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Larsen, Torben

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SEA OUTFALL DESIGN BASED ON A STOCHASTIC TRANSPORT/DISPERSION MODEL

by

TORBEN LARSEN
Department of Civil Engineering
University of Aalborg
Sohngaardsholmsvej 57, 9000 Aalborg, Denmark

ABSTRACT

This paper describes a numerical model of the dilution and disappearance of sewage discharged to the coastal zone. The model is based on the Monte Carlo (or random walk) principle. A cloud of particles is released at discrete time steps and the 3-dimensional path of every particle is simulated. The stochastic element of the movement is controlled by random numbers. It is shown how the model can simulate the unsteady case of the dilution from a sea outfall where both wind-induced and tidal currents are taken into consideration. The intention of the paper is to present how complex transport/dispersion phenomena can easily be modelled by the stochastic approach without going into advanced methods as finite differences or elements. The advantage of this approach is the simple programming and low need of computer memory. The disadvantage could be the need for excessive computing time.

NOMENCLATURE

az	wind direction, rad
c	concentration, number of bacteria per m^3
c_d	air/sea drag coefficient, dimensionless
D	diffusion coefficient, m^2/sec
D_x, D_y, D_z	diffusion coefficients, m^2/sec
h	water depth, m
M	mass, kg
m	actual number of bacteria in particle
m_o	initial number of bacteria in particle
rnd	random number between -1 and 1, equally distributed
t	time, sec
T_{90}	bacterial decay factor, sec^{-1}
U, V, W	resulting mean velocity of particle, m/sec
u_f	friction velocity, m/sec
u_n	net longshore current, m/sec
u_t, v_t	longshore and onshore tidal current, m/sec
u_o, v_o	longshore and onshore tidal current amplitudes, m/sec
u_w, v_w	longshore and onshore wind drift, m/sec
w_{10}	wind velocity 10 m above sea surface, m/sec
x, y, z	cartesian coordinates of particle, m
Δt	step in time, sec
Δx	step in length, m
ρ_a	density of air, kg/m^3
ρ_w	density of water, kg/m^3
ϕ_u, ϕ_v	phases of the longshore and onshore tidal current, rad

INTRODUCTION

The design of sea outfalls is most often based on estimates of the bacterial pollution of the bathing waters along the coasts. Primarily it has to be proved that the concentration of the indicator bacteria, Escherichia coli, do not exceed 1000 pr 100 ml more than 5% of the time. From a given source of sewage in the coastal zone, the concentration of E.coli will depend on dilution, sedimentation and mortality. This paper emphazises only on the dilution, although the effect of sedimentation and mortality formally is included in the model described later on.

In practice (at least in Denmark) the estimates of the dilution are often based on very simplified assumptions. It is e.g. assumed that a steady wind-driven current, with a fixed layer-thickness (1-3 m) is going with a velocity of 2% of the wind, and in this layer a 2-dimensional Gaussian dispersion formula is applied using a relatively large dispersion coefficient on the order of magnitude of 1 m^2/sec. This practice of design is mostly used in relation to smaller outfalls. Hydrographic and tracer investigations are often employed to the design of larger outfalls.

Experience from tracer experiments in relation to sea outfalls shows that the dilution only can be treated as a 3-dimensional problem. The most important physical factors governing the transport/dispersion process are often the tidal current, the net current and the wind drift. But also wave induced currents as Stokes drift and surf current can in some cases be significant. Density gradients are almost always present and have a strong influence on the structure of the turbulence.

Numerical modelling of 3-dimensional transport/dispersion processes are connected with serious theoretical problems. In principle, pure transport can not be simulated by use of finite differences or finite elements without generation of numerical dispersion. Advanced models, as e.g. described by Hinrichsen [1]are available but seem not applicable in practice yet.

The Monte Carlo (or random walk) principle, which is revived here, is well suited for the simulation of the transport and dispersion of matter. This Lagrangian simulation approach is basicly related to tracer or float experiments and as the programming is simple, this type of model is easily adapted to specific problems.

