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Reliability Analysis of A Mono-Tower Platform

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Abstract - In this paper, a reliability analysis of a Mono-tower platform is presented. The failure modes, considered, are yielding in the tube cross-sections, and fatigue failure in the butt welds. The fatigue failure mode is investigated with a fatigue model, where the fatigue strength is expressed through SN relations. In determining the cumulative fatigue damage, Palmgren-Miner's rule is applied. Element reliability as well as systems reliability is estimated using first-order reliability methods (FORM). The sensitivity of the systems reliability to various parameters is investigated. It is shown that the fatigue limit state is a significant failure mode for the Mono-tower platform. Further, it is shown for the fatigue failure mode that the largest contributions to the overall uncertainty are due to the damping ratio, the inertia coefficient, the stress concentration factor, the model uncertainties and the parameters describing the fatigue strength.

Key words: Reliability analysis, first-order reliability methods (FORM), sensitivity analysis, yielding failure, fatigue failure, offshore, Mono-tower platform.

1 INTRODUCTION

For a Mono-tower platform and other flexible and dynamically sensitive offshore structures, fatigue failure is often found to govern the overall configuration of the structures. However, calculation of fatigue life is subjected to large uncertainty due to uncertainties in the computation of loads, the dynamic response of the structure, fatigue strength and damage accumulation. In order to analyse these uncertainties a reliability analysis, which provide the tools for efficient uncertainty analysis, can be used. Reliability methods have been extensively applied in the last decade, where considerable progress has been made in the area of structural reliability theory. Especially, the development of the so-called first- order reliability methods (FORM) and the second-order reliability methods (SORM) have been very important, see e.g. Madsen, Krenk & Lind [1] and Thoft-Christensen & Murotsu [2]. These methods are especially developed to estimate the reliability level of structural elements and systems. The reliability methods are also an excellent tool to determine important sources of uncertainty.

In this paper, a reliability analysis of a Mono-tower platform is performed. The Mono-tower structure, considered, has been described in Petersen, Lyngberg, Eskesen & Larsen [3], where data for the environmental conditions also have been stated.

Originally, the structure had been designed as an attractive solution for a marginal oil and gas field (Rolf field) in the Danish Sector of the North Sea, but the plans for this field were changed to a traditional 4 - legs jacket structure. A short description of the Mono-tower platform is given in section 2. Then, in section 3, first-order reliability methods (FORM) are briefly summarized. Next, in section 4, modelling of two failure modes is performed. The failure modes, considered, are yielding in the tube cross-sections, and fatigue failure in the butt welds. The fatigue failure mode is investigated through a model, where the fatigue strength is expressed through SN relations. In determining the fatigue damage, Palmgren-Miner's rule is applied. In the fatigue model, the structural response is calculated on the basis of a modal spectral analysis, where the structure is modelled as a one-dimensional, lightly damped, linear, continuous single degree of freedom system. Finally, in section 5, results of the reliability analysis are presented. Reliability calculations are performed by first-order reliability methods. The sensitivity of the reliability to various parameters is calculated, and important sources of uncertainties are determined.

The reliability calculations in this paper are performed with the computer program PRADSS (Program for Reliability Analysis and Design of Structural Systems), see Sørensen [4].

2 DESCRIPTION OF MONO-TOWER PLATFORM

The single pile platform, Mono-tower, investigated, throughout this paper, is a remotely operated platform, with provision for four wells, designed for 33.7 m. of water in the Danish part of the North Sea. The platform is a single steel cylinder driven into the seabed, supporting a topside facility deck.

The structure consists of three different sections

- A cylindrical section driven 28 m. into the seabed and ranging up to 7 m above mudline. This section has an external diameter of 4.5 m.
- A tapered section from 7 m above mudline to 3 m above still water level (SWL), elevation (el.) 0.
- A cylindrical section from 3 m above SWL up to main deck located at el. +19 m. This section has an external diameter of 2 m.

The wall thickness of the Mono-tower platform is 80 mm; except for a 7 m long, 100 mm thick, section from el. -4 to el.+3.

The topside structure consists of an emergency deck at el. +15.6, a main deck at el. +19.0, a mezzanine deck at el. +21.7 and a helideck at el. +26.0. The total weight of the topside is 200 tonnes including the deck structure and all the equipment necessary for four wells. The total weight of the tower is approximately 700 tonnes. The well conductors have been placed inside the pile, while an oil export riser, a ladder, a boat-landing and anodes have been placed outside the pile.

3 RELIABILITY ANALYSIS METHODS.

A reliability analysis is based on a reliability model of the structural system. The elements in the reliability model are failure elements, modelling potential failure modes of the structural system, e.g. yielding of a cross-section or fatigue failure of a weld. Each failure element is described by a failure function

$$g(\overline{x},\overline{p}) = 0 \tag{1}$$

in terms of a realization $\overline{x}$ of a random vector $\overline{X} = (X_1, X_2, .., X_n)$, and deterministic parameters $\overline{p}$, i.e. deterministic design parameters and parameters describing the stochastic variables, (e.g. expected value and standard deviation). $\overline{X}$ is assumed to contain n stochastic variables, e.g. variables describing the loads, strength, geometry, model uncertainty etc. In general, the basic variables $\overline{X}$ are correlated and non-normally distributed. Realizations of $\overline{X}$ where $g(\overline{x},\overline{p}) \leq 0$ correspond to failure states in the n-dimensional basic variable space, while $g(\overline{x},\overline{p}) > 0$ correspond to safe states.

