Identification of Time Varying Civil Engineering Structures using Multivariate Recursive Time Domain Models

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Publication date:
1996

Document Version
Early version, also known as pre-print

INSTITUTTET FOR BYGNINGSTEKNIK
DEPT. OF BUILDING TECHNOLOGY AND STRUCTURAL ENGINEERING
AALBORG UNIVERSITET • AUC • AALBORG • DANMARK

FRACTURE & DYNAMICS
PAPER NO. 79

To be presented at ISMA21, Leuven, September 1996

P. H. KIRKEGAARD, P. S. SKJÆRBÆK & P. ANDERSEN

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JUNE 1996 ISSN 1395-7953 R9619

Printed at Aalborg University

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Identification of Time Varying Civil Engineering Structures using Multivariate Recursive Time Domain Models

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ABSTRACT

This paper presents system identification results of an RC-structure using multivariate auto-regressive moving-average vector models (ARMAV) on a running window, recursive ARMAV models (RARMAV) and Kalman filtering of a simple spring-mass-dashpot system. The identification techniques are used on simulated data generated by the non-linear finite element program SARCOF modelling a 6-storey 2-bay concrete structure subjected to amplitude modulated Gaussian white noise filtered through a Kanai-Tajimi filter. The estimated natural frequencies and mode shapes of the two first modes are compared with the smoothed quanties which have been obtained from SARCOF. The results show the usefulness of the techniques for identification of a time varying civil engineering structure. It is found that all the techniques give reliable estimates of the frequencies of the two lowest modes and the first mode shape. Only the Kalman filtering gives reliable estimates of the second mode shape.

1. INTRODUCTION

During severe dynamic excitation such as major earthquakes the response of reinforced concrete structures is influenced by non-elastic stress-strain behaviour (hysteresis). Any plastic cyclic deformation implies that the structure suffers local or global damages, ranging from harmless cracking of hitherto uncracked cross-sections to bond deterioration at the interface between reinforcement bars and concrete, crushing of the concrete in compression zones, rupture of reinforcement bars and stirrups etc. It has become common practice to instrument important buildings which may be exposed to excessive dynamic excitations in order to control the damage accumulation as measured by the sequential stiffness and strength deterioration.These damages are displayed in the dynamic response of the structure in terms of e.g. increased eigenperiods. The so-called Maximum Softening damage indicator is based on this principle, relating the global damage state of the structure to the relative decrease of the fundamental eigenfrequency, Di Pasquale et al. [1] and Nielsen et al. [2]. Alternatively, the global damage can also be detected using other damage indicators such as Park & Ang's index, maximum deformation, normalized cumulative dissipated energy, ductility ratios, a low cycle fatique model formulated by Stephens, Roufaiel & Meyers global damage index, slope ratio and flexural damage ratio, see Stephens [3]. Local damage can be estimated using the above mentioned indices and furthermore interstorey drifts and a relatively new local maximum softening damage index, see Skjærbæk et al. [4]. This local maximum softening index is estimated from time-varying eigenperiods and eventually mode shapes, if available. However, in order to use this local maximum softening damage index robust estimates of the time-varying eigenperiods and mode shapes of the equivalent linear structure are required. Among many

techniques proposed for system identification application adaptive techniques such as the recursive prediction error methods (RPEM) and Kalman filtering seem to be available tools to identify time-varying civil engineering structures.

