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# Buffeting

## Buffeting Response of Suspension Bridge Girder with Separate Control Flaps

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# Buffeting Response of Suspension Bridge Girder with Separate Control Flaps

Truc HUYNH<sup>\*</sup> & Palle THOFT-CHRISTENSEN<sup>†</sup>

**Abstract:** This paper presents the calculation of the root mean square (RMS) response of a suspension bridge using separate control flaps (SCF) in turbulence conditions. It is assumed that the mean wind velocity is not large enough to cause coupled vibrations and that single mode buffeting response is of interest. The RMS response is determined on the basis of the equation of motion, which is formulated stochastically according to the wind random turbulence components. It is further assumed that the sum of the motion-induced forces and the buffeting-induced forces from the girder and the flaps is computed on the basis of independent flutter derivatives and independent aeroelastic coefficients from the girder and from the flaps. The theory is demonstrated by a numerical example based on a long-span suspension bridge model with the Great Belt girder.

**Key words:** Aeroelastic Forces, Buffeting Forces, Wind Spectra, Stochastic Modal Analysis, Suspension Bridges.

## 1. Introduction

Several short-span cable-supported bridges built in the 19th century have been oscillating in both purely vertical and purely torsional modes due to the wind, William [3] and Scruton [6]. Assuming that the mean wind velocity  $U$  is constant along the span, the flutter wind velocity can be considerably increased when aeroelastic forces of the separate control flaps attached along the girder are used. Further, depending on the flap lengths along the girder and the flap configurations in different locations, control spillover can be regulated or omitted in multimode coupled flutter, Huynh [1] and Huynh & Thoft-Christensen [7].

However, the wind buffeting effect due to the natural wind flow can induce vibrations of the bridge at a wind velocity lower than the flutter wind velocity.

The girder response to turbulence buffeting in a *single mode* is addressed in this paper for several reasons. Firstly, to be able to reduce the complications related to the forces from the control flaps. Secondly, since the buffeting vibration occurs at a lower mean wind velocity than flutter, the modal coupling effects due to wind action are usually not strong compared to those of damping. Thirdly, to ensure that the single-mode vibrations do not develop a catastrophic vibration amplitude. Finally, multimode coupled buffeting analysis for the girder with SCF can be developed from the analysis in the present paper and from a series of papers by Scanlan and his associated workers for a traditional suspension bridge.

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## 2. Formulation of Generalised Forces and Equation of Motion

Let  $M_i$  denote the generalized inertia of a full-span bridge in vibration mode  $i$ . The equation of motion for mode  $i$  is

$$M_i \left[ \ddot{\xi}_i + 2\zeta_i \omega_i \dot{\xi}_i + \omega_i^2 \xi_i \right] = F_{iae}^{tot}(t) + F_{ib}^{tot}(t) \quad (1)$$

where  $\xi_i(t)$  is the generalised coordinate in mode  $i$ .  $\omega_i$  is the radian natural frequency and  $\zeta_i$  is the damping ratio without wind in mode  $i$ . The total generalised forces on the right-hand side of Eq. (1) consist of the aeroelastic forcing term “ $ae$ ” and the buffeting term “ $b$ ” of the girder and of the flaps in mode  $i$ , respectively. They are defined by:

$$\begin{bmatrix} F_{iae}^{tot}(t) \\ F_{ib}^{tot}(t) \end{bmatrix} = \int_{deck} \begin{bmatrix} L_{ae}^{deck} + L_{ae}^{le} + L_{ae}^{tr} & M_{ae}^{deck} + M_{ae}^{le} + M_{ae}^{tr} \\ L_b^{deck} + L_b^{le} + L_b^{tr} & M_b^{deck} + M_b^{le} + M_b^{tr} \end{bmatrix} \begin{bmatrix} \phi_i(x) \\ \psi_i(x) \end{bmatrix} dx \quad (2)$$

where  $\phi_i(x)$  and  $\psi_i(x)$  are the vertical and the torsional mode shapes in mode  $i$ .  $L_{ae}^{deck}$ ,  $L_{ae}^{le}$  and  $L_{ae}^{tr}$  are the motion-induced lift per unit span of the girder, the leading and the trailing flap, respectively.  $M_{ae}^{deck}$ ,  $M_{ae}^{le}$  and  $M_{ae}^{tr}$  are the corresponding motion-induced moment per unit span. It is assumed that the lifts depend on the vertical motion only and that the moments depend on the torsional motion and its velocity only. Then, Simiu & Scanlan [5]

