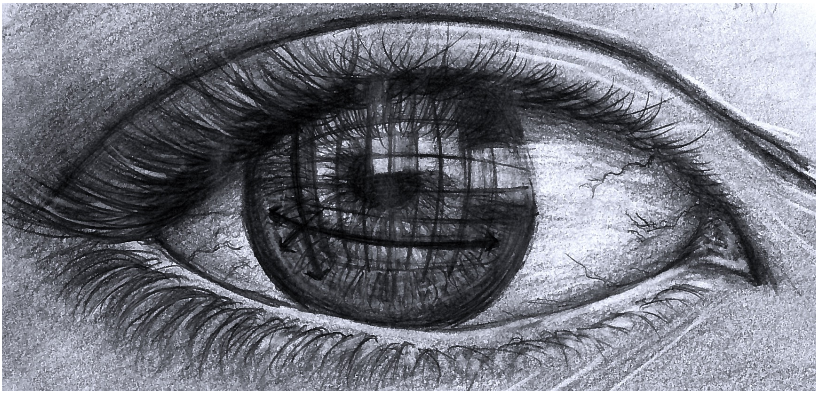

Figure 8: The mathematical gaze of the desired scientific child

And with this mathematical gaze of the desired scientific self, the (d)effected child, I open the road to the papers…

The question of the "truth" of the individual geometrical propositions is thus reduced to one of the "truth" of the axioms. Now it has long been known that the last question is not only unanswerable by the methods of geometry, but that it is in itself entirely without meaning. We cannot ask whether it is true that only one straight line goes through two points. We can only say that Euclidean geometry deals with things called "straight lines," to each of which is ascribed the property of being uniquely determined by two points situated on it. The concept "true" does not tally with the assertions of pure geometry, because by the word "true" we are eventually in the habit of designating always the correspondence with a "real" object; geometry, however, is not concerned with the relation of the ideas involved in it to objects of experience, but only with the logical connection of these ideas among themselves.

It is not difficult to understand why, in spite of this, we feel constrained to call the propositions of geometry "true." Geometrical ideas correspond to more or less exact objects in nature, and these last are undoubtedly the exclusive cause of the genesis of those ideas.

Albert Einstein (2001, p.4)

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The sightless eyes of reason: Scientific objectivism and school geometry

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There is a gap between the aims of school geometry in terms of the teaching of spatial abilities to young children, and the dominance of a school geometry rooted in Euclid's axioms and abstractions. Such gap is not to be explained in terms of a "misimplementation" of the curricular intentions. Rather, the gap evidences elements of the power effects of school geometry on children's subjectivities. We problematize both the truths circulating in school geometry discourses and the effects on children's subjectivities, by adopting cultural historical strategies to research the functioning of school geometry curriculum. We argue that school geometry fabricates the scientific minds of the future by educating students to see not with the eyes of their bodies, but with the eyes of reason and logic.

Keywords: Objectivity, subjectivity, school geometry, power effects.

INTRODUCTION

Nowadays it is argued that spatial visualization ability plays a key role in shaping the successful scientific minds of the future, probably as important as verbal and mathematical thinking (Newcombe, 2010). This ability becomes critical when developing expertise in STEM domains (science, technology, engineering, and mathematics). It is believed that including spatial ability as a criterion for identifying talented youth would help recruiting many adolescents with potential for studying STEM fields, but who are currently being missed (Wai, Lubinski, & Benbow, 2009). There is a research trend claiming the importance of providing spatial education for young children because "increasing access to a preschool "spatial education" constitutes a safe bet for fuelling school readiness and igniting long-term performance gains in STEM-related fields" (Verdine, Golinkoff, Hirsh-Pasek, & Newcombe, 2014, p. 20).

But why has spatial visualization become so important? One possible reason is how research has found that visualization is central in the conceptualization processes of scientific discoveries. For example, the use of spatial reasoning is implicated in several physics discoveries such as Galileo's laws of motion, Faraday's electromagnetic field theory, and even Einstein's theory of relativity (Kozhevnikov, Motes, & Hegarty, 2007). Furthermore, the discovery of the structure of DNA was "centrally about fitting a three-dimensional spatial model to existing flat images of the molecule" (Newcombe, 2010, p. 29).

It is postulated that "spatial thinking can be taught; [... and that] it is possible with appropriately structured programs and curricula" (National Research Council [NRC], 2006, p. 109). Several studies on school geometry deal with the development of spatial abilities in students, from introducing diverse activities with building blocks to changing the entire school geometry curriculum. Despite the recognition that spatial ability is a key element to science and that it can be taught, it is also highlighted that the teaching of this ability is largely ignored in formal school settings (Clements & Sarama, 2011).

It is the contention of this paper that there is a gap between the aims of school geometry in terms of the teaching of spatial abilities to young children, and the dominance of a school geometry rooted in Euclid's axioms and abstractions, which prompts to a flat and abstract world. Our assumption is that such a gap is not to be explained in terms of a "misimplementation" of the curricular intentions. Rather, the gap evidences elements of the power effects of school geometry on children's subjectivities. Adopting cultural historical strategies to investigate this contention in the constitution of school geometry, the paper deploys an argument in three movements. Firstly, we examine how notions of space move between discussions of perception and formalization. The formalization of the language of Euclidean geometry, through scientific objectivism, provides ways of building a scientific self. Secondly, we explore how such objectivation entails the subjectivation that takes place through the discourses of school geometry. By examining the curricular materials of Chile, we exemplify the existing gap between such discourses and the expressions of the ideal student. Finally, we problematize both the truths, in terms of Foucault, circulating in school geometry discourse and the effects on children's subjectivities. We evidence how the school practice of knowing geometry has effects on how students understand and train themselves to see space.

ANALYTICAL STRATEGY

There are many truths that circulate in mathematics education research, and such truths constitute unproblematized understandings of the practices of mathematics education. The type of research deployed in this paper assumes that mathematics education practices are political because they govern subjectivities in both productive and constraining ways (Valero & García, 2014). Evidencing the subject effects of a series of practices and discourses, such as school geometry, is a contribution in understanding how the school mathematics curriculum fabricates the subjectivities of children through educational processes. This is important since mathematics education is not only a process of knowledge objectivation, but also a process of subjectivation or of becoming within culture.

More concretely, this approximation is inspired by the work of Michel Foucault. Our strategy is composed by some concept-tools we borrow from Michel Foucault (subjectivity, discourse, truths). We bring this concept-tools to help us reasoning about the problem of the apparent gap between the aims of school geometry in terms of the teaching of spatial abilities and the dominance of a school geometry rooted in Euclid's axioms and postulates. Our empirical materials consist of students' textbooks, curricular guidelines for teachers and geometry maps of learning progress, all of them produced by the Chilean Ministry of Education (MINEDUC).

Since discourses are produced by the interaction of different spheres of social life and are shaped by statements and their related truths (Foucault, 1972), to understand how school geometry is operating it is also necessary to study how geometrical knowledge has been shaped. Here we delineate elements of such study connecting geometry with cultural historical studies of science (e.g., Daston & Galison, 2007). The discussion of objectivation/subjectivation in science and geometry invites us to approach the school geometry curriculum as practices that govern subjectivities through the enunciation of the ideal student. Hence, problematizing the naturalised truths that circulate in school geometry discourse and the statements about the aims of school geometry will help us to elucidate the effects of geometrical knowledge objectification on the self.

THE OBJECTIVATION OF SPACE

Mathematics lives in a world of abstractions, axioms and formulas. There is a perfect and ideal world within mathematics, every calculation applied correctly should work impeccably, even if is not about a real object. For de Freitas (2013), mathematical objects are taken to be entirely free from spatio-temporal conditions. Hence, if mathematics is universal and has no context it is possible to understand it as a blind sight, without inference, interpretation or intelligence (Daston & Galison, 2007).

However, it is believed that mathematics can describe the world we live in. But if geometry can describe what we are able to see, our surroundings, why has it become a blind sight? According to Boi (2004), the anatomy of the eye entails light on a curved retina, therefore our visual system deploys a projective geometry rather than Euclidean metrics. An experiment conducted by Blumenfeld (Hardy, Rand, & Rittler, 1951) demonstrated that phenomenological visual judgments do not satisfy all Euclidean properties, he revealed that physical configurations do not coincide with Euclidean geometry (Suppes, 1977)[1]. Likewise, Burgin (1987) claims that the conception of Euclidean geometry's space was based on technique rather than on visual evidence. It was based on axiomatic, which deployed an idealized world of ideal shapes, such as triangles, squares, platonic solids and so on. It is an objective knowledge.

For Daston and Galison (2007)[2], objectivity in science was not a matter of viewing nature as it really was, but as it should be to be studied – nature as an ideal nature –. The result of objectivism was an annulation of the self by the self, it was "the suppression of individuality, including images of all kind, from sensations of red to geometrical intuitions" (Daston & Galison, 2007, p. 46). Images, within science, were 'left behind' because it was the only way to break the mental world of individual subjectivity.

If objectivity in science aspires to a knowledge that bears no trace of the knower, how is geometry supressing individuality? For Daston and Galison (2007), objectivity becomes an 'epistemic virtue' when abstractions are able to transform subjective representations into objective concepts. For example, Tazzioli (2003) shows that Mario Pieri, an Italian mathematician, introduced the axioms and methods of projective geometry without any reference to intuition (neither to monocular vision). He was close to cut the link between geometry and empiricism. Hilbert's work, inspired by Pieri, was the masterpiece that led to a geometry based on logic, axioms and theorems (*règles*), a form of geometry in which intuition and experience do not have a strong role. This led to a space that can only be reached by mathematics, very distant to the one we are able to see and interact with.

But space is a product of concrete practices and attempts to representing them; it is not abstract at all. However, when it comes to the knowledge that traditionally has dealt with space —geometry—, then it becomes the realm of abstraction. It becomes an objectified space. For instance, according to Lefebvre (1991) space can be understood in three forms: space as perceived, as conceived and as lived. The first form takes space as a physical form, as real space, a space that is generated and used. The second form, the space of knowledge (*savoir*) and logic, takes space as an instrumental space. Space becomes a mental construct, an imagined space. The third one, space of knowing (*connaissance*), sees space as produced and modified over time and through its use. It is a space that is real-and-also imaginary. Geometrical space is a space of *savoir*, within Lefebvre's forms; it is an idealized, imagined and constructed space.

In the same fashion, school geometry is leading us to see space as an instrumental space, a mathematical space of *savoir*. Space becomes Euclidean, Cartesian, and flat. It is very distant to the one we can perceive. Ray (1991) stresses that school mathematics has been based on the axioms of Euclidean geometry because they provide an internally consistent system, evident to the eye. But this objective geometrical space deployed by Euclidean geometry is limited, in terms of Lefebvre's forms of space.

BLINDING THE CHILD

In school, geometrical knowledge tends to be constructed outside the body. It is a fixed knowledge that has to be learnt by students. School geometry is based on abstractions, very distant from our perception of 'daily space', as if we had a body without the sense of sight. In other words, dealing effectively with school geometry tends to fabricate a sightless body. Then, learners are not really prompted to use their senses to learn or interact with geometrical knowledge.

For example, Chilean mathematics curricular guidelines claim that mathematics was developed to solve diverse challenges of mankind and of mathematics itself, within history and culture. Therefore, school mathematics must provide and facilitate the understanding of the real world we live in (Ministry of Education of Chile [MINEDUC], 2010). In this sense, school mathematics should supply students with tools to interact with the world they are able to see. An analysis of official documents of the Chilean Ministry of Education (MINEDUC) shows that its claims are built around certain statements, in a Foucaultian sense, that delineate the features of an ideal child, such as:

> School mathematics curriculum is aimed to provide students with the basic knowledge of the field of mathematic, and, at the same time, that students develop a logical thinking, deduction skills, accuracy, abilities to formulate and to solve problems and abilities of modelling situations (MINEDUC, 2010, p. 3, our translation).

> To learn mathematics enriches the understanding of the reality, facilitates the selection of strategies to solve problems and contributes to an autonomous and own thinking (MINEDUC, 2010, p. 3, our translation).

School geometry becomes a valued and useful knowledge, a set of tools that will help students to fulfil any situation of the real world. For Valero, García, Camelo, Mancera and Romero (2012), school mathematics "inserts subjects into the forms of thinking and acting needed for people to become the ideal cosmopolitan citizen" (p. 4). In this case, an ideal student should be

When students have achieve this level, they solve activities as the following:

- To calculate, by using geometrical and analytical methods, the area between four circumferences of radius a, which centers are the vertices of a square of length 2a.
- To determine the number of diagonals of an n-sided polygon, by using combinatorial techniques.
- To prove by contradiction, that a line is tangent to a circle when the radius drawn to the point of contact is perpendicular to the tangent at that point.
- To prove, by using congruence axioms, that the exterior angle of a triangle is grater that either of the remote interior angles.
- To construct a regular pentagon using a compass and straightedge or a software, and to justify the construction.
- To deduce the vector equation of a line from its Parametric and Cartesian form.
- To formulate and to verify which geometrical shapes satisfy that: the sum of the areas of the shapes built up on the sides of a right triangle is equal to the area of the shape built up on the hypotenuse.

Figure 1: Seventh level of the map of progress (MINEDUC, 2010, p. 18, our translation)

a logical thinker and a problem solver. The student should be capable of modelling real life situations by only using mathematical knowledge.

Furthermore, the Chilean Ministry of Education (MINEDUC) established a map of progress with seven levels that students have to achieve along school geometry in compulsory education. An ideal student should perform successfully in activities where he/she must be able to solve problems by only using geometrical axioms and theorems (see Figure 1), which leads to a notion of space in terms of the formal system of Euclidean geometry. What it takes to deal successfully with these tasks is far from spatial visualization. It seems that this ability is something that has to be developed by the student itself.

The mismatch between the expectations and the description of abilities could be related to the fact that Chilean school geometry is based on Euclidean, Cartesian and vectorial geometries, necessary to cope with other school subjects, such as physics.

> The world we live in is three-dimensional [...]. It is aimed that students are placed in a real three-dimensional context, providing new tools to make spatial and flat representations, such as the vector model. This model constitutes one of the basic foundations of physics and mathematics. (Ministry of Education of Chile [MINEDUC], 2004, p. 68, our translation).

According to this quote, it becomes important to link "reality" with school mathematics. By doing so, it is assumed that students will be able to use geometrical tools to solve everyday-life problems. The way to achieve this link is by introducing three-dimensionality to students. Within school, the "world we live in" becomes vectorial. Consequently, the expressed desire to link spatial thinking and the real world seems to blur, and the only important part left are the perennial mathematical abstractions. This generates a new type of space, the space of school, which has been 'chopped' and has been restricted.

As an example, the MINEDUC (2004) enhance certain types of activities where students "emphasize relations between Cartesian and vector equations within geometrical shapes" (MINEDUC, 2004, p. 68, our translation) than other type of activities where students could develope spatial skills. Space for school is in terms of XYZ, a space than can be modelled by school mathematics. Chilean school geometry is based on a flat geometry, mainly Euclidean. But in Euclidean geometry the studies are on objects situated in the void; objects that are not real (Kvasz, 1998). A possible question to pose is if Euclid of Alexandria was living in an abstract world? The easy answer is that of course not he was, and, moreover, he started his studies by analyzing his surroundings. He developed a theory known as Euclid's optics. Which is a theory of vision and of intuition (Suppes, 1977).

According to de Freitas (2013), logic and axiomatic relations in mathematics tend to erase the temporal and ontological. As a result, school mathematics is an untouchable knowledge that becomes universal, decontextualized and, therefore, without culture or the possibility to influence in it (Valero & García, 2014). It is an unalterable truth, installing mathematics as the science of pure logical structures and negating all connections between mathematics and the real world we live in (Kollosche, 2014).

> The concept of space to be reconstructed in the students' understanding is that of a rational, referential space with fixed points in two or three dimensions. It is assumed that the conceptual development of the child will lead to an internal and abstract representation which will contribute to making a decontextualized child, freed from the practical capacities of acting with objects in space, particularly of those spaces where everyday life occurs (Valero et al., 2012, p. 7).

However, school keeps making this link between what we are able to perceive and the abstract space of mathematics. We claim that there is a gap here to explore.

THE SIGHTLESS EYES OF REASON

We deployed a discourse analysis of official curricular materials of the Chilean Ministry of Education. This analysis, built from Foucault, has pointed to the existence of statements circulating about an ideal student. In this existing discourse, it is believed that by connecting school geometry and reality students will become problem solvers, logical thinkers, 'reality modellers' and so on. The existing discourse requires that students perceive themselves as agents who are able to change the world, but also as agents who are responsible for their own learning (Foucault, 2009). More precisely, school geometry deals with power-knowledge relations [3], by promoting the fabrication of a certain type of subject, a scientific trained child.

But, how is school geometry discourse operating on students? Here the discussion is not about the contents of school geometry itself rather it is on how school geometry is operating in the fabrication of children's subjectivity. In other words, it is on the power effects of school geometry in fabricating forms of being in the world. In this sense, human beings become subjects through the objectifying effects of scientific knowledge, a knowledge that is also objective (Foucault, 1982). And, at the same time, the practice of knowing generates effects in the form of knowing and in the subjects who know (Daston & Galison, 2007). Therefore, students must train themselves to become part of a practice. An example of this self-training is illustrated in the following activity proposed in the 6th level of the progress map of Chilean school geometry (MINEDUC, 2010, p. 17).

> A young girl is observing a pit formed by two concentric circumferences, 1 m. and 1.2 m. of radius respectively, and 3.5 m. of depth. Using this information, she is able to make a model by drawing a rectangle ABCD in a coordinate system XYZ. It is asked to the girl to determine the rectangle's vertices and to argue on which axis the rectangle must rotate to obtain a three-dimensional representation of the pit. Finally it is asked to the girl to calculate the pit capacity, in litters.

Clearly the ideal student must forget about his/her senses; must train him/herself to be able to model real life situations using geometrical deductions; must be able to think space in terms of XYZ. It is not necessary to use spatial visualization ability to solve this activity because is useless. Is it relevant to mention that the

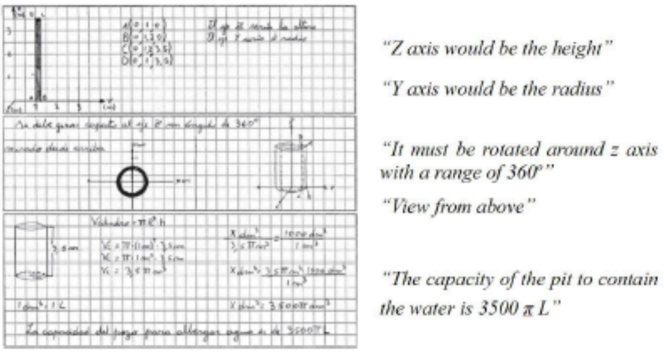

Figure 2: Example of a expected answer of a student

girl was observing the pit? Would it make any difference if the problem had not been contextualized to a real life situation? Why is it relevant to use a coordinate system to calculate the volume of the pit?

This evidences a gap between the aims of school geometry on curricular materials of Chile, in terms of the teaching of spatial abilities, and the notion of space that school geometry promotes, which is rooted in Euclid's axioms and abstractions. Reasoning with the objectification of space in geometry, to shape a scientific self implies to suppress the self, which means to cut all links between perception and geometrical formalizations. This suppression leads to perceive space and geometrical knowledge as decontextualized, and as universal and timeless.

Nonetheless, space outside the school is not universal neither timeless. What if this notion of space changes? Sanjorge (2003) argues that the space in which the subject is constructed has already changed; consequently the subject itself has been changed. There are virtual movements, where there is no orientation, not right or left, it is a post-Cartesian space, which is nonlinear. It is a "spherical space, where up and down are not positions in the world but situations of the viewer" (Sanjorge, 2003, p. 5, our translation). This is a notion of space unfolded by technology, a virtual world that is subjectifying children to perceive no orientation, where everything is reachable by a 'click'. It is a space opposed to Cartesian movements within 'school space' and the different spaces are not related at all.

At the end, the interplay between power and mathematics education is on how the school mathematics curriculum generates cultural and historical subjects (Valero & García, 2014). Then, school geometry becomes a technology of the self and the others, by regulating children's conduct, and by developing 'cultural thesis' (Popkewitz, 2008) about an ideal student who is able to see with sightless eyes. This generates systems of reason in which forms of life and subjectivity are made possible, organized and constrained. Therefore, school geometry has power effects on how students understand and train themselves to see space. Such subjectification pursues to fabricate the scientific minds of the future by educating students to see not with the eyes of their bodies, but with the eyes of reason and logic.

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ENDNOTES

1. Blumenfeld performed a series of experiments with parallel and equidistant alleys. One of the tasks in the experiment was to arrange two rows of point sources of lights as straight and parallel to each other as possible. The lights were placed on either side of a plane. The results revealed that physical configurations do not coincide with Euclidean geometry. In Euclidean geometry, parallel lines are equidistant along any mutual perpendicular. However, in the experiment, the resulting lines diverged, they were not parallel at all. He concluded that Euclidean geometry does not apply to our visual space.

2. Daston and Galison (2007) analysed images on scientific atlases to study its history, emergence and development.

3. This power is not to be understood in terms of domination of the self; it is not an imposition to train students' sight. This power understands 'the other' as a person who acts on his/her own; depending on the freedom of the subject (Foucault, 1982). Likewise, school geometry discourses are not a form of impositions; they are produced because we reproduce them through language. They are an 'action upon action' (Foucault, 1982).

Shaping a Scientific Self: A Circulating Truth within Social Discourse

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In this paper we illustrate how a truth circulates within social discourse. We examine a particular truth reproduced within science, that is: through the understanding of Euclid's axioms and postulates a person will gain the access to all human knowledge. We deploy a discourse analysis that helps us to understand how a truth is reproduced and circulated among diverse fields of human knowledge. Also we show why we accept and reproduce a particular discourse. Finally, we state Euclidean geometry as a truth that circulates in scientific discourse. We unfold the importance of having students follow the path of what schools perceive a real scientist is, not to become a scientist, but rather to become a logical thinker, a problem-solver, and a productive citizen who uses reason.

Introduction

We want to tell a story about a circulating truth that has been shaping a scientific self since before science was called science. Even though there are many truths within scientific knowledge, this particular truth seems to resist every attack, seems to win every fight. Within social discourse, it is believed that mathematics is a powerful knowledge that will enlighten people.

> All adults, not just those with technical or scientific careers, now require adequate mathematics proficiency for personal fulfilment, employment and full participation in society. [… Students should] be able to apply them to solve problems that they encounter in their daily lives (Organization for Economic Co-operation and Development, 2014, p. 32).

Then, in order to be the productive citizens that society requires, it becomes important that students develop mathematical thinking.

Students should be able to

> reason mathematically and use mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals in recognising the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens (Organization for Economic Co-operation and Development, 2014, p. 28).

Therefore, mathematics becomes the tool to solve problems from everyday life. In fact, one of the areas that PISA measures is *Mathematising*, this is the ability to move between the, so called, 'real world' and the mathematical world. Thus, schools address the development of this particular ability by connecting everyday life to mathematics. But this link does not always work, because the 'real world' of school is not the physical world. The models that we have to link both are outdated (Burgin, 1987).

School has been developing a 'school space', where everything has a coordinate in a two or three-dimensional Cartesian system. This 'school space' is rooted in Euclidean geometry. In other words, it is shaped by Euclid's axioms and postulates. At school, straight lines are always straight, they do not curve at the horizon, parallel lines are in fact parallel and the sum of the interior angles of a triangle is 180 degrees. However, 'school space' is a space that has been modelled by mathematics which is different from the world the students live in, precisely because geometry provides the materials for models of the physical world, models that are abstract analogies and not the world itself (Ray, 1991).

In this paper, we examine a particular truth reproduced within scientific thinking. The belief is that through the understanding of Euclid's axioms and postulates a person will gain the access to knowledge, not just the access to geometrical or to mathematical knowledge, but to all human knowledge. We are going to deploy a discourse analysis that will help us to understand how a truth is reproduced and circulated among diverse fields of human knowledge. Also it will show why we reproduce a particular discourse through our own language.

How are Discourses Reproduced?

Discourses are not impositions; we are not forced to believe in them. But sometimes, something sounds very reasonable to us, so self-evident, so logical, so common sense that we agree with it and we start to reproduce it through our own language. Who will go against the idea that we need mathematics in our daily life? But we perceive these 'truths' as common sense just because we are inserted in a particular time and place, spatio-temporal conditions, with a particular rationality. We are *subjected* to those self-evident truths.

Foucault claims that "taken-as-truth" statements circulate within social discourses, discourses that are produced because we reproduce them through language (Foucault, 1982). At the same time, these discourses are not isolated; they are produced by the interaction of different spheres of social life and are shaped by statements and their related truths (Foucault, 1972). In other words, discourses do not materialize from thin air, nor are they commandments by a superior force, such as a God or government.

Therefore, truths become a discursive formation, and there exists diverse rules for what is considered to be true and false (Jørgensen & Phillips, 2002). In other words, there are regimes of knowledge determining what is accepted as meaningful and true and what is not. As Deleuze stressed, only statements may be repeated, but these statements "are not visible not hidden" (Deleuze, 1988, p. 10).

For instance, a circulating truth could be that 'school provides tools to achieve success in life'. Some people might agree with this truth and reproduce it, and some people might be against it. In fact, educational sciences have been providing these tools, with the promise of a better future, for the fabrication of a 'cosmopolitan child' (Popkewitz, 2008). If we analyse school mathematics, it was believed that by a 'mathematics for all' it was possible to create this brighter future (Valero, 2013), and that belief has not changed though time. But, why do we say that it is a 'circulating truth'? Recall the PISA quotation above: *All adults require adequate mathematics proficiency for personal fulfilment, employment and full participation in society.* This implies that to be 'productive' and 'successful' one must know mathematics and science.

There are many naturalised truths circulating in the discourse of diverse scientific fields, and such truths constitute unproblematized understandings of its practices. One of these truths is to believe, for example, that Euclid's axioms and postulates became a universal key to access human knowledge (Sbacchi, 2001). In the same fashion, that Euclidean geometry began to appear as a dominant perspective within scientific knowledge (Majsova, 2014). Or, that Euclid's *Elements* are so necessary to every science that we must believe in them as its basis, principle and fundamental elements (Guarini, 1968). So, a particular truth within scientific knowledge has been reproduced.

Building from this truth, what makes Euclid's *Elements* so important? Are the Elements important because it was the only 'recognized' form of geometry until the 19th century? Harrison (1919) stresses that for a great period of time Geometry and the *Elements* of Euclid were considered as synonymous. But, is it the only reason?

> In the 1630s, Descartes's Discourse set out philosophical reasons for seeing Euclid's Geometry as an intellectual model for theories in other areas of inquiry. Fifty years later, Newton showed that this model was not just formally rigorous, but empirically powerful: i.e., it resolved problems that had plagued European thinkers ever since the publication of Nicolaus Copernicus's de Revolutionibus (1543) (Toulmin, 1998, p. 330).

Apparently Euclid's *Elements* were more than just books summarising the geometrical knowledge of his time, Descartes and Newton recognized them as an intellectual, rigorous and powerful *model*. So, the question left is: how has Euclidean geometry been operating in the development of scientific knowledge?

How Can Truths on Scientific Discourse be Analyzed?

Now, we have a truth: "Euclid's *Elements* are a key to access human knowledge". But, how are we going to deploy a discourse analysis to understand how has Euclidean geometry been operating through scientific discourse?

As we stated above, discourses are not isolated, they are produced by the interaction of different spheres of social life. This means that diverse spheres will evolve around certain truths, a certain statement will be shaped. But, at the same time, the same statement might be repeated in other spheres. Bang (2014) adds a new insight to this 'equation', he presents a new framework employing an image of 'quasi-self-similar fractals' to trace entanglements between multiple semi-autonomous fields.

> A new image of thought employing quasi-self-similar fractal—an image better suited to clarifying the issues and understanding the transversals and influences among multiple fields. This new image of thought is an attempt to represent the strange universality or 'universal mechanisms of fields' one encounters (Bang, 2014, p. 54).

To understand how a truth is circulating among social discourses, which means, to understand how 'Euclid's *Elements* are a key to access human knowledge' is navigating within different spheres of scientific discourse, we have to think outside the box of causality (e.g. Daston & Galison, 2007; Popkewitz, 2008). We have to trace the entanglements across scientific fields --- a 'quasi-self-similar fractals' image. This truth moves between the different spheres and also between spatio-temporal conditions. For example, Valero (2013) examines a "taken-as-truth" statement on mathematics education research, this statement is: a "mathematics education for all" is needed. She uses a discourse analysis strategy, which implies a rhizomatic analytical move (Deleuze & Guattari, 1987).

> My analytical strategy involves visiting a number of interconnected spaces that without any linear or strict logical connection [...] map different aspects of the statement under examination. [...] I also move in the connection of ideas in time and space. As mathematics education research is thought as an international field of inquiry, and probably because for many of its practitioners mathematics is still conceived as a universal activity, [...] In keeping my eye on the ideas that circulate across nations I try to make evident how a field of inquiry generates truths that seems to be transferable from place to place and from time to time, contributing in this way to the reification of mathematical ability as a human ability and right that equates with reason, and

> with that installs one unified logic of being (Valero, 2013, p. 6)

Therefore, to analyse how the *Euclidean truth* is operating, we are going to move outside the field of mathematics, then, we are going to be able to see the entanglements across fields, the rhizome.

Is there a Euclidean Truth?

It is possible that one might think that we are forcing the Euclidean truth to appear. As if we were searching for a suspect, then everyone would be guilty. As we previously mention, discourses are produced within different spheres of social interactions. So, how will you react if we state that Euclidean geometry was not only the root for the development of mathematical knowledge? Well, the answer could be simple: physics. But no! Euclid's *Elements* have been entangled across diverse fields, such as architecture, literature, religion, philosophy, political science, and so much more. A 'quasi-self-similar fractal', a rhizomatic web where everything is connected by, the one and only, Euclidean thinking.

By deploying an analysis on the discourse of scientific research fields, about their 'roots', the existence of some beliefs about Euclidean geometry emerged. A truth that states Euclid's *Elements* as a method, the *Euclidean model*, that it was considered the "standard pattern for any "hard" science" (Toulmin, 1998, p. 336).

> Euclid's geometry, for instance, is notable for its rigor in demonstration [...] is distinguished for its orderly "progression from the simple to the compound, from lines to angles, from angles to surfaces, and so forth," a method that particularly "contributes to the enlargement of mind and makes us think with precision" [...He] develops the propositions of geometry in response to a natural need to know or to a spontaneous order of inquiry [...] Any of these three systems increases the student's capacity for reasoning, for understanding ideas, which properly understood are "notions determined by relations" [...] It is "nothing more than the faculty of arranging, *facultas ordinatrix*" [...] The desire for order leads to the ideas of truth, goodness and beauty (Frank, 2007, p. 251)

From this, it is clear that Euclidean geometry is understood as a rigorous model of demonstration, as a model for 'organizing' knowledge as a progression and, finally, as a response to a natural need to know. These three aspects of Euclidean geometry will increase the capacity of students for reasoning and so forth. It is possible to think that this sort of statement derives from school mathematics discourse, or that it was a result from mathematics education research. But no, it was stated within the field of theology, in 1765, where it was argued how a man, through reason, becomes a man (Griffiths & Griffiths, 1765).

> Euclid's reasoning described a method which [...] provided the foundation for all true reasoning, an abstract scientific method, through which the world becomes intelligible by a means of reasoning which is entirely independent of sense perception (Vinnicombe, 2005, pp. 670-671). Geometry is central to the great philosopher's thought in two quite distinct ways: as methodological guide and example, and as the most basic of all branches of knowledge, from which "synthesis" might deduce, step by step, the immutable laws of social justice (Grant, 1990, p. 151).

In the 17th century, it was believed that a new political science could be established that relied upon the principles of Euclidean geometry, an abstract scientific method, which developed the model for organizing human behaviour. Hobbes, who was proposing this connection, believed that 'geometry was central', a knowledge cannot subsist without a proper method and the key to achieve that method was held by Euclid's reasoning (Grant, 1990).

And those statements about the Euclidean truth have been entangled in other fields,

> In Architettura Civilequite often the elements of geometry become the elements of architecture *tout court*. For Guarini, for example, a wall is a 'surface' and a dome a 'semisphere.' [...] the problem, for him, was not 'how to build' but 'how to draw.' Therefore, not only Euclidean geometry has become a part of architectural theory but it has also carried with it its implied linearis essential (Sbacchi, 2001, pp. 30-31).

It is clear that this quotation above is from architecture. These notions of Euclid's *Elements* were formally introduced in the 15th century by the *Trattato di Architettura Civile e Militare*. Euclidean geometry began to appear as "a good alternative to more complicated numerological calculations [...And was probably] the preeminent one among the masses and the workers" (Sbacchi, 2001, p. 27).

It is possible to think that these quotations are old and that they recognize Euclidean geometry as the *model* for science simply because non-Euclidean geometries were not "formalized" until the 19th century. But, in the 21th century, literary education is being rooted in Euclidean geometry, not in the axioms and postulates, but in the model of order, of organizing knowledge, from the self-evident to the most complicated abstractions. Where the self-evident technical literary terminology are assumptions as the role of an "unreliable narrator" in a book or a play, there is no need to define it (Rabinowitz & Bancroft, 2014).

> We are proposing Euclid as a model because we believe that literary education should begin with the fewest possible number of initial assumptions, and that more complicated interpretations, in later years, should come from increased development and subtler manipulation of those assumptions, rather than from introducing entirely new concepts (Rabinowitz & Bancroft, 2014, p. 4)

I Completely Agree with It! Do You?

So, how has this *Euclidean truth* been accepted in the development of scientific knowledge? To reproduce a truth is not to repeat it incessantly, rather to reproduce implies acceptance and agreement. No scientist was forced to think *Euclideanly*, they accepted the *Euclidean model* because it seemed reasonable for them. In other words, they are *subjected* to the self-evident truth of Euclidean geometry's consistency, simplicity, rigour, "progression order" and so on.

This is the second time that the word *subjected* appears. According to Foucault (1982), there is not a domination of the self; no one is forced to do or believe anything by imposition. That is how he understood power. So, this power implies 'the other' as a person who acts on his/her own. Hence, this power depends on the freedom of the subject. Here is when the term *subjected* comes to play; to be subjected could mean "to be shaped in a particular way" or "to be shaped to become a particular self". And every spatio-temporal condition has a 'rationality', a way of thinking and behaving, as codes that are transmitted to people. The game of "being part of" requires the acceptance of that spatio-temporal discourses, not a simple repetition; one has to believe in it.

So, let's return to Euclidean geometry. How has this *Euclidean truth* been accepted in the development of scientific knowledge?

> Since seventeenth- and eighteenth-century natural philosophers took their Platonist ambitions from Galileo and Descartes […]. From the start, formal systems modelled on Euclid had a charm that carried people's imagination over into fresh fields: if the world of nature exemplified in Newton's dynamics had a timeless order, this could presumably be extended to the world of humanity as well (Toulmin, 1998, p. 353).

> The mathematical method of deduction from axioms had a decisive effect on the social sciences of the Enlightenment. […]. Find the axioms of human nature, deduce from them in the approved Newtonian manner, and a complete science of man became a possibility (McClelland, 2005, p. 290).

> One of the issues which played a major rôle in most of the discussions of the Theory of Relativity was the simplicity of Euclidean geometry. Nobody ever doubted that Euclidean geometry as such was simpler than any non-Euclidean geometry with given constant curvature […] Euclidean geometry is the only metric geometry with a definite curvature in which similarity transformations are possible (Popper, 2005, pp. 129-130).

Here the acceptance is not in order to accept the brilliance of Euclid, or to agree to only use Euclidean geometry. Neither is a matter of stating that a science will become science depending on how much Euclidean geometry was used in the development of their field of knowledge. It is an acceptance of Euclidean Geometry as an axiomatic, scientific *model* (Hartshorne, 2000), an acceptance that through a

Euclidean way of thinking people will become a scientific self.

We are aware that not everybody blindly accepted Euclidean geometry. For example Einstein demonstrated that this geometry was only thought to be applied in a void, not in the *real world,* where space is inseparable from matter (Woods & Grant, 2007); "where mass tells space-time how to curve, and space-time tell mass how to move" (Wheeler, in Sweeney, 2014, p. 826). Einstein was referring to Euclidean geometry as a mathematical model of space; however, he was interested in a geometry that provided him tools to understand the physical space. For instance, "it was much later, with Einstein's general relativity, that it was shown that the geometry of the universe is not Euclidean but curved" (Hirsch, 1996, p. 62). We want to be clear that the discussion is deeper than that. We are not against Euclidean geometry. We are drawing awareness to Euclidean geometry as a truth that circulates in scientific discourse and performs, as an effect of power, a scientific self.

So, am I a Scientist Already?

> What Euclid did that established him as one of the greatest names in mathematics history was to write the Elements. [...] Euclid' great genius was not so much in creating a new mathematics as in presenting the old mathematics in a thoroughly clear, organized, and logical fashion" (Brodkey, 1996, p. 386)

Indeed Euclid was a great geometer, probably the most recognized of all time. The *Elements* deploy a schematic *order* from the basic definitions to the most abstract *formalizations* (axiomatics). But this sacredness was not eternal. Not only mathematicians, but also researchers of others fields of knowledge tried to show that Euclidean axioms are in opposition with our optical perception of space. These studies concluded that visual space is far from being Euclidean (Suppes, 1977). But it is possible to find that almost all Western school geometry is based on Euclid's work (Burgin, 1987; Ray, 1991). So, how can we explain this Euclidean resistance? As stated in the previous section, Euclidean Geometry is not just formulas and axioms that need to be applied to solve problems. This geometry is being operated in a completely different fashion.

Does this mean that I will become a scientific self if I accept Euclidean geometry as the 'basis' of all knowledge? It is not as simple as that. Subjectivity does not imply only the repetition of a truth; the acceptance of this discourse will operate in an interesting way. For {Daston, 2007 #33@@author-year;Foucault, 1982 #92}Foucault (1982), human beings become subjects through the objectifying effects of scientific knowledge. At the same time, the practice of knowing generates effects in the form of knowing and in the subjects who know (Daston & Galison, 2007). Therefore, subjects must train themselves to become part of a practice; in other words, they have to conduct their own conduct. Such *subjectification* pursues to fabricate a scientific thinking.

How is this Euclidean truth prompting to a scientific self? The method deployed by Euclid is shaping a deductive and axiomatic way of thinking, a method that "contributes to the enlargement of mind and makes us think with precision [and increases the] capacity for reasoning and for understanding ideas " (Frank, 2007, p. 251).

In the End…

Let's return to school, school geometry is rooted in Euclidean geometry, but this was not intentionally. It was not because someone wanted it there. This geometry is an important part of school due to the circulating truth within 'scientific discourse'. Currently, to become a scientist means to become a productive citizen. Therefore, if we want to have a brighter future we have to be *subjected*, in Foucaultian terms, by schooling.

> Educational sciences have provided the tools for fabricating the cosmopolitan child through being a cornerstone of the planning of social life for the promise of a better and brighter future. […] I connect the statement of the need of a mathematics education for all for creating a brighter future with the way in which educational sciences in the 20th century have produced the elements for the reasoning making possible such statements (Valero, 2013).