The aim of this paper is to evaluate system identification techniques based on windowed multivariate Auto-Regressive Moving-Average (ARMAV) time domain models, recursive ARMAV (RARMAV) and Kalman filtering for on-line estimation of natural frequencies and mode shapes of an RC-structures subjected to an earthquake. ARMAV models have primarily been developed by control engineers and applied mathematicans, see e.g. Ljung [5] and Söderström et al. [6]. However, in recent years the application in civil engineering of ARMAV to the description of structural systems subjected to stationary ambient excitation has become more common, see e.g. Gersch et al. [7], Pandit et al. [8], Kozin et al. [9], Hamamonton et al. [10], Li et al. [11], Hoen [12] , Prevosto et al. [13], Kirkegaard et al. [14] and Andersen et al. [15]. All these references are related to identification of linear time-invariant structures. However, in order to deal with time-varying systems a recursive implementation of a multivariate ARMAV model can be used, see e.g. Ljung [5], Söderström et al. [6], Kirkegaard et al. [16]. In this paper two different recursive implementations of multivariate ARMAV models used for identification of a time-variant civil engineering structure. The two implementations of 1) recursive pseudolinear regression (RPL/RARMAV) and 2) the recursive least square (RLS/RARV) method, that both can be considered as the special forms of the RPEM method for particular forms of the ARMAV model. The different between the RPL and the RLS technique is that the RPL estimates the moving average parameters in the ARMAV model while the RLS does not. The frequencies and mode shapes estimated using RPL and RLS are compared with estimates obtained by Kalman filtering using a simple spring-mass-dashpot system. The identification techniques are used on simulated data generated by the non-linear finite element program SARCOF modelling a 6-storey 2-bay concrete structure subjected to amplitude modulated Gaussian white noise filtered through a Kanai-Tajimi filter, Mørk [17]. SARCOF, has been verified to be able to predict accurately the response of deteriorating RC-structures with well-defined structural parameters (bending stiffness of cracked and uncracked of all beam-elements must be specified) when the structure is exposed to different levels of peak ground excitation.The program estimates the two lowest eigenfrequencies and mode shapes of the equivalent linear structure at each time step. The results show the usefulness of the techniques for identification of a time varying civil engineering structure. Especially, the results using an ARMAV model used on a running window seems to give the most robust estimate of the fundamental eigenfrequency.

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2. BASIC EQUATIONS FOR ARMAV

This section describes the relationship between an Auto-Regressive Moving-Average Vector model (ARMAV) and the governing differential equation for a linear n-degree of freedom elastic system. Further, it is explained how the ARMAV model can be calibrated to output time series using recursive techniques.

2.1 Continuous Time Model

In the continuous time domain an n-degree linear elastic viscous damped vibrating system is described to be a system of linear differential equations of second order with a constant coefficient given by a mass matrix $\boldsymbol{M}$ (n x n), a damping matrix $\boldsymbol{C}$ (n x n), a stiffness matrix $\boldsymbol{K}$ (n x n), an input matrix $\boldsymbol{S}$ $(n \times r)$ and a force vector $\boldsymbol{f}$(t) (r x 1). Then the equations of motion for a linear multivariate system may in the time domain be expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) = \boldsymbol{S}\boldsymbol{f}(t) \tag{1}$$

where $x(t)$ is the displacement vector. The state space model corresponding to the dynamic equation (1) is

$$\dot{z}(t) = Ax(t) + Bf(t) \quad , \quad A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ -M^{-1}S \end{bmatrix} \tag{2}$$

where $z(t)$ is the state vector. It is assumed that the system matrix A is asymptotically stable and can be eigenvector-eigenvalue decomposed as

$$A = U\mu U^{-1} \ , \quad U = \begin{bmatrix} u_1 & \cdots & u_{2n} \\ \mu_1 u_1 & \cdots & \mu_{2n} u_{2n} \end{bmatrix} \quad \mu = diag\,[\mu_i] \ , \ i=1,2,\ldots,2n \tag{3}$$

U is the matrix which columns contain the scaled mode shapes $\mathbf{u}_i$ of the ith mode. $\boldsymbol{\mu}$ is the continuous time diagonal eigenvalue matrix which contains the poles of the system from which the natural frequency ω_i and the damping ratio ζ_i of the ith mode can be obtained for under damped systems from a complex conjugate pair of eigenvalues as

$$\mu_i, \mu_i^* = -\omega_i \zeta_i \pm \omega_i i \sqrt{1-\zeta_i^2} \tag{4}$$

2.2 Discrete Time ARMAV Model

For multivariate time series, described by an m-dimensional vector $y(t)$, an ARMAV(p,q) model can be written with p AR-matrices and q MA-matrices

$$y(t) + \sum_{i=1}^{p} A_i y(t-i) = \sum_{j=1}^{q} B_j e(t-j) + e(t) \tag{5}$$

where the discrete-time system response is $y(t) = [y_i(t), y_2(t) \ldots y_m(t)]^T$. A_i is an $m \times m$ matrix of auto-regressive coefficients and B_j is an $m \times m$ matrix containing the moving-average coefficients. $e(t)$ is the model residual vector, an m-dimensional white noise vector function of time. Theoretically, an ARMAV model is equivalent to an ARV model with infinite order.The ARV is often preferred because of the linear procedure of the involved parameter estimation. The parameter estimation of the ARMAV model is a non-linear least squares procedure and requires some skill as well as large computation effort. A discrete state-space equation for equation (6) obtained by uniformly sampling the structural responses at time t is given by, e.g. Pandit et al. [8]

$$Z_t = FZ_{t-1} + W_t \tag{6}$$

with the state vector Z_t and the system matrix F, respectively, are given by

$$Z_k = \{y(t)^T\ y(y-1)^T\ y(t-2)^T \ldots y(t-p+1)^T\}^T \quad , \quad F = \begin{bmatrix} -A_1 & -A_2 & \ldots & -A_{p-1} & -A_p \\ I & 0 & \ldots & 0 & 0 \\ . & . & \ldots & . & . \\ 0 & 0 & \ldots & I & 0 \end{bmatrix} \tag{7}$$