$$L_{ae}^{deck}(x, t) = \frac{1}{2} \rho B^2 \omega_{i0} H_1^* \phi(x) \dot{\xi}_i(t) \quad (3)$$

$$M_{ae}^{deck}(x, t) = \frac{1}{2} \rho B^4 \omega_{i0} A_2^* \psi(x) \dot{\xi}_i(t) + \frac{1}{2} \rho B^4 \omega_{i0}^2 A_3^* \psi(x) \xi_i(t) \quad (4)$$

$$\begin{bmatrix} L_{ae}^{le}(v_z, r_{ae}^{le}) \\ L_{ae}^{tr}(v_z, r_{ae}^{tr}) \end{bmatrix} = \frac{1}{2} \rho B'^2 \omega_{i0,c} H_5^* \begin{bmatrix} a_{le} \\ a_{tr} \end{bmatrix} \phi(x) \dot{\xi}_i(t) \quad (5)$$

$$\begin{bmatrix} M_{ae}^{le}(v_z, r_{ae}^{le}) \\ M_{ae}^{tr}(v_z, r_{ae}^{tr}) \end{bmatrix} = \frac{1}{2} \rho B'^4 \omega_{i0,c} A_6^* \begin{bmatrix} a_{le} \\ a_{tr} \end{bmatrix} \psi(x) \dot{\xi}_i(t) + \frac{1}{2} \rho B'^4 \omega_{i0,c}^2 A_7^* \begin{bmatrix} a_{le} \\ a_{tr} \end{bmatrix} \psi(x) \xi_i(t) \quad (6)$$

where  $\rho$  is the air density,  $B$  is the girder width,  $B'$  is the flap width,  $\omega_{i0}$  is the vibration frequency of the bridge when the motion-induced forces take place,  $\omega_{i0,c}$  is the frequency when this motion is affected by the control flaps.  $H_1^*$ ,  $A_2^*$  and  $A_3^*$  are the uncoupled flutter derivative of the girder depending on the actual frequency of the bridge under wind action.  $H_5^*$ ,  $A_6^*$  and  $A_7^*$  are similarly the uncoupled flutter derivatives of the flaps determined by the Theodorsen circulatory function (also frequency dependent).  $a_{le}$  and  $a_{tr}$  are the rotational amplification factor of the leading and the trailing flaps. For  $a_{tr} = a_{le} = 1$ ,  $r_{ae}^{le} = r_{ae}^{tr} = r_x$  where  $r_x$  is the rotation of the girder. The buffeting-induced lift and moment per unit span of the girder and the flaps are, Simiu & Scanlan [5]

$$L_b^{deck}(x, t) = \frac{1}{2} \rho U^2 B [2C_L u(x, t)/U + (C_L' + C_D) w(x, t)/U] \quad (7)$$

$$M_b^{deck}(x, t) = \frac{1}{2} \rho U^2 B^2 [2C_M w(x, t)/U + C_M' w(x, t)/U] \quad (8)$$

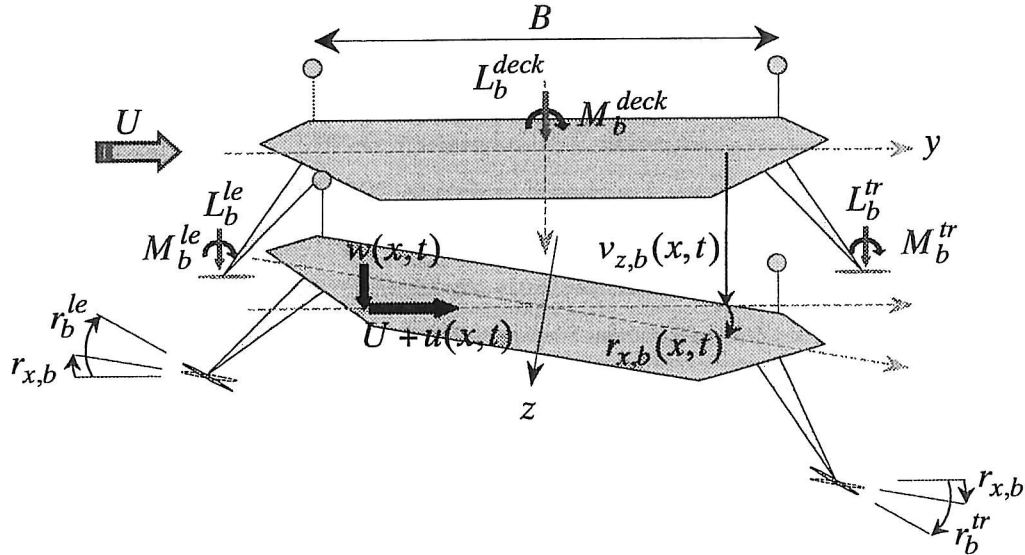


$$\begin{bmatrix} L_b^{le}(x,t) \\ L_b^{tr}(x,t) \end{bmatrix} = \frac{1}{2} \rho U^2 B \left( 2C_L^f \frac{u(x,t)}{U} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + (C_L'^f + C_D^f) \begin{bmatrix} r_b^{le}(x,t) \\ r_b^{tr}(x,t) \end{bmatrix} \right) \quad (9)$$