The failure probability P_f is generally calculated from

$$P_f = \int_{g(\overline{x},\overline{p})\leq 0} f_{\overline{X}}(\overline{x})d\overline{x} \tag{2}$$

where $f_{\overline{X}}(\overline{x})$ is the joint probability density function of the basic variables.

Numerical methods for calculation of P_f have been developed during the last decade and first-order reliability methods (FORM) have been recognized as being both efficient and sufficiently accurate, if the failure functions are not too non-linear.

First-order reliability methods

In first-order reliability methods (FORM) a transformation $\overline{T}$ of the generally correlated and non-normally distributed variables $\overline{X}$ into standardized, normally distributed variables $\overline{U} = (U_1, U_2, .., U_n)$ is defined. Let $\overline{U} = \overline{T}^{-1}(\overline{X},\overline{p})$. In the $\overline{u}$-space the reliability index β is defined as

$$\beta = \min_{g(\overline{T}(\overline{u}),\overline{p})=0} (\overline{u}^T\overline{u})^{\frac{1}{2}} \tag{3}$$

The solution point $\overline{u}^\star$ of the optimization problem (3) is the point on the failure surface, defined by (1), closest to the origin in the $\overline{u}$-space, and is called the design point. Linearization of the safety margin M in the design point gives

$$M = g(\overline{T}(\overline{U}),\overline{p}) \approx \frac{\nabla_u g^T}{|\nabla_u g|}\overline{U} + \beta = -\overline{\alpha}^T\overline{U} + \beta \tag{4}$$

where $\nabla_u g$ is the gradient of g with respect to $\overline{u}$ in the design point $\overline{u}^\star$.

If the failure function is not strictly non-linear, the probability of failure P_f of the failure mode can with good accuracy be determined from $P_f \approx \Phi(-\beta)$, where $\Phi(\cdot)$ is the standard normal distribution function.

If the whole structural system is modelled, as a series system, by m failure elements, and failure of the system is defined as failure of one failure element, then a generalized systems reliability index β^s of this series system can be estimated from, see e.g Thoft-Christensen & Murotsu [2]

$$\beta^s = -\Phi^{-1}(1 - \Phi_m(\overline{\beta}; \overline{\overline{\rho}})) \tag{5}$$

where $\Phi_m(\cdot)$ is the m-dimensional normal distribution function. $\overline{\beta} = (\beta_1, \beta_2, .., \beta_m)$ are the reliability indices of the failure elements determined by the FORM analysis. The elements in the correlation coefficient matrix $\overline{\overline{\rho}}$ are determined by $\rho_{ij} = \overline{\alpha}_i^T \overline{\alpha}_j \quad i,j = 1,2,..,m$.

To estimate the series systems reliability in (5) a number of methods can be used , see e.g. Thoft-Christensen & Murotsu [2]

- Hohenbichler approximation
- Average correlation coefficient approximation
- Simple bounds
- Ditlevsen bounds
- Simulation

Sensitivity analysis

Besides the absolute values of the element reliability indices β_i and the systems reliability index β^s , it is often of interest to know the sensitivity of the element reliability indices and the systems reliability index to variations of parameters $\overline{p}$.

The derivatives of β_i and β^s with respect to $\overline{p}$ become, see Sørensen [4]

$$\frac{\partial \beta_i}{\partial p_j} = \frac{1}{\beta_i} \sum_{l=1}^{n} u_{il}^{\star} \frac{\partial \{T_l^{-1}(\overline{x}_i^{\star}, \overline{p})\}}{\partial p_j} \tag{6}$$

$$\frac{\partial \beta^s}{\partial p_j} \approx \frac{1}{\varphi(\beta^s)} \sum_{i=1}^{s} \Phi_{s-1}(\overline{\beta}_i^a; \overline{\overline{\rho}}_i^a) \varphi(\beta_i) \frac{\partial \beta_i}{\partial p_j} \tag{7}$$

where it is assumed that the s significant failure modes are numbered $1, 2, ..., s$. $\overline{\beta}_i^a$ and $\overline{\overline{\rho}}_i^a$ are the conditional reliability indices and correlation coefficients, respectively, see Sørensen [5]. $\varphi(\cdot)$ is the normal density function.

4. RELIABILITY MODELLING OF MONO-TOWER

Two different failure modes are investigated

- Yielding failure of the tube cross-sections due to an extremely high wave.
- Fatigue failure in the circumferential butt welds along the Mono-tower structure.

It is assumed that fatigue will occur at the welded joints. Further, it is assumed that buckling failure modes are not significant.

For the two failure modes one yielding failure element and one fatigue failure elements are formulated.

4.1 Yielding failure element

Failure function

The failure function for the yielding failure element is assumed to be

$$g(\overline{x},\overline{p}) = Z_1 - (\frac{N}{N_F})^2 - \frac{|M|}{M_F} \tag{8}$$

where the variable Z_1 is a model uncertainty variable. The yield capacity in pure bending is $M_F = \sigma_y(d^3 - (d-2t)^3)/6$ and in pure axial loading $N_F = \sigma_y \pi(d^2 - (d-2t)^2)/4$. σ_y,d and t are the yielding stress, the tube diameter and tube wall thickness, respectively. The load effects in the cross-section are axial force N and bending moment M.