W_t includes the MA terms of the ARMAV model. It is assumed that F can be decomposed as

$$F = L\lambda L^{-1} \ , \quad L = \begin{bmatrix} l_1\lambda_1^{p-1} & l_2\lambda_2^{p-1} & \ldots & l_{pm}\lambda_{pm}^{p-1} \\ l_1\lambda_1^{p-2} & l_2\lambda_2^{p-2} & \ldots & l_{pm}\lambda_{pm}^{p-2} \\ . & . & . & . \\ l_1 & l_2 & \ldots & l_{pm} \end{bmatrix} \ , \quad \lambda = diag\ [\lambda_i] \ , \ i=1,2,\ldots,pm \tag{8}$$

The discrete state space model can now be used to identification of modal parameters and scaled mode shapes as follows, see Andersen et al. [15]. First, the discrete system matrix F is estimated by minimizing a quadratic error criterion $l(\epsilon)$ using a damped Gauss-Newton optimization algorithm and analytically gradients,

$$l(\epsilon) = \frac{1}{2}\epsilon^T\Lambda^{-1}\epsilon \quad , \qquad \epsilon(t,\theta) = y(t) - \hat{y}(t|t-1) \tag{9}$$

Λ and ϵ are the weighting matrix and the prediction errors, respectively. By solving this optimization problem the matrices in (5) are estimated implying that F can be established, see Andersen et al. [15].

Next, the discrete eigenvalues of F are estimated by solving the eigen-problem $\det(\mathbf{F}-\lambda\mathbf{I}) = 0$ which gives the pm discrete eigenvalues λ_i.The continuous eigenvalues can now be obtained by $\lambda_i = e^{\mu_i\Delta}$ which implies that the modal parameters can be estimated using (4). The scaled mode shapes are determined directly from the columns of the bottom $m \times pm$ submatrix of L. The number of discrete eigenvalues in general are larger or different from the number of continuous eigenvalues. Therefore, only a subset of the discrete eigenvalues will be structural eigenvalues. This means that the user has to separate the physical modes from the computational modes. The computational modes are related to the unknown excitation and the measurement noise processes. The separation can often be done by studying the stability of e.g. frequencies, damping ratios and mode shapes, respectively, for increasing AR model order. Often, it is also possible to separate the modes by selecting physical modes as the modes with corresponding damping ratios below a reasonable limit for the modal damping ratios. However, satisfactory results obtained using ARMAV models for system identification require that appropriate models are selected and validated. A throughout description of the problem of model selection and validation is given in e.g. Ljung [5].

2.3 Discrete Time Recursive Model

In order to estimate a time-variant system using an ARMAV model the parameters of the model have to be estimated on-line by using a method for parameter estimation known as the Recursive Prediction Error Method (RPEM). Such an approach has two main advantages: 1) it requires much less memory in the computer since the calculations are done sequentially using only the latest segment of

data, and 2) it can detect time varying characteristics at each time step. The RPEM algorithm for a multivariate ARMAV model can be formulated as, see e.g. Ljung [5]

$$\begin{aligned} \theta(t) &= \theta(t-1) + \boldsymbol{L}(t)\epsilon(t,\theta(t-1)) \\ \boldsymbol{L}(t) &= \boldsymbol{P}(t-1)\psi(t)[\lambda(t)\Lambda_t + \psi(t)^T\boldsymbol{P}(t-1)\psi(t)]^{-1} \\ \boldsymbol{P}(t) &= [\boldsymbol{P}(t-1) - \boldsymbol{L}(t)\psi^T(t)\boldsymbol{P}(t-1)]/\lambda(t) \end{aligned} \tag{10}$$