$$\begin{bmatrix} M_b^{le}(x,t) \\ M_b^{tr}(x,t) \end{bmatrix} = \frac{1}{2} \rho U^2 B^2 \left( 2C_M^f \frac{u(x,t)}{U} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_M'^f \begin{bmatrix} r_b^{le}(x,t) \\ r_b^{tr}(x,t) \end{bmatrix} \right) \quad (10)$$

where  $u(x,t)$  and  $w(x,t)$  are the along-wind and the vertical turbulence components.  $C_L$ ,  $C_M$ , and  $C_D$  are the non-dimensional lift, moment and drag coefficient. They depend on the angle of attack  $r_b$  of the wind to the girder.  $C_L' = dC_L/dr_b$  and  $C_M' = dC_M/dr_b$  are the slope of  $C_L$  and  $C_M$ , respectively. Coefficients with superscript  $f$  refer to the flaps. The angles of attack of the leading and trailing flap can written (see Fig. 1)

$$\begin{bmatrix} r_b^{le} \\ r_b^{tr} \end{bmatrix} = \begin{bmatrix} a_{le} \\ a_{tr} \end{bmatrix} \frac{w(x,t)}{U} = \begin{bmatrix} a_{le} \\ a_{tr} \end{bmatrix} r_b(x,t) \quad (11)$$



**Figure 1:** Buffeting-induced wind loads and positive definition of deformation direction.

By transferring the aeroelastic forcing terms given by (2) to (6) on the left-hand side of Eq. (1) and by assuming that the stochastic modal response is given by  $\xi_i(t) = \xi_i(\omega)e^{j\omega t}$  (Simiu & Scanlan, (1996)), where  $\omega = 2\pi f$  is the actual frequency,  $j = (-1)^{1/2}$ , one gets

$$M_i \left( -\omega^2 + 2\gamma_{i,c}\omega_{i0,c}j\omega + \omega_{i0,c}^2 \right) \xi_i(\omega)e^{j\omega t} = F_{ib}^{deck}(t) + F_{ib}^{flap}(t) \quad (12)$$

where

$$2\gamma_{i,c}\omega_{i0,c} = 2\xi_i\omega_i - \frac{\rho B^4 L}{2M_i} \omega_{i0,c} \left( \frac{H_1^*}{B^2} \Phi_{ae} + A_2^* \Psi_{ae} \right) - \frac{\rho B^4 L}{2M_i} (a_{le} + a_{tr}) \omega_{i0,c} \left( \frac{H_5^*}{B^{1/2}} \Phi_{ae} + A_6^* \Psi_{ae} \right) \quad (13)$$

$$\omega_{i0,c}^2 = \omega_i^2 - \frac{\rho B^4 L}{2M_i} \omega_{i0,c}^2 A_3^* \Psi_{ae} - \frac{\rho B^4 L}{2M_i} (a_{le} + a_{tr}) \omega_{i0,c}^2 A_7^* \Psi_{ae} \quad (14)$$

The aeroelastic modal integrals  $\Phi_{ae}$  and  $\Psi_{ae}$  in (13) and (14) are defined by

$$\Phi_{ae} = \int_{L_1}^{L_2} \phi_i^2(x) dx, \quad \Psi_{ae} = \int_{L_1}^{L_2} \psi_i^2(x) dx \quad (15)$$

The generalised buffeting forces on the girder and on the flaps are (see (2) and (7) to (10))

$$\begin{bmatrix} F_{ib}^{deck}(t) \\ F_{ib}^{flap}(t) \end{bmatrix} = \frac{\rho U^2 L_{deck}}{2} \int_{deck} \begin{bmatrix} Ba(x)u(x,t)/U + Bb(x)w(x,t)/U \\ B'a_c(x)u(x,t)/U + B'b_c(x)w(x,t)/U \end{bmatrix} \frac{dx}{L_{deck}} \quad (16)$$

where

$$a(x) = 2(C_L \phi_i + C_M B \psi_i), \quad b(x) = (C'_L + C'_D) \phi_i + C'_M B \psi_i \quad (17)$$

$$a_c(x) = 4(C_L^f \phi_i + C_M^f B \psi_i), \quad b_c(x) = (C_L^f + C_D^f)(a_{le} + a_{tr}) \phi_i + C_M^f (a_{le} + a_{tr}) B \psi_i \quad (18)$$