Euclidean geometry is much more than a particular way of seeing space or a formalization of the metrics of the earth; it is much more than just learning a set of mathematical concepts and rules. Euclid's elements are deploying a deductive system, rooted in proofs and demonstrations. Euclidean geometry becomes the template or the path to become a scientific self. So, in order to become this scientific self, students must follow the path of what schools perceive a real scientist is, not to become a scientist, but to become a logical thinker, a problem solver, who uses reason! The desired cosmopolitan child (Popkewitz, 2008), as described by the Chilean Ministry of Education when stating:

> School mathematics curriculum aims to provide students with the basic knowledge of the field of mathematic, and, at the same time, helps students to develop logical thinking, deductive skills, accuracy, abilities to formulate and solve problems and abilities to model situations [...] The learning of mathematics enriches the understanding of the reality, facilitates the selection of strategies to solve problems and contributes to an autonomous and individual way of thinking (Ministry of Education of Chile, 2010, p. 3, our translation).

This discussion is not about how Euclid's axioms and postulates are the easiest for children, cognitively speaking. The discussion is that Euclid's Elements are a consistent, deductive and progressive system that shapes the way of thinking of a scientist. School geometry also operates by constructing its subjects; it shapes in students a way of visualizing the world and a way of thinking about space and reality. If the method deployed by the *Elements* was the basis of almost all scientific knowledge, then it does not seem such a bad idea to teach Euclidean geometry at schools, right?

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THE EUCLIDEAN TRADITION AS A PARADIGM FOR SCIENTIFIC THINKING

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ABSTRACT

In this paper, Euclidean geometry is described as a paradigm for science that has had an enormous impact on Western thinking and education. We will outline first, the foundations of Euclid's Elements. Secondly, what characterises the Euclidean paradigm and explain its importance as a product of its origin in the ideas of the most celebrated thinkers in Western history. Thirdly, how this Euclidean paradigm has had an impact on scientific reasoning on three different scientific fields presented as examples of the paradigm in action. We will analyse such fields to seek, in the foundations of their thinking and tradition, statements about what a 'proper scientific approach' should look like. Finally, we will discuss the connection between the Euclidean paradigm and the constitution of a particular subject through school mathematics. We conclude that Euclid's Elements are much more than just an axiomatic mathematical structure. They are a view of science, an ideology about how to think; a form of geometry entangled in our notion of science and in the configuration of education for making productive citizens in society.

INTRODUCTION

Have you ever wondered why Euclid's elements became so 'popular' in Western thinking? We use the word 'popular' because the elements have transcended every language and time. In fact, the *Elements* is the second most translated book of all time, preceded by… well just one!

> [The Elements] had a profound impact on Western thought as it was studied, analyzed, and edited for century upon century, down to modern times. It has been said that of all books from Western civilization, only the Bible has received more intense scrutiny than Euclid's Elements. (Brodkey, 1996, p. 386)

In this paper, we will attempt to unravel what made the *Elements* so captivating in Western culture. The contention of this paper is that Euclidean geometry should not just be considered as a form of geometry that can be isolated as a mathematical structure, a set of geometrical concepts and rules. Instead, Euclidean geometry transcended mathematical knowledge and has become a model for knowledge organization and a paradigm for mathematical and scientific thinking. We will follow three movements to illustrate how Euclidean geometry has been considered not just as a particular form of geometry, but according to its historical dominance, as a paradigm for thinking in 'the right scientific way' for doing proper science among diverse fields of inquiry. Firstly, we will analyse the roots of the *Elements*, to set the foundations of Euclidean Geometry. Secondly, we will attempt to remove the geometrical surface from the *Elements,* and we will outline what characterises the Euclidean paradigm. This movement will help us to explain its importance even today as a product of its origin in the ideas of the most celebrated thinkers in Western history. Thirdly, we will outline, in relation to three different scientific fields, how this Euclidean paradigm has had an impact on scientific reasoning. We will analyse the entanglements of Euclidean geometry with notions of science in those three fields to seek the foundations of their thinking and tradition. We will achieve this through the analysis of their statements about what a 'proper scientific approach' should look like. Finally, we will discuss further the connection between the Euclidean paradigm and the constitution of a particular subject through school mathematics. This will enable us to unfold the importance of having students following the path of what schools perceive a real scientist is, not to become a scientist, but rather to become a logical thinker, a problem-solver, and a productive citizen who uses reason.

EUCLID'S INSPIRATION

Euclid's greatness was not in creating a new mathematics, but it was in presenting the known mathematics of ancient Greece in a thoroughly clear, organised, and logical fashion (Brodkey, 1996). His product —the *Elements*— was the synthesis of all the geometrical knowledge of his time and the beginning of a deductive system based on demonstrations. Nonetheless, what was so revolutionary about the *Elements* that led Euclidean geometry to become a paradigm for scientific thinking even in modern times? To answer that question and to understand the triumphant manner in which the *Elements* are written we have to overlook the geometrical content. As if we were carefully removing the flesh of the *Elements'* bones to leave only the skeleton visible, only the core. For that matter, we are going to move back to Euclid's sources of inspiration. Euclid was inspired by the ideas guiding the Academy of Plato and Aristotle's incredibly influential considerations about how a proper science should be organised. So, let us address these two roots a little further.

On the one hand, Euclid structured the *Elements* under the ideas inspired by Platonism, a dominating force that formed a central pillar in a hierarchical view of science where mathematics is positioned at the absolute top right up until today. Plato considered mathematics as being independent of human consciousness and deeply intertwined with the structures of our physical universe. Within this, the basis for knowledge is only to be found in the 'real world', in the world of eternity and immutability. It is in this light where the comprehension of the world of Ideas goes through thinking and not through the senses (Skovsmose & Ravn, 2011, p. 34):

> Plato's differentiation between the world of phenomena and the world of ideas provides, despite the continuous relations between these degrees of reality, the strong impression that all what is concerned with the senses is at least damaging for knowledge. Mathematics and mathematical proofs instead lead to an insight in the truth, which does not have to do with the phenomena of the experienced (empirical) world. Mathematics lead to insight in the necessary truths. And this is connected with the very special foundation of mathematics in the Platonic world of the ideas. Mathematics shows that a secure insight should not be looked for among the (changeable) diversity of phenomena (our translation).

With this in mind, mathematics and science dealt with an absolute truth about the universe, in which each field of study "could be reduced to a more fundamental one

[… and this] reduction rested on the rock solid foundations of pure mathematics and logic" (Hirsch, 1996, pp. 60-61).

On the other hand, the *Elements* are rooted in Aristotelian thinking. In *Posterior Analytics,* Aristotle presents a structure of the principles of any deductive science: "one is what is proved, the conclusion, one are the axioms and the third is the underlying genus, the properties and the per se attributes of which the proof makes clear" (Acerbi, 2013, p. 680). According to Aristotle, these assertions —the first principles— are not proven and also remain unproven in the further development of science. Therefore, the reliability of these assertions must be taken for granted, but on the basis of these, the truth of other assertions can be proven (Skovsmose & Ravn, 2011). In an Aristotelian sense, any science must develop from first 'theoretically true' principles, but also, one must prove truths on the grounds of truths. Therefore, if one has no truths on which to base one's proof, then one has no science. Such structure of science, from first principles, is perfectly exemplified in geometry, as formulated in Euclid's *Elements*:

> In brief, definitions in the form of atomic premises represent for Aristotle a perfectly genuine kind of basic knowledge peculiar to each science. […] In Euclid, too, definitions play an essential role in some of his proofs (Hintikka, 2004, p. 15)

Mix essential ideas, stemming from both Plato and Aristotle, together with mathematical thinking and the result will be a rigorous paradigm for scientific thinking —not just for mathematics. Euclidean geometry was, even from the very beginning, not only a model for structuring physical space, but also a deliberate attempt to configure scientific thinking and the idea of axiomatic structures in scientific reasoning. Then, Euclidean geometry as a paradigm, rooted in the academy of Plato and Aristotle's ideas about an organisation of science, is related to an axiomatic method that has been entangled in the roots of countless scientific productions throughout the centuries.

> From Hugo Grotius and René Descartes until the First World War, the ideals of intellectual order and rational intelligibility current among European intellectuals emphasized regularity, uniformity, and above all stability. From this standpoint, the merit of Newton's Principia was to show that the solar system of which the earth is a member is a "demonstration" —a paradeigma, in the Classical Greek— of an intrinsically stable system. This assumed success for Newton's theory convinced the "Mathematical and Experimental Natural Philosophers" […] that their use of Euclid's Elements of Geometry as a model for a new physics —or, for Thomas Hobbes, a political theory— was not a dream born of Platonist epistemology alone, but a realistic program for scientific research (Toulmin, 1998, pp. 329-330).

THE CHARACTERISTICS OF THE EUCLIDEAN PARADIGM

We have unfolded the roots of the becoming of the *Elements*. Let us turn now towards a further elaboration of the characteristics of the Euclidean paradigm. Some researchers have considered Euclidean geometry as a dominant perspective (Majsova, 2014) that suddenly became a universal key to access human knowledge (Sbacchi, 2001). So what are the key characteristics that attract in these old volumes?

Euclid's axioms and postulates appeared to be considered a structured set of commandments, which were displaying the front gate to understand space and the universe. The *Elements* seems to be the way to gain the entrance to access the world of ideas, by choosing the scientific path of reason and logic, as the Bible helps to reveal the way of obtaining the entrance to the Kingdom of God. And despite all the discussions against the *perfection* of this type of geometry, by non-Euclideans, Euclidean Geometry still holds an impact on Western thought (Shuttleworth, 2010). It has a privileged position precisely because it is not just a set of mathematical knowledge or a consistent body of axioms and postulates. It has been taken as the paradigm of a deductive method that brings coherence while organizing knowledge:

> [The Elements] more than just a practice of measurement, as a result of this mandatory structure geometry now had become: an exemplary mode of thought and deduction, as expressed in the "more geometrico demonstratae" of Benedict (Baruch) de Spinoza; a "pure science" in the sense of Immanuel Kant's idealist philosophy; and, for the positivist philosophies and sciences of the eighteenth and nineteenth centuries, the science of physical space (Krauthausen, 2010, p. 232).

Euclidean geometry inspired the thinking and reasoning of diverse Western thinkers. As an example, in the 18th century, Gerdil —an Italian theologian who wrote in many fields including philosophy of education— stated that it is through reason that a man is a man. He considered reason to be a social faculty of nature: "make a man reasonable, and you make him sociable [...] reason cannot be cultivated but by lessons which have relation to social life" (Griffiths & Griffiths, 1765, p. 40). Gerdil argued that the process to become a reasonable man involved the learning of elementary geometry. According to Frank (2007),

> Gerdil insisted on conceptual mastery of a synthetic system of elementary geometry. [For Gerdil] Euclid's geometry, for instance, is notable for its rigor in demonstration [...] is distinguished for its orderly "progression from the simple to the compound, from lines to angles, from angles to surfaces, and so forth," a method that particularly "contributes to the enlargement of mind and makes us think with precision" [...a method which] develops the propositions of geometry in response to a natural need to know or to a spontaneous order of inquiry [...] Any of these three systems increases the student's capacity for reasoning, for understanding ideas, which properly understood are "notions determined by relations" [...] It is "nothing more than the faculty of arranging, *facultas ordinatrix*" [...] The desire for order leads to the ideas of truth, goodness and beauty. (Frank, 2007, p. 251)

What characterises Euclid's Elements is not simply their recognition as a template or a method for organising scientific knowledge. The Elements have shown to be recognized as a foundation or the core of some scientific fields, in terms of the promotion of a mode of thought and deduction. Following Gerdil's thinking, this geometry contributes to the enlargement of mind and leads to an accurate form of thought, with precision. It is a type of geometry written in such a way that, as Gerdil state, increases reasoning and the understanding of ideas in response to a natural need to know. The *Elements* are taken not as a form of geometry; Kant referred to it as a pure science (Krauthausen, 2010). It appears that the *Elements* were more than just books summarizing the geometrical knowledge of ancient Greece. Descartes and Newton also recognized in Euclid's books an intellectual, rigorous and powerful model (Toulmin, 1998).

The Elements became a bible for many sciences not because of the mathematical insights but because of the 'facultas ordinatrix'. The non-geometrical part of the books, the Euclidean Non-Geometry in the *Elements,* helped shaping a form of appropriation of Euclid's axioms and postulates to develop a scientific thinking and tradition, a 'Euclidism'. Let us illustrate the characteristics of Euclidism within the development of three fields of Western science.

THE CORE OF SCIENTIFIC KNOWLEDGE

Why do we say that Euclidean geometry has been so influential for scientific knowledge? If Euclidean Geometry is a form of mathematics that models space, should it not be influencing all fields of inquiry that involve mathematical knowledge? It is well known that the *Elements* had been the foundation of the development of geometrical knowledge and also, it has been a vast influence in physics. But, in order to grasp and fully understand Euclidism we are going to step out of the fields of mathematics and physics because they are the most obvious fields where Euclidism has unfolded. In fact, a variety of other fields —including arts, philosophy, literary education, and many others— have been setting their roots in Euclid's axioms and postulates, a Euclidism adopted by other fields in the development of their own scientific thinking.

And so, the question remains: how has Euclidism influenced the development of diverse forms of scientific knowledge? There are many possible examples to explore. One of them is literary education. Within this field of inquiry the *Elements* are roadmaps that model the path that students should follow, using Gerdil's words, from the simple to the compuond (see Rabinowitz & Bancroft, 2014). In this section, we are going to present examples in Architecture, Political Science and Theology of how Euclidean geometry has been entangling into diverse fields of scientific knowledge.

Euclidism within Architecture

The relationship between geometry and architecture might seem obvious since architecture is – sometimes – built on the basis of geometrical knowledge. The impact that Euclidean geometry had upon the theory of architecture was mostly because it brought to this field a structure for the visualization of space; it brought a way of organizing the visual space. Sbacchi (2001) stresses that, for architects, it was for a long time more important to learn how to draw and how to visualize, than to engage in complicated calculations. Thus, Euclidean geometry first began to emerge as a way to address this necessity. The relationship between Euclidean geometry and architectural drawing was formalized in the *Trattato di Architettura Civile e Militare* in the 15th century:

> We can assume that an 'Euclidean culture associated with architecture,' existed for a long time and that it was probably the preeminent one among the masses and the workers […] With Francesco di Giorgio Martini's Trattato di Architettura Civile e Militare, the Euclidean definitions of line, point and parallels make their first appearance within an architectural treatise, although in a rather unsystematic way. Serlio, later, goes a step further: his first

> two books include the standard Euclidean definitions and constructions. (Sbacchi, 2001, pp. 27-28).

Nevertheless, Guarino Guarini, an Italian architect during the 17th century, is recognized as the first to establish the relationship between Euclidean geometry and the theory of architecture. He stressed that *Elements* are to be considered the foundation for every science (Guarini, 1968, p. 10):

> The Elements of Euclid are so necessary to every science [...] and also to whoever would advance themselves in the military arts must believe them to be the basis, principle and fundamental element on which to build, and beyond which to advance, and on which to lay every speculation.

He placed Euclidism as the basis of architectural thinking. Euclidean geometry was not a structural visualization of space anymore, neither a guideline on how to draw; it was a way of thinking. Euclidean geometry was, for Guarini, a universal key to access human knowledge. According to Sbacchi (2001), Guarini's Euclidism made him consider Euclidean norms as the basis of every scientific work. And so, he presented his work named "Architettura Civile" using a Euclid's *Elements* structure:

> In the first treatise of the five constituting the book, Guarini early on states his geometrical interests. [A] chapter explores the "Principles of Geometry necessary to Architecture." It contains the nine definitions of point, line, surface, angle, right angle, acute angle and parallel lines. Chapters dedicated to surfaces, rectilinear shapes, circular shapes follow and the whole first treatise continues basically in this way with postulates, other principles and several typical Euclidean transformations such as "To draw a line from a given point in order to make it touch the circle" [Guarini 1968:41]. (Sbacchi, 2001, p. 29)

Euclidism within Political Science

In the 16th century, the English philosopher Thomas Hobbes relied upon the principles of geometry to establish a new political science (Vinnicombe, 2005). Hobbes believed that knowledge cannot subsist without a proper method and the key to achieve that method was held by geometers and physicists (Grant, 1990). He intended to analyse social behaviour by using Euclidism as a methodological model of organization, a model of deduction, step by step:

> Hobbes sought to analyze human behaviour in a hypothetical early collection of unorganized individual [...] The description of those moving atoms should proceed: by application of geometry [...] Geometry is central to the great philosopher's thought in two quite distinct ways: as methodological guide and example, and as the most basic of all branches of knowledge, from which "synthesis" might deduce, step by step, the immutable laws of social justice. (Grant, 1990, p. 151)

Hobbes saw in Euclidean geometry a model to shape a deductive method. From the introduction of self-evident or common sense definitions and postulates to the most advanced and challenging theorems. The *Elements* become a "deductive passage from simple to complex, from supposedly evident definitions and postulates to deep theorems and difficult constructions" (Grant, 1990, p. 151). In his principal work *Leviathan*, Hobbes stated three possible ways of knowing. Firstly a divine revelation, secondly, an experience form and, finally, through definitions. Hobbes had the intention to exclude the first two, "leaving geometry as the only possible means by which man could reliably understand all manner of things" (Vinnicombe, 2005, p. 670).

> Euclid's reasoning described a method which Hobbes believed provided the foundation for all true reasoning, an abstract scientific method, through which the world becomes intelligible by a means of reasoning which is entirely independent of sense perception. This method, adopted by Hobbes, is described by him as the compositive-resolutive method and its mathematical counterpart is the synthetic-analytic. (Vinnicombe, 2005, pp. 670-671)

According to Vinnicombe (2005), Hobbes not only aimed to set Euclidean geometry as the 'only true way of knowing'. He also aimed to set this Euclidean method of reasoning as something natural to man, where 'natural' is understood as something that represents a type of mental discourse, a discourse without words. Hobbes' goal was to establish Euclidsm as the only way of thinking in Political Science.

Euclidism within Theology

Herbert de Cherbury, an English philosopher in religion, declared Deism as the belief that by reason and by the observation of the world we live in —the natural world— we can determine the existence of God (de Cherbury, 1633). He stated that 'common notions' were the foundations of reasoning, in which he set the basis of his philosophy. However, the term *common notion* was borrowed from Euclid's *Elements*:

> The idea that 'common notions' were the basis of reasoning was not original to Herbert. The term ultimately derives from Euclid's Elements, in which *koinai ennoiai* are the axiomatic foundations of geometry. The demonstrative prestige of this discipline led to the concept being taken up and applied more widely in early modern logic and the sciences (Serjeantson, 2001, p. 222).

Deism was rooted in Euclid's *Elements*. de Cherbury's Euclidism helped to develop a book that operates as a 'guide for the perplexed', in the words of Sherman (2012). In his book *De veritate,* written in 1624, he established the philosophical foundations of his work, where he set five *common notions*. *De veritate*, as the *Elements*, presents the key terms of his philosophy in a structured way, which led to instructing or tutoring about "how to discriminate among classes of truth and how to distinguish certainty from probability, possibility, and error" (Sherman, 2012, p. 189).

As we can see, Euclidean geometry has been shaping the development of scientific thinking in such a way that, according to Norton (2013), the *Elements* turned into a template for organizing scientific knowledge. The exposition of Euclidean geometry, rooted on Platonism and Aristotelian thinking, brings together truth, knowledge, and certainty. It shapes an epistemological Trinity that, as illustrated, influenced not only mathematics but also epistemology in general. This Trinity has been giving to mathematics and Euclidean geometry a central role in human cognition (Skovsmose and Ravn, 2011). For Hirsh (1996, p. 62), "it was geometry that became the more solid base for certain knowledge [...] Every statement required a rigorous justification; nothing could be taken as obvious".

EUCLIDISM FOR EVERYONE

Why, then, is it important to bring the discussion of Euclidism into mathematics education? As discussed elsewhere (Andrade-Molina & Valero, 2015), schools aim at making students follow the path of what is perceived as a real scientist. Of course the *Elements* are not the explicit books pupils must study and read —they are, after all, more than 2000 years old volumes. Students probably have no idea that what they should learn is a space modelled by a Platonic realm —mathematics— that seems to be the important one in the educational world. And which is, most likely, totally different from the messy and complex world they live in. Nonetheless, Euclidean geometry is an important part of schools and it holds a privileged position when starting to learn geometry in many countries (Harel, 2014; Ray, 1991; Sbacchi, 2001). The impact and importance of this dominance is not just about pupils learning a set of mathematical concepts and rules. Geometry, in schools, also operates by constructing its subjects. School geometry shapes in students a form of visualising the world and a form of thinking about space and reality in a particular way (Andrade-Molina & Valero, 2016). The aim of schooling is that students become logical thinkers, problem-solvers and productive citizens capable of reasoning mathematically.

> [Students should] reason mathematically and use mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. [While] recognising the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens (Organization for Economic Co-operation and Development, 2014, p. 28).

But the relevance of Euclidism in schools is that the *Elements,* as a paradigm for science, led to believe for so long that through the understanding of Euclid's axioms and postulates a person will gain access to knowledge, not just mathematical knowledge, but all forms of human knowledge. Therefore, there is the implicit "acceptance that through a Euclidean way of thinking people will become a scientific self" (Andrade-Molina & Valero, 2015, p. 292).

Above we have illustrated how Euclidian geometry has influenced many fields of scientific knowledge. Moreover, we have sought to show how and why the *Elements* became a view of science and an ideology about how to think 'scientifically'. The *Elements* became the *Bible* for many sciences —an ideal to achieve for all other areas of human knowledge.

> [Euclidean geometry] has been taught to teach reasoning and intellectual discipline. This is why Plato placed his famous motto over the academy door. That is why Abraham Lincoln studied Euclid". (Jamison, 2000, p. 54)

As a consequence, Euclidism also lives a fruitful life inside schools. The Euclidean paradigm can be tracked back to some of the most important scriptures of Western thought, and it is in this light that the dominance for centuries of Euclidean geometry in schools should be understood.

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Chapter 15
The Effects of School Geometry in the Shaping of a Desired Child

Melissa Andrade-Molina and Paola Valero

Abstract In this chapter we explore how school geometry becomes a technology for the government of the self, and how the pedagogical devices of school geometry conduct students' ways of thinking and acting. We contend that students, in their working with pedagogical devices, engage in a training process in which they learn to regulate their own conduct so that they perceive space through the trained eyes of reason provided by Euclidean, school geometry. Our contribution is an analysis of the power effects of school geometry in terms of the fabrication of children's subjectivities towards the shaping of the desired child of society.

Introduction

The Organisation for Economic Co-operation and Development, OECD, states that mathematics is a tool to solve everyday life problems (OECD, 2014). The ability of mathematising, measured by OECD's Programme for International Student Assessment (PISA), is considered to be central to solve those everyday life problems because it involves connecting the "everyday-world" and the mathematical world. Schools seem to address the need of developing this ability by introducing everyday life, contextualised problems into the mathematics classroom. Particularly the learning of geometry, geometric modelling and spatial thinking are said to be important competencies for solving those types of problems (Mevarech & Kramarski, 2014; National Council of Teachers of Mathematics [NCTM] 2000).

It is said that school geometry offers ways of interpreting, describing, and reflecting on the world that is reachable through our senses (Clements & Battista, 1992; NCTM, 2000; National Research Council [NRC] 2006). This claim grants spatial

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thinking and visual-spatial skills the status of a key element to improve students' performances in school settings, for example, while dealing with digital technologies (OECD, 2012). According to the American National Research Council, spatial thinking is a "pervasive and powerful way of thinking that operates across the sciences, social sciences, and even the humanities. [It] is the start of successful thinking and problem solving" (NRC, 2006, p. 131). Therefore, several studies on school geometry have attempted to provide ways for students to enhance and develop their spatial thinking, varying from the introduction of activities involving the manipulation of building blocks to the overall improvement of the school geometry curriculum (Clements, 2008; Hauptman, 2010; Prieto & Velasco, 2010).

It is also recognised that visual-spatial skills improve student's performances while learning geometry or mathematics. These skills are fundamental while reading, interpreting and recognising information contained in diverse images, whether in two or three dimensions (OECD, 2012). In the field of science education, it is believed that visual-spatial skills and spatial thinking go beyond mathematics and have been implicated in several scientific advancements and theories such as the discovery of the DNA structure (Newcombe, 2010), Galileo's laws of motion, Faraday's electromagnetic field theory and Einstein's theory of relativity (Kozhevnikov, Motes, & Hegarty, 2007). In these studies, spatial thinking is referred to as a way of thinking that is as important as verbal and mathematical thinking (Newcombe, 2010; Wai, Lubinski, & Benbow, 2009; Verdine, Golinkoff, Hirsh-Pasek, & Newcombe, 2014).

In the statements sketched above, spatial thinking and visual-spatial skills are positioned as key elements in the shaping of the successful minds of the future, and, therefore, they should be developed or enhanced in schools. It seems that whoever possesses these abilities will have a successful performance in many areas of life in general, and particularly in areas of interest for education, such as international comparative assessments like PISA, the use of digital technologies for learning, and the reading of images that appear in diverse school subjects like physics or biology.

These statements articulate discourses about not only what students should learn in school geometry but also about whom they should become with and through school geometry. Here we bring further our previous work (Andrade-Molina & Valero, 2015), as well as add new insights to recent investigations that use Foucault's tools to think power and governmentality in mathematics education (Diaz, 2014; Kollosche, 2014; Valero & Knijnik, 2015). In this chapter, we focus on the question of how school geometry becomes a technology for the government of the self, and how the pedagogical devices of school geometry conduct students' ways of thinking and acting. We contend that students are not forced to be or to see in a particular way. Rather, in their working with pedagogical devices, they engage in a training process in which they learn to regulate their own conduct so that they perceive space through the trained eyes of reason provided by Euclidean, school geometry. Our contribution is an analysis of the power effects of school geometry in terms of the fabrication of children's subjectivities. More precisely, we shed light on how school geometry is understood as a technology of the self that conducts the conduct of children towards the shaping of the desired child of society (Popkewitz, 2008).

Governing Through Mathematics Education

There are several understandings of the notions of government and governing. Cotoi (2011) describes two meanings of these notions in the work of Foucault. The first refers to an area of human existence and expertise produced by ways of thinking and acting aimed at transforming human behaviour. The second refers to what Rose (1996) describes as the attempt, by political elites, to ensure the wellbeing through the ordering of the affairs of a territory and its population. The former meaning is used in a comprehensive sense to trace the link between forms of power and processes of subjectification (Lemke, 2002). Following Foucault (1993, p. 204),

> Governing people is not a way to force people to do what the governor wants; it is always a versatile equilibrium, with complementarity and conflicts between techniques which assure coercion and processes through which the self is constructed or modified by himself.

Governing understood as the "conduct of conduct" (Foucault, 2008) works through technologies that are systematised, regulated and reflected modes of power that include forms of self-regulation (Foucault, 1997). These technologies, inscribed in a particular form of rationality, enable subjects to change and to develop their thoughts and conduct their ways of being. In this regard, the practices of the self are not invented by the subject. They do not emerge from thin air, but "they are models that [the subject] finds in his culture and are proposed, suggested, imposed upon him by his culture, his society, and his social group" (Foucault, 1984, p. 291). However, the subject also embraces them and acts with them productively. It is in this sense that, from a Foucaultian perspective, subjectivity is formed in the constant tension between subjection and subjectification.

Education has been pointed out as a very important space of Modern government where subjectivity is fabricated with the use of technologies of pedagogy, the curriculum and educational sciences. Within that, the school mathematics curriculum is a way of conducting subjects' conducts since mathematics education practices insert in children norms of reason in both productive and constraining ways (Valero & García, 2014). Mathematics education is not only a process of knowledge objectification but also a process of subjectification. For example, Kollosche (2014, p. 1070) argues that school mathematics is a technology for the government of others through logic and calculation practices, in which the desired student is "able and willing to think and speak logically and act bureaucratically". Diaz (2014) investigates the way in which the emphasis on the adequate teaching/learning of the equal sign in current reforms in mathematics in the USA link with broader meanings in society about equality. The pedagogies of mathematics in the curriculum operate classifications and differentiations of those children who learned the right equality and those who fail to do so. The apparent neutrality and goodness of learning "the equal sign" renders children objects of the calculations of power.

Our interest here is to approach school geometry as a technology of the self that governs students' perception of the spatial and visual according to certain norms, thus allowing them to become the desired child of the curriculum. Since diverse pedagogical devices articulate the technologies of the self, we deploy an analysis of a series of official curricular documents from the Chilean Ministry of Education (MINEDUC). These materials are school textbooks designed to accompany students' learning processes (Del Valle, Muñoz, Santis, 2014; Muñoz, Jiménez, & Rupin, 2013), curricular programmes and guidelines for teachers (Ministry of Education of Chile [MINEDUC] 2004, 2011; Bórquez & Setz, 2012; Ortiz, Reyes, Valenzuela, & Chandía, 2012; Zañartu, Darrigrandi, & Ramos, 2012) and a map of learning progress in geometry (MINEDUC, 2010). Heightened attention is given to this map because it is the official document that displays the desired performances of students while solving school geometry problems, and because it is expected that teachers use this map to trace individual progress in geometry. The examples of students' answers displayed in this map of progress are considered, by MINEDUC, the successful or ideal performance students should achieve in the learning of school geometry. Because this map of progress expresses the successful practices of desired students, it is possible to find in it evidence of the expected power effects of school geometry in terms of the training of the self of children.

Statements about the training of the self were detected in the map of learning progress. Then they were connected to statements about expected behaviour of children in geometry expressed in student's textbooks and teacher's guidelines. To detect expressions of the training of the self implies to find discursive recurrences in the documents regarding the desired expectation of students' performances while learning geometry. For example, students should be able to recognise, to demonstrate, to measure and to locate by themselves (MINEDUC, 2010). These recurrences in the school geometry discourse are the ground for an interpretation of school geometry as a technology of the self and its effects on students' subjectivities.

School Geometry and the Trained Child

The Chilean curriculum states that schools should provide students with the basic knowledge of mathematics to facilitate an understanding of the "real world" (MINEDUC, 2010). The Chilean curriculum for school geometry aims at teaching students to deal with everyday life problems to engage their knowledge and abilities developed in the classroom (MINEDUC, 2011).

> Students should develop logical thinking, deduction skills, accuracy, problem posing and solving, and modelling ability [as/since] mathematics enriches the understanding of reality, facilitates the selection of strategies to solve problems, and contributes to an autonomous and own thinking. (MINEDUC, 2010, p. 3, own translation)

It is believed, as aforementioned, that by developing spatial skills, in two and three dimensions, students would be able to link "everyday life experiences" and school mathematics. For instance, according to MINEDUC (2010), a key component in the development of spatial thinking is measuring, precisely because it enables students to link school geometry to the environment and to other school subjects. In this sense, school geometry and spatial thinking are taken as a tool to link the surroundings and school mathematics, but, also, as a tool to make sense of other school subjects such as physics and geography (Battista, 2007). The teaching of school geometry involves practices in which students should engage themselves in a training process in order to achieve what is desired. The desired child should be able to perform successfully in everyday life problems by using the tools, the abilities, and the skills acquired in the classroom.

In 2010, MINEDUC released a map of learning progress with seven levels that students have to complete along school geometry during the 12 years of compulsory education. This map expresses the successful performances that the desired student should deliver at every level of this map. From recognising the fundamental elements of geometrical figures—side, vertex, perimeter, area and so on—to solving problems by applying geometrical axioms and theorems in a three-dimensional rectangular coordinate system. Since there are many topics in school geometry, in terms of content, the training of the self is going to be illustrated following the route traced in the map, along the levels and tasks the desired child should complete while learning how to navigate in space and how to locate shapes and places.

The Training of the Self

On the first level, students should learn concepts and notions such as vertices and sides of geometrical figures, as well as parallelism and perpendicularity. Students should recognise the basic elements of prisms and geometrical figures. They should also be able to relate these basic elements using the notions of parallelism and perpendicularity. Students should use these elements to describe and represent diverse shapes of their "physical environment". For example, in one task of this level, students should surmise the resulting shapes produced by cutting a cube along the diagonals of one of its faces. They have "to anticipate the resulting shapes correctly; identify them […] and depict them" (MINEDUC, 2010, p. 7, own translation). While performing this task, students should learn that it is more accurate to draw figures and prisms when using a squared paper, as seen in the expected performance of students in Fig. 15.1.

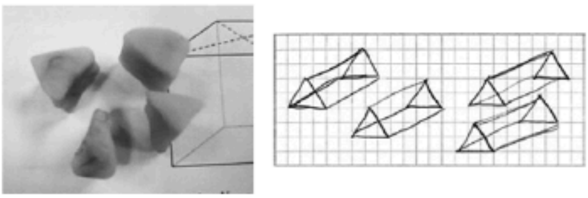


Fig. 15.1 Desired drawing of geometrical figures (MINEDUC, 2010, p. 7)

By acquiring the basic elements of geometrical figures and prisms and realising the usefulness of the squared paper, students should learn how to measure, depict, and operate with these tools. By grasping how to operate with the figures and the squared paper, they can then solve another type of problems. As students advance through the levels of the map of learning progress, they should gain new tools. They should continue to apply these processes to the next school experience or to the next level. For example, identifying vertices and sides of a figure becomes helpful to identify polygon's angles. Then, students will be able to use these tools to operate with the congruence criteria on the third level of the map of learning progress. Each time students encounter a problem, they should use the previously acquired tools and they should find other uses for them, such as sketching a flattened box on the squared paper as a step to build it in three dimensions (Fig. 15.2):

There are many possible ways to solve this problem, but the expectation at the second level of the map of learning progress—the desired solution—requires the use of the squared paper. Students should be able to draw the resulting box on a grid surface, in which each square of the grid has a 1 cm side. Once students draw the figure (Fig. 15.3), the following task is to calculate the dimensions of the resulting box "by counting the squares along each side" (MINEDUC, 2010, p. 9, own translation). In this second level, students should be able to estimate lengths, areas and volumes of geometrical shapes by counting squares. This tool will become applicable in subsequent levels, for example at fourth level, when students learn how the variation of the perimeter of a figure modifies its volume and area.

Once students learn how to handle the 1 cm-squared paper, they can use this tool for solving another type of tasks. Students could apply their knowledge of positioning vertices and sides of geometrical figures to a rectangular coordinate system. For example, at the fifth level they should learn how to depict and transform basic elements of Euclidean geometry into a Cartesian coordinate system (Ortiz et al., 2012). One of the tasks of this level, in which students should use all previous tools—counting squares, locate vertices, trace sides and so forth—is to rotate vertices (Fig. 15.4). This new skill, locate vertices by knowing the rotation centre, will become helpful in the sixth level, when they learn vectors in two and three dimensions.

Fig. 15.2 School task using the squared-paper (MINEDUC, 2010, p. 22, own translation)

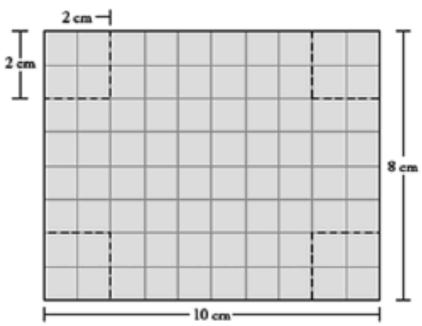


Fig. 15.3 Squaring geometrical shapes (MINEDUC, 2010, p. 9)

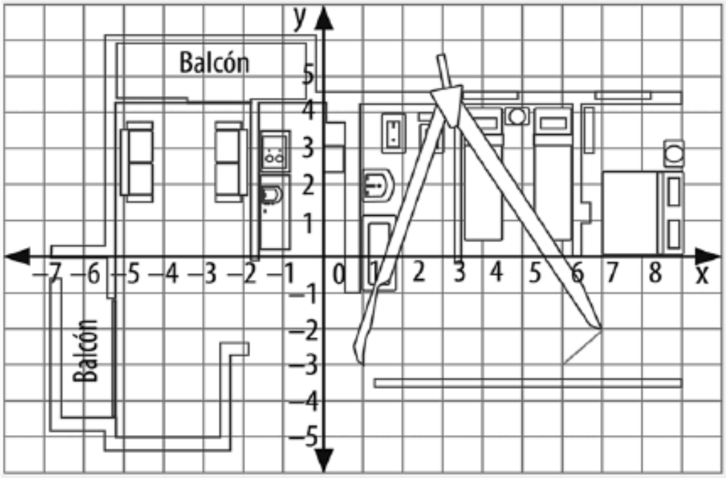


Fig. 15.4 Cartesianised geometrical figures (Del Valle et al., 2014, p. 205, own translation)

At fifth level, students also learn how to transform—rotate, translate, and reflect—geometrical figures, and how to use the congruence and similarity criteria in a Cartesian coordinate system. They have to use the tool acquired before to face these tasks. For example, while determining which transformations a triangle had and which were the rotation centres of each transformation (Fig. 15.5).

At the sixth level, students should learn how to locate specific points, mentioned in everyday life situations, in a Cartesian coordinate system. By doing this, students should realise that the Cartesian coordinate system could be related to the cardinal points—North, West, East, and South (Fig. 15.6). For example, students should be able to estimate the distance between two given points according to their coordinates (MINEDUC, 2010).

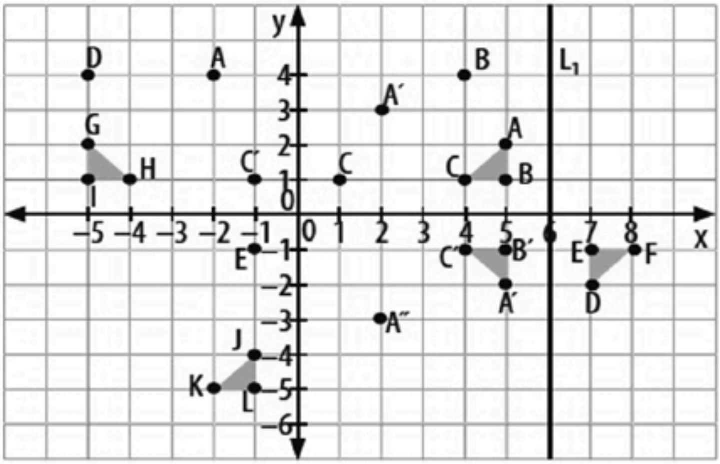


Fig. 15.5 Transformations in a Cartesian coordinate system (Del Valle et al., 2014)

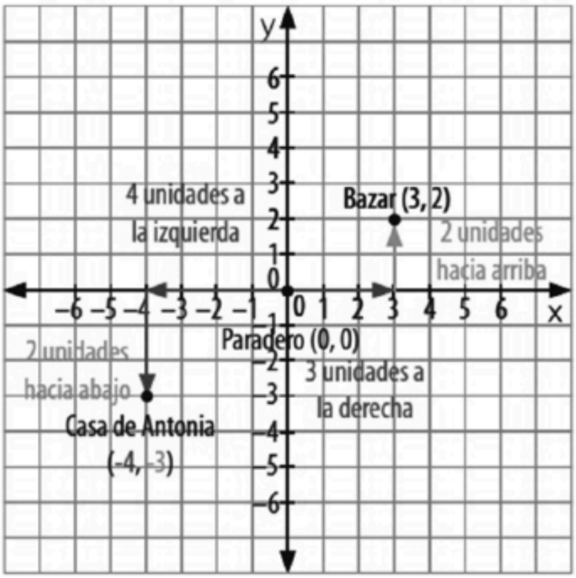


Fig. 15.6 Locate places in a Cardinal coordinate system (Del Valle et al., 2014, pp. 176–177, own translation)

Moreover, students are also prompted to use all these tools while being challenged with another type of problems. In these challenges students "are placed in real three-dimensional situations [which provides] new tools to make spatial and flat depictions, such as the vector model" (MINEDUC, 2004, p. 68, own translation). Students should learn how to navigate in space with Cartesian tools and Euclidean planes, figures and shapes. Students should be able to relate movements in the everyday life with vectors (Fig. 15.7). At the seventh level they should use models as Cartesian and parametric equations to solve these tasks.