The parameter vector θ is given as

$$\theta = col(\{\boldsymbol{A}_1,\ldots,\boldsymbol{A}_p,\boldsymbol{B}_1,\ldots,\boldsymbol{B}_q\}) \tag{11}$$

where $col(\boldsymbol{X})$ means stacking of all columns of a matrix $\boldsymbol{X}$ on top of each other. $\psi(t)$ is the gradient of the prediction $y(t|t-1)$ with respect to θ . Λ_t is a matrix that weighs together the relative importance of the components of ϵ. $\lambda(t)$ is a forgetting factor, a number somewhat less than 1. This means that one can assign less weight to older data that are no longer representative for the system. The choice of the forgetting factor is often very important. Theoretically, one must have $\lambda(t) = 1$ to get convergence. On the other hand, if $\lambda(t) < 1$ the algorithm becomes more sensitive and the parameter estimates can change quickly. For this reason it is often an advantage to allow the forgetting factor to vary with the time. A typical choice is to let $\lambda(t)$ tend exponentially to 1, see e.g. Söderström et al. [6].
In order to start the recursion initial values of the parameters need to be specified. It can be shown that for stable systems the effect of initial values diminishes very rapidly with time, thus they can be assumed zero. Further, one also has to specify initial values for the covariance matrix $\boldsymbol{P}(t)$.
If this matrix is initialized with small values the parameter estimates will not change too much from the initial estimates of the parameters. On the other hand, if $\boldsymbol{P}(t)$ is initialized with large values, the parameter estimates will quickly jump away from the initial values of the parameter vector.
The intermediate steps of the derivation of (10) are given in e.g. Ljung [5] where it also is shown that the RPEM represents a general family of recursive system identification methods.There are several other methods, such as e.g. the recursive pseudolinear regression (RPL), maximum likelihood estimation (ML), and the recursive least square (RLS) method that all can be considered as the special forms of the RPEM method for particular forms of the ARMAV model. The RPL (RARMAV) technique is obtained by replacing the gradient $\psi(t)$ in (12) by the multivariate regression matrix $\phi^T(t)$ of dimension $m \times (p+q)m^2$ defined as

$$\begin{aligned} \phi^T(t) &= \varphi^T(t)\otimes I_m \\ \varphi^T(t) &= \{-y^T(t-1),\ldots,-y^T(t-p),e^T(t-1),\ldots,e^T(t-q)\}^T \end{aligned} \tag{12}$$

where $\boldsymbol{I}_m$ is an $m \times m$ identity matrix, and $\otimes$ is the Kronecker product. By using $\phi^T(t)$ the one step-ahead predictor $\hat{y}(t|t-1)$ of the ARMAV model (6) is given by

$$\hat{y}(t|t-1) = \phi^T(t)\theta \tag{13}$$

The RLS (RARV) technique is obtained by replacing the gradient $\psi(t)$ in (12) by the multivariate regression matrix $\phi^T(t)$ of dimension $m \times (p+q)m^2$ defined as

$$\begin{aligned} \phi^T(t) &= \varphi^T(t) \otimes I_m \\ \varphi^T(t) &= \{-y^T(t-1), \ldots, -y^T(t-p)\}^T \end{aligned} \tag{14}$$

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3. BASIC EQUATIONS FOR KALMAN FILTERING

The following section describes the basic equations for Kalman filtering, see e.g. Juang [18]. To apply Kalman filtering for system identification, the state vector is augmented to include the parameters to be identified. Then this new state vector consists then of two parts as follows

$$\boldsymbol{Z}_1(t) = \begin{bmatrix} \boldsymbol{Z}_s(t) \\ \boldsymbol{Z}_p(t) \end{bmatrix} \tag{15}$$

where the subscript s denotes the system state vector component and p denotes the parameters to be identified. In short notation the differential equations of the system to be identified are written:

$$\dot{\boldsymbol{Z}}_1(t) = g(\boldsymbol{Z}_1(t), t) \tag{16}$$

where the observation equation takes the following form

$$y = \boldsymbol{H}\hat{\boldsymbol{Z}}_1 + \nu \tag{17}$$

where $\boldsymbol{H}$ is the observation matrix and ν represents the measurement noise which is assumed to be zero-mean Gaussian white noise with a covariance matrix $E[\nu(k)\nu(j)] = \boldsymbol{R}\,\delta(k - j)$ at any discrete time k. $\delta(k - j) = I$ when $k = j$ else zero.

The uncertain initial conditions are modelled as Gaussian Random variables with a mean vector $\boldsymbol{Z}_1(0)$, and an error covariance matrix of $\boldsymbol{D}(0)$, i.e.

$$\boldsymbol{Z}_1(0) \sim N(\hat{\boldsymbol{Z}}_1(0), \boldsymbol{D}(0)) \tag{18}$$

Thus the *a priori* information needed for the identification process is $\boldsymbol{Z}_1(0)$, $\boldsymbol{D}(0)$ and $\boldsymbol{R}(0)$. In the used Kalman Filter the information collected in the measurment vector (17) is used to update the state-vector $\boldsymbol{Z}_1$. The updating is done in two steps, i.e. a prediction phase and a update phase, respectively.