Calculation of loading

The weight of the topside, tube, appurtenances and marine growth are assumed to contribute to the axial force.

Wave, wind, current from one direction and deflection are taken into account in the calculation of the bending moment, by a quasi-static analysis. Dynamic exciting of the Mono-tower by higher order components of the wave loading is neglected. Wave plus current loads are computed by Morison's equation, as the current and wave particles velocities are added together. Hydrodynamic coefficients used for the combined tube and riser have been estimated in Jacobsen, Hansen & Petersen [6]. The wave kinematics of the 50 year extreme wave is calculated from linear Airy wave theory. In order to extend the linear Airy wave theory up to the free surface, the wave particle velocity in the wave crest is set equal to the value at the mean water level (MWL). The wave particle acceleration is assumed to decrease constantly from MWL to the free surface, where the acceleration is assumed to be zero.

Vortex shedding is not taken into account, see Jacobsen, Hansen & Petersen [6].

The current velocity and the wind velocity profiles are calculated as stated in the Danish offshore Code, DS-449 [7].

It is assumed that the water depth at storm is increased with 1m.

The maximum overturning moment for the Mono-tower structure is assumed to be located below mudline and is 12.5 % higher than the mudline moment, see Petersen, Lyngberg, Eskesen & Larsen [3].

Stochastic variables

In table 1, the statistical characteristics of the basic stochastic variables are fully enumerated. Further, there is shown the deterministic design parameters, which are investigated in a sensitivity analysis. In this paper, statistical characteristics of the basic variables for both the failure elements are mainly from published information. In Enevoldsen & Kirkegaard [8], the stipulation of the statistical characteristics has been discussed in details. The SI units system is used.

Variable	Designation	Distribution	Expected value	Coeff. of var.
H	Extreme wave height	EX1	17.1	0.08
C_D	Drag coefficient	N	1.85	0.10
C_M	Inertia coefficient	N	2.92	0.10
TM	Mass of topside	N	275000	0.10
t	Wall thickness	N	1.0*	0.05
σ_y	Yield stress	LN	316000000	0.10
V_{tide}	Current velocity	N	0.85	0.10
V_{Wind}	Current velocity	EX1	0.73	0.12
θ	Angle of deflection	N	3.0	0.10
$Wind$	Wind velocity	EX1	1.0*	0.10
Ecc	Eccentricity of topside	N	1.0	0.10
Z_1	Model uncertainty	N	1.0	0.10
Z_2	Model uncertainty	N	1.0	0.05
d	Tube diameter	D	1.0*	
d_1	Marine growth	D	1.0*	
G	Acceleration of gravity	D	9.82	
ρ_w	Density of sea water	D	1025	
h	Water depth	D	34.7	

Table 1: Statistical characteristics (EX1 : Extreme type 1, N : Normal, LN : Lognormal, D : Deterministic).

Expected values represented by 1.0* indicate that the expected value varies along the structure. In the reliability calculations, the expected value 1.0* is multiplied with the real expected value of the stochastic variable at the given level.

The expected value of TM includes permanent loads and live loads. The basic variables V_{wind} and V_{tide} are the current velocities at the SWL, due to wind and tide, respectively.

The statistical characteristics of H, $Wind$, V_{wind} and V_{tide} have been estimated for extreme environmental conditions with a 50 year recurrence interval. The model uncertainty variable Z_1 models the model uncertainty connected with the use of the failure function (8). Z_2 takes into account uncertainties by the models, used to estimate the loads, i.e wave-model, current-model, hydrodynamic force-model etc. The wave height H is assumed to be fully correlated with V_{Wind} and $Wind$, respectively. C_D and C_M are calculated mutually correlated with the correlation coefficient $\rho =$ -0.9. All the others stochastic variables are assumed to be independent.

4.2 Fatigue failure element

In this section, it is assumed that Palmgren-Miner's rule in combination with SN-curves provides a fairly good fatigue model to establish a fatigue failure function.

Of the dynamic loads, which produce stress fluctuations and with that fatigue damages in the Mono-tower, only the load due to wave action is taken into account. Contribution of large long period storm waves to fatigue is excluded.

Wind loads are ignored, because for fixed offshore structures, these represent only a contribution of about 5 % to the total environmental loading, Watt [9].

Current loads are also ignored, because the frequencies of current loads are not sufficient to excite the structure.

According to Jacobsen, Hansen & Petersen [6] vortex shedding is not taken into account.

In Petersen, Lyngberg, Eskesen & Larsen [3], it is stated that the Mono-tower will only sustain minor damage in a collision with a supply boat, wherefore this contribution to the damage is neglected.

Failure function

The failure function for the fatigue element is written

$$g(\overline{x},\overline{p}) = D_{Fail} - (D_{Driving} + D_{wave}) \tag{9}$$

where D_{Fail} is the value of Palmgren-Miner's sum at failure. $D_{Driving}$ is the damage from the driving of the Mono-tower into the seabed and D_{Wave} is the damage from wave action.