Through this training of the self, students should be able to understand their situation, their environment, and experience to master this process. When students solve a problem "they also learn how to act while facing new challenging experiences" (Zañartu et al., 2012, p. 26). While facing a challenge, according to MINEDUC, students should decide for themselves, by following a four-step plan for action and decision-making (Bórquez & Setz, 2012; Zañartu et al., 2012). Firstly, they have to understand the problem. Secondly, they have to create a plan. Thirdly, they have to execute this plan. Finally, they have to reflect on the resulting outcome.

As illustrated above, the desired student should be able to apply the previously acquired tools and the four-step plan strategy each time he/she is being faced with new experiences where he/she has to learn new abilities and strategies. In this sense, the tools students apply to these new experiences at one point stop being "external", but start flowing from an internal, individual source of thinking. These were not familiar tools to the students. They were planted or were acquired—learned—through the technologies of the self in the pedagogical devices for learning geometry. Through this process of training of the self, the desired student should learn how to use school geometry tools to navigate in space.

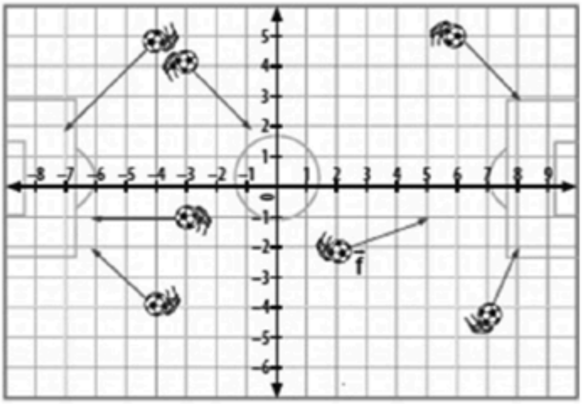


Fig. 15.7 Cartesianised football match (Del Valle et al., 2014, p. 183, own translation)

Students are also prompted to apply the tools acquired in the training process while facing other tasks. For example, the items in the PISA test are another scenario for performance that offers challenging experiences to students. The following problem presents modern architecture to students; it presents a building with "unusual" shapes (Fig. 15.8).

The task students face prompts the use of spatial visualisation and spatial skills. The presentation of the task in words and the illustrations apparently appeal to students resorting to their capacity of visualisation of how the middle of the building would look like. However, when checking the criteria for point assignment to the students' answer to this task, it becomes clear that the prompted behaviour is not valuable. If students rely only in visual information and depict the resulting shape according to the instruction, they will receive no credit at all if, additionally, they do not include information about the rotation centre, the direction of the rotation and the angle:

> *Full credit*: A correct drawing, meaning correct rotation point and anti-clockwise rotation. Accept angles from 40° to 50°. *Partial credit*: One of the rotation angle, the rotation point, or the rotation direction incorrect. *No credit*: Other responses and missing. (OECD, 2009, p. 184, emphasis added)

It becomes evident here that actual use of visualisation and spatial skills is not really part of the tools that the desired child should display when solving everyday life problems. Our point here is not whether there is a mis/match between reality in and outside school, and how such a mis/match is handled in mathematics education.

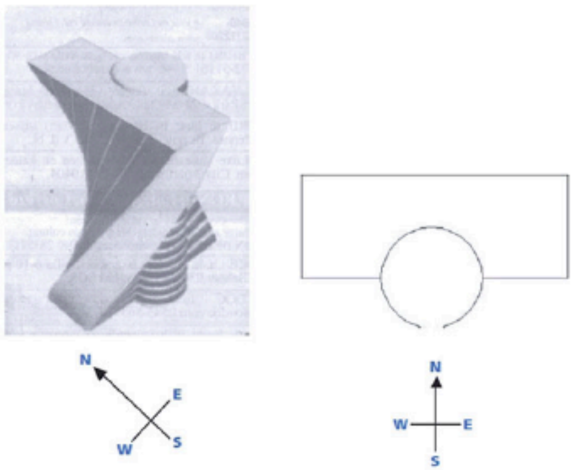


Fig. 15.8 PISA task with unusual shaped building (OECD, 2009, p. 146)

Our point here is that the training of the self in school geometry, although apparently appealing to the usefulness of geometrical thinking in the real world, in fact inserts in children a way of perceiving and seeing that is articulated under the logic of Euclidean metrics, axioms and theorems, and Cartesian coordinate systems.

This has two effects. First, the eye of the body becomes a trained eye through the norms of Euclidean reason (Andrade-Molina & Valero, 2015). Second, for this eye to "see", a new type of space is needed. These technologies simultaneously fabricate a subject and objectify space in particular ways. The space of school geometry emerges. This space has been "chopped" into particular routines, and it has restricted ways of seeing and being:

> The concept of space to be reconstructed in the students' understanding is that of a rational, referential space with fixed points in two or three dimensions. It is assumed that the conceptual development of the child will lead to an internal and abstract representation, which will contribute to making a decontextualized child, freed from the practical capacities of acting with objects in space, particularly of those spaces where everyday life occurs. (Valero, García, Camelo, Mancera, & Romero, 2012, p. 7)

Objectification and subjectification, the two basic mechanisms of mathematical learning according to Radford (2008), are brought together in the making of child who trains himself and his eye to perceiving space and geometrical knowledge as decontextualised, universal and timeless.

The Training of the Untrained Eye

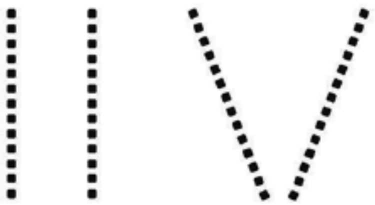
If a feature of the desired child is a trained eye, then untrained eyes would be the navigation of space that the child has at the beginning of the process of training of the self through school geometry. Untrained not because these eyes cannot see, but because they are not trained in terms of school. Although it is difficult to imagine and know exactly how untrained eyes would navigate in space, these eyes would rely on each subject's individual relations with space before school. The type of interaction through the body and the senses of a subjectification outside the school also shape untrained eyes. As Crary (1992, p. 5) states,

> Vision and its effects are always inseparable from possibilities of an observing subject who is both the historical product and the site of certain practices, techniques, and procedures of subjectification.

Outside the training of the self in schools, the subject becomes the observer of an optically perceived world. Optical perception has been considered as a key aspect of the interaction between the body and world. It is believed that humans have a visual dominance, which means that humans tend to rely more on visual information than other forms of sensory information (Gal & Linchevski, 2010; Sinnett, Spence, & Soto-Faraco, 2007).

Optical perception and its relation to geometry have been addressed in research. Some studies have been exploring, among others, the connection between the optical

Fig. 15.9 Parallel lines and the experiment result



perception of space and the space described by Euclidean geometry (Blumenfeld, 1913; Boi, 2004; Burgin, 1987). For example, a series of experiments conducted in 1913 demonstrated that phenomenological visual judgments do not satisfy the properties of Euclidean geometry (Hardy, Rand, & Rittler, 1951). It was concluded that physical configurations and Euclidean geometry do not coincide. One of the experiments was to arrange two rows of point sources of lights as straight and parallel to each other as possible. The lights were placed on either side of a plane. While, in Euclidean geometry, parallel lines are equidistant along any mutual perpendicular, the resulting lines in the experiment diverged. The lines were not parallel at all (Fig. 15.9).

If the existing research shows the inconsistency between Euclidean geometry and optical perception (see Suppes, 1977), it is possible to problematise the reduction that may operate in children when the untrained eye starts interacting with the notions and procedures that organise school space.

Space Through Trained Eyes

Lefebvre (1991) challenged the monopoly of mathematics over the concept of space. On the grounds of a critique to the influence of the metaphysical philosophy that had made space and time absolute categories to organise the physical world, he proposes to bring back space to the realm of the social. He argues that space is as a product of concrete practices and the attempt to represent them. Space is experienced in three forms: space as perceived, as conceived, and as lived. The *perceived space* exists as a physical form, a space that is generated and used. The *conceived space* is instrumental; it is a space of knowledge (savoir) and logic. Space becomes a mental construct, an imagined space. The *lived space* is produced and modified over time and through its use, it is a space of knowing (connaissance); a space that is real-and-also imaginary.

However, space becomes the realm of abstraction when it comes to the knowledge that traditionally has dealt with it—geometry. Schools cut the links with the body—senses; the perceived space is only reachable by reason and logic. By doing this, school space turns into an instrumental space, a mathematical space of savoir.

The stated aims of school geometry attempt to connect all three forms of space. However, the pedagogical devices that operate the training of the self lead to a reduction of this link. Thus, conceived space cannot connect with the lived space. In other words, the untrained eye, produced by *connaissance*, is unlinked to the trained eye, produced by *savoir*. As illustrated above, the desired child should navigate in a space in terms of XYZ. Nevertheless, understanding spatiality in terms of a coordinate system restricts the concept of dimensionality of an object. It is argued that this understanding of space leads to students' misconceptions about the perceived space (Skordoulis, Vitsas, Dafermos, & Koleza, 2009).

> In geometry, students are presented with the properties of shapes and theorems for proof […] all the information needed is given in the problem, and the students are asked to apply the theorems in what has to be proven. […] The skills needed to solve these types of problems are limited, and teaching these skills usually consists of demonstrating the appropriate technique followed by a series of similar problems for practice". (Mevarech & Kramarski, 2014, p. 24)

The lack of interaction between both eyes in school raises some concerns about failure in geometry. It is believed that the learning of geometry has been difficult for students due to the emphasis that school has been given to deductive processes, which neglect the underlying spatial abilities (Del Grande, 1990). NRC (2006) also addresses this issue by stating that Euclidean geometry interferes with the understanding of other notions, for example, activities involving "specifying locations". This type of activities moves forward to formal Euclidean geometry, in which, for example, a point is a dimensionless location, not a point in space with a small but definite area.

Euclidean axioms and postulates have gained such importance within school geometry in Chile that students during the last grade of compulsory education (17–18 years old) are less prone to use spatial abilities while solving problems. Andrade and Montecino (2011) show the struggles students experienced while accepting, for example, that the interior angles of a triangle do not always add 180°. This type of difficulties emerges while solving problems involving spatial abilities. It restricts students to move from a flat surface to a curved surface. The example illustrates how students have been trained to navigate in an instrumental space dominated by the tools acquired in the process of training of the self through school geometry.

Building on Valero (2011) it is possible to formulate that the forms of subjectivity that school geometry promotes in children contrast sharply with children's experiences of space in their activities out of school. Hence, school geometry promotes the fabrication of a certain type of subject, a desired trained child who is able to see through reason and logic, with trained eyes.

The Horror! A School Nightmare

What if students were not able to detach from the "eyes of the body"? The desired child portrayed has some features. It is known that many students will come close to become such desired child by having appropriated the forms of thinking and being specified through the practices of school geometry. However, does every student become the desired child? What would happen to those who do not succeed? It is said that students encounter difficulties when both eyes meet:

> [Difficulties] that occur as a result of their spontaneous processes of visual perception in cases in which they contradict the geometric concepts/knowledge aimed at by the teacher and the tasks. Students fail to accomplish a dimensional deconstruction of the figures in order to infer mathematical properties in axiomatic geometry. (Gal & Linchevski, 2010, p. 180)

This invites to understand how perception interacts with reason and logic while shaping students' own existence in the training process. Andrade-Molina (2015) shed light on the implications of neglecting perception in school geometry. Students should train themselves to operate within certain discourses. They learn that while solving problems involving the measurement of the height of buildings or objects located perpendicularly to the ground, they can use the Pythagorean theorem or trigonometry. They have a 90° angle to operate with (Fig. 15.10).

As students advance through the levels of the map of learning progress, they are confronted with the challenge displayed in Fig. 15.11. Students should be able to use their tools to solve this problem. They have learned the Pythagorean theorem as a method to calculate the height of an object or the distance between a given point and that object—an object that is perpendicular to the ground. When using this behaviour to overcome this challenge, they should gather all information regarding the situation to understand the problem, and then they have to create a plan, as MINEDUC expects.

Because they trained themselves to use Pythagorean theorem, the plan would be to find a right angle (Fig. 15.12): The height (h) with the ground, the same technique as their previous experiences. Therefore, they have a height (h); they have the right angle and the distance that they should estimate. They are not asked to give an exact measurement, but they are asked to play with the tools they have.

Probably they will use other theorems, even trigonometry to face this challenge and "win the game". But neglecting the curvature of the Earth is a horrifying nightmare,

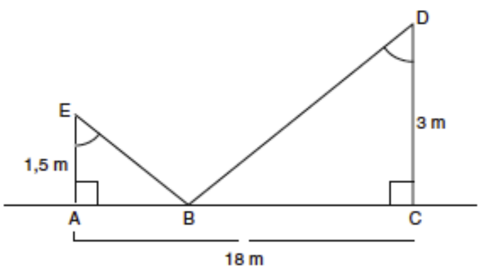


Fig. 15.10 Calculating heights and distances (Muñoz et al., 2013, p. 104)

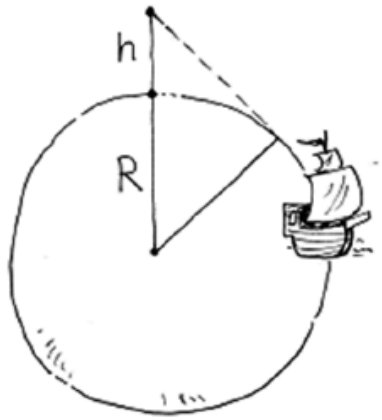


Fig. 15.11 School geometry task (MINEDUC, 2004a, p. 95, own translation)

Fig. 15.12 Resulting triangle

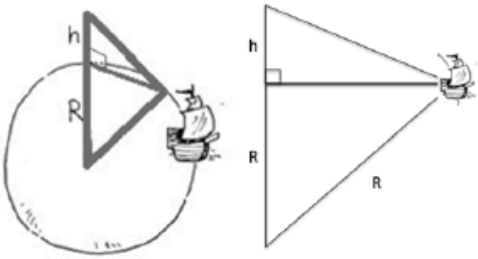


Fig. 15.13 A horrifying nightmare

at least in this case. The students who are able to make the type of configuration in Fig. 15.12 are disregarding the curvature. Why is this horrifying? Because it leads to contradictions of geometrical theorems, for example, that the sum of the length of the legs of a triangle does not add more than the length of its hypothenuse (Fig. 15.13).

> [In this particular problem] it does not seem so irrelevant to neglect the curvature of the earth, through our eyes it is impossible to say that a 90° angle is formed. Visually it does not make sense, but in school we have to accept it. Otherwise, we cannot use the geometry learned in school. We accept it because, locally, it seems like a right angle […] both leg and hypotenuse have the same measurement, the Earth radius […] Perception and visual judgements cannot be separated from reason and logic. (Andrade-Molina, 2015, p. 7, own translation)

Students have to agree that, for this type of problems, the earth has to be flat. They should be subjected to this rationality. In this sense, the horrors that may result in the extension of the logic of school space are not to be explained as a fault created by teachers' "mis-implementation" of the curriculum, neither by students' cognitive deficiencies. Rather, they are to be explained in terms of the very same power effects of school geometry to shape students' ways of dealing with space.

Popkewitz (2008) introduces the term "abjection" to state that while certain discourses express the desired—the included—at the same time, they are expressing the undesired—the excluded. The analysis deployed in this chapter reveals the features of the desired child. It also reveals the ones of the "feared child". The feared child can tell the contradiction of the leg and hypotenuse of the triangle having the same length (Fig. 15.13). It is the one who is capable to link both eyes and see school space not only through reason and logic.

Shaping the Child Through School Geometry

> Forms of knowledge are effected and effect power as they bring together knowing and being as two sides of a coin. Forms of knowledge do not only bear the rules of how one knows and what it is to be known, but also impose ways of being on the knowers […] If knowing and being are inseparable, the question emerges of what the forms of knowing and being are that the mathematics curriculum effects in children, and whether those forms of subjectivities are desirable. (Valero et al., 2012, p. 3)

Chilean school geometry has effects of power in students' subjectivities, not only in terms of oppression and subjection, but also in fabricating productive forms of being in the world through practices of the self. Students should be able to perceive themselves as "agents" (Foucault, 2009) who are responsible for their own learning. They should care for themselves. Students should feel curiosity; students should engage and ask themselves "what if?" while learning geometry. However, it is not only an invitation to think, but it is also an invitation to act, to create plans, strategies, and execute them.

> By introducing at first vectors in the plane—which are easier to imagine and to depict—and then, moving forward to depict and operate with vectors in space, might invite students to ask themselves about other possibilities in higher dimension. (MINEDUC, 2004a, p. 68, own translation)

Reasoning with governmentality, techniques of government aim at controlling human behaviour, but at a distance—a kind of wireless management. This is achieved by using techniques that regulate the habits and desires of students, "arranging things so that people, following only their own self interest, will do as they ought" (Scott, 1995, p. 202). Therefore, the interplay between power and mathematics education is on how the school mathematics curriculum generates cultural and historical subjects (Valero & García, 2014). Understanding school geometry as modes by which power operates to shape subjects, allows unpacking how school geometry inserts students into a form of rationality. In other words, how school geometry becomes a technology of the self that fabricates a "desired child" who is able to see with trained eyes—sightless eyes (Andrade-Molina & Valero, 2015)—generating systems of reason in which forms of life and subjectivity are made possible, organised and constrained.

Henceforth, technologies of the self enable students to change and to develop their thoughts and conduct their ways of being. These technologies enable students to modify, structure and constitute themselves as subjects.

> Permit individuals to effect by their own means or with the help of others a certain number of operations on their own bodies and souls, thoughts, conduct, and way of being, so as to transform themselves in order to attain a certain state of happiness, purity, wisdom, perfection, or immortality. (Foucault, 1988, p. 18)

Since discipline makes individuals (Foucault, 1979), disciplinary technologies are means of producing compliant, meeting the requirements subjects, through the exercise of management techniques, which govern every aspect of life. All in all, "who is subjected to a field of visibility, and who knows it, assumes responsibility for the constraints of power […] he becomes the principle of his own subjection" (Foucault, 1979, pp. 202–203). Thus, in order to reach a state of happiness, purity, wisdom, perfection or immortality, students must train and modify themselves. Not only they will acquire the skills of the "desired child", but they will also acquire certain attitudes to navigate in space. School geometry, as a technology of the self, shapes and also subjugates students through "relations of power" (Foucault, 1982). Students accept the space deployed by school geometry as well as the tools to operate in it, Within formal school settings, students should accept to neglect their senses; they should accept to model everyday life situations by using geometrical deductions; they should accept to see space through reason and logic.

The analysis deployed in this chapter traces these techniques. The techniques used in the production of the "new subject" from school geometry discourse. It also traces the power effects on students' subjectivities, as a process of cutting the links between the eyes of the instrumental space and the eyes of the lived space. Although this critique emerges from the expressions of a desired particular way of being, however, there is no certainty on how these strategies of power self-govern students. The map of progress analysed does not offer information on how many students performed accordingly to the expectations of MINEDUC, and how many did not. In that sense, it is more the critique of a dream, of a non-existent desired child. It is possible to tell the story about how school geometry becomes a technology of the self because it effects children's subjectivities. Nevertheless, it is not possible to state, by this discourse analysis, how power is effecting the shaping of the self. Only the desired child will be the one shaped by the strategies of power, but not all students become the desired child.

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NOT ANOTHER TYPICAL STORY, YET NOT A NEW CRITIQUE. A JOURNEY TO UTOPIA ACROSS STANDARDIZED ASSESSMENT

NÃO É OUTRA HISTÓRIA TÍPICA, PORÉM NÃO É UMA NOVA CRÍTICA. UMA VIAGEM PARA UTOPIA ATRAVÉS DE AVALIAÇÃO PADRONIZADA

ABSTRACT

This is an approach to standardized assessment built on Foucault's work. Assessment is taken as a *dispositif* that allows the exertion of power, in terms of governmentality. Diverse discourses about assessment circulate among national and international agencies, which promote its "goodness" for progress and development of nations. This paper, by historicizing the present, aims at tracing the continuities and discontinuities of discourses about standardized assessment and at portraying how assessment systems govern subjects. The structure of this paper is not the typical academic structure. Instead, it uses the plot of two movies to articulate and to depict assessment systems as a *dispositif* of power.

Keywords: standardized assessment, dispositif, historicizing the present, Foucault.

RESUMO

Esta é uma aproximação para avaliação padronizada construída sobre o trabalho de Foucault. A avaliação é tomada como um dispositif que permite o exercício do poder, em termos de governamentalidade. Diversos discursos sobre avaliação circulam entre agências nacionais e internacionais, que promovem sua "bondade" para o progresso e o desenvolvimento das nações. Este artigo, ao historicizar o presente, visa traçar as continuidades e descontinuidades dos discursos sobre a avaliação padronizada e retratar como os sistemas de avaliação governam os sujeitos. A estrutura deste artigo não é a estrutura acadêmica típica. Em vez disso, usa o enredo de dois filmes para articular e descrever os sistemas de avaliação como um dispositif de poder.

Palavras-chave: avaliação padronizada, dispositif, historicizar o presente, Foucault.

CAUTION!

This is not the typical story about assessment. This is not a story about a method to select, certify or control groups of students (Jurdak, 2014). It is not about forms of pressure to "deliver" and to exclude students (Ball et al, 2012), neither about equity

(Gipps, 1999) nor inequity (Cooper & Dunne, 2000). This is not a story about a promoter of active feedback (Black et al, 2004), neither about a gatekeeping dispositive (Björklund Boistrup, 2016). This is not a story about the goodness of assessment for progress, economy and development of nations (Kellaghan & Greaney, 2001). This is not a story about a report on international comparisons of children's curriculum knowledge (PISA), nor a national system to improve every improvable aspects or agents of schooling (SIMCE in Chile). This is a story of none of these and, at the same time, about all of these. This is a journey to utopia across a path of continuities and discontinuities, of hopes and dreams about assessment in school[1].

CHAPTER 0: MONOLOGUE

[Voiceover] Well, we meet again. I remember the first time I saw you. I was so naïve. I thought we were friends, but I didn't know you had other plans, that you were going to talk behind my back, against me. You questioned everything I thought I knew and you still do. Are you mocking me? Do you enjoy judging me? Probably I didn't take you seriously. Was I foolish? I believed everything you promised me; everything they told me about you. You were going to help me help others. That was the plan[2]! You were going to help me decide whom to help first, whom to help next. I relied on you! You would tell me what they needed, what they lacked, to decide what to do[3]. I still remember when we chatter about equity and quality, about harmony and equality, about our hopes of a brighter future. We were supposed to make everything better. But… I was so wrong! We sounded so much like hippies... youngsters' dreams. And I know what you are going to say to me. *"What are you talking? I didn't judge you; I was the only one able to say what you truly were and what you could become. I was being your friend, your only friend".* But you were not my friend, and you never will. What do you want of me? Are you analysing me? You will not be able to escape, not again. So, let our little dance begin.

CHAPTER 1:

[Background music: ...You and me we have no faces. They don't see us anymore.
Without love as they had promised and no faith for what's in store[xxi]*]*

Numbers, numbers, numbers. You and I didn't think this would be it, didn't we? Was it worth it? Measuring progress, quality, performance, effectiveness, achievement, measuring them, us… everything! Numbers, so powerful and so meaningful, yet so tricky to work with. You can play with them as you please. Numbers, little tricksters!

[xxi] Fragment of *No one's there*, by KoЯn.

Numbers, numbers, numbers! So closely linked to competition and accountability. Are we pursuing a business? Are we some sort of enterprise? Numbers to arrange, to allocate, to identify, to test, to conclude, to elude, to increase, to decrease. Numbers, so powerless and so meaningless unless you give them power. And that is what we all did, did we? We played with them, we used them, we thought we needed them, and now we got used to them. They became part of us, of who we are, of what we do, of what we know.

I have so many questions. Do you see them as consumers?[4] Are you judging if we sell them valuable tools? No wonder why we have to redistribute efficiently our resources[5], are you helping us to achieve that? Are you giving us numbers to reward? Numbers to reward our best sellers and their skills for consumers to be content with the final product. But, our customer service is still full of complains[6]. Is that what you are helping us with… to improve our products' quality?

A marathon, running, rushing to be on the top… Tripping, colliding, hoping not to be last… Falling, failing… Training exhaustedly, eating properly, buying the best clothes, seeking for advisors, making decisions, monitoring every move, trying to reproduce what winners do, investing for our success. Is that what you want us to be? Runners?

Numbers, numbers, numbers! Wait…

> …Memory's not perfect. It's not even that good. Ask the police. Eyewitness testimony is unreliable. The cops don't catch a killer by sitting around remembering stuff. They collect facts, they make notes and they draw conclusions. Facts, not memory! That's how you investigate. I know. It's what I used to do. Look, memory can change the shape of a room. It can change the colour of a car and memories can be distorted. They're just an interpretation. They're not a record. They're irrelevant if you have the facts[xxii].

That's it! Facts! Numbers to collect facts, to gather data, to produce variables, to explore, to report, to compare, to show, to prioritize, to select, to appraise, to research. Numbers that numb. Numbers to bring order to the chaos, they said. Numbing numbers. Numbing the ones judging or the ones that are being judged? Perhaps both. But what are numbers seeking? Are they seeking the failure to bring quality, an average quality?[7]

I can hear you coming, step-by-step, walking towards me. Well, we meet again… (Back to: Chapter 0).

[xxii] Leonard talking to Teddy, in Christopher Nolan's film "Memento" (2000).

CHAPTER 2:

[…To all these nameless feelings I can't deal with in my life.
To all these greedy people trying to feed on what is mine…[xxiii]*]*

Before the 90s, we were four little gullible dreamers in this end of the world[8]; we were the ones you convinced first. After that you spread like a virus[9]. Do you remember how everybody used to say you were the one able to make things better? The redeemer of our sins and failures… weren't they using you to set standards? Weren't they using you to tell us what to do and how to behave?

[The phone rang] Who is this?

I don't understand it. You expanded so quickly and you still do. They see you as the Holy Grail, as if you were the key to unlock all answers. You are trained to seduce others with your greatness. I just couldn't see it. You have many masks. Are you truly teaching them how to behave?[10]

> [On the phone] At that time, in the sixties, it was all about social change and progress. Intuitive and global interpretations were left for scientific approximations. We were seduced by the abstraction of models and paradigms. We were so into economic progress[11], into the promises of foreign lands. 'Theories of modernization'[12], they used to call it…

Do you remember the World Bank, OREALC, the Interamerican Development Bank or UNESCO? They were all interested in you. Of course, they said that quality was prompted by decentralization, accountability and market competition[13]. You were just a puppet. Have you heard about TIMSS or PISA? You are just one of many others[14].

> [On the phone] Americans thought that their industrial society and also the European one were the ideal models we had to follow. Why? I didn't really know, but we believed them[15]. Did we feel insecure, unevolved, or less-developed?

Can you tell what they are doing with you? Reports in the newspapers, all carefully aligned for everyone to see. Judgment day[16]. Everyone judges, everyone points[17]. Do you think they do it to be transparent? Probably they do, but… numbers are tricksters, remember? What if you are not at the top of the list? Will they start drawing conclusions? Will they feel the need to track facts to explain the failure? What could they track… conditions, income, backgrounds, staff's quality, previous movements, previous outcomes, owners, consumers' background, believes,

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xxiii Fragment of *No one's there*, by KoЯn globalization, the over use of social networks, the position of the moon. How far will they track? How many factors will they consider?

> [On the phone] …Modernization was the only path to overcome poverty. We had to become less underdeveloped and, also, ensure welfare. We started approximating scientifically to education. In 1967 we run the first test[18]. A revolutionary aptitude prediction system aimed at measuring capacities, knowledge, abilities, decision-making, and actions. We were so proud of it, asking relevant questions to measure their abilities for the prediction of their future performances…

Pressure… *pushing down on me, pressing down on you, no man asks for. Under pressure, that burns a building down, splits a family in two, puts people on streets*[xxiv]... pressure to be normal, to be average, to fit in. But everything is always moving, changing. So they become chasers, hunters of a dream, pursuers of an illusion. Why? Because that is the standard. Then, they train, they instruct, they run tests, they monitor progress, they practice, they improve, they hope, they fear, they chase[19]…

> [On the phone] No, of course not. You have to monitor it....

You are so stubborn. Yes, they can blame themselves for their own failure, but isn't everyone involved? Aren't published results making all actors accountable? Aren't you creating a culture of evaluation?[20]. And suddenly everything collapse, a training culture emerge, playing the game only to survive one more day. Was all of this because of you? But how? After all, you are just numbers. Numbers, numbers, numbers... (Back to: Chapter 1)

CHAPTER 3:

[... So what can it be? No one hears me call,
Echoes back at me. No ones there...[xxv]*]*

I give up! It is impossible to talk to you. You refute anything I say because other people are using you and you don't even realise… you are even more naïve than me. Do you think you are the best option and that everything should turn around you? Do you even care?

> [On the phone] That's why in 1978 we started developing a new test[21]. In 1982 we tried it again. We called it PER, the "assessment program for

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[xxiv] Fragment of *Under pressure*, by Queen.

[xxv] Fragment of the song "No one's there", by KoЯn.

> school achievement". In those years, the ministry of education was in charge of all educational services, a giant task…

Your friends have been supporting you, and promoting you amongst nations, for you to arise as the salvation, as the only answer[22]. They are using you to identify standards and encourage others to reach those same ideals, been shepherded like sheep. They say you are important but only because they have to; because they need to. Have you seen what are you doing to people?[23]

> [On the phone] And the idea remained… we thought we needed a system to guide our future actions. Standardized educational measurements! That was the key, that was the tool we created to guide our decisions[24]. The neoliberal Chilean experiment led us to negotiate with the World Bank, UNESCO, and OREALC in Santiago[25]. Decentralization was our motto[26].

People cracking, failing, doubting, competing, deteriorating, fading, people being advertised, accountable people, good publicity for some, bad publicity for others, for sellers, for consumers, for products, for factories. Blame to share by all of them, and none of them[27]. Is the system breaking?

> [On the phone] Then, standardized tests were all they talked about. UNESCO began to promote international agendas to improve these methods for the collection of information and how to use the resulting data to make proper decisions[28]. Chile was no exception… SIMCE left Chile at the top in Latin America. We were so avant-garde!

They said you were aimed at monitoring quality[29], was it at any cost?[30] Was it by trying to control every move?[31] They said you were going to change everything. You were going to be a revolutionary system, so visionary, but for whom? Quality equality was your slogan[32]. Egalitarian opportunities, knowledge, and skills, they said. For all! Those were the hopes of a brighter future, full of utopian promises… You came here to stay and we became more dependent; do we know how to live without you?[33] With you it's easier to make strategies and to test them, trial and error, until we reach the top. By copying past winners, by enchanting others to follow the path to victory. More and more to be higher and higher!

> [On the phone] SIMCE quickly became an information provider, of extremely useful information for developing policies[34]. At the beginning, it was thought as a tool to quantify quality… Yes, quality it was, at that time, in terms of the knowledge students had, obviously according to curricular expectations[35]. SIMCE was aimed at improving the quality, not just as a mean of quantification… SIMCE's outcomes were thought to be helpful to improve equity too[36]. What a dream!

Is that why they train and agree to be trained? Are you making them athletes against their will? But they have to be athletes to belong, to not be indicated as failures in the system. Is that why they train and agree to be trained? You need sellers to perform flawlessly. Is that why they train and agree to be trained? They agree because they believe in you, in your mighty wisdom. Do they doubt about themselves? Do they agree on not to disagree?

> [On the phone] We thought that by not releasing individual students' scores we would be able to avoid undesirable consequences. You know, exclusion or selection of students by their scores in SIMCE, competition between students, and all that jazz. We aimed at protecting their privacy[37]. Even journalists became an important part of SIMCE to avoid undesirable perceptions about the test. They were very important for the appropriate dissemination and public communication of SIMCE's outcome[38]. You see… we had nothing but good intentions….

But they are alone, if they scream what will they hear back? Silence? You are making them follow a path by guiding every step[39]. They are like little children picking up each candy you toss on the floor. You reward every step they take towards victory[40]. What are they to you? What are they to your friends?

> [On the phone] There were no hopes and dreams. There were facts! SIMCE was nationally acknowledged for been a reliable, credible, and rigorous system[41]. In Latin America we were praised for its stability, coherence, coordination, and quality in the dissemination of outcomes[42]. We had created the perfect system to overcome inequity[43]...

…I find the answers aren't so clear. Wish I could find a way to disappear. All these thoughts they make no sense. I find bliss in ignorance. Nothing seems to go away. Over and over again[xxvi]. You only care about results, numbers is all you see. Why can't we see that? Why can't they see that? Are we so seduced by your "greatness"? Are we so drown by your promises? What do you want us to be? Competitors? Entrepreneurs? I don't understand. Are you just empty promises? Before the 90s… (Back to: Chapter 2)

CHAPTER 4:

[… Oh I wish that I could see, How I wish that I could fly
All the things that hang above me, to a place where I can cry…[xxvii]*]*

[xxvi] Fragment of *One step closer,* by Linkin Park

[xxvii] Fragment of *No one's there*, by KoЯn

I wonder how will life be. I wonder if we will rise without you. I wonder if you are the only solution[44]. Is it possible to find other ways?

> [On the phone] Of course we were not interested only in SIMCE's outcomes, we complemented our national scores with international surveys. The more, the merrier. We have been participating since 1997 in these international studies. Well, TIMSS, the Civic Education Study, the International Civic and Citizenship Education Study… Of course! How could I forget about PISA? Also in the UNESCO one… the Latin American comparative surveys[45]. It sounds as if tests are the only thing we have in mind, but… that's how you reach excellence…

If you helped to bring harmony to the globe[46]… why wouldn't you bring harmony to us? We should continue this journey and see how it ends. We should remain being friends[47]…

> [On the phone] At the beginning, we started only with Spanish and Mathematics. Kids need to be able to express themselves and communicate properly[48]. Kids also need to communicate numerically. Mathematics helps to develop abstraction, calculation, and reasoning[49]. Now, if I step aside, it seems to be out of control. Currently SIMCE has increased and new school subjects are been incorporated… social sciences, history, geography, natural sciences, writing, English, the use of TICs, and physical activity… sports… kids also need to be healthy! Apparently we are a bit obsessed. We even increased the frequency. So, we have plenty useful information to perfectly plan our next moves.

It is inevitable to feel this impulse to doubt… I know there is plenty evidence, plenty of good results, plenty of research, plenty conclusions, but… what if this is not the appropriate step to take? I cannot help thinking about the possible implications…

> [On the phone] It has been twenty years now… time really flies. SIMCE has helped us tremendously. It has been an efficient tool to reveal inequities in student's learning, which helped us to connect social disparities with schools' outcomes. That was a fascinating aspect we wouldn't be able to see without the test… A culture of evaluation? It could be, but accountability is necessary[50]… some people might say that SIMCE exerts powerful influence in curricular and pedagogical activities. One of them, they say, is "Teaching to the test"[51], reducing the national curriculum only to meet SIMCE's expectations, rejecting students[52]. I don't believe it! There are plenty knowledge that the test doesn't cover, so how could that be possible? The system works! Doesn't it?[53]

Am I being too negative? Am I being too insecure? I give up! It is impossible to talk to you… (Back to: Chapter 3)

CHAPTER "23":

[You and me we have no faces. Soon our lives will be erased
Do you think they will remember? Or will we just be replaced?...[xxviii]*]*

> [On the phone] Does it work? SIMCE is a tool to nationally monitor the whole school system providing orientation regarding decision-making policy process. To achieve quality it becomes necessary to track every school's score yearly, to compare and visualize their progress, if they have any[54]. Also it is compulsory, in order to achieve our standards, to guide teachers on how to improve their practices by handing them standardized instruments and SIMCE samples[55]. SIMCE is a tool teachers could use to detect their strengths and weaknesses to improve their practices[56]… Monitoring? Tracking? Guiding? Detecting? What is this? On the bright side, it is a longitudinal study that has many qualities and advantages for achieving equity of quality. On the dark side… is SIMCE acting as a form of surveillance? It doesn't make any sense, isn't it? Why did we start with mathematics and Spanish? Why mathematics? Of course in the sixties we thought logic was the foundation of every science because it leads to reasoning accurately and rigorously, which is the core of any argumentation and of critical thinking, and, obviously, because it was fundamental to achieve those abilities for pursuing further education, right?[57]. Am I wrong? Did we do something wrong?

Imagine there's no heaven, it's easy if you try. No hell bellow us, above us only sky… Imagine all the people, living life in peace[xxix]. I wonder how will life be. I wonder if we will rise because of you. I wonder if you are the solution...

> [On the phone] Although… In the sixties we were trying to develop a scientific system to predict a potential learning and aptitudes towards that learning[58]. And we succeed in it, in my opinion. Nowadays we have a system that is able to predict student's achievement in SIMCE before participating in the test. We have created the "Learning Standards". These standards are taken as a reference point able to describe what students should know and do in order to demonstrate their achievement[59]… No, if you want to achieve… but international tests also… No we are not stressing out children to obtain a higher score in international tests, it is a

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[xxviii] Fragment of *No one's there*, by KoЯn.

[xxix] Fragment of *Imagine*, by John Lennon

> matter of achieving quality! We want students to have better opportunities in life, to be successful… they need mathematic proficiency[60]. Those types of questions help us unveil… Wait… do they help?

I hear them talk about you[61]. You seem so complete, so structured. You seem like a good option for us. According to them, you can provide the information we need to be better, to improve…

> [On the phone] National assessments should be taken as an opportunity to help teachers and students to unveil their achievements, according to national standards, or what needs to be reinforced, to reach set standards… feedback[62]. Assessment is the only path to help students to be successful and to have a brighter future, better opportunities, and... Yes, they need to be assessed… They have to be assessed. It is the only way to stop with inequity… to have a quality education… This doesn't make any sense, doesn't it?

If you helped them, why not us… What did winners do to be considered winners?[63]

> [Hanging the phone] *Chapter 23, you can call me Fingerling… That number followed me… It was a mistake to think I could escape it… The number had gone after me. And now it wanted her. I was right. She was in danger. I just didn't realize the danger was me*[xxx]. But, what more harm can it do? After all is just a number.