3.1 Prediction Phase

Based upon the uncertain initial conditions the state of the system can be estimated which include estimates of a new mean and new error-covariance At time step k the prediction of the state and error-covariance at time step $k+1$, $\mathbf{Z}(k+1|k)$ and $\mathbf{D}(k+1|k)$, respectively, can be be estimated by solving the following differential equations, Juang [18].

$$\begin{aligned} \dot{\boldsymbol{Z}}_1(t) &= g(\boldsymbol{Z}_1(k|k),t) + \dot{w}(t), \qquad k \le t \le k+1 \\ \dot{\boldsymbol{D}}(t) &= F(\boldsymbol{Z}_1(k|k),t)\boldsymbol{D}(t) + \boldsymbol{D}(t)\boldsymbol{F}^T(\boldsymbol{Z}_1(k|k),t), \qquad k \le t \le k+1 \end{aligned} \tag{19}$$

where $\boldsymbol{F}(X(k|k)$ is the gradient of the system dynamics as follows

$$\dot{\boldsymbol{F}}(\boldsymbol{Z}_1(k|k),t) = \frac{\partial g(\boldsymbol{Z}_1(k|k),t)}{\partial \boldsymbol{Z}_1(t)}\Big|_{\boldsymbol{Z}_1(t) = \hat{\boldsymbol{Z}}_1(t)} \tag{20}$$

w(t) represents the modelling error which is assumed to be zero-mean Gaussian white noise with a covariance matrix $E[w(k)\ w(j)] = \boldsymbol{Q}\ \delta(k-j)$. It is assumed that the two processess $\nu(k)$ and $w(k)$ are stationary and independent, i.e. $E[\nu(k)\mathbf{w}(k)] = 0$ for all k and j.

3.2 Update Phase

The observation that becomes available at $k+1$ is then used to update the state vector and error covariance matrix, Juang [18]

$$\begin{aligned} \dot{\boldsymbol{Z}}_1(k+1|k+1) &= \boldsymbol{Z}_1(k+1|k) + \boldsymbol{K}(k+1)\left[\boldsymbol{Y}(k+1) - \boldsymbol{H}\hat{\boldsymbol{Z}}(k+1|k)\right] \\ \boldsymbol{D}(k+1|k+1) &= [\boldsymbol{I}-\boldsymbol{K}(k+1)\boldsymbol{H}(k+1)]\boldsymbol{D}(k+1|k)[\boldsymbol{I}-\boldsymbol{K}(k+1|k)\boldsymbol{H}(k+1)] + \boldsymbol{K}(k+1)\boldsymbol{R}\boldsymbol{K}^T(k+1) + \boldsymbol{Q} \end{aligned} \tag{21}$$

where $\boldsymbol{K}(k+1)$ is the so-called Kalman Gain matrix calculated as

$$\boldsymbol{K}(k+1) = \boldsymbol{D}(k+1|k)\boldsymbol{H}^T(k+1)[\boldsymbol{H}(k+1) - \boldsymbol{D}(k+1|k)\boldsymbol{H}(k+1) + \boldsymbol{R}]^{-1} \tag{22}$$

and $\boldsymbol{R}$ is the covariance matrix of the measurement noise.

In order to treat the parameters as time varying parameters, it is necessary to add noise in $\boldsymbol{Q}$ at the positions corresponding to the covariance of these parameters to prevent the gain from tending towards zero. The equations are then repeatingly used until the end of the time series is reached and a series of time-varying stiffnesses is obtained. The mode shapes Φ of the identified system are then determined by solving the eigenvalue problem $\det(\boldsymbol{K}(t) - \omega_i^2(t)\boldsymbol{M})\Phi_i(t) = 0$ of the system where $\boldsymbol{K}$(t) and $\boldsymbol{M}$ are the stiffness and mass matrix, respectively.

4. EXAMPLE

In this example the system identification techniques described in section 2 and 3, respectively, will be investigated in a simulation study. The investigations will be based on time series simulated by a non-linear finite element program SARCOF, Mørk [17], which has been verified to be able to predict accurately the response of deteriorating RC-structures with well-defined structural parameters (bending stiffnesses of cracked and uncracked of all beam-elements must be specified).