### 3. Mean Square of Response in Frequency Domain Stochastic Analysis

Let the Fourier transform of a modal response  $\xi_i$  be defined by  $\bar{\xi}_i(\omega) = \int_0^\infty \xi_i(t) e^{-j\omega t} dt$ . Take the Fourier transforms on both sides of (12), using (16), and multiply both sides by their complex conjugates. Then, multiply the final equation by  $2/T$  and go to the limit  $T \rightarrow \infty$  to obtain the spectrum of modal response in the form, Huynh [1]

$$S_{\xi_i \xi_i}(\omega) = |H(\omega)|_c^2 \left[ \frac{\rho U L_{deck}}{2M_i} \right]^2 (J_u(C, f) S_u(z, f) + J_w(C, f) S_w(z, f)) \quad (19)$$

where  $S_{\xi_i \xi_i}(\omega) = \lim_{T \rightarrow \infty} (2/T) \bar{\xi}_i \bar{\xi}_i^*$  is the spectrum of the modal response  $\xi_i$  in mode  $i$ , and

$$\lim_{T \rightarrow \infty} \frac{2}{T} \begin{bmatrix} \bar{u}(x_a, \omega) \bar{u}^*(x_b, \omega) \\ \bar{w}(x_a, \omega) \bar{w}^*(x_b, \omega) \end{bmatrix} = \begin{bmatrix} S_u(x_a, x_b, \omega) \\ S_w(x_a, x_b, \omega) \end{bmatrix} \equiv \begin{bmatrix} S_u(z, f) \\ S_w(z, f) \end{bmatrix} e^{-C \frac{|x_a - x_b| f}{U}} \quad (20)$$

is the cross-spectrum of the wind turbulence component  $u$  and  $w$ , both measured in the bridge longitudinal direction at  $x_a$  and  $x_b$ , respectively. The cross spectra  $S_{uw}$  and  $S_{wu}$  are neglected.  $S_u(x_a, x_b, \omega)$  and  $S_w(x_a, x_b, \omega)$  are assumed to take the real forms in (20).  $z$  is the girder elevation,  $f$  is the frequency of the wind fluctuation.  $C$  is a non-dimensional decay constant that determines the spatial extent of the correlation in the turbulence (experimental determined). The wind spectra from Simiu & Scanlan [5] are given by:

$$S_u(z, f) = 200 z u_*^2 / (U [1 + 50 f z / U]^{5/3}), \quad S_w(z, f) = 3.36 z u_*^2 / (U [1 + 10 (f z / U)^{5/3}]) \quad (21)$$

where  $u_*(z) = 0.4 U(z) / \ln(z/z_0)$  is the friction velocity,  $z_0$  is the roughness length.



$J_u(C, f)$  and  $J_w(C, f)$  are the joint acceptance functions defined by:

$$\begin{bmatrix} J_u(C, f) \\ J_w(C, f) \end{bmatrix} = \iint_{deck} \left[ \frac{(Ba(x_a) + B'a_c(x_a))(Ba(x_b) + B'a_c(x_b))}{(Bb(x_a) + B'b_c(x_a))(Bb(x_b) + B'b_c(x_b))} \right] e^{-C \frac{|x_a - x_b|f}{U}} \frac{dx_a}{L_{deck}} \frac{dx_b}{L_{deck}} \quad (22)$$

which describes the interaction of the actual mode shapes (in (17) and (18)) and the wind load fluctuations measured at two joints  $x_a$  and  $x_b$  along the girder, Dyrbye & Hansen [4].