Calculation of damage

The cumulative fatigue damage D_{wave} due to wave action is assumed to be given by the Palmgren-Miner rule

$$D = \sum_{i=1}^{q} \frac{n(S_i)}{N(S_i)} \tag{10}$$

where $n(S_i)$ is the number of stress cycles of stress range S_i in the stress history and $N(S_i)$ is the number of stress cycles of stress range S_i necessary to cause failure. The summation is over all stress ranges q. Experimentally determined SN-curves are used to calculate the fatigue strength $N(S_i)$ as a function of the hot-spot stress range S_i. The SN-curve is written, see e.g. Moan [10]

$$N(S_i) = (S_i)^{-k} K \tag{11}$$

where k and K are the fatigue parameters to be determined from experimental data.

The mean fatigue damage $\overline{D_i}$ per cycle for each sea state is given by

$$\overline{D_i} = \int_0^{\infty} D_i(\hat{s}) f_{\hat{s}}(\hat{s}) d\hat{s} \tag{12}$$

where $D_i(\hat{s})$ is the damage from one stress cycle. $f_{\hat{s}}$ is the probability density function of the nominal stress maxima $\hat{s}$ (stress amplitude) as a function of the significant wave height H_s.

It is assumed that the stress range at a time is double of the stress amplitude. Further, it is assumed that the stress variation is a zero-mean narrow-band Gaussian process, see next section. For a narrow-band process it can be shown that the probability distribution of the stress amplitudes is a Rayleigh distribution, see e.g Sarpkaya [11]

Under these assumptions the total wave induced fatigue damage D_{wave} is calculated by summing up the mean fatigue damage $\overline{D}_i$ over the service lifetime T_L, which is assumed to be 25 years, of the structure, and weighting the mean fatigue damage for each sea state according to the long-term sea state probability

$$D_{wave} = \frac{T_L}{K}(2\sqrt{2})^k \Gamma(1 + \frac{k}{2}) \int_0^{\infty} \int_{-\pi}^{\pi} \frac{(\sigma_s(h_s))^k}{T_0(h_s)} f_{H_s}(h_s) f_{\Phi_s}(\varphi_s) dh_s d\varphi_s \tag{13}$$

where $\Gamma(\cdot)$ is the gamma function. $\sigma_s(h_s)$ is the standard deviation of the stress response and $T_0(h_s)$ is the zero-upcrossing period of the stress cycles. $f_{H_s}(h_s)$ is the long-term probability density function for the significant wave height, assumed to be well represented by a Weibull density function

$$f_{H_s}(h_s) = \frac{C}{B}(\frac{h_s}{B})^{C-1} \exp(-(\frac{h_s}{B})^C) \quad (14)$$

The coefficients in the Weibull distribution are estimated from a Wave-scatter diagram for the Danish part of the North Sea $f_{\Phi_s}(\varphi_s)$ is the probability density function for the predominant wave direction.

In order to reduce the degree of the non-linearity of (13), the failure function (9) is written, bearing in mind that the hot-spot stress range is obtained by multiplying the nominal stress range by a stress concentration factor SCF

$$g(\overline{x}, \overline{p}) = \ln(D_{Fail} - D_{Driving}) + \ln(K) - \ln(T_L) - k\ln(SCF2\sqrt{2}) - \ln(\Gamma(1 + \frac{k}{2}))$$

$$- \ln(\int_0^\infty \int_{-\pi}^{\pi} \frac{(\sigma_s(h_s))^k}{T_0(h_s)} f_{H_s}(h_s) f_{\Phi_s}(\varphi_s) dh_s d\varphi_s) - \frac{k}{4}\ln(\frac{t}{22}) \quad (15)$$

Since the fatigue strength of welded joints decreases with increasing plate thickness, see Berge [12], equation (15) has been corrected (the last term in (15)) for thicknesses larger than 22 mm. Here, two different SN-curves are chosen by using criteria stated in Lotsberg & Andersson [13]. A so-called C-curve is used in the cone/cylinder transitions and below level -25.7. Otherwise, there is used a F2-curve. The SN-curves, used, have been intended for joints exposed to sea water and cathodic protected.

The stress concentration factor SCF is assumed to be 1; expect at the cone/cylinder transitions, where SCF is calculated by a formula stated in API RP 2A [14].

Calculation of structural response

In order to estimate the statistical measures of stress variations, variance of the stress response $\sigma_s^2(h_s)$ and the zero-upcrossing period of the stress cycles $T_0(h_s)$, modal spectral analysis method is applied. It is assumed that the long-term sea state can be accurately modelled as a piecewise zero-mean stationary Gaussian process $\eta(t)$. Such a process is completely characterized by its spectral density function. Here, the one-dimensional Pierson-Moskowitz sea spectrum is utilized as the spectrum characterizing a single sea state. For a given direction, the PM-spectrum $S_{\eta\eta}(\omega)$ is described by the significant wave height H_s and the spectral peak angular frequency ω_p (rad/sec)

$$S_{\eta\eta}(\omega) = \frac{1}{2\pi}\frac{5}{16} H_s^2 (\frac{\omega_p}{2\pi})^4 (\frac{2\pi}{\omega})^5 \exp(-\frac{5}{4}(\frac{\omega_p}{\omega})^4) \qquad \omega > 0 \quad (16)$$

ω_p is related to H_s and acceleration of gravity, see e.q. DS-449 [7].