4.1 Test Structure

The computer model, which is a 1:5 model of a laboratory test model consists of two 6-storey, 2-bay frames working in parallel with storey weights, uniformly distributed, attached in between.The total height of the structure is 3.3 m and all storey heights are uniformly distributed. The columns and beams in the structure are 0.06 m wide, 0.05 m deep for the columns and beams, respectively. Furthermore, all columns and beams are symmetrically reinforced. The following values are used for the density $\rho = 2500$ kg/m^3, the stiffness $E = 3.5 \cdot 10^{10}$ N/m^2 and the damping ratio $\zeta = 0.05$. The stiffness and strength deterioration are modelled using a Roufaiel-Meyer hysteretic model. The first and second natural eigenfrequencies of the undamaged structure are 1.93 Hz and 6.13 Hz, respectively.

The acceleration process at the ground surface is determined as the response process of an intensity modulated Gaussian white noise filtered through a Kanai-Tajimi filter, see Tajimi [19] implying that the negative part of the ground surface acceleration is estimated. The damping ratio in the Kanai-Tajimi filter is chosen as 0.3 and the circular frequency is chosen as 10.0.s^{-1}. The excitation has maximum acceleration after 3 sec. and the duration of the excitaion is 15 sec. The integrated dynamic system is in SARCOF solved by a 4th order Runge-Kutta Scheme. The time step is selected as 0.002 sec., where it has been proved that no drift occurs in the simulated signal. It is assumed that measurements are performed at all storeys. During the simulations SARCOF gives the instantaneous frequencies and mode shapes of the lowest two modes of the structure. These estimates will vary rapidly as the structure enters and leaves the plastic regime. Therefore, it is necessary to perform a smoothing of the measured eigenperiod which corresponds to time-averaging the structural degradation. A time-averaging method of the instantaneous period has been proposed by Rodriquez-Gomez [22] and is based on the principle of a moving averaging window in the following way. The smoothed value of the ith frequency $<f_i(t)>$ at the time t is evaluated as

$$<f_i(t)> = \frac{1}{T_a}\int_{t-\frac{T_a}{2}}^{t+\frac{T_a}{2}} f_i(t)dt \qquad \textbf{(23)}$$

where T_a is the length of the averaging window, which should be sufficiently large, so that the local peaks are removed. On the other hand, T_a should not be selected so large that intervals of increased plastic deformation are not displayed in $<f_i(t)>$. The value $T_a = 2.4\,T_0$ is recommended as a reasonable compromise, Rodriquez-Gomez [22], where T_0 is the 1st eigenperiod of the equivalent linear structure. The mode shapes show the same fluctuating behaviour as the frequencies, and are therefore smoothed as the frequencies.

4.2 Kalman Filter Implementation

In this study it is assumed that the dynamics of the 6-storey RC-frame can be described by a 6 degree-of-freedom oscillator as illustrated in figure 1.

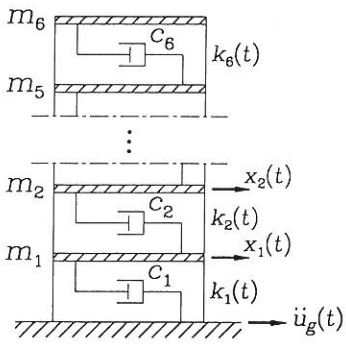

Figure 1 Dynamic model for the frame

Only the spring stiffness are assumed to vary with time. The masses and dampings are assumed to be known and constants during the motions and it has been assumed that all storeys have identical masses m and damping c. The initial stiffness of the springs is assumed to be known from static analysis.The first order differential equations used for Kalman Filtering of the system illustrated in figure 2 are