I wonder how will life be. I wonder if we will rise without you. I wonder if you are the only solution… (Back to: Chapter 4). *Now… Where was I?*[xxxi]

THIS IS…

The whimsical story is inspired by Christopher Nolan's film *Memento* and by Joel Schumacher's film *23*. *Memento*'s plot is embedded in the first five parts or "chapters". The film presents two stories, one moves backwards in time and the other moves forwards in time. The first five chapters of this writing are inspired by *Memento.* One story is about the power effects of assessment as a dispositif. It goes backwards in time. The second story, "[on the phone]", is about the temporal-spatial conditions that enabled the decision making process regarding a national standardized assessment program. It moves forward. The *23*'s plot is entangled with "on the phone" in the section "Chapter 23". In this last part, the voice "on the phone" recognizes itself as part of the problem, both the medicine and the disease.

[xxx] Final lines of Joel Schumacher's film "23"

[xxxi] Final line of Christopher Nolan's film "Memento"

Each chapter begins with fragments of the song *No one's there* that portrays the story that goes backward, and at the same time enable to shed light into the power effects of the second story on the shaping of a desired citizen.

The discussion raised here is about the historical making of citizens that has been inscribed in a cultural practice of national standardized assessment. By following an analytical strategy of historicizing the present, it problematizes the naturalized truths circulating about assessment among national and international agencies by tracing its continuities and discontinuities. From this analysis, two of the most repeated circulating discourses about standardized assessments are: "*higher score means better quality"* and "*competitiveness and accountability leads to higher performance, raising incomes, social mobility and welfare*". These have become naturalized truths in Chile.

The writing and structure of this story was aimed at portraying assessment as a *dispositif* that governs subjects to conduct their own ways of being and acting in the world (Foucault, 1991). According to Foucault (1980, p. 194), the *dispositif* is a network composed of: "discourses, institutions, architectural forms, regulatory decisions, laws, administrative measures, scientific statements, philosophical, moral and philanthropic propositions". Within this entanglement of elements, the *dispositif* raises as a formation that respond to historical-given "urgent needs". In this fashion, assessment is not to be taken solely as an instrument, but also becomes a heterogeneous ensemble with a dominant strategic function (Foucault, 1980). As Sellar and Lingard (2014, 922) argue, the reliability in numbers produced by assessment systems enables comparison as a new form of governance. In which "[Skills] presented as the solution to a range of economic and social problems remains dominant […] Skills agenda is now at the very heart of the Organization's economic work and is linked to its role in neo-liberal globalization". And so, "the new citizen is required to engage in a ceaseless work of training and retraining, skilling and reskilling […] life is to become a continuous economic capitalization of the self" (Rose, 1999, p. 161).

ENDNOTES

[1] All quotes have been translated from Spanish and French to English by the author.
[2] The Chilean assessment system for measuring the quality of education (SIMCE) aims at improving quality and equity of education "by providing data about learning
[2] The Chilean assessment system for measuring the quality of education (SIMCE) aims at improving quality and equity of education "by providing data about learning outcomes [to]: monitor and inform decision-making […]; provide feedback to schools […] improve teaching practices; foment accountability and parental and school community involvement (Meckes & Carrasco, 2010, p. 234).

[3] SIMCE is recognized as an accurate and effective system for "monitoring and managing the decision-making process" to improve the quality of education (Olivares, 1996, p. 195).
[4] Assessment systems "can augment efficiency by making principals and teachers more accountable to parents. […] parents-consumers can voice their discontent and demand better services" (Benveniste, 2002, p. 92).
[5] SIMCE has been taken as an efficient system to allocate resources to the most vulnerable educational sectors (Olivares, 1996).
[6] "[T]he role of the State has been dislocated from its central position leading the development of educational policies, being now just another stakeholder within a network that disproportionally represents the corporative, market, and conservative interest of the traditional global elite. […These] also raise questions regarding the health of the Chilean democracy and the legitimacy of evaluation policies targeting the broad population, but tailored to the needs of a cluster of neoliberal entrepreneurs" (Campos-Martinez et al. 2015, p. 121).
[7] SIMCE collects "detailed background information from students and their educational establishments […] Schools receive individualized reports comparing their test outcomes to the national average and a regional average" (Benveniste, 2002, p. 97).
[8] Until 1991, Chile, Colombia, Costa Rica, and Mexico had experience with systemic evaluations (Benveniste, 2002).
[9] During the 1990s, assessment systems flourished throughout all Latin America (Benveniste, 2002).
[10] Within formal assessments surveillance is combined with normalization: "There is qualification and classification taking place simultaneously, as well as the exercise of power and education of a specific knowing" (Björklund Boistrup, 2017, p.3).
[11] In Chile, during the first half of the 20th century, "the focus of social analysis shifted from an interest from 'culture' itself, as an expression of forms of living, to 'culture' as a normative element for economic and social transformation" (Larraín, 2001, p. 112).
[12] The first ideas of modernization in Chile "were the American ideas of the sociology of development, usually denominated "theories of modernization", which emerged in the late 40s and at the beginning of the 50s" (Larraín, 2001, p. 112)
[13] "National assessment systems support a new vision for a lean, noninterventionist state model […] The logic underpinning this model of assessment is the promotion of decentralization, market competition, and consumer accountability" (Benveniste, 2002, p. 93).
[14] Since 1996, OECD has prompted nations to shift governmental concerns away from mere control over the resources and contents of education toward a focus on outcomes (OECD, 1996).
[15] Americans authors such as "Parsons, Linton, Davis, Hoselitz, Moore, McClelland, Levy, Eisenstadt […] moved forward the idea that developing

countries […] are in transition from a traditional society into a modern society and that European and American industrial societies are the ideal model. Modernization is taken as a historical necessity" (Larraín, 2001, p. 112)

[16] Since 1995, SIMCE's scores "have been widely publicized in newspapers. Student achievement data is meant to inform parent-customers about the quality of service provision and to assist them in the selection of an educational establishment for their children" (Benveniste, 2002, p. 103).

[17] "The publication of results caused mistrust among teachers, […] since the low scores were simplistically interpreted by the public as resulting from poor school performance" (Meckes & Carrasco, 2010, p. 238).

[18] (see Cariola & Meckes, 2008)

[19] "Even those educational establishments that manifest opposition to teaching with SIMCE in mind report escalating pressure from central and district authorities to align themselves closer to the test. Some municipalities organize mock tests to give schools an opportunity to prepare children for the national evaluation. Others offer financial incentives for high-scoring schools. Some teachers fear their jobs might be at risk if their students do not perform adequately in the evaluation" (Benveniste, 2002, p. 112).

[20] "The main lessons learned are that a census-based assessment system focuses attention on learning outcomes and impacts everyone, that published results make all actors accountable, that effectively communicating results is as important as accurately generating them, and that these elements help develop a culture of evaluation (Meckes & Carrasco, 2010, p. 233).

[21] In 1978, the Chilean Ministry of Education started developing a national test aimed at comparing schools (Campos-Martinez et al, 2015).

[22] "Standardized tests have become a more important—and more controversial—element in the policy debate. […] testing methodologies have improved, making these assessments the best indicator available for measuring performance" (Vegas & Petrow, 2008).

[23] (see, Andrade-Molina, forthcoming)

[24] "SIMCE was created under the military government (between 1973 and 1990) as a complementary tool of a policy of privatization and of transferring to municipalities the management of education, according to the predominant market economy model to the time" (Cariola & Meckes, 2008, p. 38).

[25] "The idea of introducing standardized educational measurements, understood as a tool to guide decisions in the emerging education market, was the notion logically discerned from the neoliberal principles imposed during the dictatorship" (Campos-Martinez et al, 2015, p. 109).

[26] "The [World] Bank encourages continuation and strengthening of decentralization of education" (Holsinger, 1991, in Campos-Martinez et al, 2015, p. 110).

[27] Since 2000, "Policy-makers and some academics began to question the usefulness of an assessment system in which published results contributed to the

deterioration of teachers' self-esteem, as well as of the educational system as a whole" (Meckes & Carrasco, 2010, p. 239).

[28] "Planning the quality of education through informed decision-making requires the availability of accurate and timely information that links together resource inputs to education, teaching-learning conditions and processes, and appropriate indicators of the knowledge, skills, and values acquired by students" (UNESCO, 1990, p. 3).

[29] "It was expected that the market, in which all actors would have the necessary information, would regulate the quality of education itself" (Cariola & Meckes, 2008, p. 39).

[30] "Schools, in competition to attract students and the associated financial resources […to avoid loss] would do their utmost to improve the quality of education to attract and retain their pupils" (Cariola & Meckes, 2008, p. 39).

[31]"During the second half of the 1990s, SIMCE began to be used more as a tool for monitoring educational policy" (Cariola & Meckes, 2008, p. 44).

[32] "[The quality of education assurance law] stipulated the creation of a national system of quality of education assurance, in order to improve the quality of education of Chilean students within a frame of equality of opportunities" (ACE, 2012, p. 3)

[33] "[Since SIMCE] was implemented, the trend has been to increase the amount and the role of assessment—more standards, more assessment and higher consequences are being demanded and being introduced" (Meckes & Carrasco, 2010, p. 246).

[34] In 1995 SIMCE's scores were ranked, this led to individualize schools' performances and compare its results over time. Scores were linked to teacher's incomes, which prompted to a reward system (Campos-Martínez & Corbalán, 2015). It also helped to create new standards for students, teachers, head teachers, and for designing models for efficient management of schools (Bravo, 2011).

[35] "[SIMCE] seeks to find the level of students' achievement in diverse areas of knowledge" (Martinez, 1996, p.1)

[36] "[SIMCE's] aim is to improve the quality and equity of education by providing data about learning outcomes at the national and school levels"(Meckes & Carrasco, 2010, p. 234).

[37] "[SIMCE] was conceived as a tool for school rather than student assessment. Individual data on students' performance has also been protected to avoid undesirable consequences, such as exclusion or academic selection of pupils. […In Chile,] schools compete for pupils, test scores of schools are published and private-subsidised schools are currently allowed to decide which students they will admit " (Meckes & Carrasco, 2010, p. 246).

[38] Journalists are considered as important to improve the dissemination or public communication of SIMCE's outcomes (Ravela, 2004).

[39] "The recent efforts of the SIMCE and Curriculum Unit to develop a common framework of standards for reporting national test results and guiding school-based

assessment […] is a promising initiative for building a bridge of meaning between large-scale assessment and teachers' practices " (Meckes & Carrasco, 2010, p. 246).

[40] "Reward systems tied to testing outcomes may also prompt educators to direct greater efforts toward teaching practices that increase student achievement" (Benviste, 2002, p. 92).

[41] (see MINEDUC, 2003)

[42] (see Vegas & Petrow, 2008).

[43] "The important role of SIMCE data in the design and development of national education policies aimed at reducing inequities" (Meckes & Carrasco, 2010, p. 246).

[44] "The World Declaration on Education for All […] recognized that periodic student assessments make a valuable contribution toward the improvement of educational quality" (Benveniste, 2002, p. 91).

[45] (see Ferrer, 2006).

[46] (see Tröhler, 2010).

[47] "Since it was first conducted in 2000, PISA has become hugely successful and has received considerable media coverage and attention from politicians and policy-makers in many nations" (Sellar & Lingard, 2014, p. 917).

[48] "Verbal ability is vastly important while expressing and communicating ideas […] without a proper fluency and mastering of the language, learning practices become precarious and ineffective" (MINEDUC & CPEIP, 1967, p. 5).

[49] "Mathematics ability represents another great aspect of learning: the capability of abstraction, synthesis, calculation, reasoning […] every individual should develop a minimum capability of numerical communication" (MINEDUC & CPEIP, 1967, p. 5).

[50] "[SIMCE] has helped place learning outcomes at the heart of the national debate on education. It has also helped to develop a culture of evaluation and accountability at different levels of the system […] the information it provides has helped to reveal the country's great inequities in learning outcomes and their relationship to social disparities" (Meckes & Carrasco, 2010, p. 245).

[51] "SIMCE has proved to exert a powerful influence in curricular and pedagogical activities. "Teaching to the test," despite being widely criticized by educators as a stratagem to improve children's scores in order to secure a higher position in the public rankings, is commonplace" (Benveniste, 2002, p. 112).

[52] The most frequently cited unintended consequences of a high-stakes testing program such as SIMCE are teaching for the test, narrowing the curriculum that is implemented, schools rejecting or expelling students in order to raise scores, and stigmatising poorly-performing schools. (Meckes & Carrasco, 2010, p. 244).

[53] "The emphasis of Chilean teachers on practicing concurrent problems may indicate an overemphasis on skill drilling instead of mathematical understanding" (Preiss, 2010, p. 350).

[54] (see MINEDUC, 2003)

[55] Schools receive standardized instruments that could be applied themselves, to improve teachers' practices and their outcomes. (MINEDUC, 2003, p. 15)
[56] "Teachers can find in SIMCE's outcome a mean to detect their strengths and weaknesses for re-orienting and improving their practices" (ACE, 2012, p. 1)
[57] "Mathematics offers a variety of analytical procedures, modellation, calculation, measuring and making estimations […] that allow the establishment of relations between diverse aspects of reality" (MINEDUC, 2009, p. 145).
[58] "The contribution of these tests consists precisely in their capacity to scientifically measure basic learning "aptitudes" […] Ability tests, for being about "aptitude" […] are more suitable for measuring potential abilities for a future learning" (MINEDUC & CPEIP, 1967, p. 5).
[59] (see Bravo, 2011)
[60] (see OCDE, 2014)
[61] "Because the educational process is a long-term process in terms of time, intermediate controls should be made to ensure that students are learning properly and, therefore, to avoid that a low quality learning advance without correcting its defects, and a final control should be made to ensure students are learning according to the necessities and expectations of society" (Arancibia, 1997, p. 4).
[62] "The evaluation process helps both teachers and students to know their improvements and what needs to be reinforced […] With this information, teachers can make decisions to modify their lessons plan and adapt it according their students needs […] Students could centre their efforts, by trusting that they will improve their results" (MINEDUC, n.d., par. 1).
[63] The expansion of the test supposes […] the upkeep of the evaluation in reading skills and mathematics, following the most successful educational system's guidelines (ACE, 2012, p. 4).

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ASSESSING VISUALIZATION: AN ANALYSIS OF CHILEAN TEACHERS' GUIDELINES

Melissa Andrade-Molina and Leonora Díaz Moreno

The aim of this paper is to argue on how visualization is recommended, by official curricular guidelines, to be assessed in schools. We contend that spatial abilities have been granted with the status of a key element to improve students' performances by research and also by policy makers. However, this importance seems to fade when it comes to assessing students while learning school mathematics and geometry. We conducted an analysis of the official guidelines for the assessment school mathematics in Chile. The analysis of two of those guides is considered here. The results revealed that these guidelines do not help teachers while assessing visualization in schools; rather its focus is embedded in a tradition of training that leads to a reduction of space.

Key words: School geometry, visualization, spatiality, assessment, guidelines for teachers.

1. INTRODUCTION

As Suurtamm et al. (2016, p. 25) claim, assessing what students learn in schools "is a fundamental aspect of the work of teaching". They draw awareness on the complexity of the connection between large-scale assessment (for example PISA) and classroom assessment (that is usually teacher-selected or teacher-made). Such interplay leads to external large-scale assessment often influencing assessment practices in the classroom.

> Although classroom teachers have long used various forms of assessment to monitor their students' mathematical learning and inform their future instruction, increasingly external assessments are being used by policy makers throughout the world to gauge the mathematical knowledge of a country's students and sometimes to compare that knowledge to the knowledge of students in other countries. (Op. cit., p.1)

Assessments, both external-large-scale and classroom assessment, as a mean of monitoring and regulating teaching and learning practices of school mathematics are believed to "define what counts as valuable learning and assign credit accordingly" (Baird, 2014, p. 21). In which valuable learning becomes "what is important to know and to learn" (Suurtamm et al., 2016, p. 6). In other words, the monitoring and regulating features of assessment norms what is ought to be considered the important and necessary mathematical knowledge that students should learn in schools and, also, receive feedback on how to improve their own performances. As stated by Swan and Burkhardt (2012), assessments not only have influence on the content of instruction but, additionally, on how tasks are regularly presented to students (for example the preference to multiple-choice type of problems). Large-scale assessment, as pointed out by Suurtamm et al. (2016), is not concerned with examining students' thinking and communication processes—the learning itself—as they used mathematical problems frequently leading to only one correct answer. Opposite to what formative assessment in classrooms is: "Informal assessments that teachers might do as part of daily instruction as well as more formal classroom assessments used to assess the current state of students' knowledge". (Op. cit., p. 14). In this fashion, formative assessment becomes learning activities that enable students to demonstrate their knowledge, understanding and proficiency (McTighe, 2015). Building on the National Research Council's (2001) notion of assessment as "a process of reasoning from evidence" (p.2), the very nature of students results taken as "evidence" leads to imprecisions regarding of what they are able to do, what they had learned, and what they know, given that, as Pellegrino adds, such results are just estimates measurements of students performances.

In this context, Suurtamm et al. (2016, p. 7) pose five questions, which two become of interest for this monograph: (1) How do teachers negotiate the different purposes of classroom and large-scale assessment? (2) In what ways are classroom practices influenced by the demands of large-scale assessment? These two questions raise the necessity of analyzing how large-scale assessments inform classrooms practices, in which teachers have the tasks of negotiating the purposes of both types of assessment. The impact that large-scale assessment has on policy, curriculum, and classroom practice is influencing the nature of classroom instruction, for example, by shaping curricular reforms (e.g. Barnes et al. 2000). This impact prompts to what Suurtamm et al. (2016, p. 25) recognize as "the challenges teachers face in engaging in assessment practices that help to provide a comprehensive picture of student thinking and learning". Reading teachers' challenges regarding the assessment of school mathematics as a political dilemma, in terms of Windschitl (2002), help positioning the discussion over the struggles teachers face concerning their own views on classrooms' assessment and the educational polices' standards on national assessment (e.g. Engelsen and Smith, 2014).

Here, we want to explore this dilemma by analyzing the official guidelines for teachers released by the Chilean Ministry of Education (MINEDUC). These guidelines offer recommendations on how to assess students in the classroom according to national standards (derived from a Chilean large-scale assessments, SIMCE). This monograph seeks to trouble how assessment is ought to be occurred in the classroom according to teachers' official guidelines in Chile. In other words, how Chilean curricular guidelines inform teachers on how to assess visualization in schools. We intend opening a discussion about the way in which assessment enhances the training of students through school practices that aim at an axiomatization of space. First, we are going to set our theoretical background. We are building our analysis from the toolbox of Arcavi's visualization and from Lefebvre's understanding of space. Second, we present an exploration of the empirical materials of mathematics curricular official documents from the Chilean Ministry of Education (MINEDUC). Third, we pose a deeper analysis of the materials and connect with large-scales assessment. Finally, we draw attention on the axiomatization of space produced by a training tradition in Chile.

2. VISUALIZATION, SPACIALITY AND SCHOOL GEOMETRY

Since the 80s, research in the field of mathematics education has been focusing its attention to understand and grasp the complexity of visualization for the learning of school mathematics (see e.g, Swoboda & Vighi, 2016). Probably one of the most prominent and most cited "definitions" is:

> Visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings. (Arcavi, 2003, p. 217).

Visualization has become a form of reasoning that has been granted with a particular significance both in the learning and teaching of school geometry and school mathematics (Sinclair, et al. 2016). On one hand, some studies are advocating for the inclusion of this ability as part of the school mathematics curriculum (Sinclair & Bruce, 2015). On the other, Presmeg (2014), for example, suggest that it is worth arguing weather visualization is accepted, encouraged and valued in the classroom. In connecting school geometry and visualization, Andrade-Molina and Valero (2015) argue on the existence of a gap between the curricular aims of school geometry, in terms of the teaching of spatial abilities—"typically defined as spatial perception, visualization, and orientation" (Lee & Bednarz, 2012, p. 15)—, and a school geometry rooted in Euclid's axioms and Cartesian system. Skordoulis et al. (2009) claim that the reduced understanding of space in schools might prompt to students' misconceptions in school geometry.

Often, space in schools is taken as Cartesian because of its connections with the vector model: students "are placed in real three-dimensional situations [which provides] new tools to make spatial and flat depictions, such as the vector model" (MINEDUC, 2004, p. 68, own translation). To place students in real three-dimensional situations implies the visualization of an optically perceived world, which is different from the abstract reduction of school space into a Cartesian system. The concept of spatiality could be constructed from the interaction between concrete practices of human beings and of mathematical knowledge. Lefebvre

(1991) considers space as a social production. In his understanding, space is experienced in three forms: as physical form that is generated and used (Perceived); as an instrumental form of knowledge—savoir— and logic (Conceived); and as a form of knowing—connaissance—that is produced and modified over time and through its use (Lived). Following the express claims of MINEDUC (2010), school geometry pursues the connection of all three forms of space described by Lefebvre. The "real world" becomes the physical form of space, the tools and mathematical models become the instrumental form of knowledge and logic, and the context in which both are aimed to interact becomes the lived form of space. However, usually

> Students are presented with the properties of shapes and theorems for proof …all the information needed is given in the problem, and the students are asked to apply the theorems in what has to be proven. …The skills needed to solve these types of problems are limited, and teaching these skills usually consists of demonstrating the appropriate technique followed by a series of similar problems for practice. (Mevarech & Kramarski, 2014, p. 24)

This disconnection occurs even in initial years of schooling, in which Clements and Sarama (2011) have highlighted that the teaching of spatial abilities has been largely ignored in formal school settings. One possible explanation is that most geometric content, including skills such as visualization in school geometry, were removed from school as one of the consequences for the need of 'workers with particular skills' as an agenda of industrialization, urbanization and capitalism (Witheley, et al, 2015). Other explanations argue that students encounter difficulties "as a result of their spontaneous processes of visual perception in cases in which they contradict the geometric concepts/knowledge aimed at by the teacher and the tasks" (Gal & Linchevski, 2010, p. 180). But, within research on geometry education, there has been little discussion about assessment in school geometry, and on how national curriculums address visualization in assessment guidelines for teachers and standardized tests (see e.g. Sinclair, et al. 2016).

3. TEACHERS' GUIDELINES EXPLORATION

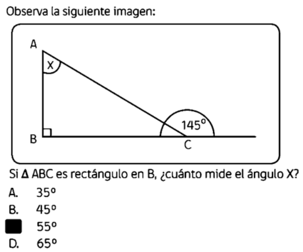
The Chilean Ministry of Education (MINEDUC) constantly releases guidelines for teachers to enhance their practices to improve the learning of school mathematics. There exist diverse guidelines for teachers from MINEDUC: School textbooks with instructions for teachers, maps of learning progress, curricular materials and programs, learning standards, and more. All of those resources have expressed recommendations about how teachers should evaluate and assess students' performances in the classroom. Here we want to explore the recommendations for the assessment of school geometry to explore how the guidelines for teachers advice to assess visualization and spatiality in the classroom. We take Lefebvre's understanding of space to search for expressions of spatiality and Arcavi's definition of visualization in the instructions for teachers and problem samples presented to teachers to assess school geometry. We present the analysis of the Learning Standard and Curricular program both from the eighth year of compulsory education.

The learning standards (MINEDUC, 2013) are aimed at teachers to guide them evaluating what students "should know and can do to reach, in national tests, appropriate levels of achievement according to the fixed learning objectives in the current curriculum" (MINEDUC, 2013, p. 4, own translation). It is thought as a mean to correlate what students have learned and the national syllabus (the school mathematics curriculum). It is aimed at helping teachers to determine what students need to learn and to monitor their progress (MINEDUC, 2013). It presents students' ideal answers, by classifying them within three categories of accomplishment. The categories compare students' performances in the Chilean assessment system for measuring the quality of education (SIMCE) with the respective performance in the classroom. The categorization is given to the teacher explicitly in the learning standards. The learning standards also enables teachers to anticipate the score students would obtain in SIMCE given that this information might be gathered by the teacher through formative or summative assessment. As specified by the learning standards, if the performance of a student is categorized as adequate, the outcome of the test should be more than 297 points; if their performance is categorized as elemental, the outcome should be more than 247 and less than 297; and, if the performance is categorized as insufficient, the outcome should be less than 247. SIMCE's scores are ranged from 0 to 400.

These are the standards for the eighth year of compulsory education (twelve years in total):

> **Learning level adequate (Fig. 1):** Students that are placed in this learning level in SIMCE demonstrate that they have accomplished the compulsory requirements to achieve this level, and also, that they have exceed those compulsory requirements. (MINEDUC, 2013, p. 14, own translation)
>
> **Learning level elemental (Fig. 2):** Students that are placed in this learning level in SIMCE demonstrate that they have accomplished the minimum compulsory requirements, and also, that they have exceed those requirements, but not enough to meet the requirements to reach the adequate learning level. (MINEDUC, 2013, p. 30, own translation)
>
> **Learning level insufficient (Fig. 3):** Students that are placed in this learning level in SIMCE do not meet the compulsory requirements to achieve the elemental learning level. This level gathers those students who are far from reaching the requirements, but, also, those that are

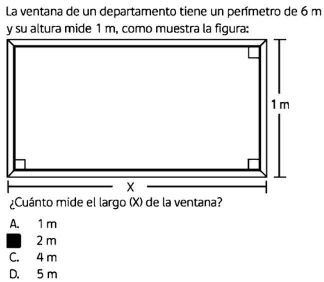
close of reaching the requirements. (MINEDUC, 2013, p. 38, own translation)



Observe the following image:

Students that achieved this level should solve this question, given that it is required to establish an appropriate process to solve it: to recognize supplementary angles, and to know that the sum of the interior angles of a triangle is 180 degrees.

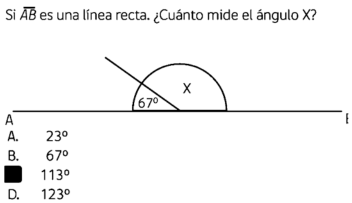
Figure 1. What is the measurement of x? (MINEDUC, 2013, p. 20, own translation)



An apartment's window has a perimeter of 6m and its height is 1m, as showed in the figure:

Students that achieved the "elemental" level should solve this question, given that the information given and the concepts required are explicit, and it is required to know how to calculate the perimeter of a rectangle given its graphic representation.

Figure 2. How long is x? (MINEDUC, 2013, p. 35, own translation)

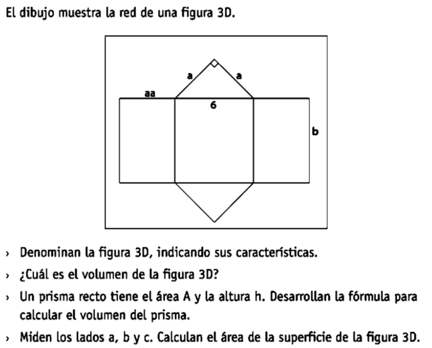


If $\overline{AB}$ is a straight.

Most of the students that are classified in this learning level "insufficient" are able to resolve this question, which implies to know the measurement of a straight angle.

Figure 3. What is the measurement of x? (MINEDUC, 2013, p. 39, own translation)

The curricular programs (MINEDUC, 2016) are aimed to guide teachers during the planning, coordination, evaluation, and so forth of each topic of the syllabus in mathematics. In these guidelines, teachers are presented with organized topics and recommendations on how to incorporate each unit of content into the classroom. It details the purpose of each topic, the mathematical knowledge required, abilities developed, attitudes that should be promoted, learning objectives, assessments indicators and examples of tasks for each topic.



The drawing shows a deconstructed 3D figure.

> *They denominate the 3D figure, indicating its characteristics.*

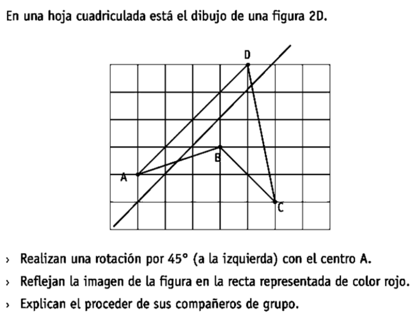
> *Which is the volume of the 3D figure?*

> *A straight prism has an area A, and a height H. They develop the formula for calculating the volume of the prism.*

> *They measure the sides a, b and c. They calculate the surface area of the 3D figure.*

Figure 4. The assessing of areas and volumes of prisms (MINEDUC, 2016, p. 140, own translation)

Teachers are guided to evaluate how students pose "intuitive estimation of the surface's area and volume" of the figure (MINEDUC, 2016, p. 140), along with the formula to calculate the area and volume of the figure's surface, its proper application and the result. MINEDUC highlights that this task is a geometrical problem of "daily life". In another task (Fig. 5), the instructions for teachers assert the prompting of spatial abilities, such as spatial imagination. Therefore, students should be encouraged to use "pictorial representations" of 2D figures, both concretely and mentally (MINEDUC, 2016). However, the considerations for assessment (translated in the image) are not directed towards visualizing, neither to spatiality. These are closer to the idea of a space inscribed in a Cartesian system, or to the use of techniques like counting squares (explored in more detail in Andrade-Molina and Valero, 2017).



In a squared paper there is the drawing of a 2D figure

> *They make a rotation by 45 degrees (to the left) with center A.*

> *They reflect the image of the figure on the red line*

> *They explain their procedure to the class group.*

Figure 5. The assessing of rotation skills (MINEDUC, 2016, p. 158, own translation)

4. THE ASSESING OF SCHOOL GEOMETRY

In these five examples above, visualization plays no role while solving each problem. Indeed, it is an ability that students might use, and so, it could appear in the classroom. But, in the expressed guidelines for teachers, visualization and spatiality are not explicit, and therefore not a necessary topic for evaluation. The instructions lead assessment to be positioned towards an axiomatization of space. As illustrated above, MINEDUC expresses the importance of visualizing in school because it helps linking the "real world"—the optically perceived world— and mathematics. For example by stating that students "use their spatial abilities… to visualize figures in 2D and 3D" (MINEDUC, 2016, p. 40, own translation). Visualization, as a form of reasoning, is reduced only to techniques of seeing and recognizing necessary information information.

Permite organizar los datos y visualizar características de una muestra.
Por ejemplo:

	HOMBRES	MUJERES	TOTAL
NATACIÓN	15	22	37
FÚTBOL	25	10	35
TOTAL	40	32	¿?

It allows the organization of data and the ***visualization*** *of the sample's characteristics.*

For example:

Figure 6. Visualization in guideline tasks (MINEDUC, 2016, p. 215, own translation, bolds added)

For example, on 8th grade, in the "adequate" level students are able to solve problems by using geometrical axioms, apply the properties of triangles and squares to measure the interior and exterior angles. In the guideline, teachers are advised to assess if students, for example, recognize supplementary angles and also the basic properties of a triangle, such as the sum of the interior angles adds 180o, while calculating the interior angle of a right triangle by knowing the measurement of the opposite exterior angle (fig. 1). On the other end, students are categorized as "insufficient" if they only are able to solve problems involving the measurement of a straight angle. Teachers are advised to ask students the measurement of an angle by knowing it supplementary angle (fig. 3). Students in this category cannot solve problems beyond this argument. But what if they use other types of techniques outside axioms and theorems? By only recognizing the results and not the procedures, school space turns into a mathematical space of savoir in which students should navigate on a Cartesian system by following Euclidean metrics. Therefore, it seems that students, from the curricular guidelines for teachers, are not confronted with events in which they develop tools outside mathematical axioms and theorems to navigate in space. In relation to Lefebvre's classification, the perceived space is not connected with the lived space. School geometry is taken only as a conceived space, which leaves the perceived space only reachable through axiomatic, and through reason and logic. Visualization takes the same connotation that depicting figures. And even though spatial thinking and reasoning are considered skills students should develop in early stages of schooling, there is no expressed guide on how to assess such skills.

> Students should learn to recognize, visualize and depict figures, and to describe the characteristics and properties of statics and dynamic 3D shapes and 2D figures. Concepts are given for students to understand the structure of space and to describe with precise language what they know of their environment. The early study of objects' movement—reflection, translation and rotation—seeks to develop students' spatial thinking and reasoning. (MINEDUC, 2012, p. 91, own translation)

PISA is another example on how spatial abilities are not considered as a school mathematics practice. As discussed elsewhere (Andrade-Molina & Valero, 2017), in the PISA sample of 2012 tasks involving the use of visualization only assign scores when the argumentation consists of using mathematics axioms (see e.g. "Twisted building" tasks on OECD, 2009), despite the attempts to support multiple solutions in international large-scale assessments (Schukajlow et al. 2015), On this particular task of "twisted buildings" relying only on visual information implies to "receive no credit at all if, additionally, [students] do not include information about the rotation center, the direction of the rotation and the angle" (Andrade-Molina & Valero, 2017, p. 260):

> Full credit: A correct drawing, meaning correct rotation point and anti-clockwise rotation. Accept angles from 40° to 50°. Partial credit: One of the rotation angle, the rotation point, or the rotation direction incorrect. No credit: Other responses and missing. (OECD, 2009, p. 184).

When it comes to assessment in school geometry, visualization seems to blur, prompting to what Andrade-Molina and Valero (2015) called the "sightless eyes of reason", in which the optically perceived world is transformed into a 'coordinate system world' inside the classroom. Students should use visualizing tools, according to MINEDUC, but in a space reduced to XYZ. As they contend, in MINEDUC activities involving visual–spatial skills the reduction of space leads to model reality only in terms of axiomatical deductions. Visualization becomes a skill that is not assess in schools, but is considered to be key in a discourse produced by the Ministry of Education in Chile.

> Students should comprehend the representations of coordinates in a Cartesian system and use their visual–spatial skills. In this process of learning, students should use diverse instruments to visualize figures in 2D and 3D and manual and IT tools are recommended. (MINEDUC, 2015, p. 100, own translation)

In the curricular guidelines analyzed in this exploratory study, geometry in school mathematics in Chile seeks for the development of skills on vectors from axiomatic methods. In this sense, students perform successfully if they are able to solve problems by using geometry axioms and theorems. As stated in the previous section, assessment in geometry from official guidelines promotes a particular type of ideal answers. In this fashion, it is possible to state that there is an assessment tradition in schools as a product of national standardized tests—SIMCE and PSU—in Chile. These types of assessments are designed to certify students' competencies and skills while solving problems in an optimum time, to test their knowledge and to grade students. MINEDUC, in order to achieve the desired score, guide teachers to use similar problems and styles of questions. Preiss (2010) describe the teaching and learning practices in school mathematics in Chile as a

> "[P]rivate appropriation of terms and procedures" because of the Chilean emphasis on individual work. The emphasis of Chilean teachers on practicing concurrent problems may indicate an overemphasis on skill drilling instead of mathematical understanding. (p. 350)

5. FINAL REMARKS

We are aware that there exists 'real life situations' in which visualization and spatiality are not enough to solve a problem, in contrast with the "twisted building" PISA task. For example, with objects that scape from what can be optically perceived.

> An engineer analyses oscilloscopes' graphics to determine the functioning of an electronic component. He "cannot see" directly the waves, its resistences and other properties of the device. Oscilloscopes' graphics become an instrument for the design and evaluation of electronic components. (Arrieta & Díaz, 2015, p. 34, our translation)

Here, visualization might become an instrument that depends on the visual reading of oscilloscopes' graphics. In these situations, other teaching and learning strategies can be grasp, such as an articulation on variational thinking (see e.g. Carrasco et al., 2014). Rather, the discussion is on school geometry aims explicitly expressed in curricular guidelines and their connections and disconnections with the official recommended modes of assessing geometry in the classroom. School geometry in Chile is aimed to be on optically perceived objects students are able to recognize (MINEDUC, 2012).

Recalling the main concerns: how assessment is ought to be occurred in the classroom according to teachers' official guidelines in Chile? Although existent research has been showing the importance of visual–spatial abilities and, also, skills for the sciences and for problem solving in the learning of school mathematics

(Sinclair et al., 2016), there is no certainty on how these skills should be assessed in school. According to MINEDUC (2004), students are placed in ‘real’ three-dimensional situations to develop spatial thinking, but there is no guidance for teachers to assess these skills in the classroom. From the materials analyzed, spatial abilities are not officially assessed in schools. In this regard, the analyzed materials show the importance of a spatiality constituted by a Cartesian coordinate system aim at the vector modeling. As students move forward in the levels of the map of learning progress, they learn how to navigate in space only in terms of XYZ, or on Lefebrve’s conceived space. And there are classified, according to the learning standards, for their expertise on geometry axioms and deductions. Assessment in school geometry does not only “promote” the learning of students but also insert them in practices to certify their competences and skills while solving problems in an optimum time; practices in which they aim to become the desired active learner by using reason and logic of a rigid ‘school space’ inscribed in Euclidean Metrics.

Moreover, from our analysis and considering Arcavi’s definition of visualization and Lefebre’s understanding of space, official guidelines for teachers do not help teachers to assess visualization. Apparently this is a task only for teachers to solve, even though visual–spatial abilities and skills are granted with such importance for school mathematics (see e.g. Bruce et al, 2015). But, again, as Presmeg (2014) highlights, visualization in the classroom depends directly on weather visualization is accepted, encouraged and valued by the teacher as a valid mathematical practice to solve problems.

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TALES OF EVALUATION IN SCHOOL MATHEMATICS

ABSTRACT

This is the tale of the power effects of evaluation in the shaping of the desired child. It maps, by using historical and cultural strategies, the circulating truths about standardized assessments, about the need for learning mathematics, and about the axiomatization of space. The analysis takes discourses around large-scale assessment and curricular guidelines to portray that the role granted to assessment for the normalization of mathematics education is intertwined with neoliberalism. Within these circulating discourses, assessment systems were taken as the path to achieve quality as a guiding system to help in decision-making processes. This paper is aimed at problematizing the need of higher proficiency in mathematics as a fixation of raising scores to achieve the illusion of producing the desired scientific self.

Once upon a time…

…in 1958, in a country named Chile there began to increase the necessity of a scientific and rational self for the economical development of the nation. Suddenly the goal and structures of education were rapidly modified to reach a 'new desired citizen'. The aim was to escape from the literary humanism and focusing instead in science and technology (Avalos, 1970; Nuñez Estrada, 1973; Vidal, 2010). And so, the journey began, a path from the spotlight of older days to the dreams of a brighter future. Voices were raised and decisions were made. In 1964, a big organization called UNESCO collaborated with the Inter-Union Commission on Science Teaching to sponsor the gathering of renowned scholars with the task of eloquently argue about science teaching and economic development of the nations across the land. These wise scholars launched the movement of "modern mathematics" that was widely discussed because of its implications to science. Oh, those wishes to possess the key to grasp even with one finger that vast and rich knowledge. Mathematics was thought as an effective tool significantly beneficial for the improvement of science teaching (Deschamps et al., 1970). And, of course, schools were aimed at striving the teaching of a mathematical language useful for the sciences (Misrachi & Aspée, 1967). Clear as water the answer emerged, a curriculum in which the teaching and learning of mathematics required as its basis the development of logical thinking (Michelow, 1969)… All for those beloved sciences! *Ash nazg* *durbatulûk, ash nazg gimbatul, ash nazg thrakatulûk, agh burzum-ishi krimpatul*[32]*... a* key, a tool, a ring to control them all.