Finally, the *frequency response function* is given by

$$|H(\omega)|_c^2 = \omega_{i0,c}^{-4} \left[ \left( 1 - [\omega/\omega_{i0,c}]^2 \right)^2 + (2\gamma_{i,c} \omega/\omega_{i0,c})^2 \right]^{-1} \quad (23)$$

For a purely vertical mode  $i$ ,  $\psi_i(x) \equiv 0$ ,  $J_u$  and  $J_w$  are given by (see (17), (18), and (22))

$$\begin{bmatrix} J_{u,i}(C, f) \\ J_{w,i}(C, f) \end{bmatrix} = \begin{bmatrix} (2BC_L + 4B'C_L^f)^2 \\ [B(C'_L + C_D) + B'(C_L^f + C_D^f)(a_{le} + a_{tr})]^2 \end{bmatrix} J_i(C, f) \quad (24)$$

where  $J_i(C)$  is the integral ( $L_m$  is the main span length)

$$J_i(C, f) = \iint_{L_m} \phi(x_a/L_m) \phi(x_b/L_m) e^{-C \frac{|x_a - x_b|f}{U}} d(x_a/L_m) d(x_b/L_m) \quad (25)$$

where the vertical mode shape  $\phi(x/L_m)$  is distinctly between the symmetrical mode, the asymmetrical mode, the main span and the side span (see Huynh [1]). The integral (25) is solved on the assumption that the correlation only depends on the distance  $|x_a - x_b|$  and not on each of the coordinates (by two equivalent single integrals, see Dyrbye & Hansen [4]).

For a purely torsional mode  $i$ ,  $\phi_i(x) \equiv 0$ ,  $J_u$  and  $J_w$  are given by (see (17), (18), and (22))

$$\begin{bmatrix} J_{u,j}(C, f) \\ J_{w,j}(C, f) \end{bmatrix} = \begin{bmatrix} (2C_M B^2 + 4C_M^f B'^2)^2 \\ [C'_M B^2 + C_M^f (a_{le} + a_{tr}) B'^2]^2 \end{bmatrix} J_j(C, f) \quad (26)$$

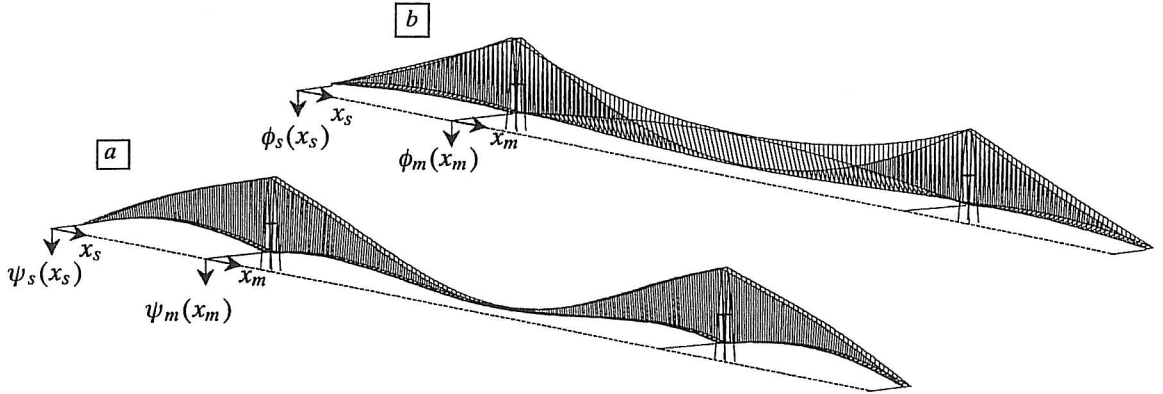
where  $J_j(C, f)$  are similarly given by (25), but the torsional mode shape  $\psi$  now replaces the vertical mode shape  $\phi$ . Finally, the mean square values of the vertical and the torsional response at position  $x$  on the main span are ((24) and (26) are inserted)

$$\begin{aligned} \sigma_{v_i}^2(x) &= \phi_i^2(x) \int_0^\infty S_{z_i z_i}(x, f) df \\ &= \phi_{m,i}^2 \left( \frac{x}{L_m} \right) \left[ \frac{\rho U L_m}{2\omega_{i0,c}^2 M_i} \right]^2 \left[ \left( (2BC_L + 4B'C_L^f)^2 \left[ 6u_*^2 + \frac{\omega_{i0,c} S_u(z, f_{i0,c})}{8\gamma_{i,c}} \right] \right. \right. \\ &\quad \left. \left. + [B(C'_L + C_D) + B'(C_L^f + C_D^f)(a_{le} + a_{tr})]^2 \left[ 0.175u_*^2 + \frac{\omega_{i0,c} S_w(z, f_{i0,c})}{8\gamma_{i,c}} \right] \right) J_{m,i}(C, f) \right] \end{aligned} \quad (27)$$