Wave spreading is ignored, while the probability distribution of the direction of wave propagation is taken into account.

From the PM-spectrum for the waves per sea state, the cross-spectral density for the wave loads is given by

$$S_{ff}(z_1, z_2, \omega) = H(z_1, z_2, \omega) H^\star(z_1, z_2, \omega) S_{\eta\eta}(\omega) \tag{17}$$

where z_1 and z_2 are positions along the structure and ω is the angular frequency. $H(z_1, z_2, \omega)$ is the transfer function from water elevation to wave loads. $\star$ denotes the complex conjugate. The transfer function is calculated by using linear Airy wave theory and Morison's equation, where the non-linear drag term is linearized by the "minimum square error method". Hydrodynamic coefficients for the combined tube and riser have been estimated in Jacobsen, Hansen & Petersen [6]. In order to take diffraction into account, the basic value for the inertia coefficient C_M is changed as function of the wave length. The connection between the wave load process and the stress process are established by a structural analysis, modal spectral analysis, where the structure is modelled as a one-dimensional, lightly damped, linear, continuous single degree of freedom system. The analysis is divided into three steps

- From wave load f to modal loading p
- From modal loading to modal coordinates q
- From modal coordinates to stress s

Accordingly, the cross-spectra for p, q and s are respectively obtained from

$$S_{p_m p_n}(\omega) = \int_L \int_L \Phi^{(m)}(z_1) \Phi^{(n)}(z_2) S_{ff}(z_1, z_2, \omega) dz_1 dz_2 \tag{18}$$

$$S_{q_m q_n}(\omega) = H_m(\omega) H_n^\star(\omega) S_{p_m p_n}(\omega) \tag{19}$$

$$S_{s_k s_l}(z_k, z_l, \omega) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} T^{(m)}(z_k) T^{(n)}(z_l) S_{q_m q_n}(\omega) \tag{20}$$

where $\Phi^{(m)}$ is the m'th eigenmode shape and L is the length of the structure.

$H_m(\omega)$ is the complex frequency response function of the system for the m'th mode shape

$$H_m(\omega) = \frac{1}{M_m(\omega_m^2 - \omega^2 + i2\zeta_m \omega_m \omega)} \tag{21}$$

where ζ_m, ω_m and M_m are the modal damping ratio, the natural angular frequency and the modal mass of the m'th mode, respectively. The modal damping ratio, the natural frequency and the modal mass are assumed constant through the lifetime of the structure. The mass of topside, tube, appurtenance, marine growth, hydrodynamic mass and internal water are taken into account at the calculation of modal mass.

The transfer function $T^m(z_k)$ is calculated by

$$T^m(z_k) = \frac{E(z_k)I(z_k)}{W(z_k)} \frac{d^2\Phi^m(z_k)}{dz_k^2} = \frac{\omega_m^2}{W(z_k)} \int_{z_k}^{L} (z - z_k)\mu(z)\Phi^m(z)dz \qquad (22)$$

where E, I, W and μ are the modulus of elasticity, the moment of inertia, the section modulus and mass along the structure, respectively.

Once the spectrum of the stress process is obtained, its variance and zero-upcrossing period are calculated, at a given level (z_k)and for a given H_s, by

$$\sigma_s^2(z_k) = \int_0^\infty S_{s_k s_k}(z_k, z_k, \omega)d\omega \qquad (23)$$

$$T_0(z_k) = \frac{2\pi\sigma_s(z_k)}{\sqrt{\int_0^\infty \omega^2 S_{s_k s_k}(z_k, z_k, \omega)d\omega}} \qquad (24)$$

where the calculation of $T_0(z_k)$ presupposes Rayleigh distribution of the stress amplitudes.

The structural analysis, described above is generally a very time-consuming process, so the total computing time required for the reliability analysis tends to be long. To reduce the computing time, only the first mode is taken into account. This is assumed to be reasonable, as the structure is assumed lightly damped and the second lowest natural frequency is not coinciding with the peak of the sea- state spectrum. Further, the two lowest natural frequencies, $f_1 = 0.49$ Hz. and $f_2 = 2.19$ Hz, have been well-separated. The frequencies have been estimated by modelling the structure, including the soil, in a finite element program. The eigenvalue analysis, which in principle has to be performed for each calculation of the failure function $g(\overline{x}, \overline{p})$, is not included in the reliability analysis. Here, the estimated eigenmode shape vector is used for all the calculations of the failure function, i.e. that the variation in the eigenmode shape to variation in the safety problems parameters is disregarded. On the other hand, the variation in the natural frequency to variation in the mass of the structure is taken into account by using Rayleigh's quotient to calculate an equivalent stiffness $Equi$ for the Mono- tower.

Stochastic variables

Deterministic design parameters, which are taken into account in a sensitivity analysis, and stochastic variables for the fatigue failure element are shown in table 2.