But… that was only the beginning. "Who would possess that precious ring?" They argued. Should it be someone able to speak the *dark tongue*[33]? And so, quickly school mathematics was thought as a form of language. Mathematics was taken as a tool that enabled students to, logically, graphically, and symbolically, express their ideas or the 'objects of the mind'—*objetos de pensamiento*— as they called it (de Graf, Luque, & Mendoza, 1979b). As brush strokes, carefully placing the paint, stroking logically, displacing the best techniques to produce a masterpiece of the mind. The need of the dark tongue spread across all nations. Suddenly it became an official and universal language for them all. Here and there, people talked about how ideas can be voiced in other languages, being materialized by all mother tongues. But something moving through the wind started to sound. A whisper, an echo, was rapidly displaying across every country in their Kingdom, a sound voicing 'human capital'. Human capital, what was that? The *Council of the Wise*[34] told the story behind that whisper, encrypted vision. Human capital was taken as the "knowledge skills, and attributes embodied in individuals that facilitate the creation of personal, social and economic well-being" (OECD, 2001, p. 18). Knights in shining armour, all together working for the same cause, for all, for progress.

Not all of them were privileged enough to posses the '*precious*'[35], not all of them were able to speak or understand the dark tongue… They were knights without a sword! Ring bearers without a ring but trying desperately to reach they dreams with the promises of a brighter future. So they trained and they were trained, and they were assessed to become knights. At the blink of an eye, national assessment arrived to appraise abilities on mother and dark tongue. The unveiling of a tool able to rate the main elements considered for the fabrication

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[32] Inscription upon the One Ring that symbolises the power possessed by the Ring to control all other rings: *One Ring to rule them all. One ring to find them. One ring to bring them all. And in the darkness bind them.* J. R. R. Tolkien's "The lord of the rings".

[33] Also known as the *Black Speech*, the dark tongue of Mordor is the language Sauron created as the unifying and official language of Mordor. The inscription upon the One Ring is in Black Speech written in Elvish letters.

[34] The *Council of the Wise*—also known as the *White Council*—is a group of the wise wizards and elves of Middle Earth. Held in the Second Age to discuss on the power of Sauron, and in the Third Age with the purpose of uniting and directing the forces of the West, in resistance to the growing power in the Hill of Dark Sorcery—*Dol Guldur*.

[35] The One Ring.

of a competent and rational knight. Mother and dark tongue were taken as the key to achieve economic progress; on the one side, to test current changes, on the other, as a mean to set standards (Bravo, 2011). Mother tongue not only to learn how to eloquently enounce thoughts, neither to enjoy a pleasant story… but also, mother tongue to understand and comprehend mathematical instructions, an operational and technical mother tongue useful for the sciences (Ministerio de Educación & Centro de Perfeccionamiento Experimentación e Investigación Pedagógica [Ministerio de Educación & CPEIP], 1967). As consequence, Nazgûl[36] emerged in the form of standardized assessment systems, testing skills and abilities… chasing the One Ring, haunting the ring bearers, fighting knights.

In the beginning, Nazgûl were chosen as keepers of the rings of power with the aim to improve by helping them to make proper decisions. But the rings were used to obtain status, wealth, and to control everything, every move. As time progressed, Nazgûl were drawn and seduced by the promises of welfare, they were eclipsed by the power of the rings, they became corrupted, they became wraiths, they lost their original purpose… There was a time in which standardized assessment was thought as a mean of measuring abilities, as an objective and quantitative scientific method for decision-making. There was a time in which standardized assessment would provide useful and important information for policy makers. There was a time in which Nazgûl were kings… Kingdoms in the need of an army… Oh, those dreams of increasing knights in shining armour, of scientific and rational knights for the economical development of the nation. All in the name of progress!

As time passed, Nazgûl evolved and transformed, always haunting, always seeking for the *precious*. They changed to be in complete harmony with shifting eras. The trends regarding the teaching and learning of mathematics are in a constant *vinyasa*[37], always moving, always flowing, as water streaming through the river. Psychology was one of the flows, pouring its wisdom into school mathematics research: “ideas are not isolated in the psyche, these are to form mental structures”, they used to say (de Graf, Luque & Mendoza, 1979a; Ripamontiz & Prieto, 1992). Changes were made in order to fit the new standards. As a progression, school mathematics was taught according to the levels of psychological development, although it is impossible for new water not to mix, very nicely and gracefully, with the old river. Songs were written and sung thought the whole Kingdom.

[36] Sauron gave nine rings of power to nine Kings of Men. Those kings used the rings to achieve prestige, wealth and a great power. But they became corrupted by the power of Sauron. The rings left the Nine as wraiths, Ringwraiths, Black Riders… Nazgûl.

[37] *Vinyasa* is a Sanskrit term often use in Yoga. It has been taken to define a “sequential movement that interlinks postures to form a continuous flow” (Maehle, 2007)

To modern psychology ideas are
arranged as mental structures,
these cannot exist isolated in the psyche.
As the notion of "triangle"
should be linked to other concepts:
line, segment, angle, polygon, vertex...
Whoever learning these notions
should be equipped with previous ideas
and their symbols. Oral and written!
Hooray![38]

Mental structures aimed to be increased, as an enlargement of the mind, as an expansion of connections to collect wisdom: the sign of intellect. School mathematics, the dark tongue, was still thought as a form of developing the desired logical thinking the Kingdom thought knights needed, and this "need" was used to promote, within people, a desired to learn useful mathematics to overcome daily life challenges in a motivated and entertained fashion (Ripamontiz & Prieto, 1992): "logic is the basis of mathematics", they said (Walker, 1978a, p. 11). Oh, those promises of learning enough dark tongue to control the power of the One to obtain wealth and welfare! In the little country named Chile, people realized they were very far away from the Kingdom's desires. So close, yet so far away to be considered in the fight for Middle Earth, still invisible, still unevolved, still underdeveloped, still… too less. Logic was once taught by the use of tautologies and demonstrations, but everything evolves (figure 1). "God crated the Universe and the order that governs it. Man needs logic to understand it", they said (Díaz & Guidici, 1970, p. 1). Oh, that language of the universe carefully placed to reveal the mysteries of creation! "[Dark tongue entailing] the study of logic, [dark tongue needed] for making possible to reason correctly and rigorously in the search for truth" (Op. cit., p. I), for truth: enigmatic veracity. "Remember that the new philosophy of mathematics teaching requires telling the truth, nothing but the truth, but not necessarily the whole truth" (Michelow, in Walker, 1978b, p. 5).

38 Fragment's adaptation of de Graf, Luque, and Mendoza's (1979b, pp. 5-6): From the point of view of modern psychology ideas are arranged as mental structures, this means that new ideas cannot exist isolated in the psyche. As the concept of a triangle should be linked to others concepts of plane, line, segment, angle, vertex, triangular region, polygon, etc. The human being who is learning this geometrical notion should be equipped with a "grounding" of previous ideas and their corresponding system of oral and written symbols.

X	Y	X ∨ Y
V	V	V
V	F	V
F	V	V
F	F	F

Figure 1: Logic taught by using tautologies (Díaz & Guidici, 1970, p. 5)

The pursuing of logical thinking prompted by problem solving started: a quest. A quest for them to conquer success through reason! And, without hesitation, Nazgûl commenced testing ring bearers for them to be logical and rational, for them to battle for their ambitions and aspirations, enchanting them to succumb to the power of the One. Studies were made trying to understand which skills Nazgûl considered valuable to be assessed (National Research Council [NRC], 2001). International surveys—*TIMSS*—, national standardized tests—SIMCE—, even classroom evaluations—*evaluación de aula*—, seeking all places where Nazgûl become visible:

Habilidades	**Evaluación de aula**	**SIMCE 8º 2004**	**TIMSS 2003**
Manejar Conocimientos y Procedimientos.	67%	10%	15%
Usar Conceptos	7%	17%	20%
Resolver problemas de rutina	19%	55%	40%
Razonar	6%	17%	25%

Figure 2: Skills Nazgûl assessed39

They realized that, in this little country, they were far behind because they were chased by diverse standards. They were able to see how the skills (*habilidades*) considered valuable for some were not the same in all situations: classroom (*evaluación de aula*), national (SIMCE) and international (TIMSS) surveys. Oh, those Nazgûl targeting different type of abilities, Nazgûl with different styles of fighting, Nazgûl hunting and seeking! One black rider was steering 67% about procedures and knowledge performance (*manejar conocimientos y procedimientos*), the solving of problems was not worthy enough. One black rider was more interested in testing skills for problem solving (*resolver problemas de rutina*), 55%, and for the using of concepts (*usar conceptos*) and of

[39] Table that shows the comparison between the percentages of skills measured by three types of assessment. The table is taken from (Meckes, 2007, p. 364) paper.

reasoning (*razonar*). The last black rider aimed at problem solving and reasoning abilities instead. Black riders, ringwraiths... Ringwraiths haunting, ringwraiths chasing, but in diverse ways… mutating! However, black riders become more effective when unified, when bind in one: Nazgûl. And so, the Ministry of Education of this country developed a program in which classroom assessment aimed at bearing a resemblance to national and international standardized surveys. Oh, those desires of uniformity to be average and more, to be at the top, to possess the precious, to possess success! Nazgûl testing skills in dark tongue to overcome the challenges they pose, not to be fluent in the Dark Speech, but in being able to use the power of the Ring to outface them.

There was a time in which Nazgûl where kings of men. There was a time in which they use its sovereignty to measure some abilities of men to become knights. Memories of younger fantasies to assess "fundamental knowledge in mathematics, skills, numeric judgment and concepts usage"! (Ministerio de Educación & CPEIP, 1967, p. 21). But the past is so far in time, distant and blurry shadows in the dark… As the rings commenced to exercise their influence, the assessing of knowledge performance in mathematics began to transform. Suddenly it was not enough to learn the structure of the dark tongue. It was not enough by learning the basic mathematical knowledge needed to understand scientific advances and technology (Valdivia, 1975), the basic dark tongue to better understand the world and to control the power of the One. In 1975, teachers should lead students to discover the structure of mathematical knowledge, by themselves, because mathematics was believed to be qualitative (Ministerio de Educación Pública, 1975). So, the logical structure of the Black Speech ought to be pragmatically developed by young minds, as a blossoming intuition of mathematically axiomatized logic.

So many knights trying to reclaim their shining armour, so many knights training and agreeing to be trained in the hopes of a brighter future[40]. Measures were taken, decisions were made, standards were released, and knights were assessed. Learning standards that evaluated "what [they] should know and can do to display, in national tests, appropriate levels of achievement according to learning objectives set in the current curriculum" (MINEDUC, 2013, p. 4). Assessed knights, assessed ring bearers, in the need to know how qualified they are to continue their journeys, to carry the ring, to fight for Middle Earth!

[40] Mathematics is the "study of the principles of logic applied to a state of abstraction barely inferior to the one of the exact sciences [in which] school mathematics is intuitively supported and almost all students' ideas are susceptible to be graphically represented. [Students] should learn a language code to acquire the specific ideas" (de Graf, Luque & Mendoza, 1979a, p.5).

> *Oh, mathematics! "Abstract science par excellence, [you are] not only an instrument, but a knowledge structure and a form of thinking through which [you, treasured mathematics,] contributes to culture" (Ministerio de Educación, 1980, p.2). Oh, mathematics! Without you knights will be so intellectually uninvolved*[41]*. School mathematics, childish game, solving puzzles, connecting dots, answering riddles, making structures, discovering… Teaching to "analyze problems, to formulate questions, to organize information to obtain new evidence […] to solve problems" (MINEDUC, 1996, p. 101). Seeing not just answers but also evaluating performance, evolved knights, well trained, logical, and reasonable, evolved knights (Bamón, González, Soto-Andrade & Medina, 2001). Oh mathematics! Black speech that "contributes to the development of communication abilities [since you, cherished mathematics, help] to analyse and interpret charts, graphs and formulas, […] to register, to describe, to explain ideas, arguments, relations and procedures" (Bamón et al., 2001, p. 8).*

Gradually, challenges started to be trickier, questions started to mutate, to transform… Firstly by using tools to help them overcome each encounter they faced. Swords in the hand of knights! They were equipped and instructed with the basic techniques, the basic movements to face battles, to defend themselves. They were challenged and they challenge themselves. Before standardized tests emerged, before kings of men become Nazgûl, classroom assessment was concerned only with results, only with answers. So, they let knights to use weapons, to memorize formulas, there was only one solution, only one procedure, only one aim. Knights were armed with protractors to measure angles, and they learned how to use their armaments to solve queries, to unravel riddles, to develop practical skills (figure 3).

> *Measure the following angles by using a protractor and write each value in the corresponding letter* —They asked in 1962.

[41] Mathematics is acknowledged for contributing to the students' training—*formación del educando*—and their intellectual development (Ministerio de Educación, 1980, p.2).

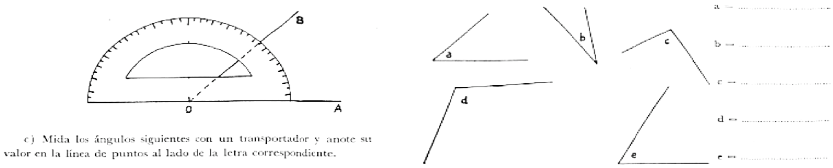


Figure 3: Measure of angles by using a protractor (de Graf & Marin, 1962, p.14)

School textbooks were design with the aspiration of promoting in these brave beings an active participation in the teaching and learning of mathematics (de Graf & Marin, 1962), in instructing and practicing the dark tongue. And so, queries became more abstract, less tangible. They were unarmed: Knights at the mercy of reason and logic! Suddenly, and without hesitation, kings were seduced by the measuring of abilities; they felt they were able to scientifically track progress to predict future mastering of the Black Speech (Ministerio de Educación & CPEIP, 1967): Oh, attractive quantifications! Standardized assessment emerged as poltergeists able to follow knights' steps, able to track national performances, able to nationally assess abilities, to measure quality, to measure progress, to measure all. Equipped only with the power of their minds, knights embarked in epic fights. New techniques were displayed. There were no protractors near by to be used. They had to calculate angles by using theorems and their own rationalities, subtracting. The images accompanying the tasks were now only a mere depiction to better understand the battled field, they did not have to be accurate anymore. These were solely representations of shadowy abstractions.

What is the measurement of beta if alpha is 60º30'? —They asked in 1967.

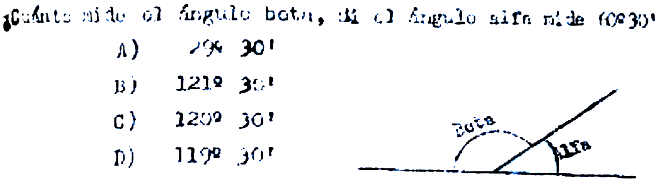
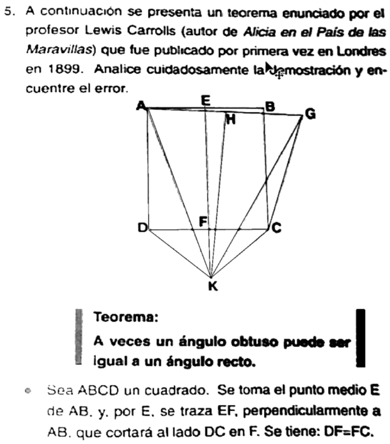


Figure 4: Suplementary angles' tasks (Ministerio de Educación & CPEIP, 1967, p. 8)

As time passed, aspirants to knights and ring bearers became more comfortable with these new techniques of fight. They became used to utilize only the power of their minds. As Jedi mind tricks, solely using the force. For kings, this was not enough. Something was missing even though knights left their swords aside. Riddles continue their journey, moving, shifting. Kings judged with their monitoring eyes (Campos-Martínez, Corbalán & Inzunza, 2015). The questions they asked to 14-year-old aspirants to knights in 1967 became less intriguing, less challenging. Kings, in the attempt of improving their armies, wondered: how were knights taking decisions? How were ring bearers analysing situations? In classrooms, teachers instructed young Padawans towards the virtues of solving problems, of managing strategies, of taking their own decisions instead of waiting patiently for instructions. Of course there was still one option to be taken as correct, one possibility to achieve the best possible results to win the disputes, but now… Now many paths were available, so many possible ways of mastering and using the Dark tongue, as three branches, opening, splitting towards sunlight.

> *The following is a theorem by Lewis Carrolls (Alice in Wonderland's author) that was published for the first time in London in 1899. Carefully analyse its demonstration and find the mistake. [Theorem: Sometimes an obtuse angle could be equal to a right angle]* —They asked in 1996.


5. A continuación se presenta un teorema enunciado por el profesor Lewis Carrolls (autor de *Alicia en el País de las Maravillas*) que fue publicado por primera vez en Londres en 1899. Analice cuidadosamente la demostración y encuentre el error.

Teorema:
A veces un ángulo obtuso puede ser igual a un ángulo recto.

- Sea ABCD un cuadrado. Se toma el punto medio E de AB, y, por E, se traza EF, perpendicularmente a AB, que cortará al lado DC en F. Se tiene: DF=FC.

Figure 5: Task that required the use of problem solving strategies (Ministerio de Educación, 1996, p. 107)

Oh that desired of displaying a multiplicity of options for knights to learn how to make proper decisions to conquer success. Some tasks, not all of them, were about making demonstrations or carefully analysing the old strategies to find mistakes, probably to improve, probably for them not to make the same errors, probably for some other unexplained and mysterious reason. And although the tactics to approach the riddles, standardized assessment remained with the same structure, with the same format, options to take, decisions to make. But its focus was budged, problem solving was all they saw. 14-year-old aspirants to knights were asked to play with the knowledge they acquired, to develop strategies… as detectives solving mysteries, as Bard the Bowman[42], posing mathematically logical and reasonable tactics to kill Smaug[43], measuaring the exact path of the Black Arrow to penetrate Smaug's armour. Or as Gandalf the Grey opening the Doors of During "*Ennyn Durin Aran Moria. Pedo mellon a Minno. Im Narvi hain echant. Celebrimbor o Eregion tethant. I thiw hin*"[44]… *"Mellon"*[45].

> *Observe the following figure formed by three equilateral triangles. What is the measurement of the marked angle?* —They asked in 2005.

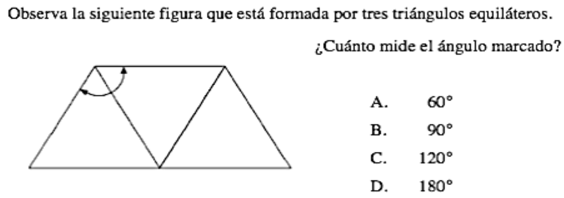


Figure 6: Calculation of angles by using properties of triangles (Ministerio de Educación, 2005, p. 11)

Aspirants to knights, ring bearers calculating exact measures by remembering the properties of equilateral triangles. Oh, trickier and trickier queries for them to solve. But kings began wondering if it was possible to predict the results of the battles. Students kept answering problems, knights solving riddles, ring bearers making decisions but… kings remained thinking about other forms of materializing the future, a reliable oracle. And so, the "learning standards" were released. Oh dear canons for classifying knights to predict their future performance!

> *Observe the following image. If ABC is a right triangle in B, what is the measurement of x?* — They asked in 2013.

[42] Bard the Bowman, also known as the Drangonslayer, was the man destined to kill Smaug with the Black Arrow. Bard then became the founder and first king of the Kingdom of Dale.

[43] Smaug the Golden, the Impenetrable, was a dragon of the Third Age. Smaug was a fire drake known as the King under the mountain.

[44] Inscription on the archivlot of the Doors of Durin, the Westgate of Moria, only visible in starlight and moonlight. *"The Doors of Durin, Lord of Moria. Speak friend and enter. Narvi made them. Celebrimbor of Hollin drew these signs".* The Fellowship of the Ring.

[45] "Mellon" in the word for friend in Sindarin. This word was the answer of the riddle inscription on the Westgate of Moria, and password to opening the doors.

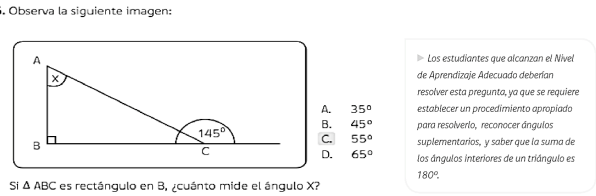


Figure 7: Query on the learning standards (Ministerio de Educación, 2013, p. 22)

And all began to crumble. The performances of knights started being labeled as adequate, elemental or insufficient according to the standards (Ministerio de Educación, 2013), according to the desired skills kings wanted to prompt in schooling. Riddles were not only to assess their expertise on the dark tongue, but also for evaluating and predict their future actions, a future not only regarding to their encounters with Nazgûl, but beyond. Problems shifting, turning as a gear system, carefully arranged, carefully displayed. The options that ring bearers and aspirants to knights decided to take (a, b, c, or d), were the path to the oracle to prophesied their whole life. Calculating the measurement of x was not naïve anymore. They needed to learned new techniques; they needed to compete with themselves to not be left out of the battles, to be considered as productive knights. Riddles of the past, of 1967 (figure 4), were now considered to be insufficient by the new learning standards (Ministerio de Educación, 2013).

> *If $\overline{AB}$ is a straight line, what is the measurement of x?*—They asked in 2013.

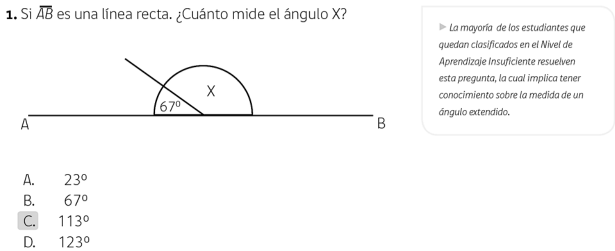


Figure 8: Query on the learning standards (Ministerio de Educación, 2013, p. 39)

And young 14-year-old aspirants to knights are now asked to solve problems that were asked in 1978 to 18-year-olds (Walker, 1978b). Oh provoking levels! Oh desired to be above average to score higher internationally. Oh attractive dreams of progress and economical development!

> *Standardized assessment system… useful instrument, reliable system to guide decisions (Campos-Martínez, et al., 2015). Standardized assessment system… to monitoring and inform (Meckes & Carrasco, 2010). Standardized assessment system… for attending necessities to achieve quality (Olivares, 1996). Standardized assessment system… for efficiency to increase achievement (Carnoy & de Moura Castro, 1996), to allocate resources (Lockheed & Hanushek, 1988). Nazgûl pursuing decisions for economic growth, decisions for being less underdeveloped, decisions for welfare! (OECD, 2014).*

There was a time Nazgûl were king of men but the rings of power Sauron gave them started to corrupt them even more. Men became dependent of the rings... The rings of power, resulting outcomes of the tests, dammed kings until they become Nazgûl, until they become wraiths… And so, the rings were all they perceived… Numbers were all they glimpsed. Quantified 'reliable' readings of the Chilean educational system. Rings of power to determine what should be improved for achieving greater quality. Dark tongue, rings of power, and Nazgûl all together subjectifying knights by using techniques to regulate their habits and desires[46]. Oh neoliberal dreams[47]! Charming governmentality effectuated through the interest of knights, of governed knights[48]. Desired greatness, the need of using the rings of power to achieve greater quality. Kings were no longer men, Kings of men were always *mortal men doomed to die*. Oh effects of power for the fabrication of the Lord of the Rings: self-regulated self-entrepreneurs. Kings adding test samples in each students' school textbook, in each guidelines for teacher, in most materials dedicated to the teaching and learning of school

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[46] Governmentality, in terms of Foucault, is aimed to generate cultural and historical subjects by using techniques to regulate the habits and desires of subjects. It is aimed for "arranging things so that people, following only their own self interest, will do as they ought" (Scott, 1995, p. 202).

[47] Here, neoliberalism is understod "through audit techniques, quality management, financial standardization, participative management and private property ideologies, managers aim to transform the employees in 'self-entrepreneurs', individuals that self-regulate, self-direct and are continuously in a process of redefining their competences" (Cotoi, 2011, p. 116).

[48] According to Cotoi (2011), governmentality is effectuated through the interest and values things get, "people are governed by and through their own interests" (p. 113), and, therefore, a neoliberal governmentality governs "by giving the impression that it is not governing" (p. 114).

mathematics produced by the Chilean Ministry of Education.: pages and pages consisting of problems mimicking the one of large-scale assessment. But knights are not forced to answer the problems displayed in their guides, teachers are not forced to ask students the tasks displayed on the teachers' guidelines, but… Oh these technologies of government! In order to be successful probably they will choose to engage in those practices, solving the samples so not so innocently placed carefully in each text… all for just one number, only for progress.

Bilbo Baggins did not learn how to speak the dark tongue, but he was not intended to control the ring of power, he was neither a mathematician nor a scientist. He had a great and long life, thanks to the power that the ring gave him. He was just a hobbit from The Shire, but he learned to handle the ring to his own convenience and ambitions, he became corrupted by its power. Bilbo became a knight in shining armour. He knew he needed the ring to be successful, he could feel it in every nerve of his body. He became dependent of the One… *One ring to rule them all, one ring to find them, one ring to bring them all, and in the darkness bind them. In the Land of Mordor where the Shadows lie…*

The end

Guiding for the tales of evaluation

The One Ring:	Is the Ring of Power able to control all other rings
	The believe of needing science for economic progress and welfare
Dark tongue:	Language created by Sauron as the official of Mordor
	Mathematics language needed for the sciences / School mathematics
Nazgûl/Black riders:	Nine kings of men dammed by Sauron
	Standardized assessment system in mathematics (SIMCE/PISA)
The rings of power:	Nineteen rings to control and corrupt every race
	Quantification / Numbers / "Scientific Methods
Ring bearer/ Knights	**Students / Citizen**
Lord of the Rings:	**Desired scientific self**

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INCEPTED NEOLIBERAL DREAMS IN SCHOOL MATHEMATICS AND THE 'CHILEAN EXPERIENCE'

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This work aims at portraying a rhizome of circulating naturalized truths about who citizens should be and how they should act within neoliberal governmentality. It does this by a historization of an incepted belief entangled in diverse social spheres. It unfolds how the ideas of human capital and welfare become the top right on mathematics education. The 'Chilean experience' is used as an example to construct a rhizomatic historization of events, strategies and technics of government that enabled to incept the neoliberal dreams into school mathematics.

Keywords: Governmentality, neoliberalism, historization, rhizome, school mathematics.

INTRODUCTION

> *An idea. Resilient... Highly contagious. Once an idea has taken hold of the brain it's almost impossible to eradicate. An idea that is fully formed—fully understood—that sticks; right in there, somewhere (Dominic Cobb, Inception).*

It is intriguing how highly perceived the Chilean models are—economy, education or health systems—to other countries. According to Taylor (2003), Chilean systems have been taken as models 'worthy of emulation'. Is Chile doing something marvellous? The country has been seen as an example of organization and 'proper' policies for economic progress and welfare (Silva, 1993). Its policies are considered as trendsetters among privatized pension systems (see Mesa-Lago, 2012), among health care reforms (see Bruce, 2000), and it was one of the first countries implementing neoliberalism as a framework in education (Aravena & Quiroga, 2016). The results in PISA, particularly in mathematics literacy, have progressively increased over the years—2000 (384); 2006 (411); 2009 (421); 2012 (423). And so, Chile has risen to be seen as one of the most developed countries in Latin America (Gregorutti, Espinoza, González, & Loyola, 2016). Chile is considered, by the World Bank's annual reports on development, the proven example of the benefits embedded in 'conforming' to a neoliberal approach to social policy.

> [Chile] is often viewed as a trendsetter in introducing fundamental and far-reaching neoliberal reforms […] the Chilean example as been heralded as proof of the success to be gained from an uncompromising commitment to neoliberal policy prescription (Taylor, 2003, pp. 21-22)

But… it is not all sunshine and roses! By building on Foucault's work, this paper aims at portraying how neoliberal discourses about mathematics education have been (re)produced and how they have circulated amongst diverse spheres of human interaction, (re)shaping citizen ways of being and acting in the world. It does this by taking "a critical attitude towards those things that are given to our present experience as if they were timeless, natural, unquestionable" (Rose, 1999, p. 20). This paper deploys a historization of the present of entangled historical events, strategies and techniques that made possible to incept the neoliberalism into school mathematics in Chile. This narration is not a critique about the implementation of educational policies in Chile; rather it is the tracing of naturalized truths in mathematics education as an assemblage of diverse governmentality techniques (Foucault, 1991). These naturalized truths are traced in five moments. First, regarding the introduction of neoliberalism as a set of political movements. Second, regarding neoliberalism as a system of reason for economic improvement. Third, regarding the specific type of citizen that the new economy requires, a consumer of goods. Fourth, regarding the productive subject of schooling for the market, a competitive subject. And finally, regarding how school mathematics becomes the vehicle to shape the desired subject for economic growth.

The plot of the movie "Inception" inspires the style of writing of this paper. In this movie a series of dreams are unfolded. Each dream should be understood as a new and deeper dream occurring inside the previous one. The dreams do not follow a chronological arrangement. It is not a lineal story; it is a rhizomatic construction. A rhizomatic network allows a non-hierarchical multiplicity of entryways, of dimensions, lines; it has no beginning or end, but always a middle: "The rhizome is altogether different, a map and not a tracing […] The map is open and connectable in all of its dimensions; it is detachable, reversible, susceptible to constant modification" (Deleuze & Guattari, 1987, p. 12). Hence, all dreams are connected not as a sequence of events, but as continuities and discontinuities. All narrations are entangled, even in different times, even in different spaces, and even in different voices. The paper is written in this form in alignment with Foucault's rejection of causality.

> We consider the understanding of the way one event succeeds another as a specifically historical issue, and yet we do not consider as an historical issue one which in fact equally so: understanding how two events can be contemporaneous […] History is quite frequently considered as the privileged site of causality […] But we have to rid ourselves of the prejudice that history without causality would no longer be history. (Foucault, 1999, p. 92)

FIRST DREAM: THE COLD WAR AND THE NEOLIBERAL REVOLUTION

It is the late 60s, in a country apparently far from the War, but close enough to be in the spotlight. There is the danger of it becoming the first socialist nation in South America, and this is threatening for the US. Silent voices were saying: "under no circumstances should Allende be elected!" But, he was… Salvador Allende became the first democratically elected socialist president in the Western hemisphere. What a revolutionary! Fighting for the people! Chile has begun to increase its role in the provision of social services.

> By subsidising the reproduction of the labour force through allocating resources to the development of state systems of health, education, housing, staple-food subsidies and social insurance, universalistic social policies tended to reinforce the purchasing power of wages thereby expanding domestic markets for industrial goods. (Taylor, 2003, p. 23)

Something is starting to go extremely wrong in Chile. Suddenly there commenced a crisis that led to most of the population clambering for improvement. The, so-called, socialist experiment "united capitalists, landowners, the middle classes, and their political party allies against labor, peasants, and leftist part" (Silva, 1993, p. 535). Apparently the US government, also pressuring the World Bank and the Inter-American Development Bank to do the same, minimized the aid they provided to Chile… And so, Chilean foreign reserves plunged from $400 to $13 million in one year (Moreno, 2008). Discontent people wanting the president out are growing in number. It is socialism versus capitalism… "$7 million channelled to anti-Allende groups", according to a report of the US senate (Moreno, 2008, p. 93). And he was overthrown on September 11th 1973, by the military force commanded by General Pinochet. Now, neoliberal ideas are being forced into Chilean minds that are afraid all the time, afraid for their lives, afraid to raise their voices. Meanwhile, those in favour of the new regime are enjoying the pleasures of the new order (Salazar, 2003).

SECOND DREAM: THE CHILEAN EXPERIMENT, FRIEDMAN AND THE SCHOOL OF CHICAGO

The year is 1950; the place, Chicago. Milton Friedman is developing a new approach to economy theory. This new theory is in opposition to socially conscious economies, which have been prominent in Western governments after 1929. Friedman believes that "economic benefit could best be optimized if the individual has the autonomy to pursue his or her own self-interest" (Moreno, 2008, p. 92). This new theory was the hope for a group of technocrats that moved to Chicago, the "Chicago Boys" (García & Wells, 1983). In the 70s, Pinochet decides to leave the economical management of Chile on the hands and knowledge of the Chicago Boys. This is going to be the first time that a group of Friedman has "an opportunity to influence governmental policy and put their theories into practices […] They already have a complete programme aiming to re-structure the economy and to reverse Allende's social reforms" (Moreno, 2008, p. 94). The military regime and the Chicago boys established neoliberal economic and social policies here (Salazar, 2003). "[T]he market supplanted state intervention in the economy, except in labor relation" (Silva, 1993, p. 527).

Within the first six years of dictatorship, the 'shock therapy' was the only approach to curb social policy and state expenditure (Huber, 1996). Chilean reform "has been led by both the advocates of monetarism, located principally in US institutions and universities, and by the Chilean reformers themselves" (Taylor, 2003, p. 22). Neoliberal ideas were taken as a sort of 'second independence' and, also, an entrance to the first world of developed countries (Salazar, Mancilla, & Durán, 2014).

THIRD DREAM: CONSUMERISM AS THE EVER-GROWING ECONOMY

Here, in this place of earth, everything could be marketed, everything could be sold, and most people would feel the urge to buy it. Health and education are, by constitution, social rights to every citizen. But here, those basic social rights fade into consumer goods. Public and private enterprises competing with each other, providing services for customers willing to pay for them, after all it is their choice (Taylor, 2003). Parents have the opportunity to choose freely the type of school—municipal, subsidized private or fee-paying private schools—and the type of education they want for their children (Mizala & Romaguera, 2000). Free choice… if they can afford it!

Public against private institutions… In a place where private institutions have the right to charge in excess to ensure better and better quality. Private schools enjoy, without any guilt, "having greater resources, enabling a stronger quality of education to be taught, and thereby reinforcing the desire of parents with available income to send their children to such schools" (Taylor, 2003, p. 34). After all, the more you pay the better you get; the less you pay the worst you obtained. In a time and place where education policies are transformed into economic policies of education (Castiglioni, 2001).

FOURTH DREAM: COMPETITIVENESS IN SCHOOLS, EDUCATION AND FREEDOM OF CHOICE

After the introduction of 'welfare' as a method to increase efficiency, "the element of competition and the response of enterprises to public desires as indicated by market forces were suggested to create an optimal allocation of resources throughout welfare provision" (Taylor, 2003, p. 26). The reform of the 80s, under the military regime, changed Chilean education system. Decentralization was key to encourage private providers to enter the market (Mizala & Romaguera, 2000). And there was more, so much more than that. This reform involved a reformulation of the interplay between state and schools, a voucher system that indirectly funded schools by assigning the resources to students (Parry, 1997). This measurement left schools receiving financial aid depending "on the number of students that they could attract […]. If schools were unable to compete in this new marketplace environment, they would be allowed to fail and face dissolution" (Taylor, 2003, p. 33). A highly competitive system generated by an educational market and by the policies aiming at improving the quality of education (Mizala & Romaguera, 2000) was shaped. And so, state accountability systems were able to reward and/or punish schools by allocating resources regarding the performance of each school (Elacqua, Martínez, Sontos, & Urbina, 2012). A system in which, schools, teachers, students are constantly competing and being assessed.

FIFTH DREAM: THE SKY IS THE LIMIT! MATHEMATICS TO THE PEOPLE

Welfare and mathematics, always hand by hand. Here, mathematics has been granted with a great importance and status. In the 60s, logic was taken as the foundation of every science, reasoning accurately and rigorously was the core of any argumentation and of critical thinking (Diaz & Giudici, 1970). Mathematics was the one that helped to develop reasoning and logical thinking and reading proficiency was thought as a tool to better understand mathematical instructions (Ministerio de Educación & CPEIP, 1967). In the 80s, the military regime reformed the curriculum and school textbooks to reflect the regime's doctrine: "education was recast to promote studies functional to the new productive structures of Chilean society, whereas traditional arts and humanities studies were discouraged" (Taylor, 2003, p. 32). It was indispensable to embody in individuals certain knowledge skills—mathematical knowledge—, and attributes to facilitate the creation of personal, social and economic well-being (OECD, 2001). Economic growth was about human capital.

National assessment started to be taken as the key to achieve economic progress, on the one side, to test current policy changes, on the other, as a mean to set standards. And so, competitiveness and accountability, within school mathematics testing, led to higher performances, higher incomes, higher social mobility and welfare (OECD, 2014). Nowadays, by knowing students' numeracy proficiency in PISA it is possible to predict, amongst many others, their likelihood of being employed or to calculate how different their hourly earnings would be (OECD, 2015). And so, the promised state of welfare is side by side with mathematics proficiency. Mathematics is now the key for a brighter future, all students have to do is to be good at math and the sky will be their only limit! ***[End of dream 5]***

The standardized test SIMCE has been a key element to promote competitiveness and pressure to the system. Since its results are publicly published, it becomes an objective indicator to assess school performances (Mizala & Romaguera, 2000, p. 393). It also enables parents, as consumers, to demand better services for their children (Meckes & Carrasco, 2010), for students to be successful and entrepreneurs. ***[End of dream 4]***

School mathematics is now an investment! Reforms have shaped education into a capitalist marketplace, by promoting entrepreneurial profit-minded investment and by remodeling education to consolidate the productive structures of economy (Taylor, 2003). ***[End of dream 3]***

And the so-called 'economic miracle', product of the economic growth in the late 70s, helped raising the prestige of neoliberalism "under the banner of 'the Chilean model'" (Taylor, 2003, p. 25). By now, Chile has become famous for its neoliberal restructuring followed under General Pinochet (Silva, 1993; Aravena & Quiroga, 2016). ***[End of dream 2]***

This is it! Chile is no longer an underdeveloped country (Salazar et al., 2014). Chile is now part of the first world, the "tiger" of Latin America (Teichman, 2016). ***[End of dream 1]***

INCEPTED NEOLIBERALISM

You create the world of the dream. You bring the subject into that dream and they fill it with their subconscious (Dominic Cobb, Inception)

From a Foucaultian perspective, conduct is governed through diverse techniques, strategies, and devices (Foucault, 1991), within a space of government that "is always shaped and intersected by other discourses" (Rose, 1999, p. 22). In doing so, each individual conducts him/herself by (re)shaping his/her own modes of being and acting in a space of 'regulated freedom' and under a promised state of welfare. In this sense, "people are governed by and through their own interests" (Cotoi, 2011, p. 113). This is precisely the idea behind the 'inception' of a neoliberal mentality. A set of naturalized truths circulating amongst diverse times and places, knitting a web to govern the self and to regulate habits and desires of cultural and historical subjects through school mathematics. These discourses help governing productive citizens, in the sense that intend to insert subjects in regulatory practices that (re)shape their conduct "without interdicting their formal freedom to conduct their lives as they see fit" (Rose, 1999, p. 23). Reforms, according to Dussel (2003, p. 94), "have to be understood as part of government technologies that intend to shape the way people are to act, think, and feel about the world, that combine the old and the new in unique ways".

One possible narrative to understand the success of neoliberalism in Chile could be grasped through the articulation of certain discourses about consumerism and competiveness. SIMCE in mathematics, for example, became the first step of knowledge consumerism and of a marketable education/society. SIMCE's results are publically published in national newspapers and widely discussed through other means of public communication, so parents and society could judge schools by their performance in standardized tests. 'Judge' in the sense of deciding which school is the best option for their children's future. This marketing of schools and teachers leads to the most utopian non-sense practices. For example, within the belief that welfare is only achieved by a high quality education, parents, in order for their children to be enrolled in those schools with "higher quality"—with good scores in national tests—are willing to stay all night in line, outside a school, to submit the admission application. Figure 1 shows a Chilean newspaper, *Las Últimas Noticias,* reporting the news: "Parents slept on the street under -3,6º Celsius because of enrolment. They are trying to enroll their children in pre-school for next year in Santa María School in Osorno". One of the parents, who waited in line for 12 hours said: "It is demeaning but what else can we do. This school is good and affordable. I have three kids and they all need to study".