$$\dot{Z}_1 = \frac{d}{dt}\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \\ x_3 \\ \dot{x}_3 \\ x_4 \\ \dot{x}_4 \\ x_5 \\ \dot{x}_5 \\ x_6 \\ \dot{x}_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} x_1 \\ -\frac{k_1(t)}{m}x_1 - \frac{c}{m}\dot{x}_1 + \frac{k_2(t)}{m}x_2 + \frac{c}{m}\dot{x}_2 - \frac{\ddot{u}_g}{m} \\ \dot{x}_2 \\ \frac{k_1(t)}{m}x_1 + \frac{c}{m}\dot{x}_1 - 2\frac{k_2(t)}{m}x_2 - 2\frac{c}{m}\dot{x}_2 + \frac{k_3(t)}{m}x_3 + \frac{c}{m}\dot{x}_3 \\ \dot{x}_3 \\ \frac{k_2(t)}{m}x_2 + \frac{c}{m}\dot{x}_2 - 2\frac{k_3(t)}{m}x_3 - 2\frac{c}{m}\dot{x}_3 + \frac{k_4(t)}{m}x_4 + \frac{c}{m}\dot{x}_4 \\ \dot{x}_4 \\ \frac{k_3(t)}{m}x_3 + \frac{c}{m}\dot{x}_3 - 2\frac{k_2(t)}{m}x_4 - 2\frac{c}{m}\dot{x}_4 + \frac{k_3(t)}{m}x_5 + \frac{c}{m}\dot{x}_5 \\ \dot{x}_5 \\ \frac{k_4(t)}{m}x_4 + \frac{c}{m}\dot{x}_4 - 2\frac{k_2(t)}{m}x_5 - 2\frac{c}{m}\dot{x}_5 + \frac{k_3(t)}{m}x_6 + \frac{c}{m}\dot{x}_6 \\ \dot{x}_6 \\ \frac{k_5(t)}{m}x_5 + \frac{c}{m}\dot{x}_5 - 2\frac{k_6(t)}{m}x_6 - 2\frac{c}{m}\dot{x}_6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \textbf{(24)}$$

where $x_i = \frac{k_i(t)}{m}$, i = 7,8,..,12 are the additional state variables.
In order to use the Kalman filter, the following intial quantities are specified

- Initial values of the parameters to be identified.
- The initial error covariance matrix $\boldsymbol{D}(0)$.
- The measurement noise covariance matrix $\boldsymbol{R}$
- The modelling error covariance matrix $\boldsymbol{Q}$.

In the case of the 6-storey RC-frame the initial stiffness/mass ratio were estimated from FEM-analysis to be as shown in table 1, where also the assumed damping/mass ratios are listed.

Storey No.	1	2	3	4	5	6
k_i / m	3950	3950	2250	2200	1950	1400
c_i / m	1.0	1.0	1.0	1.0	1.0	1.0

Table 1: Initial parameteres of the 6 - Dof system

In all the considered cases, the measurement noise covariance matrix $\boldsymbol{R}$ are modelled as a diagonal matrix with a noise variance of 10^{-6} in the diagonal corresponding to a noise-signal ratio $\frac{\sigma_{noise}}{\sigma_{signal}}$ equal to approximately 5 %.

The modelling error matrix $\boldsymbol{Q}$ are also modelled as a diagonal matrix where the variances of the displacements and velocities in all cases are set to 0.001 and the variances of the parameters to be estimated are set to q, so

$$diag(\boldsymbol{Q}) = [0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ q\ q\ q\ q\ q] \quad \textbf{(25)}$$

The initially error covariance matrix $\boldsymbol{D}(0)$ are set to be a diagonal matrix as well, also with initial variances of 0.001 for errors in displacement and velocities and the initial error covariance of the parameters to be estimated were set to p, so

$$diag(\boldsymbol{D}(0)) = [0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ 0.001\ p\ p\ p\ p\ p\ p] \quad \textbf{(26)}$$

From introductory investigations it was found that the final results are quite insensitive to the choice of variance of displacement and velocities in both $\boldsymbol{Q}$ and $\boldsymbol{D}(0)$ as well as the noise, so these can be chosen quite arbitrary. However, the choise of p and q is found to effect the results quite much. In the following $p = 0.2$ and $q = 0.0001$.

4.2 Results

Table 2 shows the estimated frequencies of the lowest two modes while figure 2 shows corresponding mode shapes for the time steps 4, 8 and 12 sec, respectively. By using different model selection criteria such as the Akaike's information criterion and pole plots etc., see e.g. Ljung [5], an RARMAV(6,5) and an RARV(6), respectively, were chosen. The parameters in the RARMAV and RARV models were estimated by using exponentially decaying weighting factors $\lambda(t)$, Ljung [5]

$$\lambda(t) = \lambda_o \lambda(t-1) + 1 - \lambda_o \quad \textbf{(27)}$$

The forgetting factor parameters $\lambda(0)$, λ_o were both selected to have the value 0.95 and 0.99, respectively. Further, the initial values of the parameter estimates θ *(0)* and covariance matrix ***P(0)*** were selected as random variables and as a unity matrix, respectively. Further, an ARMAV model was used on a window with a length of 2.4 T_0. This approach is called ARMAV+W.