The Monte Carlo method has been applied to atmospheric dispersion by Hino [2] and Thompson [3], and to the movement of oil slicks in the coastal environment by Tayfun and Wang [4].

RANDOM WALK DIFFUSION

Let us for simplicity take the one-dimensional diffusion equation without transport

$$\partial c/\partial t = D(\partial^2 c/\partial x^2) \qquad (1)$$

c is concentration (kg/m^3) and D the diffusion coefficient (m^2/s).

If the initial condition at time t = 0 is a Dirac delta function where the mass M is kept in an infinitesimal distance around x = 0, the well-known solution to (1) is

$$c = (M/\sqrt{4\pi Dt}) \exp(-x^2/4Dt) \qquad (2)$$

If $\sigma^2 = 2Dt$ is substituded into equation (2), the Gaussian distribution with zero mean and variance σ^2 appears.

From probability theory (the central limit theorem) it is known that the continuous Gaussian distribution is the limit of a symmetric binomial distribution. This binomial distribution can be estimated by the random walk of a large number of particles, which at equal time steps Δt moves randomly either Δx or $-\Delta x$, with equal probability. It can be shown that the variance of the matching Gaussian distribution is given by

$$\sigma^2 = t(\Delta x)^2/\Delta t \qquad (3)$$

Introducing D gives

$$\Delta x = \pm\sqrt{2D\Delta t} \qquad (4)$$

From this short resume of the random walk principle of one-dimensional diffusion it should be obvious that the principle without restrictions can be applied to the unsteady 3-dimensional case with arbitrary boundary and initial conditions.

At each time step the sign of equation (4) has to be found from a random experiment. In the computer it is more convenient to use a random variable rnd (-1, 1), which is equally distributed from -1 to +1. It can be shown that equation (4) now takes the form

$$\Delta x = (\sqrt{6D\Delta t}) \text{ rnd } (-1, 1) \qquad (5)$$

The heart of the computer model, where a particle with position (x, y, z) is moved from time increment i to i + 1 will be

$$x_{i+1} = x_i + U\,\Delta t + (\sqrt{6D_x\Delta t}) \text{ rnd}$$

$$y_{i+1} = y_i + V\,\Delta t + (\sqrt{6D_y\Delta t}) \text{ rnd}$$

$$z_{i+1} = z_i + W\,\Delta t + (\sqrt{6D_z\Delta t}) \text{ rnd}$$

(U, V, W) is the mean velocity of the particle.

If a particle crosses a boundary e.g. the water surface, it is reflected in the boundary.

CURRENTS AND DISPERSION IN COASTAL WATERS

In relation to Danish coasts and Danish sea outfalls, the coastal zone can be defined as a 1-3 km zone in width with water depths to approx. 15 m.

Currents in this zone are normally parallel to the coastline and during the periods in the summer where the winds are low, the tidal currents are often dominating.

Tracer experiments show that horizontal spread can vary considerably and horizontal diffusion coefficients, calculated from the growth of the horizontal variance with time, can differ orders of magnitude from one experiment to another.

The results from the two tracer experiments in the Limefjord near Nykøbing Mors are shown on fig. no. 1 and 2. The figures show the surface concentration of rodamine B released from a position of a planned sea outfall. The tracer was released near the surface and the strong influence of the wind-shear is obvious. The density difference between bottom and surface was approx. 2 kg/m^3, but from the surface down to 2-3 m depth the water was homogeneous.

Those two examples have been chosen among several cases from different locations and appear from the authors point of view as the typical pattern of the dilution from Danish sea outfalls. A qualitative description of the physical process is as follows.