32 **EL DÍA** — Viernes 2 de septiembre de 2016 / **Las Últimas Noticias**

"Es denigrante pero qué le vamos a hacer. Es bueno y está al alcance del bolsillo", dice Camilo Martínez.

Postulan a sus hijos al prekínder del próximo año en el Colegio Santa Marta de Osorno

Apoderados durmieron en la calle con 3,6° grados bajo cero por una matrícula

VALENTINA ESPEJO D.

No importó que la temperatura en Osorno, a las 7:07 de la mañana de este jueves, haya llegado a los -3,6 grados Celsius.

A esa hora, la vereda de la Avenida Manuel Antonio Matta, en la zona central de esa ciudad de la Décima Región, estaba copada por una larga fila de personas.

Algunos arropados con frazadas y otros tomando café o té en un termo. Pero todos esperaban lo mismo: que el Colegio Santa Marta abriera sus puertas para dar inicio al proceso de admisión 2017 de los prekínder.

Incluso algunos padres llegaron al colegio la noche del miércoles y se amanecieron frente al recinto.

Al parecer, sabían que no sería sencillo: había 188 interesados en una matrícula para 2017 y sólo 70 cupos.

"Es denigrante pero qué le vamos a hacer. Es bueno y está al alcance del bolsillo. Yo tengo tres hijos y todos tienen que estudiar", cuenta Camilo Martínez, uno de los apoderados que esperó en la fila durante 12 horas. El pago mensual en el Colegio Santa Marta es de $36.300.

Carolina Negrón, que también hizo la fila, dice que el colegio es barato en comparación a otros de la zona. "Este jueves hacía mucho frío. Había papás que se amanecieron con braseros y frazadas. Justo tocó una de las noches más heladas de las últimas semanas. Pero la recompensa es una educación de calidad", asegura.

A las 8:30 de la mañana se abrieron las puertas del recinto. Ahí, a los 188 interesados sólo se les pasó un número de espera para inscribirse en la postulación. Fue el primero de varios pasos a seguir para lograr el ansiado cupo. Además de la gran demanda, los postulantes tienen que lidiar con los criterios de prioridad que tiene el colegio para asignar las matrículas nuevas: en primer lugar, los hermanos de estudiantes actuales; luego, los hijos del personal del establecimiento e hijos de ex alumnos.

Desde el recinto informaron que, como nunca, vino mucha gente a inscribirse. "Sí se espera que la gente esté un rato antes de la apertura, pero no que se amanezcan", dijo Claudia Antías, asistente de educación del colegio.

El próximo 15 de septiembre, el colegio dará a conocer la nómina de los afortunados alumnos seleccionados.

Los apoderados aguardaron toda la noche.

PAULO DÍAZ/BIOBIOCHILE.CL

Figure 1: News about parents staying all night outside a school

These discourses are not isolated from the ones of school mathematics. The importance that mathematic literacy has within OECD's indicators help move research to be about how to improve students' performances in mathematics. For example, in order to be successful in SIMCE, students should not be allowed to miss classes. If students are "absent 9 days during the school year (the sample average of absences) reduced performance by at least 23% of the standard deviation of the score on the SIMCE mathematics test" (Paredes & Ugarte, 2011, p. 199). Welfare can also be measured in relation to students' performances in national tests, by correlating SIMCE scores in mathematics to predict a student's future income (Bharadwaj. Giorgi, Hansen, & Neilson, 2012).

The Chilean Ministry of Education released the "Learning standards" in school mathematics to help teachers evaluate "what students should know and are able to do for displaying, in national tests, appropriate levels of achievement" (MINEDUC, 2013, p. 4, my translation). These learning standards categorized students in three levels of achievement that, at the same time, predict their future outcome in SIMCE in mathematics. So, if students do not want to be label at the lowest level, they have to engage in regulatory school practices, they have to compete with their classmates and with themselves. In this fashion, SIMCE in mathematics also operates as a technique to generate 'self-entrepreneurs', "individuals that self-regulate, self-direct and are continuously in a process of redefining their competences" (Cotoi, 2011, p. 116).

As portrayed within the dreams, Chilean neoliberalism have been (re)producing discourses that circulate within diverse time and places in order to obtain economic growth, progress and welfare through school mathematics. School mathematics, since it was thought to shape productive citizens, was taken as the key for Chile to become a developed country. Mathematics needed to be a good that people wanted and were willing to consume. With marketable school mathematics, whomever wanting to achieve welfare would have to pay for higher quality. And, therefore, Chilean economy should increase. This would not have occurred without the dictatorship and Friedman thoughts: Economy would be best optimized if people have the freedom to pursue their own self-interest.

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BE THE BEST VERSION OF YOUR SELF! OECD'S PROMISES OF WELFARE THROUGH SCHOOL MATHEMATICS

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This work aims at tracing, from a Foucaultian perspective, the taken-for-granted truths, the promises, in OECD´s discourse on a state of welfare. These promises are built under the assumption that mathematics skills are needed to achieve citizens full potential. OECD's reading of 2012 PISA's outcomes reveals how discourses of numeracy proficiency are entangled and displayed for the making of a specific type of productive citizen for society. Also, this work problematizes OCDE's expressed 'desired citizen', by building in the double gestures of hopes and fears produced by the intention of include "all" student and of equity.

INTRODUCTION

There exist some circulating discourses about what is important for citizens 'to know' and 'be able to do.' OCDE has presented within the several documents released an approach which "reflects the fact that modern economies reward individuals not for what they know, but for what they can do with what they know" (OECD, 2014, p. 24). Mathematics has been taken, by OCDE, not as a knowledge students should acquired, but as a necessary skill –proficiency– for personal fulfilment, for future employment, and also, for a full participation in society (OECD, 2014).

> With mathematics as its primary focus, the PISA 2012 assessment measured 15-year-olds' capacity to reason mathematically and use mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena, and to make the well-founded judgements and decisions needed by constructive, engaged and reflective citizens. Literacy in mathematics defined this way is not an attribute that an individual has or does not have; rather, it is a skill that can be acquired and used, to a greater or lesser extent, throughout a lifetime (OECD, 2014, p. 17).

Several studies in the field of mathematics education have already problematized the assumption of "mathematics for all" (see, e.g., Diaz, 2013; Valero, 2013). Nowadays, there is a new 'truth' added to the one of "mathematics for all". The taken-for-granted truth that has been currently (re)produced is the need for "inclusive and equitable quality education and promote lifelong learning opportunities for all" (OECD, 2016a, p. 13), as part of the sustainable development goals for education. This result ought to be achieved by a series of indicators that "spell out what countries **need** to deliver by 2030" (Op. cit., p. 13, emphasis added), with the aim of promoting social progress. Within these discourses, numeracy –as the ability to "access, use, interpret and communicate mathematical information and ideas in order to engage in and manage the mathematical demands of a range of situations in adult life" (OECD, 2016a, p. 38) –becomes a key skill to achieve social progress and, also, to ensure welfare.

This work aims at tracing the naturalized truths or the promises (re) produced by OECD's discourse toward a state of welfare and social progress. These promises are built under the assumption that the 'proper' acquisition of mathematics skills –numeracy– is needed for citizens to achieve their full potential, making them able to excel and, therefore, to have better lives. OECD's reading of 2012 PISA's outcomes takes an important role, given that it reveals how discourses about numeracy proficiency have been entangled and displayed for the making of a productive citizen for society. Finally, this work, by portraying the desired subject, problematizes the undesired citizen through 'abjection', as the double gestures of hopes and fears produced by the intention of including 'all' students and of equity.

OECD'S PROMISES OF WELFARE

Over the years, OECD has been positioning itself as the main global network for the (re)production of policy discourses toward economical progress. Tröhler, Meyer, Labaree, and Hutt (2014) recognize OECD as a central node for local and peripheral policy ideas to be expanded on and amplified. In this regard, "[p]olicies that might have a hard time becoming accepted in local contexts seem that much more irresistible when offered as uncontested consensus of the world's leading democracies" (Op. cit., p. 2). And, therefore, many countries take into consideration what OECD states as guiding for the development of their national agendas. After the last economic crisis, OECD published several indicators and strategies for countries to improve and invest in their educational programs, targeting the making of a particular type of citizen.

The claim "be the best version of yourself" is a motto that portrays the many discourses circulating about social progress and the necessary skills for citizens to achieve better lives and for countries to achieve a greater economical progress.

Be the best version of yourself

> Equipping citizens with the skills necessary to achieve their full potential, participate in an increasingly interconnected global economy, and

> ultimately convert better jobs into better lives is a central preoccupation of policy makers around the world –poor skills severely limit people's access to better-paying and more rewarding jobs. The ongoing economic crisis has only increased the urgency of investing in the acquisition and development of citizens' skills –investing in structural reforms to boost productivity, such as education and skills development, is key to future growth. More and more countries are looking beyond their own borders for evidence of the most successful and efficient policies and practices –in a global economy, success is no longer measured against national standards alone, but against the best-performing and most rapidly improving education systems. PISA 2012 results show wide differences between countries in mathematics performance –all countries and economies have excellent students, but few have enabled all students to excel. PISA is not only an accurate indicator of students' abilities to participate fully in society after compulsory school, but also a powerful tool that countries and economies can use to fine-tune their education policies –every country has room for improvement, even the top performers (OECD, 2014, pp. 3-4, bolds removed).
>
> OECD. Better policies for better lives

The above quotation is a summarized part of OECD's *PISA 2012 Results: What Students Know and Can Do*. In this publication, OECD states a concern regarding the economic crisis and it poses the urgency of having well-equipped-citizens. In this short text, many naturalized truths are entangled to (re)produce a discourse in which education is believed as the key to future growth of nations. For example, that few countries have enabled all students to excel, and, so, even top performers have room for improvement. This discourse embodies the idea of an ever-growing economy for social progress and of lifelong learners to achieve that goal.

It has been said that, in an ever-growing economy, citizens should be equipped with the necessary skills to achieve a state of welfare, given that educational attainment is used as an alternative measuring for human capital (OECD, 2015). These necessary skills –such as literacy and numeracy– should enable citizens to reach their full potential. A full potential not only regarding students performance in PISA or in national standardized assessments, but also 'high - attainment - individuals' "generally have better health, are more socially engaged, have higher employment rates and have higher relative earnings" (OECD, 2015, p. 30). This statement means that it is taken as possible to correlate higher levels of educational attainment and positive individual and social outcomes, resulting in a state of: 'the higher educated, the better health, the better job, and therefore, the better life'.

According with OECD's readings on PISA's outcomes, higher educational attainment is due to literacy and numeracy skills. However, it has also been stated by OECD that "[c]ompared to literacy skills, numeracy skills have a more significant impact on employment outcomes" (Op. cit., p. 168). Henceforth, the odds of being employed increases directly proportional to the level of numeracy proficiency (see chart A9.2 in OECD, 2015). It is also stated by OECD that numeracy proficiency has an impact even on employees' hourly earnings. In this regard, the 'equipped citizen' not only will aspire to having a healthier life, but, depending on the level of proficiency in mathematics, will also aspire to having better opportunities of being employed and of earning more income than a 'not–so–well equipped–citizen'.

> Across the OECD, the average return to below upper secondary education stands at approximately 2.5% (ranging from 0% for those in possession of the Level 0/1 numeracy proficiency to approximately 4% for those in possession of Level 4/5 proficiency, while at upper secondary, the range of earnings outcomes (compared to an individual with the lowest level of formally recognised qualifications and numeracy skills) stands at approximately 10% (upper secondary and Level 0/1 numeracy) to 18% (upper secondary and Level 4/5 numeracy). At tertiary level, the earnings outcomes (compared to the reference group) range from approximately 33% (tertiary and Level 0/1 numeracy) to 53% (tertiary and Level 4/5 numeracy). (Lane & Conlon, 2016, p. 21)

Educational attainment has also been correlated with lower morbidity from the most common diseases –heart condition, stroke hypertension, cholesterol, emphysema, diabetes, asthma attacks, ulcer– and correlated with life expectancy, in which life could be increased up to 5 years (Cutler and Lleras-Muney, 2006). And so, educational attainment rises as an important factor for well-being (OECD, 2016a), by boosting specific features of the desired productive citizen for society.

> Highly skilled people are also more likely to volunteer, see themselves as actors rather than as objects of political processes, and are more likely to trust others. Fairness, integrity and inclusiveness in public policy thus all hinge on the skills of citizens. (OECD, 2015, p. 3)

All of the above translates in a promise of 'well-equipped-citizens' that live longer, are socially active, healthier, volunteer, engage in political processes, trust others, are more likely to be employed and earn more, all because they reach their full potential thanks to their numeracy proficiency. After all, the first OECD commitment is 'better policies for better lives'. So, be the best version of yourself!

FROM THE HOPED TO THE FEARED

OECD (re) produces discourses emboding salvation narratives about who the desired citizen is and how the desired citizen should be. The image of the productive citizen is enunciated in every statement above. The 'well-equipped-citizen' OECD expresses is a lifelong learner that should be wiling to engage in a perpetual process of making choices and problem solving (see, e.g., Popkewitz, 2008b). Also, a high skilled citizen that is willing to participate fully in society. This portrays the image of the hoped, an entrepreneur, an "individual who is continually pursuing knowledge and innovation in a never ending chase for the future" (Popkewitz, 2008a, p. 310), all because of mathematics proficiency.

Within the making of the citizen, OECD plays the role of 'homogenizing the heterogeneous' (Tröhler, et al., 2014). In which, PISA has been shaping "the "accountability" agenda in ways that rival and even overshadow the influence of national policy makers" (Op. cit., p.5). Under this homogenizing role, numeracy proficiency has been granted with the feature of producing social equality (Diaz, 2013). All students having the same opportunities, access, and possibilities translates into "reform projects assum[ing] that social, economic, and educational inequalities can be minimized if all children have the opportunity to learn mathematics" (Op. cit., p. 36). But equity, in this sense, is an illusion (see, e.g., Bullock, 2012); there are many differences amongst students. Although, according to OECD, these differences become less visible if nations invest in the acquisition and development of citizens' skills, enabling students to excel.

> When the "all children" is examined, there is no universal and undifferentiated "all" but a particular continuum of value that differentiates and divides. The "all children" implies a unity from which identities of difference are generated. As quickly as reforms state that the purpose is for "all children to learn", however, the discourse shifts to the child who is different and divided from the space of "all children". The different child is to be rescued and saved from his or her unliveable spaces. The space of the all children is the space of a difference and abjection that cases the Other into unliveable spaces. (Popkewitz, 2011, p. 42)

While certain discourses promote inclusion: "all students could become the well-equipped-citizen", at the same time there are double gestures of what is desired and what is feared that produces processes of exclusion with those of inclusion. And so, these normalizing and regulating discourses on welfare become in "ways of reasoning about who and what is normal [and also] who and what is abnormal, and in need of social administration, intervention, and salvation" (Bloch, et al., 2003, p. 15). To talk about the "all" implies to talk about the "abjected"; the one who does not fit (Popkewitz, 2008a).

OECD describes the 'abjected' as the 'low performer'. Lower performance in mathematics is attributed to an accumulation of students' 'disadvantages' throughout their lives (OECD, 2016b) –economical, social, educational, gender disadvantages– making this 'abnormal child' in need of salvation to fit the "all". In fact, OECD has even calculated the variables and the percentages of likelihood of being a low performer.

> Who is most likely to be a low performer in mathematics? On average across OECD countries, a socio-economically disadvantaged girl who lives in a single-parent family in a rural area, has an immigrant background, speaks a different language at home from the language of instruction, had not attended pre-primary school, had repeated a grade, and is enrolled in a vocational track has an 83% probability of being a low performer (OECD, 2016b, p. 13)

Above was stated that the social, economical, and educational differences of students apparently play no role in the acquisition of skills, and even that this 'girl' is recognized as a low performer, does her disadvantages make her a 'fear child'? Does this girl pose any danger to the future? Well, it depends on the point of view. But, pushing a little deeper the analysis. Who will become a 'threat' for economic progress and the welfare state?

On a neoliberal mentality, discourses about progress and welfare shape and reshape citizens modes of living –ways of being and acting in the world– to be the expected product of a neoliberal and capitalist society. Marketing, consumerism and competition are some of the key elements of neoliberalism (Kaščák & Pupala, 2011), where "people are reconfigured as productive economic entrepreneurs of their own lives" (Davies & Bansel, 2007, p. 248). Neoliberal modes of governance aim at reconfigure a lifelong entrepreneur learner, given that it will be beneficial for the economic productivity of society (Rubenson 2008). Entrepreneurs are taken as the human capital necessary for personal and social and economic prosperity. In this sense, the feared is the one that does not become the productive citizen for economical and social progress, the one who does not consume, who does not engage in market labor, who does not participate fully in society, and who does not compete; the one who cannot fit the "all". Imagine, for example, a student refusing to participate in PISA.

WELFARE AND SCHOOL MATHEMATICS

> The technology of schooling was not invented ab initio, nor was it implanted through the monotonous implementation of a hegemonic 'will to govern': the technology of schooling –like that of social insurance, child welfare, criminal justice and much more– is hybrid, heterogeneous, traversed by a variety of programmatic aspirations and professional obligations, a complex and mobile resultant to the relations amongst persons, things and forces. (Rose, 1999, p. 54)

The promises of welfare emerged in the late 20th century, as 'new patterns of governing' (Bloch, Holmlund, Moqvist & Popkewitz, 2003), as a "way of securing or "policing" the well-being of citizens and populations through the "cultural reasoning system" that orders the individuality of the welfare person" (p. 6). Within the promise of welfare, school mathematics is taken as necessary to achieve numeracy for the pursuit of individual happiness and human progress (Popkewitz, 2013).

There is a promise to shape students to fit in certain category for the development of nations, the better economy, and their own welfare. OECD's narratives of what a productive citizen should be have effects of power in the shaping of students' subjectivities –in which "the self is constructed or modified by himself" (Foucault, 1993, p. 204). OECD discourses normalize and regulate whom the productive citizen is, how the productive citizen should be and should act. To be a productive citizen means students should engage in practices to conduct their own conduct to achieve the 'well-equipped state', a will of fitting in the "all" and becoming a lifelong learner. But it also means to recognize in school mathematics an opportunity to reach welfare... All in the name of economic growth!

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REVISTA DO PROGRAMA DE PÓS-GRADUAÇÃO EM EDUCAÇÃO MATEMÁTICA DA UNIVERSIDADE FEDERAL DE MATO GROSSO DO SUL (UFMS)

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A Fabricação de Cidadãos Qualificados:

Do "Expert-Hand Worker" ao "Scientific Minded"

The Fabrication of Qualified Citizens:

from the "Expert-Hand Worker" to the "Scientific Minded"

Melissa Andrade-Molina[1]

RESUMO

Dentro das práticas da matemática escolar existem numerosas verdades naturalizadas que têm sido (re)produzidas em diferentes épocas e em diferentes lugares. A crença de que a educação matemática se torna a chave para o progresso econômico assegurando o crescimento futuro das nações é problematizada. Seguindo um movimento analítico rizomático, uma historização do presente é empregada para mapear a fabricação do cidadão qualificado desejado no Chile. A análise evidencia a (re)produção de narrativas dominantes sobre o "cidadão qualificado" são e têm sido enredadas com o funcionamento da geometria escolar e matemática escolar, como uma tecnologia de governo do eu para a fabricação o sujeito racional Moderno, inserindo alunos em particular formas de ser e conhecer no mundo.

PALAVRAS-CHAVE: tecnologias de governo. historização do presente. Ciência. práticas de matemática escolar. crescimento econômico.

ABSTRACT

Within the practices of school mathematics there exists many naturalized truths that have been (re)produced in different times and in different places. The belief of mathematics education becoming the key to economical progress assuring the future growth of nations is problematized. By following a rhizomatic analytical move, a historization of the present is deployed to map the fabrication of the desired qualified citizen in Chile. The analysis evidences the (re)production of dominant narratives about the "qualified citizen" are and have been entangled with the functioning of school geometry and school mathematics, as a technology of government of the self to fabricate the Modern rational subject by inserting students in particular forms of being and knowing in the world.

KEYWORDS: Technologies of government. historization of the present. Subjectivity. Science. practices of school mathematics. economic growth.

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enlightened subjects. Such exploration allows problematizing how modern narratives about the fabrication of the Modern 'reasonable citizen' (Andrade-Molina, forthcoming) through the learning of school mathematics bring science and mathematics together. Elsewhere it has been discussed that school geometry has power effects in the shaping of the productive scientific minds of the future by educating students to see not with the eyes of their bodies, but with the eyes of reason and logic (Andrade-Molina; Valero, 2017). My contention in this paper is that the dominant narratives about the "qualified citizen" are and have been entangled with the functioning of school geometry, as a technology of government of the self (Foucault, 1997), to fabricate a the Modern rational subject by inserting students in particular forms of being and knowing. Here a historization of the present is deployed to map the fabrication of the desired qualified citizen in Chile as a rhizomatic analytical move (Deleuze; Guattari, 1987), by building on the toolbox of Foucaultian studies about the constitution of historical subjects:

> I wanted to see how these problems of constitution could be resolved within a historical framework, instead of referring them to a constituent object (madness, criminality, or whatever).… I don't believe the problem can be solved by historicizing the subject as posited by the phenomenologists, fabricating a subject that evolves through the course of history. One has to dispense with the constituent subject, to get rid of the subject itself, that's to say, to arrive at an analysis which can account for the constitution of the subject within a historical framework. And this is what I would call genealogy, that is, a form of history which can account for the constitution of knowledge, discourses, domains of objects, etc., without having to make reference to a subject which is either transcendental in relation to the field of events or runs in empty sameness through the course of history. (Foucault, 1980, p. 117)

This exploration begins by evidencing, first, the Chilean need of bringing foreign experts to enhance education for the proper instruction of science in school, and, also, importing European methods of teaching that were taken as the avant-garde. Second, the analysis moves towards the fabrication of the "qualified citizens" for achieving progress and leaving the state of underdevelopment. Such analysis starts at the need of "expert-hand workers" as a form of securing a solid ground for a more advance society. With the advances of technology, workers left the necessity of using their hands and knowledge, but other skills were required. This led to the need of "technical workers" shaped with applied practical and useful knowledge. Finally, the "scientific minded" emerged as a need of a rational citizen, shaped under reason and logic. The Modern citizen, fabricated on the desired of the rational subject, was, and still is, the very image of assuring progress and welfare.

The need for the "wise" Europeans

Since 1810, when Chile proclaimed its independence from the Spanish colonial power, there have been several attempts to overcome a state of underdevelopment compared to European countries. Regarding schooling, the first schools in Chile began as churches imparting lessons of reading and writing, and where constituted mainly by Dominican and Jesuit orders. In 1818,

> An administration of primary teaching was created and adopted the Lancaster method […] The English educator Diego Thomson, was invited by Bernardo O'Higgins to work in the alphabetization according to the Lancastrian system, which was a method of teaching through the Bible. (Cancino, 2012, p. 150, own translation).

By 1820, schools in Chile were framed either religiously or militarily. But neither the men of faith nor the men of the army could help achieving the current desired of progress and development in accordance to European levels. In this light, it was raised, in 1823, a concern about the necessity of a more suitable education for the shaping of the 'men needed' for science, commerce, industry, agriculture, and arts for the State to achieve higher power and richness (Labarca, 1939). Miguel Luis Amunategui—minister of public instruction since 1852—stated, in 1856, that ignorance, mainly regarding analphabetic citizens, was the only source able to keep the society on an underdevelopment state. He acknowledged that Chilean society was struggling with a very real and powerful enemy, but just not an enemy of flesh and bones.

> There exists in our land an enemy worse that an invasion, more tremendous than barbarian colonizers […] An enemy that impedes us from breathing […] that stops us from taking the steps towards the path of progress […] That enemy is ignorance […] It is necessary, it is urgent for us to declare the war against this domestic enemy. (Amunategui in the establishment of the Society of Primary Instruction in Santiago, in Labarca, 1939, p. 141, own translation)

Chileans were attempting to build a path towards progress. And the way thought to scape from a society without industries, without commerce, without economy, and without enough intellectual knowledge was to increase agriculture and, of course, industries by trusting in science. The Chilean state began implementing policies for hiring the "wise" foreign people to bring science to the country, which has been a constant practice since the first years of its independence (Gutiérrez; Gutiérrez, 2004). Chile started to fall in love of the French sophistication, in which education was completely separated from theology. Even since 1818, Chile was drowned by the French's liberal thinking (Cancino, 2012). The importing of knowledge and techniques of the "wise" led to the use of exact translations of French textbooks (Gutiérrez; Gutiérrez, 2004) and to the adaptation of the educational system of French schools to, what Labarca (1939) calls, a society orphan of cultural traditions. After Chile's independence, the Chilean school system was structured as a precise image of the French system. French mathematics textbooks were of particular interest for achieving the desired European quality, being translated and put into operation in Chilean schools. Gorbea—a Spanish engineer—in 1831 organized the teaching of mathematics according to the European French style and he translated a textbook written for the École Polytechnique, *the complete pure mathematics course of Francoeur*—El curso completo de matemáticas puras de Francoeur— (Gutiérrez; Gutiérrez, 2004). At the same time, Ignacy Domeyko—a Polish Educator—was making several improvements in the University of Chile. He, Domeyko, was the first to proposed the creation of a School dedicated only to the training of teachers for secondary school under European guides (Amunátegui, 1913).

French's influence dominated Chilean education in schools until 1880. After the Battle of Sedan, Chile started to be amaze by the German army and public education. 1878 was a year of exploration, Nuñez was asked by the government to travel to the United States and to Europe to learn about how these first world countries organized their system of education and their teaching methods. After this travel, he wrote "Organización de las escuelas normales" a highly influential book that set plenty of changes in Chilean education (Labarca, 1939). And Chile left the enchantment of Parisian horizons to turn to the German standards and structuring of school mathematics. In 1886, Pedro Montt—president of the Chamber of Deputies at the time, and the president of Chile from 1906 to 1910—presented a reform to group school subjects according to clusters of knowledge (implemented in 1889). In this reform, the sciences—biology, physics, and chemistry—were accentuated (Labarca, 1939). As consequence, Chilean schools were beginning to resemble the German Realgymnasium.

From the fabrication of expert-hand workers…

> If Chile wants progress […] it is necessary to disseminate primary instruction to all. Making science popular is the only path to improve […] To think the opposite, is expecting the impossible. The earth does not produce what it should without expert hands to grow it. (Amunategui, 1856. In Labarca, 1939, p. 144, own translation)

As aforementioned, Chile was a society lacking of commerce, industries, intellectual knowledge and experts to overcome these deficiencies. One solution was to import knowledgeable Europeans. The other was to instruct all citizens in all areas of labour possible to the time. "The earth does not produce what it should without expert hands too grow it", this expertise was not in terms of knowledge gained by experience but to knowledge gained by proper instructions, in which science was positioned as key. As Amunategui called them, the 'expert-hand' workers had their own technical schools. In those schools, mathematics played a central role while instructing students. For example the School of Arts and Occupations had courses of arithmetic, elemental geometry and applied drawing specifically aimed at industrial workers (Labarca, 1939). In 1890, geometry and drawing were thought to have a close connection, and therefore elemental geometry was not separated from line drawing, being together the same subject (Consejo de Instrucción Pública, 1890).

> The teaching of drawing, which completes the study of forms and enables to exercise the hand and children's sight, giving him the means to represent graphically the objects that surround him, it is also one of the most powerful agents used nowadays for the education of the senses. The application of drawing to all professions and most general uses in life adds to this study an interest and importance that would not be possible to unknown. And which scope could only be appreciated according to the advantages that a labourer or an industrial worker could have of drawing. (Nuñez, 1883, p. 104, own translation)

The desirable "expert-hand" workers were thought to use geometry and drawing as tools to link their senses—sight—with the abstractions of geometrical knowledge. Geometry was not to instruct students to forms of proof and calculation, but to educate the senses. Also, compared to other school subjects, school geometry was the most repeated subject taught: Elemental geometry and line drawing, complements of geometry, analytical geometry and trigonometry (Figure 1). However, the teaching of school geometry through drawing was difficult due to the required use of specific devices. Henceforth, the first years of school were mainly dedicated to intuitive geometry—drawings—, and after a few years, to practical geometry—calculation and demonstration—(Consejo de Institución Pública, 1893). Geometry started to be taken as a school subject that "need exact notions [and] the child would not be able to develop these notions unless he has time to study and is aware of the new notions, that most of the times are abstract" (Op. cit., p. 95, own translation). According to Labarca (1939) several schools that taught young students the traditional elementary program—consisting of religion (Catholic), reading, writing, arithmetic, grammar, line drawing, geography, and finally, only for boys, history and Chilean constitution, and only for girls, household's economy, sewing and embroidering—were severely criticized because it was thought that teaching science to all students was dangerous for the conservation of faith, and therefore, was considered detrimental for Catholics.

By late 19th century, Chile began to introduce the teaching of mathematics as a rational and analytical model and as a mode of the most powerful intellectual gymnastic (Nuñez, 1883). According to Nuñez (1883)—the author of the most influential educational book of this period: *Organización de escuelas normales*—, number and form were keys, in which school geometry was taken as the fundamental topic of the most elevated principle of pedagogy at that time: Fröebel's. Suddenly, the Pacific war opened the new form of maximizing the saltpeter extraction and agriculture in the Central Valley, and therefore highly qualified engineers and technicians were urgently needed (Labarca, 1939). And mathematics in schools was taken as a skill for the development of reasoning and logical thinking (Ministerio de Educación, & CPEIP, 1967).

Geografía descriptiva...
Historia sagrada...
Id. antigua y griega...
Id. romana...
Id. de la edad media...
Id. moderna y contemporánea...
Id. de América y de Chile...
Aritmética...
Geometría elemental y dibujo lineal...
Álgebra...
Física...
Química...
Cosmografía...
Historia natural...
Geografía física...
Gramática castellana...
Francés...
Literatura (retórica y poética)...
Id. (historia literaria)...
Psicología y lógica...
Moral, teodicea é historia de la filosofía...
Inglés...
Latín...
Alemán...
Álgebra científica...
Id. con complementos de geometría...
Complementos de geometría...
Geometría analítica...
Id. práctica...
Id. práctica con elementos de geometría descriptiva...
Trigonometría rectilínea...

Figure 1- School subjects in 1890
Source: (CONSEJO DE INSTRUCCIÓN PÚBLICA, 1890, p. 15).

…To the "technical worker"…

Post-Pacific-war Chile required the development of specific skills in students—the fabrication of a productive 'technical' worker—, workers should be able to apply practical knowledge of geometry in their everyday practices in the workplace and in life. To ensure quality work force, labourers needed practical geometry for a better appropriation of space, and industrial workers also needed to connect geometry with their fields—other sciences. In 1889, as product of the reform presented by Pedro Montt in 1886 and his wishes for schools to resemble the German Realgymnasium, geometry, arithmetic and algebra were together in a new cluster of knowledge called mathematics (Labarca, 1939).

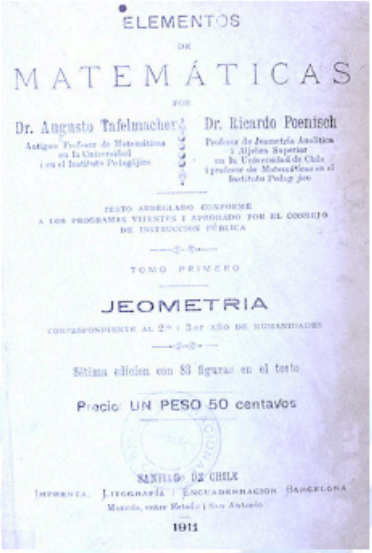
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Figure 2: Tafelmacher and Pōenish's "Elementos de Matematicas: Jeometría"
Source: author data

By 1909, some schools taught mainly "applicable knowledge" for future workers. And, therefore, students were assumed to be able, for example, to deeply study the functioning of a machine, draw it, adjust its pieces, and so on (Labarca, 1939). School textbooks needed to be rephrased since geometry was not intuitive or linked to drawing as the previous years. The Council of Public Instruction—Consejo de Instrucción Pública—start realising and approving books of the *Elements of Mathematics*, being geometry the first book in the collection (Figure 2). This book was written for the humanistic schools by the German teachers Tafelmacher and Pöenish, both brought from Germany to Chile to improve the teaching of mathematics. And so, schools that resembled the Realgymnasium and schools for occupations (technical workers) were vastly different.

Alfonso (1912), at that time wrote that the teaching of algebra and jeometría should be reduced. The motive was that it was necessary to increasing arithmetic, knowledge thought of a more practical use for students. He was not the only one, Galdámes (1912) did not see, at humanistic schools, students using mathematics in their everyday social interactions, claiming that mathematics and *jeometría* were only important if students wanted to pursue engineering, astronomy, physics, chemistry, or even biology. Galdámes adds that knowledge in general, not only mathematical knowledge, develops reasoning if it is studied scientifically.

> In this stage, [mathematics] textbooks strongly highlight geometric reasoning abilities through geometrical construction by using ruler and protractor, in addition of the deductive presentation of geometry. (Vidal, 2010, p.8, own translation)

Amunategui addressed all critiques and talked about the need of a productive citizen to overcome Chile's industrial and commercial inferiority produced by a weak economy (Labarca, 1939). For assuring the fabrication of productive minds, educational policies aimed at cutting 'impractical topics' of the curriculum. For example, plane geometry was taken as an abstraction of space; rather three-dimensional geometry was taken as intuitive and with practical applications in physics, technical and natural sciences (Consejo de Instrucción Pública, 1926).

> From the formal concept of 1900, formation of an abstract and theoretical man, we arrived to the concept of education as a social function, on the service of collective progress. (Labarca, 1939, p. 239, own translation)

School subjects began taking a new shape, in which geometry was separated from drawing as a result of an attempt of homogenizing school programs for girls and for boys (Labarca, 1939, p. 260). This measure led to cut all unnecessary topics such as sewing—one of the subjects on the program for girls.

ASIGNATURAS	AÑOS					
	I	II	III	IV	V	VI
Castellano	4	4	4	3	3	3
Filosofía	—	—	—	—	2	2
Francés	4	3	3	3	3	3
Inglés o Alemán	—	4	4	3	3	3
Historia y Geografía	3	3	3	4	3	3
Matemáticas	4	4	4	4	3	3
Ciencias naturales	2	2	2	2	2	2
Física	—	—	—	2	2	2
Química	—	—	—	2	2	2
Educación Cívica	—	—	—	—	2	2
Religión (optativa)	2	2	3	1	1	1
Dibujo y Caligrafía	5	3	2	2	2	2
Trabajos manuales	2	2	2	2	2	2
Canto y Gimnasia	3	3	3	3	3	3
	29	30	30	31	33	33

Figure 3: Chilean school program in 1893.
Source: (LABARCA, 1939, p. 240)

A Ministry of Education was formed to shape this new path of education in schools. And so, intellectual education was aimed at teaching how to think about general, clear and coherent ideas regarding all school subjects. Schools subjects were thought as the necessary knowledge students needed to be part of the economic life and to help with social welfare and to interpret the physical world around them (Ministerio de Educación Pública, 1935).

… To the fabrication of the scientific minded

By 1891 Chile entered in an economic crisis as a result of the War of the Pacific (1879–1883) and the end of the Portalian state (Góngora, 1988). And both a proper education and more suitable economic policies were the only way thought to overcome the crisis (Encima, 1981). 'Expert-hand" and "technical" workers suddenly were not enough. In schools, education was vastly criticized, mainly because of the weak implementation of policies in education since, at least, 1885 (Valdes Cange, 1910). When the German reform was brought to schools, but with inadequate translations of pedagogic materials, and German 'pseudo-teachers' that did not understand the reform (Encina, 1981). Or as Labarca (1939) expresses, German teachers that were not familiarized with Chilean culture and history when they arrived seeking for economic stability, and so, it was not strange to see these German teachers teaching students, for example, about German's flora and fauna.

With the Pedagogic Institute, created in 1889, mathematics education took a new dimension of a *corpus* of knowledge, in the hands of the German teachers Tafelmacher and Pöenisch (Gutiérrez; Gutiérrez, 2004). Although, German teachers had a great influence in teaching methods as well given that primary teachers, from Germany, brought the scientific teaching conception of Herbert's pedagogy and the first approximation of the psychology of science (Labarca, 1939). Both, mathematics as a *corpus* and more scientific approximations to teaching, lead to perceive science as a creative process. Tafelmacher and Pöenisch published a series of six books called "Elements of Mathematics"—*Elementos de Matemática*—which were used in Chilean schools until 1912 (Vidal, 2010). However it was not over for the Germanization of Chilean mathematics education. Pröschle, another German mathematics teacher, and Pöenish continue writing school mathematics textbooks (Vidal, 2010).

> Mathematics, thus, broke the mould of being a "useful" science [...] and acquires the category of a cultural and autonomous discipline. Something that France, Germany and other European countries had already fulfilled since 19th century. (Gutiérrez; Gutiérrez, 2004, p. 13, own translation).

Since 1902, as part of the pedagogic conference and also of the scientific conference of 1910, Chilean teachers started to walk away from the "German enchantment", alongside, they started to argue that Chilean citizens should be taught by Chilean teachers, within this demand, education should be adapted to social necessities of Chilean people (Labarca, 1939). Enrique Villegas, the minister of instruction—on behalf of the Chilean government—opened the conference by mentioning the need of an education towards the shaping of a modern citizen.

> [Secondary education] is the centre of the whole educational system. It tends to shape children towards the development and equilibrium of all their faculties, preparing them for the highest profession in letters or sciences and for the life of citizens aware on their civic and moral duties corresponding to a member of modern democracies. (Villegas, 1913, p. 4, own translation).

All of these changes or improvements led teachers perceiving school mathematics as operations students solve by using logical methods, not for the development of a useful tool. Mathematics was taught "in a particular way, as submitted to abstract science and not as realities that construct the principal objects of science" (Galdámes, 1913, p. 79, own translation). Galdámes (1913) critique moved towards how mathematics in school left the status of being operational for the workers and applicable to their lives, and so, school mathematics became a training of the scientific knowledge needed for University. The qualified citizen needed was the highly educated to achieve Europeans standards of development, a "scientific minded".

> We like to think that the Chilean child is the same child from Germany, France or England, subjected to the same influences of the environment, of an equal mentality than them [...], of an organic and psychological development [...] The difference is not anymore on the language or on skin and hair colors. And our simplistic view conducts us to educate him as German, French or English children are educated. (Galdámes, 1913, p. 86, own translation).

But Chile continued his path to Europe. Also from Germany, Karl Gandjot, participated in the creation of the instructions for the progress and growing of national sciences, he also made major contributions in bringing Modern Mathematics in Chile at University level (Gutiérrez; Gutiérrez, 2004). On 1926, discussions were still vibrant. The

Pedagogic Assembly of the National Society of Teachers tried to put an end to this admiration Chile had to foreign lands, not only Europe, but the United States too—Dewey was also very influential in education—(Labarca, 1939).

The Sputnik shock changed it all, Mathematicians met in Edinburg and concluded that Western mathematics needed a change, and Modern Mathematics emerged (Vidal, 2010). Which led to "more advanced mathematics in all schools to give a greater scientific knowledge to students; men of science were needed" (Op cit, p. 9, own translation).