$$\begin{aligned}
\sigma_{r_j}^2(x) &= \psi_j^2(x) \int_0^\infty S_{\alpha_i \alpha_i}(x, f) df \\
&= \psi_{m,j}^2 \left( \frac{x}{L_m} \right) \left[ \frac{\rho U L_m}{2 \omega_{j0,c}^2 M_j} \right]^2 \left( \left( 2C_M B^2 + 4C_M^f B'^2 \right)^2 \left[ 6u_*^2 + \frac{\omega_{j0,c} S_u(z, f_{j0,c})}{8\gamma_{j,c}} \right] \right. \\
&\quad \left. + \left[ C_M' B^2 + C_M^f (a_{le} + a_{tr}) B'^2 \right]^2 \left[ -1.35u_*^2 + \frac{\omega_{j0,c} S_w(z, f_{j0,c})}{8\gamma_{j,c}} \right] \right) J_{m,j}(C, f)
\end{aligned} \quad (28)$$

For  $B' = 0$  (no flaps), Eqs. (27) and (28) become the expressions given by Simiu & Scanlan [5], where the frequencies and the total damping ratios are replaced by quantities only depending on the girder.

#### 4. Numerical Example



**Figure 2:** a) Symmetric Vertical mode SV1 and b) Symmetric Torsional mode ST1.

A long-span suspension bridge based on the Great Belt Bridge girder and the corresponding flutter derivatives is designed to illustrate the outlined theory. The main span length  $L_m = 2500$  m, side span length  $L_s = 1000$  m, cable sag in the main span  $f_m = 265$  m, cable space  $B = 27$  m, girder mass (incl. cables)  $m = 23700$  kg/m, girder mass moment of inertia  $J = 2.5 \text{ E6 kgm}^2/\text{m}$ , air density  $\rho = 1.29 \text{ kg/m}^3$  and structural damping in the vertical and the torsional mode are 0.02. The SV1 and ST1 modes are considered. The associated analytical mode shapes shown in Fig. 1 are used in (27) and (28), Huynh [1].

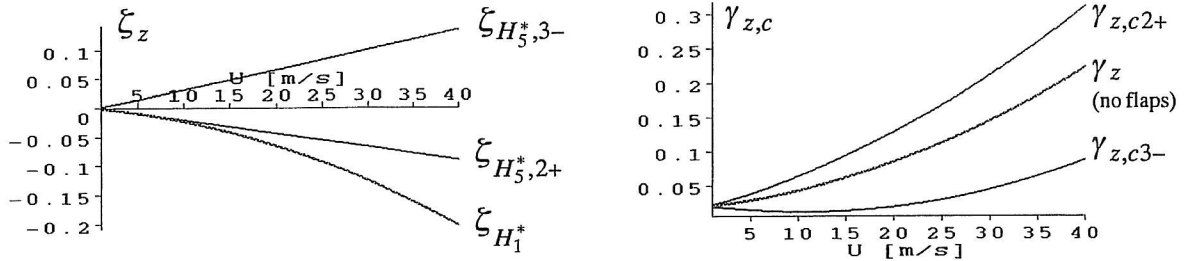
##### 3.1 Aerodynamic Damping and Frequency depending on the Flap Rotations

For a purely vertical mode in wind,  $\Psi_{ae} \equiv 0$ ,  $M_i \equiv m\Phi$  (where  $\Phi \equiv \Phi_{ae}$  assuming that the aeroelastic forces act on the full-span bridge). The aerodynamic vertical damping related to the flaps and the total vertical damping are (see (13) and (14))

$$\omega_{i0,c} = \omega_i = \omega_z \quad , \quad \gamma_{z,c} = \xi_z - \xi_{H_1^*} - \xi_{H_5^*} \quad (29)$$

$$\xi_{H_1^*} = \rho B^2 H_1^* / 4m \quad , \quad \xi_{H_5^*} = \rho B'^2 (a_{le} + a_{tr}) H_5^* / 4m \quad (30)$$





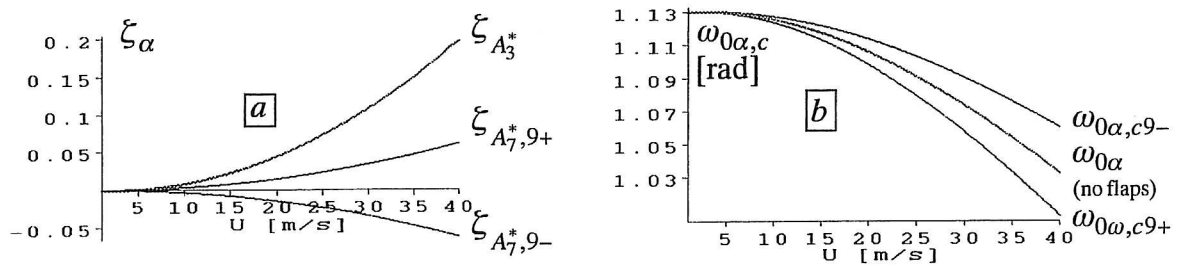
**Figure 3 :** Dependence of aerodynamic vertical damping and total vertical damping on the flap rotations (angle of attack).