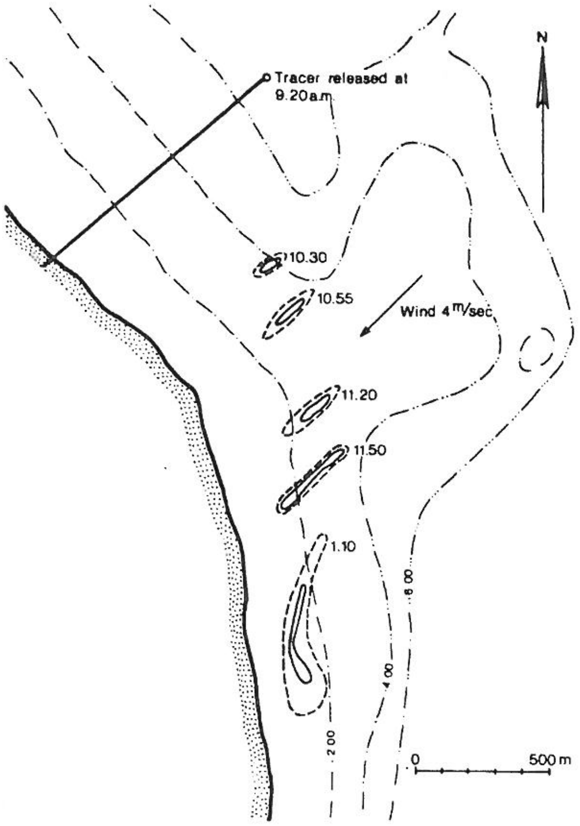

Fig. 1 Tracer experiment in the Limefjord north of Nykøbing Mors, Denmark, 05.08.1976, wind from NE.

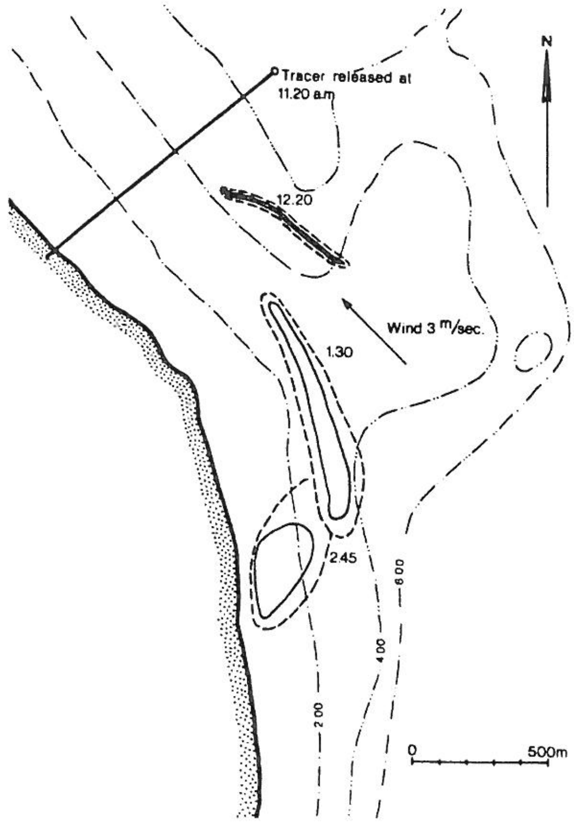

Fig. 2 Tracer experiment same location as above, 06.08.1976, wind from SE.

1. Main transport is governed by the tidal current. Dispersion in pure tidal current is negligible due to the damping effects of vertical density gradients. Meandering of the plume due to large scale horizontal turbulence can be seen.

2. Dispersion occurs primarily as the result of vertical wind-shear diffusion and the varying velocity vector in vertical direction. The velocity vector rotates with depth because of the combined effect of wind drift and tidal current.

If dispersion and dilution are the effect of wind-shear turbulence in the uppermost meters of the sea, the problem can be solved according to the same principles as used by Taylor [5], Elder [6] and Fisher [7] for the dispersion in pipes, channels, and natural streams respectively.

Churchill and Csanady [12] and [13] have recently measured the surface currents in the top 2 m of the sea by a sophisticated float-technique. Their conclusion was that the surface current could be described as a logarithmic turbulent boundary layer velocity profile as suggested by von Karman. Furthermore it was shown that the second order effect of the waves, known as the Stokes drift, also was significant in some cases.

Wind-induced currents in a laboratory flume have been measured in several cases, e.g. by Baines and Knapp [8]. Theoretical turbulence models of the so-called k-ε type have been applied to wind-induced currents e.g. Svensson [9]. The overall impression of these works confirms the assumption of the logarithmic velocity profile.