Variable	Designation	Distribution	Expected value	Coeff. of var.
C_D	Drag coefficient	N	1.0*	0.8
C_M	Inertia coefficient	N	1.0*	0.2
TM	Mass of topside	N	200000	0.1
t	Wall thickness	N	1.0*	0.05
SCF	Stress concent. factor	N	1.0*	0.1
B	Parameters in long-term	N	2.35	0.1
C	distribution of H_s	N	1.89	0.1
$Equi$	Equivalent stiffness	N	1.0*	0.1
m_1	Thickness correction	LN	1.0*	0.1
λ	Coeff. for added mass	N	0.9	0.1
$D_{Driving}$	Damage from "driving"	LN	1.0*	0.15
ζ	Damping ratio	LN	0.015	0.5
K	Constant in SN-curve	LN	1.0*	0.65
D_{fail}	Damage at failure	LN	1.0	0.3
Z_1	Model uncertainty	N	1.0	0.2
d	Tube diameter	D	1.0*	
d_1	Marine growth	D	1.0*	
G	Acceleration of gravity	D	9.82	
ρ_w	Density of sea water	D	1025	
h	Water depth	D	33.7	

Table 2: Statistical characteristics (EX1 : Extreme type 1, N : Normal, LN : Lognormal, D : Deterministic.)

The expected value of TM includes permanent loads and not live loads. $m_1 = \frac{k}{4}$ model the uncertainty with the plate thickness reduction factor. In order to take into account the uncertainty of the stiffness of the soil and structure, respectively, the equivalent stiffness $Equi$ is modelled stochastic. A direct stochastic modelling of the stiffnesses is not possible, as the eigenvalue analysis has been excluded from the reliability calculations. Uncertainties in the calculation of added mass, due to surrounding water, is modelled by λ. By modelling the modal damping ratio stochastic the uncertainties of the different contributions to the damping are taking into account. It is assumed that the damping of a Mono-tower consists of structural damping, viscous hydrodynamic damping, radiation damping and soil damping. Further, it is seen in table 2 that only K in the SN relation is modelled as a stochastic variable. This is proposed by Wirsching [15], where statistical characteristics of K are stated too. D_{fail} is a model uncertain variable, which models the uncertainty connected by Palmgren- Miner's rule. The other model uncertainty variable Z_1 models the uncertainties connected by the models, which are used to calculate the variance and the zero-upcrossing period of the stress process. The statistical characteristics of this stochastic variable have been chosen according to Wirsching [15]. C_D and C_M are assumed to be mutually correlated with the correlation coefficient ρ = -0.9. All the others stochastic variables are assumed to be independent.

5. RESULTS

For each of the failure elements, formulated in section 4, element as well as the systems reliabilities of the Mono- tower platform are estimated.

5.1 Reliability calculated by using the yielding failure element

The systems reliability of the Mono-tower structure against yielding failure is estimated by modelling the reliability model of the Mono-tower structure as a series system with twelve yielding failure elements. Eleven failure elements are placed between level +15 and -33.7 (mudline), see figure 1. Since the maximum overturning moment is located below mudline, the element reliability of failure element no. 12 is estimated with the overturning moment at mudline, increased with 12.5 %, see Petersen, Lyngberg, Eksesen & Larsen [3]. It is assumed that all the stochastic variables have been fully correlated between the failure elements.

The variation of the element reliability index β_i along the structure is shown in figure 1.

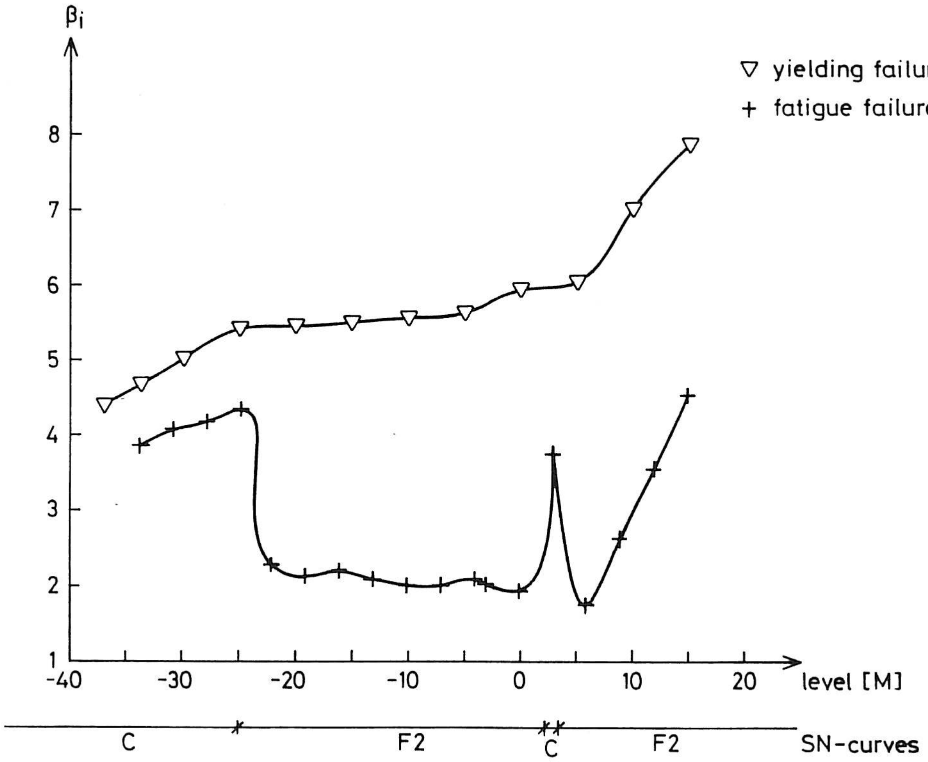

Figure 1: The variation of the element reliability index β_i along the Mono-tower platform. (Notice, the influence of the stipulation of SN-curves on β_i)

The systems reliability index of the Mono-tower platform is estimated as stated in section 3. Here, the Hohenbichler approximation is used.