The estimated natural frequencies estimated by the techniques are shown in table 2 for the time steps 4 sec., 8 sec. and 12 sec., respectively. It is seen that KF, RARMAV and ARMAW +W give results for the first frequency corresponding to the SARCOF result. For the second frequency only the ARMAV+ W technique seems to give usable resluts.
The estimated first and second mode shape for the same time steps as the first two frequencies are shown in figure 2. Here, it is seen again that the KF, RARMAV and ARMAW +W techniques give usable results for the first mode while the second mode is only estimated well by the KF technique.

Mode	SARCOF			KF			RARMAV			RARV			ARMAV+W		
time (sec)	4	8	12	4	8	12	4	8	12	4	8	12	4	8	12
1	9.8	8.5	7.3	11.5	9.0	7.1	10.9	8.5	8.0	10.8	10.2	9.4	14.4	8.1	7.8
2	34.7	32.1	29.5	33.2	28.5	23.4	27.2	23.4	22.7	23.7	22.9	21.6	33.8	28.7	28.4

Table 2: Frequencies of the lowest two modes for the time steps 4, 8 and 12 sec.

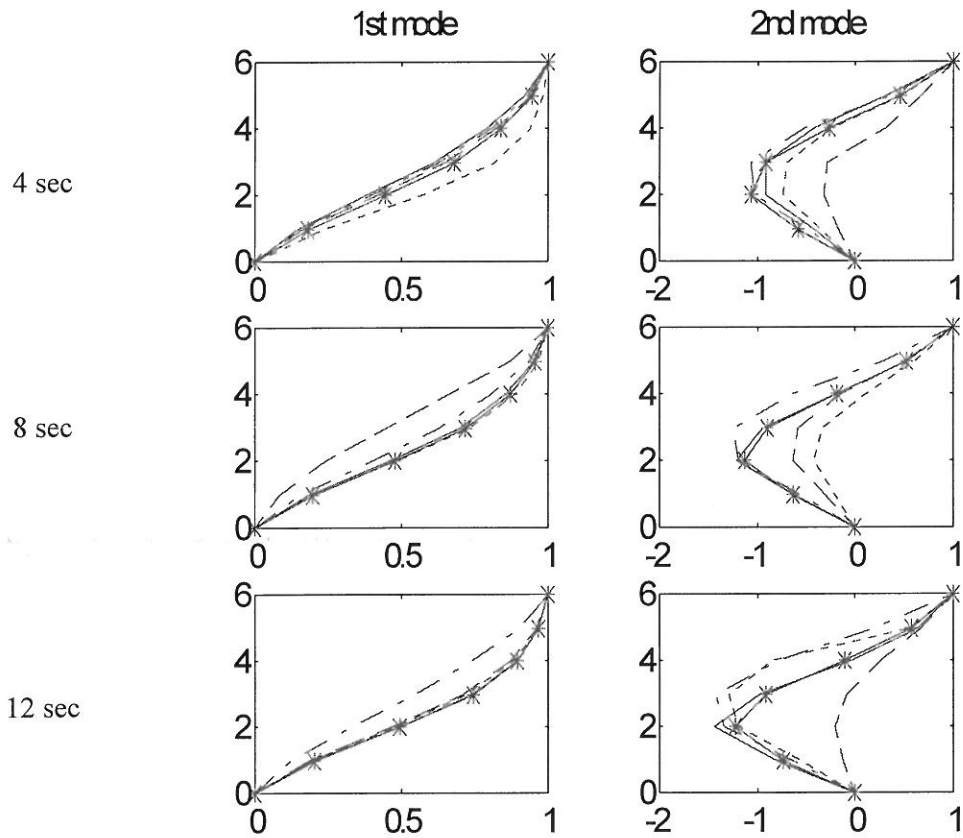

Figure 2: Mode shapes of the lowest two modes for the time steps 4, 8 and 12 sec.
[-*-*-] : SARCOF, [– - – -] : RARMAV, [– – –] : RARV, [——] : KF, [- - - - -] : ARMAV+W

5. CONCLUSION

This paper presents system identification results of an RC-structure using multivariate auto-regressive moving-average vector models (ARMAV) on a running window, recursive ARMAV models (RARMAV) and Kalman filtering of a simple spring-mass-dashpot system. The resluts show that that the KF, RARMAV and ARMAW +W techniques give results for the first frequency and mode shape corresponding to the SARCOF result. For the second mode only the ARMAV+W and the KF techniques seem to give usable results for the second frequency and the mode shape, repsectively.

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6. ACKNOWLEDGEMENTS

The Danish Technical Research Council is gratefully acknowledged.

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