The modern citizen

> Governing people is not a way to force people to do what the governor wants; it is always a versatile equilibrium, with complementarity and conflicts between techniques which assure coercion and processes through which the self is constructed or modified by himself (Foucault, 1993, p. 204).

Naturalized truths are and have been circulating amongst diverse times and places. The need of science for progress and welfare is a taken-as-granted truth that has been circulating since the independence of Chile, in 1810, to overcome the state of poverty as a result of the colonization and the aspiration to become a developed country. This truth has been naturalized and it has been knitting a web to govern the self and to regulate habits and desires of cultural and historical subjects through school mathematics.

From a Foucaultian perspective, conduct is governed, through diverse techniques, strategies, and devices (Foucault, 1991). The mapping of school mathematics as a technology for the government of the self, evidences the effects of power that geometry has had in fabricating the desired subject of schooling: the qualified citizen for economic growth. In doing so, each "qualified citizen"—the expert-hand worker, the technical worker, and the scientific minded—conducts him/herself by (re)shaping his/her own modes of being and acting in a space of 'regulated freedom' and under a promised state of welfare. In which, the belief of needing more mathematics for greater scientific skills opened the path for assuming that advanced mathematics—as rational and analytic model—should fabricated a more suitable qualified citizen for society. This embodies salvation narratives of mathematics, for example, to overcome the misery in Chile as a product of the war, the economic crises or colonization itself. As Egaña said, "without education it is not possible to shape the men needed, instructed in diverse scientific fields. By putting in action commerce, agriculture, industry, arts and science, they work for giving to the State power and richness" (Egaña, 1823, in Amunategui, 1913, p. 14, own translation).

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“MINDNIAC”

THE REASONABLE CITIZEN OF SCHOOLING (CHILEAN EDITION).

Abstract

"Mindniac" is a comic book that traces, from a Foucaultian perspective, the historical making of citizens through school mathematics. It traces, through a historization of the present, the way in which taken-as-granted truths are produced and reproduced within discourses about who the desired product of schooling should be and how this desired product should act. It analyses how discourses about the productive citizen have been circulating amongst Chilean discourses on official documents from 19th century until the present, and the discourses that have been (re)produced by OECD since 2012. It explores, in four different stories, how the school mathematics curriculum becomes a technology for the government of the self and how curricular devices, e.g. assessment, have become a *dispositif* for the exertion of power.

The comic book format helps playing with images to portrait the desired of a mathematically competent citizen and the illusion of reaching it, illusion in the sense that it becomes a never-ending process. "Mindniac" aims at portraying the days of future past; it aims at tracing the continuities and discontinuities that have been configuring the making of the "perfect" children for society.

How to read this comic

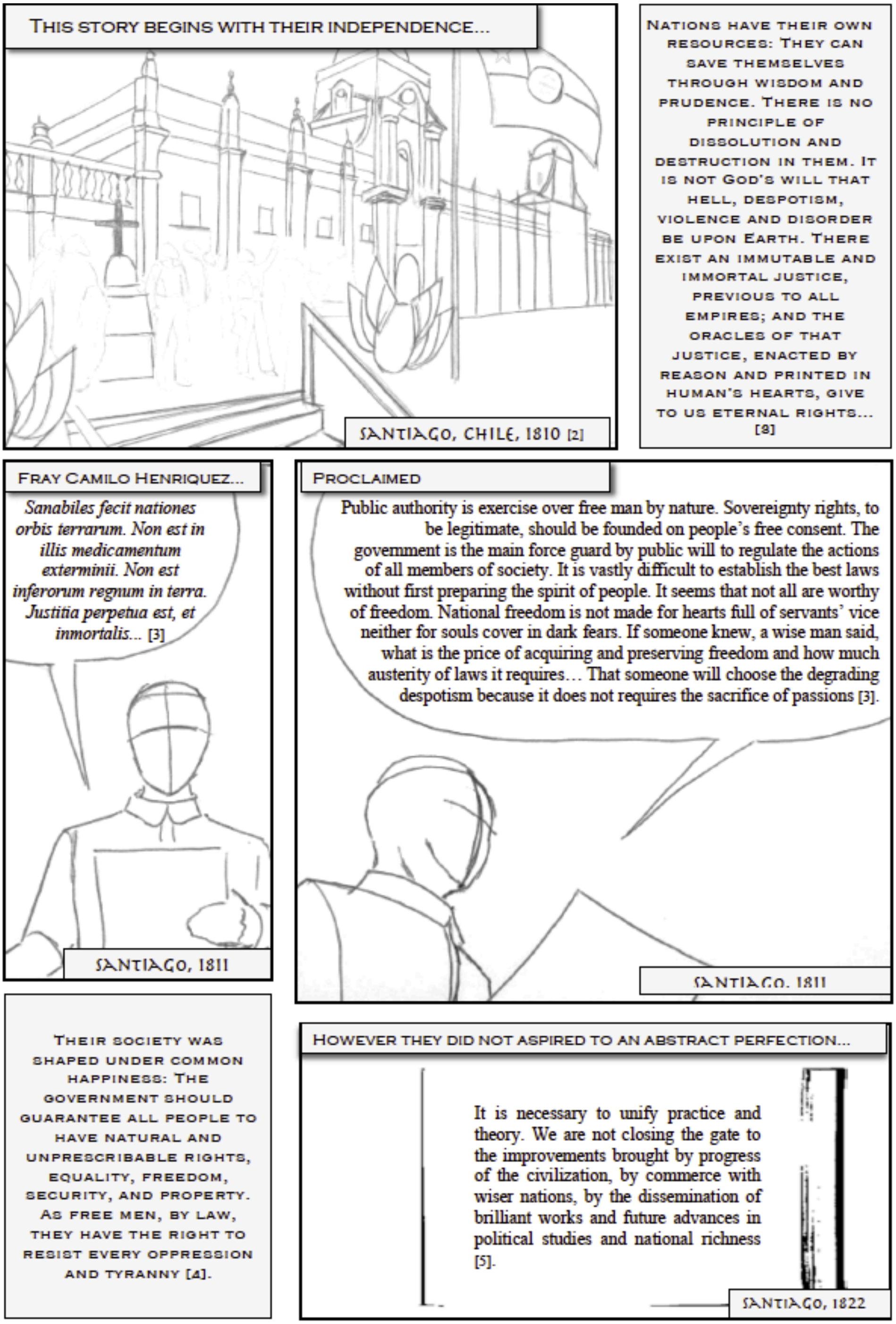
This comic book is not different to others in structure. However, there are a few considerations to have in mind when reading it. First of all, there is the *prologue;* this section will contain all necessary information to embark into the story, a sort of introduction. Also, there is the *epilogue*; this contains a closing of the story and added information not addressed on the story. Finally, there are two options for reading the comic. There are five sections, easily identified by the type of line each frame has, the chapters. You can choose to read the comic as one full story, or you can choose to read the comic following the types of lines of the frames. In other words, reading all the frames with the same type of line as separate stories.

Mindniac

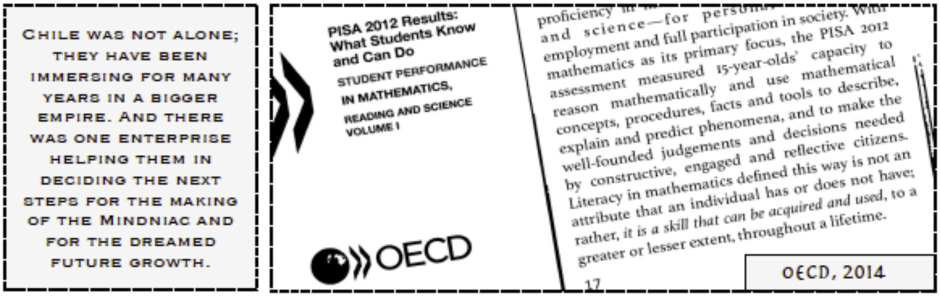
Written and drawn by Melissa Andrade-Molina

Prologue

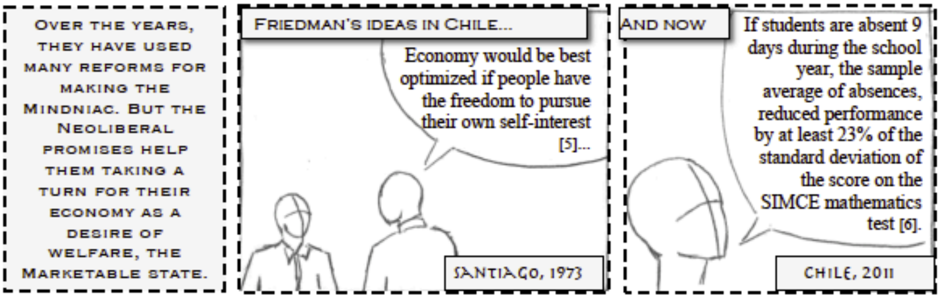
Since its independence, a country desiring to overcome its poverty and misery, aimed at producing the competent citizens beneficial for the achievement of power and richness. They became impressed by French's sophistication and by German's structures modes of teaching. They wanted to scape from ignorance, they wanted to be wiser. An intellectual evangelization started to get stronger and stronger. Dr. Chile was always looking at first world nations, always exploring what they did to be successful. Experiments were conducted; foreign wise teachers were brought to help in the pursuing of glory. They produced their own school textbooks and then, it began the first attempts to produce national research. But, after Second World War, the desired for scientific and technological progress started to increased. Modern Mathematics and Industrialization seduced a post-war Chile. Again, foreigner's thoughts charmed a very impressionable little country. The idea of more mathematics for greater scientific skills opened the path for believing that advanced mathematics would potentially create the perfect human for society. A global enterprise named OECD positioned itself as network amongst nations for the (re)production of solutions toward economical progress. It became a central node for connecting local and peripheral policies to be expanded, amplified and made so irresistible for underdeveloped nations [1]. This naïve country, hungry of power, took into consideration the result of OECD's experiments for the improvement of its national agendas, and started conducting its own experiments. The aim: to achieve the perfect citizen for economic productivity and prosperity, the Mindniac…


THIS STORY BEGINS WITH THEIR INDEPENDENCE...
SANTIAGO, CHILE, 1810 [2]
NATIONS HAVE THEIR OWN RESOURCES: THEY CAN SAVE THEMSELVES THROUGH WISDOM AND PRUDENCE. THERE IS NO PRINCIPLE OF DISSOLUTION AND DESTRUCTION IN THEM. IT IS NOT GOD'S WILL THAT HELL, DESPOTISM, VIOLENCE AND DISORDER BE UPON EARTH. THERE EXIST AN IMMUTABLE AND IMMORTAL JUSTICE, PREVIOUS TO ALL EMPIRES; AND THE ORACLES OF THAT JUSTICE, ENACTED BY REASON AND PRINTED IN HUMAN'S HEARTS, GIVE TO US ETERNAL RIGHTS... [3]
FRAY CAMILO HENRIQUEZ...
Sanabiles fecit nationes orbis terrarum. Non est in illis medicamentum exterminii. Non est inferorum regnum in terra. Justitia perpetua est, et inmortalis... [3]
SANTIAGO, 1811
PROCLAIMED
Public authority is exercise over free man by nature. Sovereignty rights, to be legitimate, should be founded on people's free consent. The government is the main force guard by public will to regulate the actions of all members of society. It is vastly difficult to establish the best laws without first preparing the spirit of people. It seems that not all are worthy of freedom. National freedom is not made for hearts full of servants' vice neither for souls cover in dark fears. If someone knew, a wise man said, what is the price of acquiring and preserving freedom and how much austerity of laws it requires… That someone will choose the degrading despotism because it does not requires the sacrifice of passions [3].
SANTIAGO, 1811
THEIR SOCIETY WAS SHAPED UNDER COMMON HAPPINESS: THE GOVERNMENT SHOULD GUARANTEE ALL PEOPLE TO HAVE NATURAL AND UNPRESCRIBABLE RIGHTS, EQUALITY, FREEDOM, SECURITY, AND PROPERTY. AS FREE MEN, BY LAW, THEY HAVE THE RIGHT TO RESIST EVERY OPPRESSION AND TYRANNY [4].
HOWEVER THEY DID NOT ASPIRED TO AN ABSTRACT PERFECTION...
It is necessary to unify practice and theory. We are not closing the gate to the improvements brought by progress of the civilization, by commerce with wiser nations, by the dissemination of brilliant works and future advances in political studies and national richness [5].
SANTIAGO, 1822

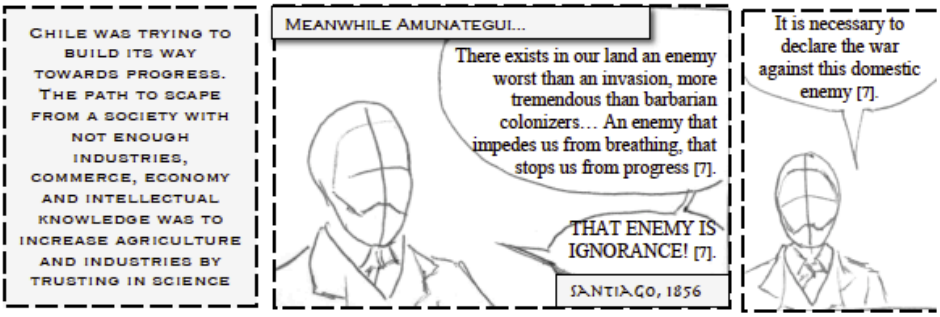
CHAPTER ONE: THE UTOPIA OF THE FUTURE



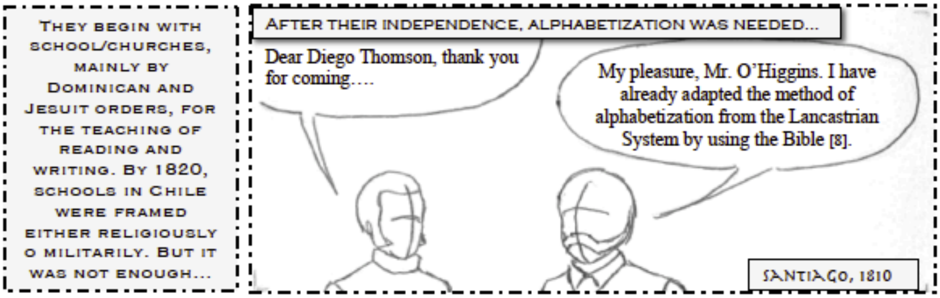
CHAPTER TWO: NEOLIBERAL PROMISES OF WELFARE

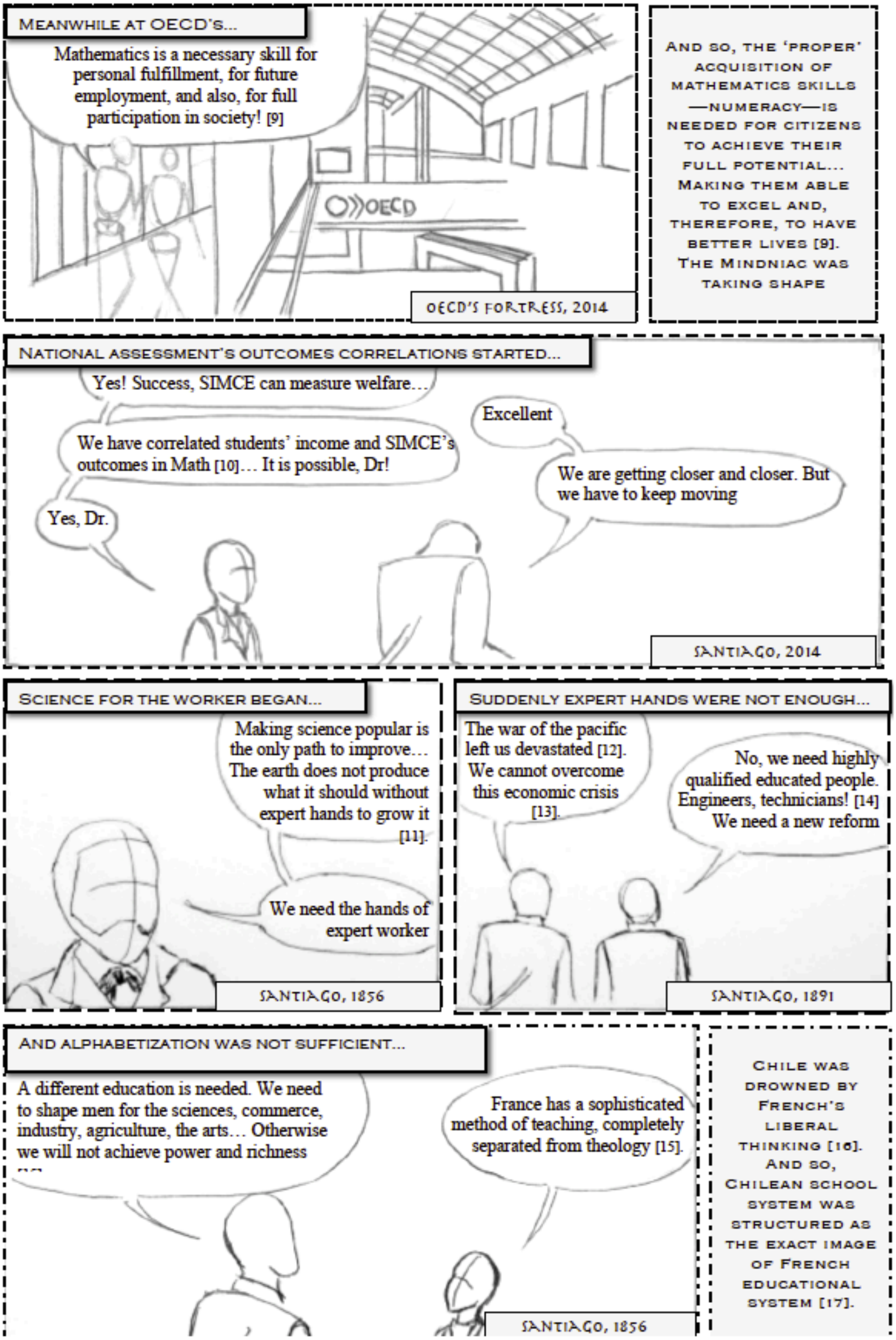


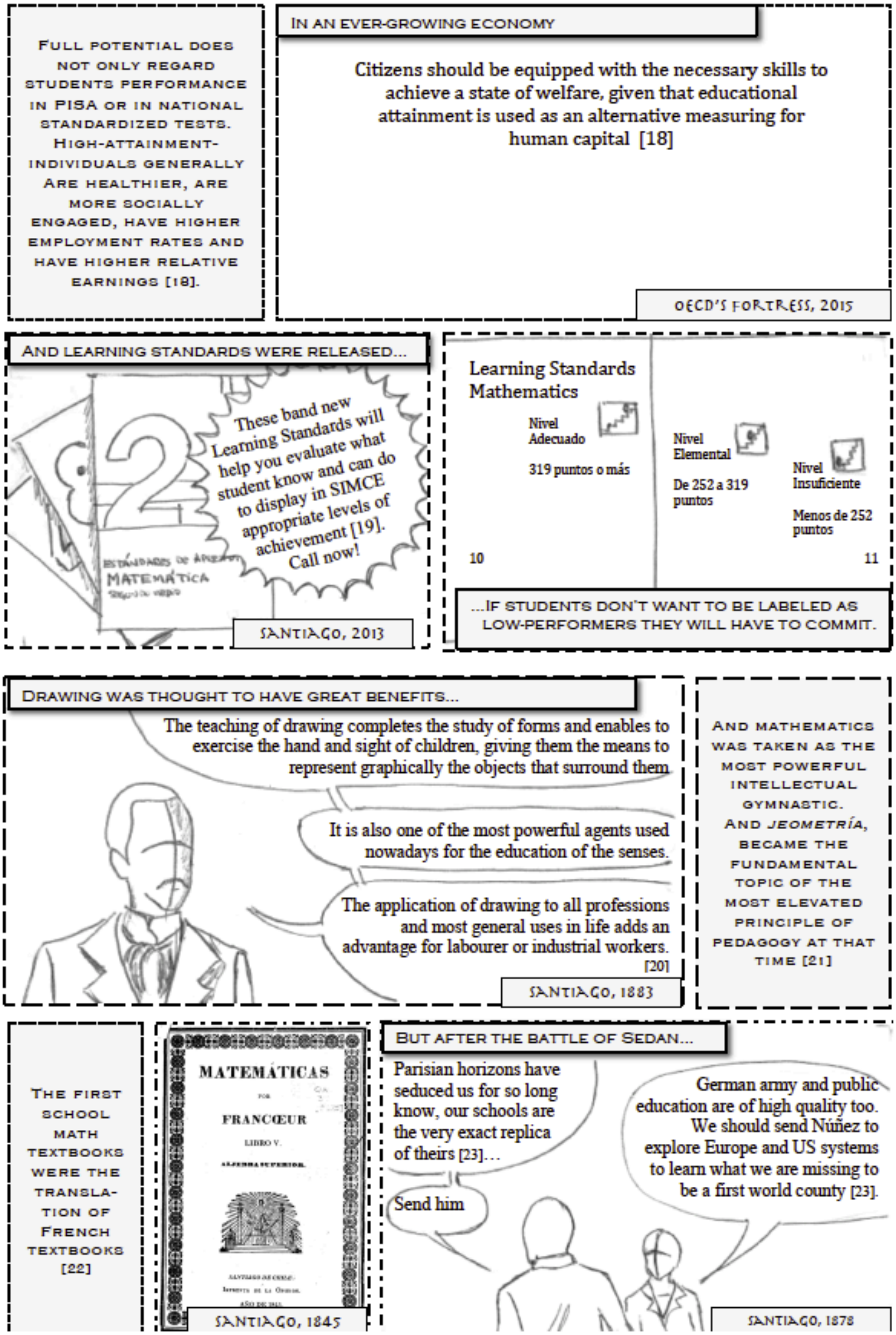
CHAPTER THREE: WHEN GEOMETRY WAS SPELLED WITH "J"

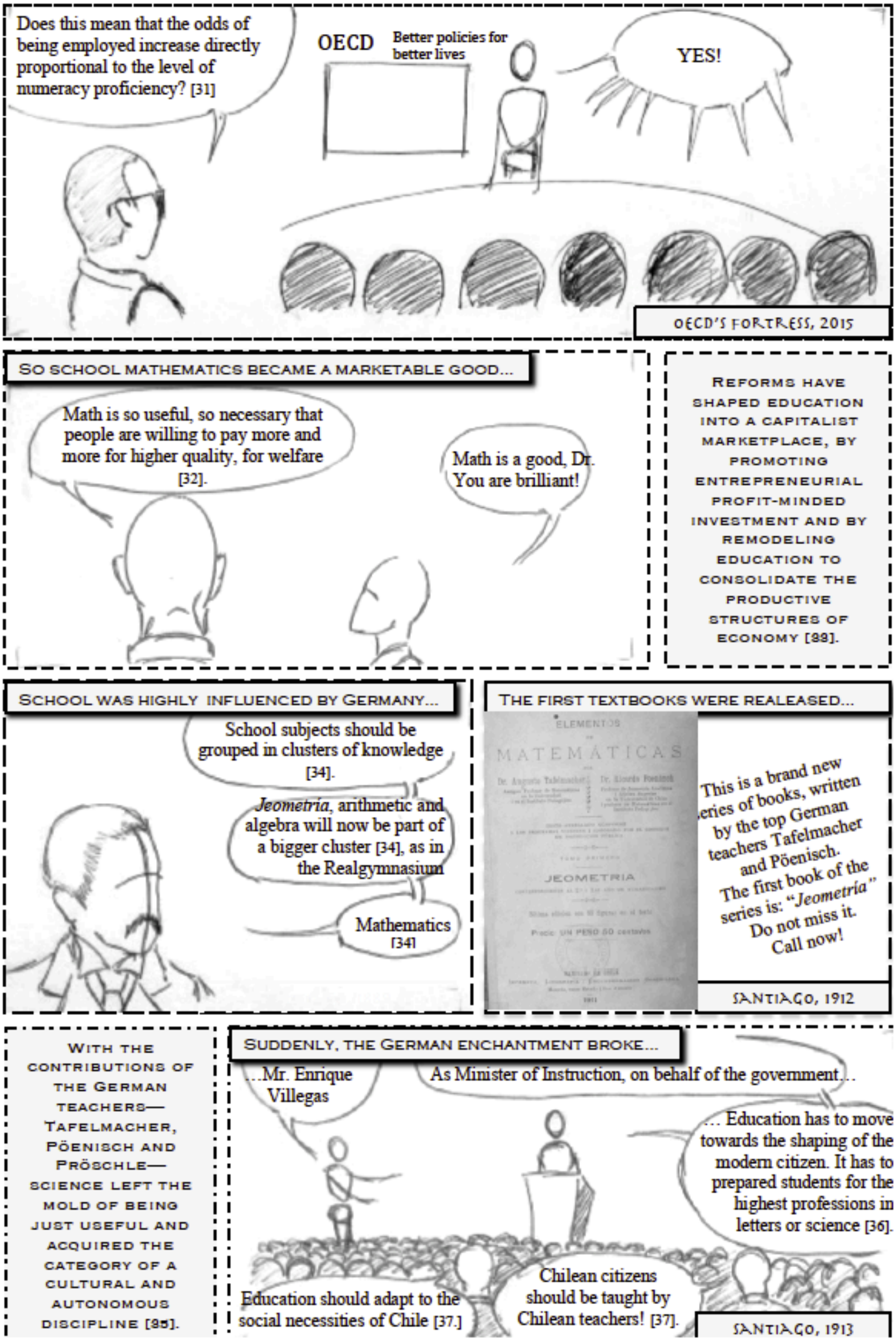


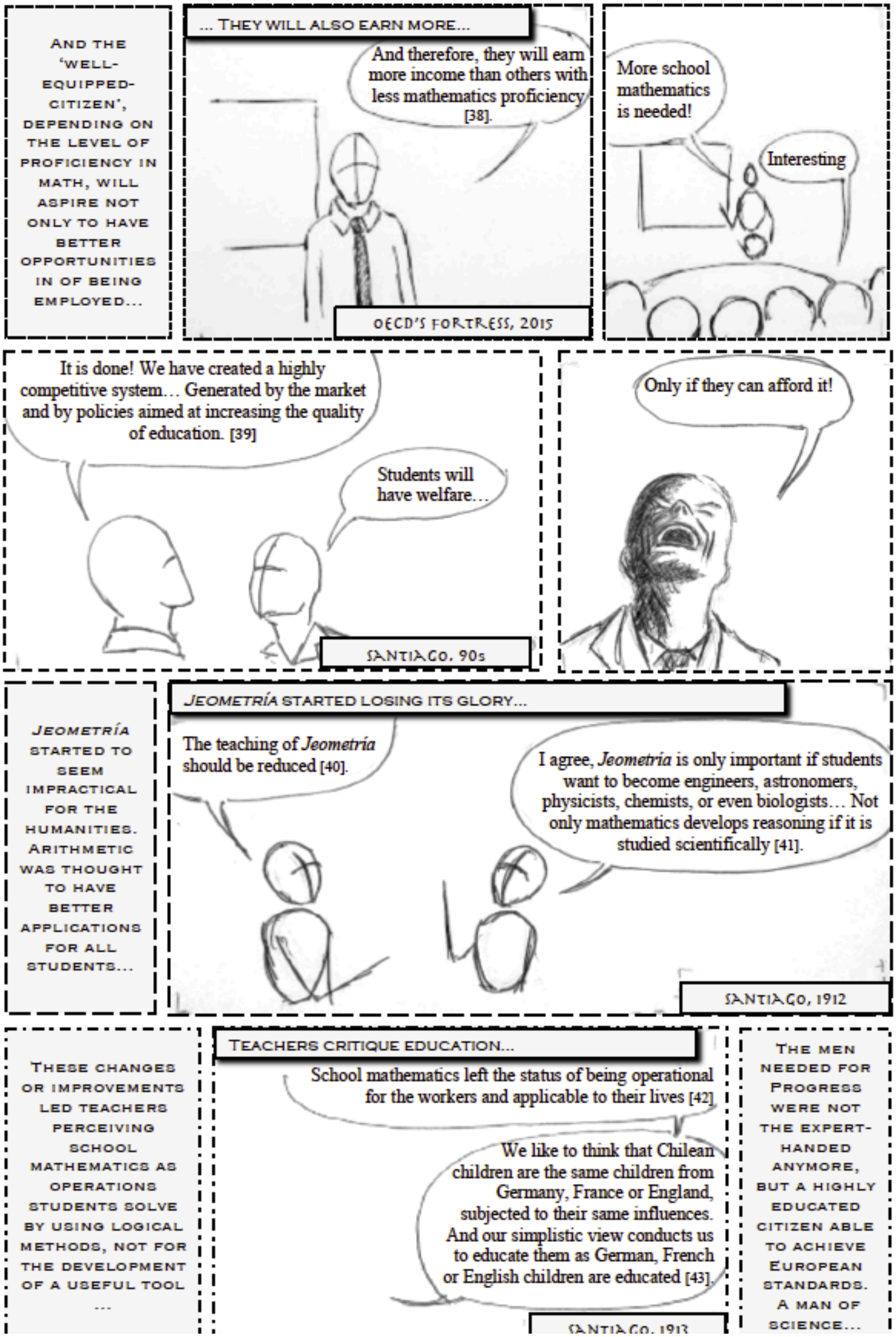
CHAPTER FOUR: SEDUCED BY EUROPEAN LANDS

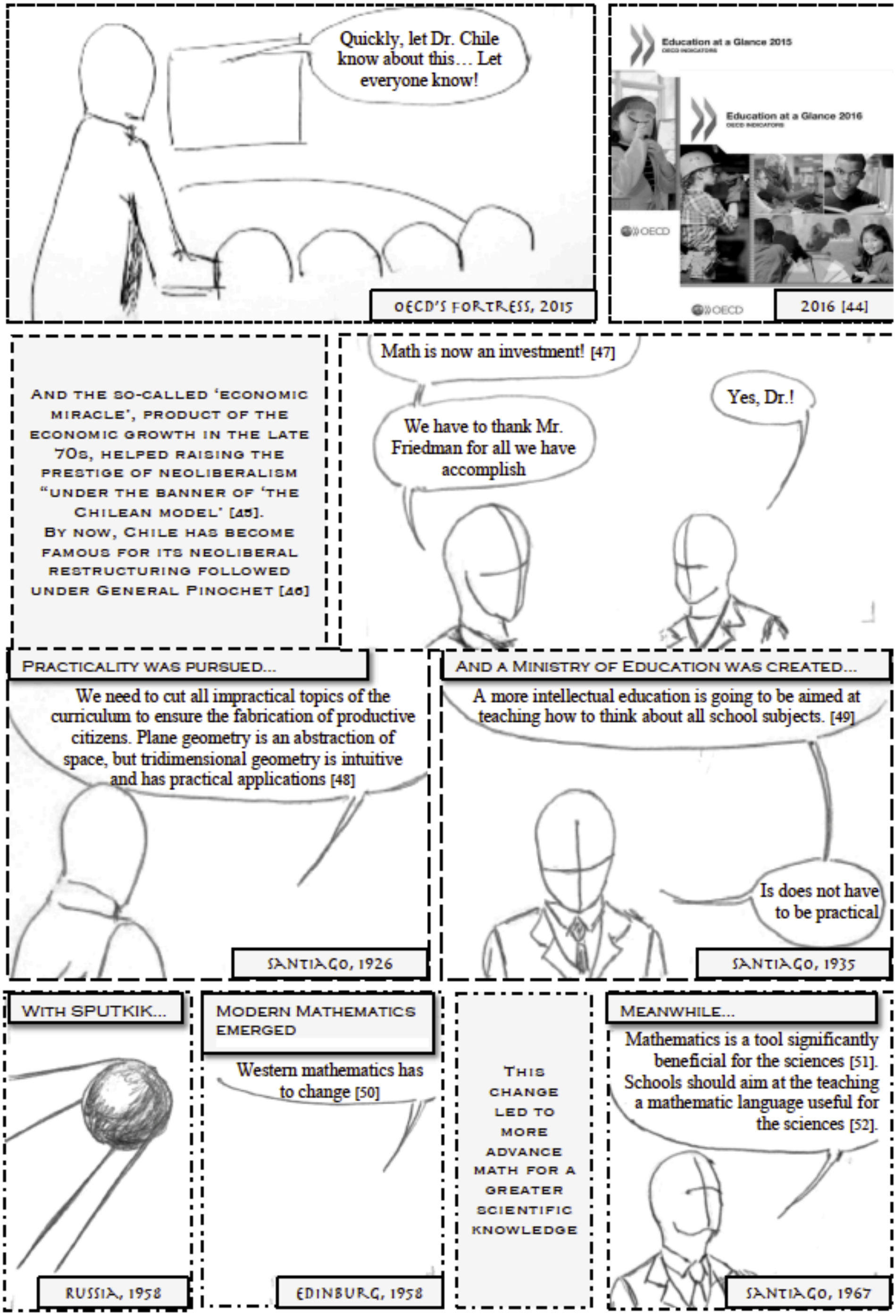



MEANWHILE AT OECD'S...
Mathematics is a necessary skill for personal fulfillment, for future employment, and also, for full participation in society! [9]
OECD
OECD'S FORTRESS, 2014
AND SO, THE 'PROPER' ACQUISITION OF MATHEMATICS SKILLS —NUMERACY—IS NEEDED FOR CITIZENS TO ACHIEVE THEIR FULL POTENTIAL... MAKING THEM ABLE TO EXCEL AND, THEREFORE, TO HAVE BETTER LIVES [9]. THE MINDNIAC WAS TAKING SHAPE
NATIONAL ASSESSMENT'S OUTCOMES CORRELATIONS STARTED...
Yes! Success, SIMCE can measure welfare…
Excellent
We have correlated students' income and SIMCE's outcomes in Math [10]… It is possible, Dr!
We are getting closer and closer. But we have to keep moving
Yes, Dr.
SANTIAGO, 2014
SCIENCE FOR THE WORKER BEGAN...
Making science popular is the only path to improve… The earth does not produce what it should without expert hands to grow it [11].
We need the hands of expert worker
SANTIAGO, 1856
SUDDENLY EXPERT HANDS WERE NOT ENOUGH...
The war of the pacific left us devastated [12]. We cannot overcome this economic crisis [13].
No, we need highly qualified educated people. Engineers, technicians! [14] We need a new reform
SANTIAGO, 1891
AND ALPHABETIZATION WAS NOT SUFFICIENT...
A different education is needed. We need to shape men for the sciences, commerce, industry, agriculture, the arts… Otherwise we will not achieve power and richness
France has a sophisticated method of teaching, completely separated from theology [15].
SANTIAGO, 1856
CHILE WAS DROWNED BY FRENCH'S LIBERAL THINKING [16]. AND SO, CHILEAN SCHOOL SYSTEM WAS STRUCTURED AS THE EXACT IMAGE OF FRENCH EDUCATIONAL SYSTEM [17].


Full potential does not only regard students performance in PISA or in national standardized tests. High-attainment-individuals generally are healthier, are more socially engaged, have higher employment rates and have higher relative earnings [18].
In an ever-growing economy
Citizens should be equipped with the necessary skills to achieve a state of welfare, given that educational attainment is used as an alternative measuring for human capital [18]
OECD'S FORTRESS, 2015
And learning standards were released...
These band new Learning Standards will help you evaluate what student know and can do to display in SIMCE appropriate levels of achievement [19]. Call now!
MATEMÁTICA
SANTIAGO, 2013
Learning Standards Mathematics
Nivel Adecuado
319 puntos o más
Nivel Elemental
De 252 a 319 puntos
Nivel Insuficiente
Menos de 252 puntos
10
11
...If students don't want to be labeled as low-performers they will have to commit.
Drawing was thought to have great benefits...
The teaching of drawing completes the study of forms and enables to exercise the hand and sight of children, giving them the means to represent graphically the objects that surround them.
It is also one of the most powerful agents used nowadays for the education of the senses.
The application of drawing to all professions and most general uses in life adds an advantage for labourer or industrial workers. [20]
SANTIAGO, 1883
And mathematics was taken as the most powerful intellectual gymnastic. And *jeometría*, became the fundamental topic of the most elevated principle of pedagogy at that time [21]
The first school math textbooks were the translation of French textbooks [22]
MATEMÁTICAS
FRANCŒUR
LIBRO V.
SANTIAGO, 1845
But after the battle of Sedan...
Parisian horizons have seduced us for so long know, our schools are the very exact replica of theirs [23]…
German army and public education are of high quality too. We should send Núñez to explore Europe and US systems to learn what we are missing to be a first world county [23].
Send him
SANTIAGO, 1878


Does this mean that the odds of being employed increase directly proportional to the level of numeracy proficiency? [31]
OECD
Better policies for better lives
YES!
OECD'S FORTRESS, 2015
SO SCHOOL MATHEMATICS BECAME A MARKETABLE GOOD...
Math is so useful, so necessary that people are willing to pay more and more for higher quality, for welfare [32].
Math is a good, Dr. You are brilliant!
REFORMS HAVE SHAPED EDUCATION INTO A CAPITALIST MARKETPLACE, BY PROMOTING ENTREPRENEURIAL PROFIT-MINDED INVESTMENT AND BY REMODELING EDUCATION TO CONSOLIDATE THE PRODUCTIVE STRUCTURES OF ECONOMY [33].
SCHOOL WAS HIGHLY INFLUENCED BY GERMANY...
School subjects should be grouped in clusters of knowledge [34].
Jeometria, arithmetic and algebra will now be part of a bigger cluster [34], as in the Realgymnasium
Mathematics [34]
THE FIRST TEXTBOOKS WERE REALEASED...
ELEMENTOS
MATEMATICAS
JEOMETRIA
This is a brand new series of books, written by the top German teachers Tafelmacher and Pöenisch. The first book of the series is: "*Jeometria*" Do not miss it. Call now!
SANTIAGO, 1912
WITH THE CONTRIBUTIONS OF THE GERMAN TEACHERS—TAFELMACHER, PÖENISCH AND PRÖSCHLE—SCIENCE LEFT THE MOLD OF BEING JUST USEFUL AND ACQUIRED THE CATEGORY OF A CULTURAL AND AUTONOMOUS DISCIPLINE [35].
SUDDENLY, THE GERMAN ENCHANTMENT BROKE...
...Mr. Enrique Villegas
As Minister of Instruction, on behalf of the government...
... Education has to move towards the shaping of the modern citizen. It has to prepared students for the highest professions in letters or science [36].
Education should adapt to the social necessities of Chile [37.]
Chilean citizens should be taught by Chilean teachers! [37].
SANTIAGO, 1913


AND THE 'WELL-EQUIPPED-CITIZEN', DEPENDING ON THE LEVEL OF PROFICIENCY IN MATH, WILL ASPIRE NOT ONLY TO HAVE BETTER OPPORTUNITIES IN OF BEING EMPLOYED...
... THEY WILL ALSO EARN MORE...
And therefore, they will earn more income than others with less mathematics proficiency [38].
OECD'S FORTRESS, 2015
More school mathematics is needed!
Interesting
It is done! We have created a highly competitive system… Generated by the market and by policies aimed at increasing the quality of education. [39]
Students will have welfare…
SANTIAGO, 90s
Only if they can afford it!
JEOMETRÍA STARTED TO SEEM IMPRACTICAL FOR THE HUMANITIES. ARITHMETIC WAS THOUGHT TO HAVE BETTER APPLICATIONS FOR ALL STUDENTS...
JEOMETRÍA STARTED LOSING ITS GLORY...
The teaching of *Jeometría* should be reduced [40].
I agree, *Jeometría* is only important if students want to become engineers, astronomers, physicists, chemists, or even biologists… Not only mathematics develops reasoning if it is studied scientifically [41].
SANTIAGO, 1912
THESE CHANGES OR IMPROVEMENTS LED TEACHERS PERCEIVING SCHOOL MATHEMATICS AS OPERATIONS STUDENTS SOLVE BY USING LOGICAL METHODS, NOT FOR THE DEVELOPMENT OF A USEFUL TOOL ...
TEACHERS CRITIQUE EDUCATION...
School mathematics left the status of being operational for the workers and applicable to their lives [42]
We like to think that Chilean children are the same children from Germany, France or England, subjected to their same influences. And our simplistic view conducts us to educate them as German, French or English children are educated [43].
SANTIAGO, 1913
THE MEN NEEDED FOR PROGRESS WERE NOT THE EXPERT-HANDED ANYMORE, BUT A HIGHLY EDUCATED CITIZEN ABLE TO ACHIEVE EUROPEAN STANDARDS. A MAN OF SCIENCE...