The vertical damping ratio  $\xi_z$  related to  $H_5^*$  (flaps) is high compared to  $H_1^*$  (girder). The total vertical damping  $\gamma_{z,c}$  is changed even for small values of  $a_{le}$  and  $a_{tr}$ , Fig. 3. For a purely torsional mode in wind,  $\Phi_{ae} \equiv 0$ ,  $M_i \equiv J\Psi$  (where  $\Psi \equiv \Psi_{ae}$  assuming that the aeroelastic forces act on the full-span bridge), the aerodynamic torsional damping related to the flaps and the total torsional damping are (see (13) and (14)):

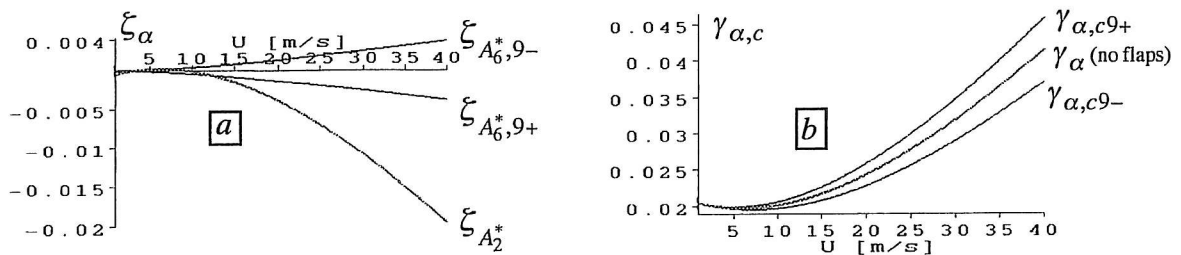
$$\omega_{\alpha 0,c} = \omega_{\alpha} / \sqrt{1 + \xi_{A_3^*} + \xi_{A_7^*}} \quad , \quad \gamma_{\alpha,c} = \xi_{\alpha} \sqrt{1 + \xi_{A_3^*} + \xi_{A_7^*}} - \xi_{A_2^*} - \xi_{A_6^*} \quad (31)$$

$$\xi_{A_2^*} = \rho B^4 A_2^* / 4J \quad , \quad \xi_{A_3^*} = \rho B^4 A_3^* / 2J \quad (32)$$

$$\xi_{A_6^*} = \rho B^{*4} (a_{le} + a_{tr}) A_6^* / 4J \quad , \quad \xi_{A_7^*} = \rho B^{*4} (a_{le} + a_{tr}) A_7^* / 2J \quad (33)$$



**Figure 4 :** Dependence of aerodynamic torsional damping and frequency on the flap rotations (angle of attack).



**Figure 5 :** Dependence of aerodynamic torsional damping and total torsional damping on the flap rotations (angle of attack).

The damping ratio related to  $A_7^*$  (flaps) is low compared to  $A_3^*$  (girder) in purely torsional mode, Fig. 4a. Consequently, the flap rotations do not modify the torsional frequency considerably, see (31). Up to  $U = 30$  m/s and for  $a_{le} = a_{tr} = +9$ , the frequency due to wind

action  $\omega_{\alpha 0, c9+}$  is reduced by only 1.5% from 1.074 rad/s to 1.058 rad/s compared to no flaps, Fig. 4b. The damping ratio related to  $A_6^*$  (flaps) is also low compared to  $A_2^*$  (girder), Fig. 5a. Consequently, the total damping ratio  $\gamma_{\alpha, c}$  given by (31) is always positive, although the rotational amplification of the flaps is strongly increased, Fig. 5b.

### 3.2 Dependence of RMS Response on the Flap Rotations

The following aeroelastic coefficients of the girder and the flaps are assumed

Table 1 : Lift, Drag and Moment coefficient for the Girder and for the Flaps					
	$C_L$	$C'_L$	$C_D$	$C_M$	$C'_M$
Girder	0.067	4.37	0.57	0.028	1.17
Flaps	0.07*	$2\pi$	0.60*	0.03*	$\pi/2$

\* Values assumed to be identical with the girder values.