Using the Hohenbichler approximation the systems reliability index becomes $\beta^s = 4.38$.

It is seen, as expected for fully correlated failure elements that the systems reliability is coinciding with the lowest estimated element reliability index β_i, here 4.38.

Figure 2 shows the sensitivity of the systems reliability index β^s to variations of the expected values of the stochastic variables $\frac{\partial \beta^s}{\partial \mu_j}$ and standard deviations $\frac{\partial \beta^s}{\partial \sigma_j}$. The sensitivities are relatively compared, as each derivative is multiplied with a hundredth parameter.The sensitivity of the systems reliability to variations in deterministic parameters is estimated by modelling the deterministic parameters as fixed stochastic variables. In tabel 1, the designation of the stochastic variables has been stated.

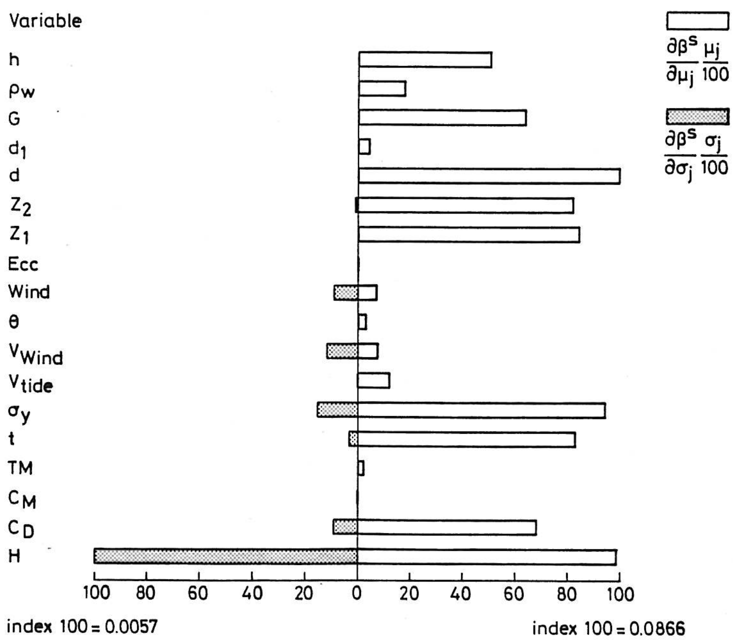

Figure 2: Sensitivity of the systems reliability to variations of the parameters of the stochastic variables, yielding failure

It is seen from figure 2, that the largest contribution to the overall uncertainty is due to the wave height H, the yield stress σ_y and the thickness of the tube wall t. Besides that, the model uncertainties turn out to be important. The systems reliability is also seen to be very sensitive to variations of the diameter of the tube.

5.2 Reliability calculated by using the fatigue failure element

The Mono-tower platform is modelled as a series system with eighteen fatigue failure elements, between level -33.7 and +15, see figure 1. Each element is assumed to model the damage at that point in the butt weld, where the greatest fatigue damage will occur. Between the failure elements, the stochastic variable K is assumed to be correlated with the correlation coefficient $\rho = 0.5$. The same assumption is also made for D_{fail}. All the others stochastic variables are assumed fully correlated between the failure elements. The variation of the element reliability index along the structure is shown in figure 1.

Using the Hohenbichler approximation the systems reliability index becomes $\beta^s = 1.432$.

The estimated systems reliability index, which is relatively low, indicates, that the fatigue limit state is a significant failure mode for the Mono-tower platform.

It is seen from figure 1 that the element reliability index, calculated by using fatigue element, is very sensitive to different SN-curves, as the reliability index is significantly changed, when the SN-curve is changed. This is also seen from the results of the sensitivity analysis, see figure 3. In table 2, the designation of the stochastic variables has been stated.

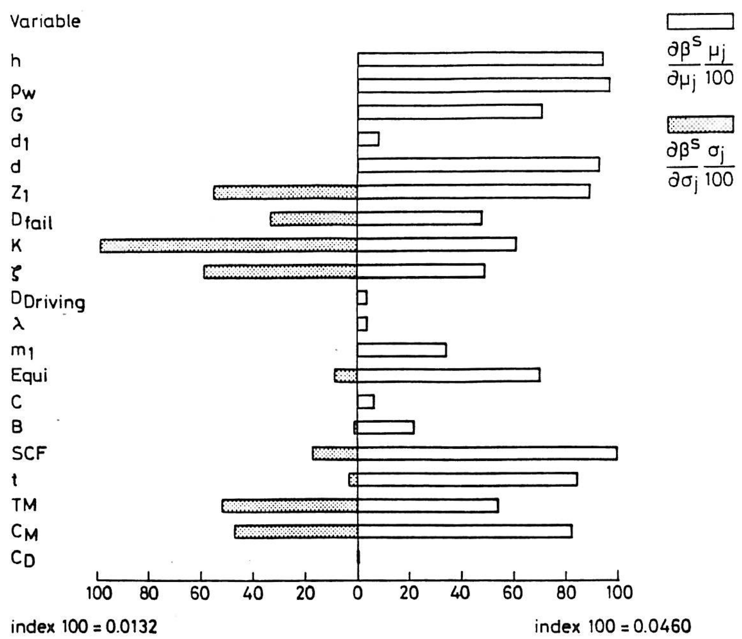

Figure 3: Sensitivity of the systems reliability to variations of the parameters of the stochastic variables, fatigue failure