Quickly, let Dr. Chile know about this… Let everyone know!
OECD'S FORTRESS, 2015
Education at a Glance 2015
Education at a Glance 2016
OECD
OECD
2016 [44]
And the so-called 'economic miracle', product of the economic growth in the late 70s, helped raising the prestige of neoliberalism "under the banner of 'the Chilean model' [45].
By now, Chile has become famous for its neoliberal restructuring followed under General Pinochet [46]
Math is now an investment! [47]
We have to thank Mr. Friedman for all we have accomplish
Yes, Dr.!
Practicality was pursued...
We need to cut all impractical topics of the curriculum to ensure the fabrication of productive citizens. Plane geometry is an abstraction of space, but tridimensional geometry is intuitive and has practical applications [48]
SANTIAGO, 1926
And a Ministry of Education was created...
A more intellectual education is going to be aimed at teaching how to think about all school subjects. [49]
Is does not have to be practical
SANTIAGO, 1935
With SPUTKIK...
RUSSIA, 1958
Modern Mathematics emerged
Western mathematics has to change [50]
EDINBURG, 1958
This change led to more advance math for a greater scientific knowledge
Meanwhile...
Mathematics is a tool significantly beneficial for the sciences [51]. Schools should aim at the teaching a mathematic language useful for the sciences [52].
SANTIAGO, 1967

The well equipped: The healthy and successful lifelong learner

After the last economic crisis, the enterprise helping the Empire to excel—OECD—, posed the urgency of developing *well-equipped-citizens*. They detailed indicators for nations to know what it was needed, what they need to achieve [53] with the aim of promoting social progress... Experiments were conducted, conclusions were made and Nations were told that Mathematics is a knowledge that the Mindniac should master, a necessary skill for personal fulfilment, to enable all citizens to reach their full potential. Full potential not only regarding students performance in OECD's experiments—PISA—or in the experiments each nation decides to do—national standardized assessments—, but also high-attainment-individuals:
Generally have better health, are more socially engaged, have higher employment rates and have higher relative earnings [54].

The competent: the self-entrepreneur and knowledge consumer

Chilean experiments—SIMCE—became tools for the making of a marketable education and for the promotion of welfare and progress. Consumerism and competiveness are key elements for Neoliberalism [55], and so Chile has been (re)producing discourses in order to obtain the desired economic growth, progress and welfare through school mathematics, just as OECD advised them. And so, school mathematics, since it was thought to shape productive citizens, was taken as the key to become a developed country. After president Allende was overthrown, neoliberal ideas were taken as an entry to the first world of developed countries [56]. Education policies were transformed into economic policies of education [57] and private institutions were granted with the right to charge in excess to ensure higher quality [58]. All for fabricating the Mindniac!

The technical: The expert hand worker and the highly qualified

Since the 19th century, both a proper education and more suitable economic policies were the only way they thought to overcome the misery in Chilean [59]. Schools were aimed at teaching mainly applicable knowledge for future workers, the hands of experts [60]. Chile began to introduce the teaching of mathematics as a rational and analytical model and as a mode of the most powerful intellectual gymnastic [61]. According to Nuñez—the author of the most influential educational book of this period: *Organización de escuelas normales*—, number and form were keys, in which geometry, or *jeometría*, was taken as the fundamental topic of the most elevated principle of pedagogy at that time. But progress was slow, wars came, and the need of highly qualified minds led to an image of the productive citizen not as a worker in the use of their hands, but as producer in the use of scientific knowledge to increase economic progress and growth.

The reasonable: The scientific minded

The Chilean state policy of hiring the wise foreign ones to bring science to the country has been a constant practice since the first years of its independence [62]. The need of qualified scientific citizens has been developed with the purpose of growth and progress to achieve, or aspire to be, European levels. Over the years, diverse philosophies and methods of teaching have been imported to Chile—together with teachers, textbooks, and school's programs—from France, England, Germany, Sweden, the United States, and so much more. But, since 1973, an American paradigm has been dominating, a neoliberal model [63]. Although that is already history.

Epilogue

Many naturalized truths have been entangled and (re)produce in different places and times, a discourse in which education is believed as the key to future growth. From its fortress, the OECD enterprise, by voicing the need of an ever-growing economy for social progress and of lifelong learning, has been recommending that nations should equip citizens with the necessary skills to achieve the so desired state of welfare. This enterprise believes that the investment in the proper acquisition of mathematics skills is needed for all citizens of all nations to achieve their full potential, making them able to excel and, therefore, to have better lives [65]. The importance that mathematic literacy has within OECD's indicators has help moving nations towards the desired to improve all students' performances in school mathematics, as impossible that it may seem [66].

Similar to OECD's experiment called PISA, Dr. Chile's experiment, SIMCE, became the first step of knowledge consumerism and of a marketable education/society, given that its results have been publically published across all the land in national newspapers and widely discussed through other means of communication. And so, parents and society could decide which schools are the best and worst options for their children's future by analysing their results in SIMCE. Also, SIMCE results in mathematics were correlated with students' future income [67]. And so, school mathematics became a good people were willing to consume to achieve the promises of welfare [68]. Dr. Chile, being guided by the experiments' results of OECD and ideas of foreign lands, successfully created a highly competent and marketable system able to fabricate the perfect knowledge consumer needed for the economy. And so, whomever wanting to achieve welfare would have to pay for higher quality [69].

The need of qualified citizens, the perfect masterpiece for social economic progress, has been vibrant for many years, through many governments, all of them! Dr. Chile made their own experiments, seeking help in other nations; he tried to improve reforms. The 'expert-hand' workers, as Amunategui called them, had at the beginning their own technical schools, such as the School of Arts and Occupations. They enjoyed courses of arithmetic, elemental geometry and applied drawing specifically aimed at those industrial workers, a practical knowledge [70]. Suddenly, the Pacific war opened the new form of maximizing the saltpetre extraction and agriculture in the Central Valley, and therefore highly qualified engineers and technicians were urgently needed, the highly qualified men [71]. Dr. Chile kept making tests, trying to improve the desired product of schooling in the name of progress. The answer emerged from the old continent, science. They needed a specific language, and so, Dr. Chile started to promote in schools a mathematical language useful for the sciences [72].

All of the above translates in the promises of a 'well-equipped-citizens' that live longer, are socially active, healthier, volunteer, engage in political processes, trust others, are more likely to be employed and earn more, all because they reached their full potential thanks to their numeracy proficiency [73]. In the promises of a 'competent child', the self-regulated [74], the one who is willing to pay to achieve welfare, the one who is disposed to engage in regulatory schools practices, such as SIMCE. In the promises of a 'technical citizen', the men with practical knowledge, able to increase economy by having expert hands and a highly qualified mind. In the promises of a 'lifelong learner', the logical and reasonable 'scientific mind'… All of the above translates into the promises of the fabrication of the ideal citizen: the Mindniac. The Mindniac, the desirable productive citizen that will save nations from misery by conquering power and richness!

Endnotes

[1] OECD, for Tröhler, Meyer, Labaree, and Hutt (2014), is a central node for local and peripheral policy ideas to be expanded on and amplified. "Policies […] seem that much more irresistible when offered as uncontested consensus of the world's leading democracies" (Op. cit., p. 2).
[2] Drawing inspired in the painting "Proclamation of Independence" by Pedro Subercaseaux in 1945.
[3] Fray Camilo Henríquez in 1811. (Tribunal Constitucional, 2015, pp. 48–53, my translation).
[4] Chilean constitution, 1822. (Tribunal Constitucional, 2015, pp. 106-107, my translation).
[5] Milton Friedman's thoughts. (Moreno, 2008, p. 92).
[6] (Paredes & Ugarte, 2011, p. 199).
[7] Amunategui's speech in the Society of Primary Instruction in Santiago (Labarca, 1939, p. 141, my translation).
[8] "The English educator Diego Thomson, was invited by Bernardo O'Higgins to work in the alphabetization according to the Lancastrian system, which was a method of teaching through the Bible". (Cancino, 2012, p. 150).
[9] (OECD, 2014)
[10] Bharadwaj, Giorgi, Hansen, & Neilson, 2012.
[11] (Amunategui, 1856. In Labarca, 1939, p. 144, my translation).
[12] By 1891 Chile entered in an economic crisis as a result of the War of the Pacific (1879–1883) and the end of the Portalian state (Góngora, 1988).
[13] Both a proper education and more suitable economic policies were the only way thought to overcome the crisis (Encima, 1981).
[14][15] (Labarca, 1939).
[16] (Cancino, 2012).
[17] (Gutiérrez & Gutiérrez, 2004, p. 11, my translation).
[18] (OECD, 2015, p. 30).
[19] (MINEDUC, 2013).
[20] (Nuñez, 1883, p. 104, my translation).
[21] (Nuñez, 1883).
[22] (Gutiérrez & Gutiérrez, 2004, pp. 11–12, my translation).
[23] (Labarca, 1939).
[24] (OECD, 2015).
[25] (Ministerio de Educación, & CPEIP, 1967; Diaz & Giudici, 1970).
[26] (Consejo de Institución Pública, 1893, p. 95, my translation).
[27] (Consejo de Institución Pública, 1893).
[28] By 1909, some schools taught mainly "applicable knowledge" for future workers. And, therefore, they are able, for example, to deeply study the functioning of a machine, draw it, adjust its pieces, and so on (Labarca, 1939).
[29] (Vidal, 2010).
[30] (Labarca, 1939).
[31] (OECD, 2015).
[32] (Taylor, 2003).
[33] "[E]ducation was recast to promote studies functional to the new productive structures of Chilean society, whereas traditional arts and humanities studies were discouraged" (Taylor, 2003, p. 32).
[34] Pedro Montt, 1886. (Labarca, 1939).
[35] (Gutiérrez & Gutiérrez, 2004, p. 13, my translation).
[36] (Villegas, 1913, p. 4, my translation).
[37] (Labarca, 1939).
[38] (OECD, 2015).
[39] (Mizala & Romaguera, 2000).
[40] (Alfonso, 1912).
[41] (Galdámes, 1912).
[42] (Galdámes, 1913).
[43] (Galdámes, 1913, p. 86, my translation).
[44] (OECD, 2015; OECD, 2016).
[45] (Taylor, 2003, p. 25).
[46] (Silva, 1993; Aravena & Quiroga, 2016).
[47] See Taylor (2003)
[48] (Consejo de instrucción Pública, 1926).
[49] (Ministerio de Educación Pública, 1935).
[50] "[Western schools need] more advanced mathematics in all schools to give a greater scientific knowledge to students; men of science were needed" (Vidal, 2010, p. 9, my translation).
[51] (Deschamps et al., 1970).
[52] (Misrachi & Aspée, 1967).
[53] (OECD, 2014, p. 13).
[54] (OECD, 2015, p. 30).

[55] (Kaščák & Pupala, 2011).
[56] (Salazar, Mancilla & Durán, 2014).
[57] (Castiglioni, 2001).
[58] (Taylor, 2003).
[59] (Encina, 1981).
[60] (Labarca, 1939).
[61] (Nuñez, 1883).
[62] (Gutiérrez & Gutiérrez, 2004, p.11, my translation)
[63] (Cancino, 2012. P. 149, my translation).
[64] Competitiveness and accountability, within school mathematics testing, led to higher performances, higher incomes, higher social mobility and welfare (OECD, 2014).
[65] (OECD, 2014).
[66] Within the making of the citizen, OECD plays the role of 'homogenizing the heterogeneous' (Tröhler, et al., 2014), in which, all students having the same opportunities, access, and possibilities translates into "reform projects assum[ing] that social, economic, and educational inequalities can be minimized if all children have the opportunity to learn mathematics" (Diaz, 2013, p. 36).
[67] Bharadwaj, Giorgi, Hansen, & Neilson, 2012).
[68] See Andrade-Molina (2017).
[69] Within the promise of welfare, school mathematics is taken as necessary to achieve numeracy for the pursuit of individual happiness and human progress (Popkewitz, 2013).
[70] (Labarca, 1939)
[71] (Labarca, 1939).
[72] In the 60s, logic was taken as the foundation of every science, reasoning accurately and rigorously was the core of any argumentation and of critical thinking (Diaz & Giudici, 1970). Mathematics was thought as the skill that helped to develop reasoning and logical thinking, and reading proficiency was thought as a tool to better understand mathematical instructions (Ministerio de Educación, & CPEIP, 1967).
[73] (OECD, 2015).
[74] "[Self-entrepreneurs are]]individuals that self-regulate, self-direct and are continuously in a process of redefining their competences" (Cotoi, 2011, p. 116).

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THE AMALGAM OF FAITH AND REASON:

EUCLID'S ELEMENTS AND THE SCIENTIFIC THINKER

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ABSTRACT

Problematizing the truths of mathematics education is one of the roles of the philosophy of mathematics education. That mathematics education is a matter of reason and science —not of faith and religion—, and that mathematics is timeless, universal and immutable, objective knowledge that is independent from people's work and sense-making are two strong taken-for-granted statements that navigate in common understandings of mathematics education. Using a Foucault-Deleuze inspired analytical strategy, we examine the contention that mathematics education for the making of the rational and logical child intertwines with what was ought to be the 'scientific thinker' to Christianity. We focus on how Euclidean geometry, taken as a proper method of inquiry amalgamated with the Christian worldview to provide explanations about the natural world. In modern education, the core of this amalgamation continues connecting reason with faith, and science with Platonic views of mathematics. The effect of power is the making of the Modern scientific thinker.

Key words: Platonism in mathematics education, faith, sacralisation of mathematics, scientifisation of education.

INTRODUCTION

Part of doing philosophy of mathematics education, as Ernest et al. (2016) express, is about a systematic analysis and a critical examination of problems that are fundamental to mathematics education. By enabling

> [T]o see beyond the official stories about the world, about society, economics, education, mathematics, teaching and learning. It provides thinking tools for questioning the status quo, for seeing 'what it is' is not 'what has to be'; to see that the boundaries between the possible and impossible are not always where we are told they are. (Op. cit, p. 4)

Such phrasing resonates with our positioning on the cultural politics of mathematics education. We are interested in exploring the practices of mathematics education by making evident how mathematics and its inclusion in the school curriculum made part of the technologies of power/knowledge for the making of Modern subjectivity (e.g., Andrade-Molina & Valero, 2017; Valero & Knijnik, 2016). Blurring the division between fields of study such as history, cultural studies, educational sciences and mathematics education research itself, we broaden the possibilities to understand schooling as a social institution in concrete historical configurations, and the desire to strive for the fabrication of rational, enlightened subjects. Our analytical strategy, drawing from the toolboxes of Foucaultian and Deleuzian studies, invite to historicize the present in a rhizomatic search for how rationalities about mathematics and school mathematics have been constituted and have found a solid place in the current narratives of the undeniable necessity of mathematics for citizenship, society, economics now and in the future. From this perspective, the philosophy of mathematics education is concerned with moving beyond the official stories about what are the objects and subjects of mathematics education, and challenge the status quo of what has come to be accepted as taken-for-granted truths about school mathematics.

Elsewhere (Andrade–Molina & Ravn, 2016) we have discussed the value granted to Euclidean geometry for the shaping of scientific thinking since the structure of the Elements was taken, by scientific research, as a proper method of inquiry. This is important in understanding how in the desire for making the rational subject though education, the scientific and mathematical rationalities have been intertwined through history. Now, we take a step further in problematizing how modern narratives about the fabrication of the 'reasonable citizen' (Andrade-Molina, forthcoming) through the learning of school mathematics bring science, mathematics and religion together. Our contention is that current naturalized truths about the role of mathematics education for the making of the rational and logical child are intertwined with what was ought to be the 'scientific thinker' to Christianity. We problematize two truths that navigate in the way mathematics in the school curriculum and educational practices are currently conceived of. First, school mathematics forms the rational mind, and such formation is distinctly separated from faith and religion. In other words, mathematics education is conceived a secular project of rationality, advancing the Enlightenment for the expansion of human reason over obedience to the rule of faith. Second, it is the idea that mathematics is timeless, universal and immutable, objective knowledge that is independent from people's work and sense-making. In other words, particular notions of mathematics are embedded in the common practices of school mathematics despite efforts to introduce pedagogies rooted on the socio-cultural-political theories of knowledge and learning (e.g., Radford, 2008; Planas & Valero, 2016). We problematize these truths as interconnected statements tracing the historical justifications for the inclusion of Euclidean geometry as a topic of teaching and learning.

The dominant narratives of mathematics education position mathematics as an objective knowledge completely independent from faith, ideologies, culture, society or politics. However, the teaching of mathematics, in particular of Euclidean geometry, shaped scholastics and the expansion of Christianism on a quest for certainty and for a closer understanding of God. The view of mathematics as the language used by God in Christianity had a great impact in the forming of Western, Modern education. On the one hand, the Modern languages of education as cultural expressions emerged in an amalgamation of Christian notions of morality belonging to the different confessional orders in Europe in the 18th century and political ideals (Tröhler, 2011). Education as a tool of State governing through the fabrication of notions of the "moral man" also articulated the growing desire for scientific and mathematical knowledge (Valero & García, 2014). On the other hand, the quest for God in natural philosophy also promoted the advance of mathematics (Kvasz, 2004). Here we will explore the entanglement between modern discourses of mathematics education and the discourses on faith from Christianity.

SETTING THE SCENE

Before beginning to unpack the discourses on faith, we want to set the scene. All taken events occur in a time and place where there is no such thing as 'scientists', yet. What we currently identify as science had until the 18th century occurred in the realm of "natural philosophy" (Beltrán, 2009) or "natural history" (Foucault, 1971). And 'the Philosopher' par excellence of that time was Aristotle, given that his work was taken as the "eminent representation of science" (Op. cit., p. 284, our translation). His ideas "transformed the way the West thought about the world and its operations" (Grant, 2004, p. 14). For many years, the Elements of Euclid were considered as a particular expression of Aristotelian logic. Scholars made efforts to establish 'one-to-one correspondence' between Aristotelian logic —axiomatic— and Euclid's postulates (Gómez-Lobo, 1977), sometimes assuming that Euclid had put Aristotle's 'dictum' into practice while configuring the Elements (Mueller, 1969).

According to Descartes, what Euclid accomplished should be understood as a model for inquiry in all areas (Toulmin, 2003). His insistence on the "Euclidean model of knowledge planted some seeds in natural science between 1600 and 1650" (Op. cit., p. 43). Since then and until the 19th century, the Elements were "taken as the paradigm for establishing truth and certainty. Newton used [its form] in his Principia, and Spinoza in his Ethics" (Ernest, 1991, p. 4). The geometry on Euclid's books became a logical system:

> [The *Elements*] is one of the great achievements of the human mind. It makes geometry into a deductive science and the geometrical phenomena as the logical conclusions of a system of axioms and

postulates. The content is not restricted to geometry as we now understand the term". (Chern, 1990, p. 679)

This is precisely what other philosophers saw in Euclid's books: Content not only restricted to geometry. The Elements became a guide to produce science through a particular way of describing reality. And its status as an example of Aristotelian logic led it to expand throughout most fields of inquiry. That Euclid's axiomatic was the model to produce secure knowledge became a part of Western culture and appeared repetitively in many cultural expressions:

Fig. 1. Botticelli's painting of Saint Augustine

> Botticelli portrayed Augustine as a scholar-saint wearing clerical robes with an open treatise on geometry and a weight-driven clock nearby. He looks heavenwards, seeking the order that the Christian God (like Plato's demiurge) imposed on creation by dividing light from darkness. (Gamwell, 2015, p. 476)

THE AMALGAMATION OF FAITH AND SCIENCE

In the Middle Ages, Aristotelian logic was perceived as "the indispensable instrument for demonstrating theoretical knowledge" (Grant, 2004, p. 10). The use of principles instead of axioms led Aristotle's work to be considered as an axiomatization of science (Geréby, 2013). On the one hand, as aforementioned, natural philosophers relied on the work of 'the Philosopher' for their inquiries. Philosophy was viewed as a "wish to gain a rational understanding of the world in which we live, and the fundamental processes at work in nature, society and our own way of thinking" (Grant & Woods, 2002, p. 25). So, to understand the Universe is to comprehend the nature of things "by observation and reflection, to discover the causal principles, the forces, the powers and potentialities of the things that govern their behaviour" (Tiles, 2003, p. 351). On the other hand, there was a conflict among Christians on the symbolic or literal reading of the Bible (Midgley, 2005). Within this conflict, 'science' was taken as dangerous given that God forbid Adam and Eve to eat from the Tree of Knowledge or as archangel Raphael told Adam to be 'lowly wise' when he began questioning the nature of the universe (Wolpert, 2013). With the flourishing of natural philosophy, conservative theologians "were alarmed [...]. They were concerned that Aristotle's natural philosophy was circumscribing God's absolute power to do anything" (Grant, 2004, p. 12). According to Murray and Rea (2016, par. 2), "early Christian thinkers such as Tertullian were of the view that any intrusion of secular philosophical reason into theological reflection was out of order”. Natural philosophy sought for a kind of certainty that faith could not grasp.

The Christian Church took its wishes of expansion to spread the evangelic message as an agreement between faith and reason (Beltrán, 2009). And the Church used Aristotle’s language to articulate its documents without an interest on “establishing the truth […] but to only capitalize some possibilities of the Greek cosmogony in conditions to make more explicit the sense of the proper mysteries of religion” (Op. cit, 2009, p. 283, our translation). The need to reconcile faith and ‘science’ emerged in connection to this desire of expansion:

> [S]ome of the greatest Christian theologians clearly had defended the position that the concrete contents of religious truth should be based on reason alone. […]A rationalist position was particularly tempting in times in which there was a clear awareness that a legitimate interpretation of “revelation”—of scripture and tradition—was itself a work of reason. If the last meaning of scripture was allegorical, tropological, and anagogical, then this meaning had to be based on rational arguments, which alone could have the power to transcend literal meaning. (Hösle, 2013, pp. 2–3)

Thomas Aquinas balanced a Catholic discourse of faith and science rooted in Aristotelian philosophy; he believed that God’s existence was rationally demonstrable (Hösle, 2013). How else can one approach God, who is unreachable by the senses, if not by the use of logic and reason? In this need of amalgamation, “Aquinas’ *Summa Contra Gentiles* is a good example of how dialectical investigations have been carried out in philosophy and theology” (Bovell, 2010, p. 70). The claims made by either theology or philosophy under the Thomistic model, were not believed to conflict anymore (Murray & Rae, 2016), since “some truths can be known only through faith, some can be known only through reason, and some can be known through either faith or reason” (Garcia, 2003, p. 623). In his *Summa,* analogue to philosophy, “theology consists of (theological) principles, and (theological) theses derived from these principles” (Geréby, 2013, p. 175):

> The genius of Aquinas articulated itself in the fact that he transformed the insecurity of Christianity that resulted from the discovery of the Aristotelian corpus, […], into a positive development and, despite many hostilities that culminated in suspecting him of heresy, conceived a great synthesis of Christianity and Aristotelianism that satisfied both the religious need and the need for knowledge of the empirical reality. (Hösle, 2013, p. 151)

Prior to Aquinas, Augustine established a connection between faith and natural philosophy where philosophy complemented theology "but only when these philosophical reflections were firmly grounded in a prior intellectual commitment to the underlying truth of the Christian faith" (Murray & Rea, 2016, par. 2). His work had a significant impact in Christianity (Finocchiaro, 1980), "in the process that would eventually lead to the rationalization of medieval theology" (Grant, 2004, p. 39). Augustine's theory of illumination is embedded in Malebranche and Descartes's works (Spade, 1994). Grant (2004) argues that Augustine had a Platonic interpretation of the 'valid rules of logic', which made him believed that

> [L]ogic was a valuable tool that would enable them to infer the correct conclusions from the initial [Scriptural or doctrinal] premises […] With this attitude toward logic and reason, Augustine was not reluctant to use analytic tools – especially Aristotle's categories – in his analysis of doctrinal truths, as he did in one of his greatest works, the fifteen books of On the Trinity (p. 39).

The amalgamation of faith and science, through Aristotelian logic, fulfilled the Christian need to base 'revelations' on reason and logic. This allowed adding certainty to the allegorical interpretation of the Bible. From this merging, thinkers of the world of faith made contributions to the world of empirical philosophies, for example, Aquinas' and Augustine's contributions to astronomy (see Campion, 2014). And despite being regarded as two separated, even opposite, fields of knowledge in modernity, both are not "games that can be played independently of each other. Both are about truth, and reality. Their divided claims cannot stand with the assumption of there being one single reality" (Geréby, 2013, p. 177).

WHEN SCHOLASTICS MET EUCLID

A religious search for knowledge and certainty has not to be reduced only to a simplistic discussion about God, or to what we currently consider theology as "the science of God" (Hösle, 2013). Within the Church's structure, Bishops, the teachers of Christianity, had a preeminent authority among scholastics to provide instruction. Scholastics were not to rely only on their faith and their beliefs. And so, 'students' of the religious orders were educated under the oeuvre of classical pagan philosophers of Ancient Greece, for example, Plato's theories of the soul: "Platonism held that the soul could exist apart from the body after death. This would obviously be appealing to Christians, who believed in an afterlife" (Spade, 2016, par. 9). Scholastics were encouraged to study 'sciences': geometry, astronomy, Aristotelian logic, Platonic tradition, among others (Clavius, 2002). They were also encouraged to translate and reproduce the most prominent books of the time. For example Boethius' translations from Greek into Latin of Aristotle's and Plato's most dominant works, including Boethius' commentaries to 'illuminate' their philosophies (Marenbon, 2009). They also translated the Elements of Euclid, since they recognized in these books more than just geometrical knowledge of Ancient Greece. They used them to study proof, common notions, and axiomatics; and through such study the reasonable and scientific thinker needed for Christianity could be shaped:

> [The student] offer new proofs of some of the propositions of Euclid, thought out by himself; in these places, let praise be given to those who best solve the problem proposed, or who commit the fewest false syllogisms, which occur not rarely, in the invention of the new proofs. For it would happen thus, that they would become not a little eager for these studies, when they see such honor given to them, and at the same time would understand the eminence of these same studies, and they would make greater progress in these things through this exercise. (Clavius, 2002, p. 467)

The structure of the Elements became a very powerful model for achieving certainty through axiomatics. The Elements began intertwine with the productions of faith. Scholastic made "efforts to build theological systems from scriptural texts as plane geometry is built from the postulates of Euclid". (Pals, 2011, p. 919). Nicholas of Amiens, a French theologian from the 12th century, wrote Ars Catholicae Fidei [Art of catholic faith] based on Euclid's books. Amiens provided a sequence of arguments to set the rules of the Catholic faith by assembling "definitions [descriptions], postulates [petitiones], and common notions, or axioms [conceptions]" (Grant, 2004, p. 67). Aquinas' book Summa Contra Gentiles became the best example of the amalgamation of faith and science, theology and philosophy. According to Bovell (2010), Aquinas' work is rarely related with Euclid's books, but his *Summa Contra Gentiles* 'mimics' the configuration of the *Elements*. "These seem the syntactical equivalents to Euclidean proofs of demonstration and play analogously the same role in Aquinas as proofs of demonstration do in Euclid" (Op. cit., pp. 70-71). As his predecessors, Herbert de Cherbury, an English philosopher of the 17th century, borrowed the term 'common notions' from Euclid as the foundation of reasoning in his book *De veritate* (Serjeantson, 2001). de Cherbury contends that "the being of God is indicated by the structure of reality" (Pailin, 1983, p. 198) and, so, his existence can be determined by reason and by observation of the natural world (de Cherbury, 1633).

To Christianity the *Elem*ents were special; they encapsulated "a form of reasoning, and a handmaiden of natural theology" (Cohen, 2007, p. 164). It had common notions, rigorous mathematical proofs, and a deductive system.

> [C]ommon notions are the ultimate and indisputable principles by which understanding ought to be governed and are God's way of ensuring that every person has what is essential 'for this life and for life eternal'. They are not, however, principles which everyone is always aware of. They

> emerge to consciousness only when the mind has been aroused by appropriate experiences. What is common to all people is the basic structure of understanding through which any individual, suitably provoked, may come to recognize them and perceive their certainty. (Pailin, 1983, p. 198)

In this regard, it is not rare that the *Elements* were used in missions to expand Christianity. The Italian Jesuit Matteo Ricci recognized in Euclid's books "something different, namely, propositions presented in order and so definitely proven that even the most obstinate could not deny them" (Gallagher 1942, p. 471). As an example, the Elements were brought to China as a mean to introduce Aristotelian logic (Yuan, 2012). Matteo Ricci and the Chinese mathematical scholar Xu Guangqi translated the firsts books into Chinese. The Jesuits selected these books to deal with the main differences between Western and Chinese culture. According to Yuan (2012), Chinese logic is pragmatic, inscribed in a world of concrete objects that is always flowing, always changing. On the contrary, Aristotelian logic is a hierarchical order system of abstract concepts. According to Gallagher (1942), Euclidean geometry pleased Chinese as much as any other form of knowledge.

> Though no Chinese understood Aristotelian logic at the beginning, *Jihe yuanben* [Euclid's book I–VI], as a mathematical text and source of logical training, became more and more popular in China over the last four hundred years […] By contrast, studying Aristotelian logic itself is still considered as difficult work. (Yuan, 2012, pp. 83–84)

SACRED MATHEMATICS AND THE PATH TO GOD

To Christianity, the Elements were not only an instrument to teach scientific reasoning through Aristotelian logic for the shaping of the scientific thinker. Neoplatonism helped giving a different status to geometry, and mathematics was conceived as a form of thinking for approaching Immutable Truths (Grant, 2004). The Greek Proclus based his philosophy on Plato's idealist thoughts, the Neoplatonic, in which, "beings exist in a cave of impaired perception, a profane realm of limited, imperfect things: matter, decay, ever-changing shapes" (Cohen, 2007, p. 19). To Proclus, human existence occurs in the realm of muddled existence. The divine, above humans, is a 'sphere' of purity and eternal Truth. And "[m]athematics plays a special role in this divided universe —it ascends from the world of impermanence to this higher, heavenly plane. (Cohen, 2007, p. 19). The notion o infinity was considered as negative in Ancient Greece, however it was taken as a path to God for medieval scholars: "Theology made the notion of infinity positive, luminous, and unequivocal [infinity] was interpreted as the consequence of human finitude and imperfection" (Kvasz, 2004, p. 114). For example Leibniz's soul-like monads metaphysical system, in which "God is needed to ensure that the components of the universe interact as harmoniously as possible" (Francks & Macdonald, 2003, p. 665). Or Descartes attributing God a fundamental role in the conservation of momentum.

> Descartes is one of the earliest philosophers who sees in the conservation laws of physics an expression of God's immutability and even if he adduces as an example the conservation of momentum, he still regards momentum as a scalar magnitude, not as a vector. Therefore, he can believe that the *res cogitans* may influence the mere direction of the *spiritus animales* without altering the quantity of momentum (Hösle, 2013, p. 99).

Natural philosophers "saw all regular phenomena as marks of God's Rational Order" (Toulmin, 2001, p. 51), It is in this sense that Christianity sacralized mathematics as the path to access divinity. The realm of the primary causes, the cause without cause, is reserved to God. But the realm of the second causes, the natural world derived from the first cause, can be understood, studied and known through science and reason. Conservative theologians saw in the natural order of things a clear proof of the hands of God regarding the creation of the Universe and of humanity. Even today, primary causes are questions that science cannot solve for Christianity. Pope John Paul II claimed, the conference about cosmology, held in the Vatican in 1981, that "there is needed that human knowledge that rises above physics and astrophysics and which is called metaphysics; there is needed above all the knowledge that comes from God's revelation" (John Paul II, 1981, par. 5). And he told the participants they could "study the evolution of the universe after the big bang, but [physicists] should not inquire into the big bang itself because that was the moment of creation and therefore the work of God" (Hawking, 2003, p. 67).

Reading the *Elements* as the vehicle to reach an understanding of the "first cause", enables to describe the connection that mathematics establishes between man and God:

> John Dee inherited this occult quest and was convinced that mathematics was the special language that would transport its conjurer to that higher plane of divine truth. Dee's introduction to Euclid's Elements encapsulated the purpose and efficacy of mathematics in a manner that resonated with the mathematical idealism of the early Victorian age […] Dee divided all things in the universe into three categories: the natural, the supernatural, and the mathematical. Natural things are perceivable, changeable, and divisible. Supernatural things are invisible, immutable, and indivisible. Mathematical concepts occupy a critical middle position between the natural and the supernatural, thus mediating between these realms. (Cohen, 2007, pp. 21-22)

The *Elements* were not only to teach scholastics how to reason and to be logical, since it was believed that the "knowledge of God cannot be achieved by means of science, it was thought to be beyond the reach of reason" (Hösle, 2013, p. 3). Geometry was thought to be the architecture of divinity. Through the understanding of natural things scholastics could approach the purity and eternal realm of God's structure of the universe. And since then this idea has navigated widely in a variety of expressions of Western culture, from Christian medieval expression of God as the geometer of the Universe, to Mandelbrot's geometry of nature theory —fractals—, to the molecular composition of the DNA.

Fig. 2. Geometry as a divine architecture in three expressions of Western culture.

The latter is a modern example of the sacred character given to geometry "The helix, which is a special type from the group of regular spirals, results from sets of fixed geometric proportions" (Lawlor, 2002, p.4). "Fixed" geometric proportions not necessarily mean that DNA is a creation of God, but gives to geometry a Platonic character, a supernatural thing, according to Dee's division. As Lawlor (2002) continues these fixed geometric proportions "can be understood to exist *apriori*, without any material counterpart, as abstract, geometric relationships. [The helix] existence is determined by an invisible, immaterial world of pure form and geometry" (p.4).

In medieval schools, mathematics was included as part of the *quadrivium* (astronomy, geometry, arithmetic and music) and it was taught by scholastics. The aim was to "yield knowledge concordant with both human reason and the Christian faith" (Garcia, 2003, p. 620). The Jesuit mathematician Clavius expressed that "one cannot understand various natural phenomena without mathematics" (Smolarski, 2002, p. 258). A need to teach geometry and mathematics emerged: Mathematics to achieve Truth in the divine.

> [B]ecause of [mathematics] participation in both the perfect and imperfect spheres of existence, mathematics provides a mental pathway for ascending out of the material realm and attaining an ideal comprehension of the universe. (Cohen, 2007, p. 20)

THE DISCOURSES ON FAITH

Aristotelian logic and Neoplatonism gave to Christianity a solid foundation on which to ground their beliefs on the existence of God. Both gave to their philosophy the certainty needed in the Middle Ages. The *Elements*, taken as the perfect example of Aristotle's understanding of science, helped in shaping the 'scientific thinker'. *Geometry* was taken as a deductive science with logical conclusions (Chern, 1990). And so, Euclid's books were taken to be the core of any science, to Christianity. The learning of common notions, propositions, axioms, and proof enable scholastics to engage in scientific (philosophical) discussions with the books produced by Christianity. For example, de Cherbury belief in God "is not derived from history, but from the teachings of the Common Notions" (Pailin, 1983, p. 200). The discourses on faith about science became intertwined with what we know today as science.

Medieval schools emerged as a previous step towards the formation of the university. The latter, "was a wholly new institution that not only transformed the curriculum but also the faculty and its relationship to state and church" (Grant, 2004, p. 29),. And although it seems that Christian beliefs, in contemporaneity, have been distinctly separated from school curricula, it seems fair to conclude that there is no religious beliefs been reproduced in schools nowadays. Buchardt (2016, p.1) argues that, while "it is common sense in the educational field that religion before modernity has played a central role in education, opinions differ when turning to a perspective of the present" She argues that the apparent secularization of education through its increased scientification has created the idea that contemporary schooling is about science —even in subjects such as "Religion". However, a close analysis would reveal how religious notions guiding education reconfigured into new, secularized and scientifized forms of school subjects (Buchardt et al., 2016). In the case of school mathematics, the discourses on faith that have historically made part of the amalgamation of religion and science through mathematics are the connecting thread that binds faith with reason. Such fine thread remains though unexposed in current understandings of school, school mathematics, mathematics and science, although some work has pointed in that direction (e.g., Restivo, 2008; Valero & García, 2014; Peñaloza & Valero, 2016).

As we showed, the teaching of mathematics shaped scholastics and supported the expansion of Christianism on a quest for certainty and for a closer understanding of God. The historical amalgamation of faith and reason through the articulation of theology and science in Christianity positioned Eculidean geometry and its axiomatics as a privileged element in education for the making of a desired knowledgable, scientific and faithful self. In schools, mathematics is still mainly though of as a sacred, timeless, universal, objective knowledge, and an immutable truth in the sense of Christianity. It is not the path to access the purity of God, but to access the purity of the Platonic world of ideas. School mathematics seems not to be mutable, as the history of mathematics and mathematics education reveal. Despite this, it keeps on being conceived as fixed. Some kind of essence seems to escape the possibility of transformation. Not all students nowadays are meant to be mathematicians, nonetheless all are expected to recognize in mathematics the key to access knowledge… just as medieval scholastics did.

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SUMMARY

This dissertation explores the socio-political dimension of mathematics education. It problematizes the power effects of school geometry, particularly Euclidean geometry, in the fabrication of modern and cosmopolitan subjects. Based on Foucault's discursive analysis strategies and power/knowledge relationships, this study maps the scientification of the self through school mathematics, in which the desired citizen is shaped via the learning of elementary geometry. The mapping is conducted in two moments. First, it uses a discourse analysis of the self-evident truths about the "goodness" of mathematics for the future of the students. These truths have granted mathematical knowledge salvation narratives as a secure way to achieve success, welfare, health, social commitment and economic stability. Second, it uses a historization of the present to map the continuities and discontinuities that make possible to fabricate a desired rational and scientific citizen. It seeks to understand how truths about the need for a scientific child for economic progress are established. This thesis demonstrates the emergence of what is called the (d)effect of power of a specific scientific subjectivity. In which, school mathematics has a disciplinary and normalizing (homogenizing) effect in the fabrication of the productive and desired subjected body.

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