The girder angle  $r_b(x, t)$  is defined positive clockwise (Plus), and the configurations of the flaps (leading + trailing) are similarly defined. The most interesting configuration of the flaps is the Configuration Minus Minus (CMM), where both the leading and the trailing flaps are rotated against the girder. Thus, a negative increase of  $a_{le}$  and  $a_{tr}$  means that the term  $B'(C'_L + C'_D)(a_{le} + a_{tr})$  in Eq. (27) reduces the RMS vertical response (although the total vertical damping reduced in Fig. 3). At  $a_{le} = a_{tr} = -3$ , the RMS vertical response at the main span centre is reduced to approximately one fourth from 0.40 m to 0.11 m at  $U = 40$  m/s compared to a no flaps situation, Fig. 6a. Similarly, the term  $(C'_M B^2 + C'_M (a_{le} + a_{tr}) B'^2)$  in Eq. (28) for torsional response also is reduced for a negative increase of  $a_{le}$  and  $a_{tr}$ , and thus also the RMS values. However, the small width  $B'$  of the flaps in purely torsional mode does not reduce the RMS response significantly since  $B^2 = 100B'^2$ . The term above is decided by  $C'_M B^2$  of the girder, where  $C'_M \approx C'_M$  is assumed in Table 1.

Contrary to the CMM, the Configuration Plus Plus (CPP) raises the RMS response in both the vertical and the torsional modes because of the two terms mentioned above. Finally, for CMP or CPM,  $(a_{le} + a_{tr}) = 0$ , there are no significant changes in response with the appearance of the terms  $4C'_L B'$  and  $4C'_M B'^2$  in (27) and (28).

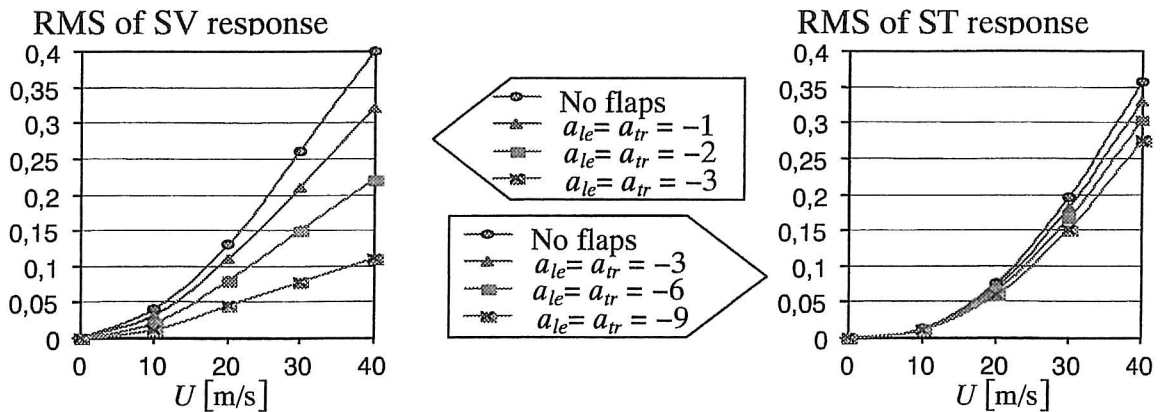


Figure 6 : Dependence of RMS response on the flap rotations (main span centre, CMM).



#### 4. Concluding Remarks

In addition to the efficiency of using the separate flaps to increase flutter critical wind velocity of the suspension bridge, the flaps are also useful to reduce the mean square of girder response to turbulence buffeting. Most important is that the flaps do not induce unexpected response in the turbulence wind loads when using CMP.

Further, by using the CMM with increasing  $a_{le}$  and  $a_{tr}$ , the mean square of vertical response (single mode) reduced considerably. Unfortunately, a similar reduction for the torsional response requires wider flaps.

The spectrum of the modal response depends on the joint acceptance function  $J(C, f)$  that expresses the correlation of the aerodynamic forces along the girder. Two functions  $J_u$  and  $J_w$  related to the alongwind turbulence component  $u(x, t)$  and the vertical turbulence component  $w(x, t)$  must be computed for each mode of the bridge subjected to turbulence wind loads. The joint acceptance function  $J_w$  is the dominant one and is referred to the slope of the aeroelastic coefficients of the girder. When using the CMM, the value of the function  $J_w$  is reduced with increased values of  $a_{le}$  and  $a_{tr}$ . Therefore the mean square of response is also reduced.

Finally, it should be noted that the a reliable determination of the mean square response of a certain bridge to turbulence buffeting requires realistic information on the wind turbulence at the actual location.

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