Figure 3 shows that many of the stochastic variables contribute to the overall uncertainty. Especially, K, C_M, SCF, ζ, D_{fail} and Z_1 contribute to the uncertainty. The systems reliability is also seen to be very sensitive to variations of the deterministic design parameters, except the marine growth. As $Equi$ and TM do not turn out to be very important, it is seen, that, in this example, the exclusion of the eigenvalue analysis from the reliability calculations does not influence the results significantly.

6 CONCLUSIONS

Based on the reliability analysis of the Mono-tower platform the following conclusions can be stated

1) It has been shown, how reliability methods can be used in an uncertainty analysis to estimate a nominal element reliability level as well as a systems reliability level. It has also been shown, how the reliability methods can be used to estimate the sensitivity of the systems reliability in order to identify the most important uncertainties, thereby pointing at problems for closer investigations.

2) Two different failure modes, yielding and fatigue, have been considered. The fatigue failure mode turned out to be the most significant for the structure considered.

3) A sensitivity analysis with respect to the systems reliability index, calculated using the yielding failure element, showed that the largest contributions to the overall uncertainty were due to the extreme wave height, the steel yield stress and the thickness of the tube wall. Using the formulated fatigue failure element, the largest contributions to the overall uncertainty were due to the damping ratio, the inertia coefficient, the stress concentration factor and the parameters describing the fatigue strength.

4) It has been revealed that the amount of computational work, used for a reliability analysis, can be rather large. The total computer time for the estimation of the systems reliability and the sensitivity factors, using the fatigue element, was 4.26 CPU-hours on a VAX 8700.

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7 REFERENCES

[1] Madsen, H.O., S.Krenk & N.C. Lind: *Methods of Structural Safety.* Prentice-Hall, 1986.

[2] Thoft-Christensen, P. & Y. Murotsu: *Application of Structural Systems Reliability Theory.* Springer Verlag, 1986.

[3] Petersen, M.J., B.S. Lyngberg, S.D. Eskesen & O.D. Larsen: *Design of a Monotower Platform for Marginal Fields in the North Sea.* In Proceedings of the Sixth International Offshore Mechanics and Arctic Engineering Symposium, Vol. 1, Chung, J.S., Ch.P. Sparks, T. Nogami, T.R. Chari & T.R. Penny (edt.), 1987.

[4] Sørensen, J.D.: *PRADSS: Program for Reliability Analysis and Design of Structural Systems.* Structural Reliability Theory, Paper No. 36, The University of Aalborg, Denmark, 1987.

[5] Sørensen, J.D.: *Reliability Based Optimization of Structural Elements.* Structural Reliability Theory, Paper No. 18, The University of Aalborg, Denmark, 1986.

[6] Jacobsen, V., N.E.O. Hansen & M.J. Petersen: *Dynamic Response of Monotower to Waves and Currents.* 17'th Offshore Technology Conference, Paper No. OTC 5031, 1985.

[7] DS-449: *Code of Practice for Pile-Supported Offshore Steel Structures.* Dansk Ingeniørforening, 1983.

[8] Enevoldsen,I. & P.H. Kirkegaard: *Reliability Analysis of a Mono-tower Platform.*(In Danish), M.Sc. Thesis, The University of Aalborg, Denmark, 1988.

[9] Watt,B.J.: *Basic Structural Systems-A Review of Their Design and Analysis Requirements.* Numerical Methods in Offshore Engineering, 1978.

[10] Moan, T.: *Overview of Offshore Steel Structures.* Fatigue Handbook for Offshore Structures, A. Almar Næss (ed.), pp. 1-37, Tapir, Trondheim, 1985.

[11] Sarpkaya,T. & M. Isaacson: *Mechanics of Wave Forces on Offshore Structures.* Van Nostrand Reinhold Co., New York, 1981.

[12] Berge, S.: *Basic Fatigue Properties of Welded Joints.* Fatigue Handbook for Offshore Structures, A. Almar Næss (ed.), pp. 157-236, Tapir, Trondheim, 1985.

[13] Lotsberg, I. & H. Andersson: *Fatigue in Building Codes Background and Applications.* Fatigue Handbook for Offshore Structures, A. Almar Næss (ed.), pp. 459-501, Tapir, Trondheim, 1985.

[14] API RP 2A: *Recommended Practice for Planning, Designing and Constructing Fixed Offshore Platforms.* Thirteenth Edition, American Petroleum Institute, January, 1982.

[15] Wirsching, P.: *Fatigue Reliability for Offshore Structures.* Journal of Structural Engineering, pp. 2340-2356, Vol. 110, No.10, Oct., 1984.

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