THE DILUTION MODEL

It is now described how the dilution of sewage from a sea outfall can be modelled by the random walk principle for the situation of a longshore nonturbulent tidal current and a crossing wind-induced turbulent drift, fig. 3.

It is postulated that the sewage, before the turbulent dispersion takes over, is spread on the surface by buoyancy to a horizontal scale of several times the scale of the vertical turbulence. Although this buoyancy spread is an important part of the dilution process, it will not be discussed further here. If the initial surface spread is considerably larger than the scale of the vertical turbulence it is possible to apply a gradient-flux or Fickian diffusion model to the problem. This is fairly analogous with the assumption of Taylor [5] and

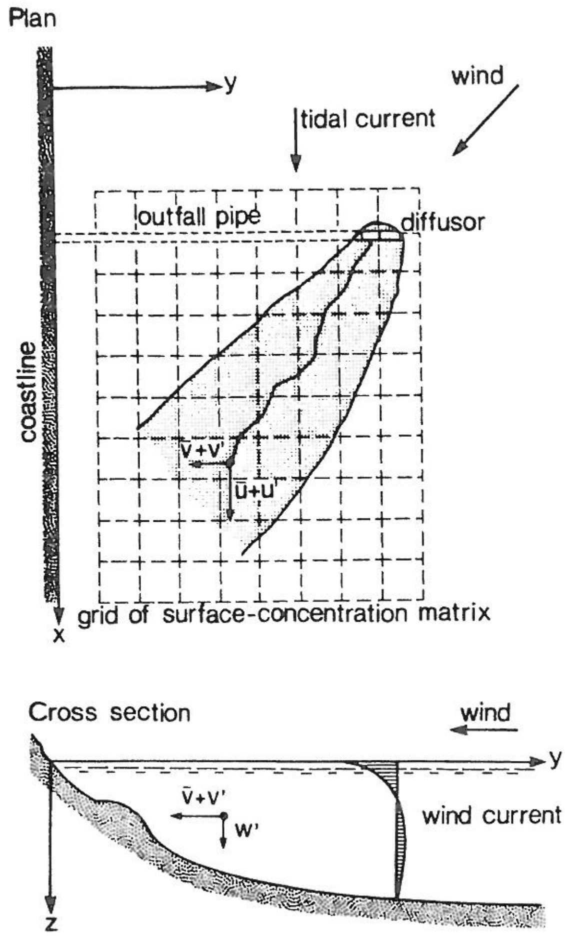

Fig. 3 Schematic principle of random walk model for the coastal zone

Elder [6] in their analysis of shear dispersion. As mentioned earlier, the random walk model in its simple form is an approximation to Fickian diffusion. From this chain of arguments it is hopefully understood that the simple random walk model is sufficient for the simulation of wind generated dispersion on the surface of the sea.

The equations used in the model are listed below.

Tidal currents

From experience it is known that the M_2-component is the dominating part of the tidal current so the longshore part u_t and the onshore part v_t are described as

$$u_t = u_o \sin (2\pi\ t/T + \phi_u) + u_n \tag{6}$$

$$v_t = v_o \sin (2\pi\ t/T + \phi_v) \tag{7}$$

where (u_o, v_o) are the amplitudes of the tidal currents and (ϕ_u, ϕ_v) the phases. u_n is the net current. T is the M_2 period equal to 12.42 hour.

Wind drift and diffusion

The wind drift profile is assumed to be logarithmic and following the wind direction, (fig. 4). The longshore part u_w and the onshore part v_w are

$$u_w = 2.45\ u_f \ln(h/z) \cos (az + \pi) \tag{8}$$

$$v_w = 2.45\ u_f [\ln(h/z) - 1] \sin (az + \pi) \tag{9}$$

The wind direction is az and h is depth of water. The friction velocity u_f is determined by

$$u_f = w_{10} \sqrt{c_d\ \rho_a/\rho_w} \tag{10}$$

where w_{10} is wind velocity 10 meters above the sea surface, ρ_a and ρ_w are density of air and seawater, and c_d is the air/sea drag coefficient.

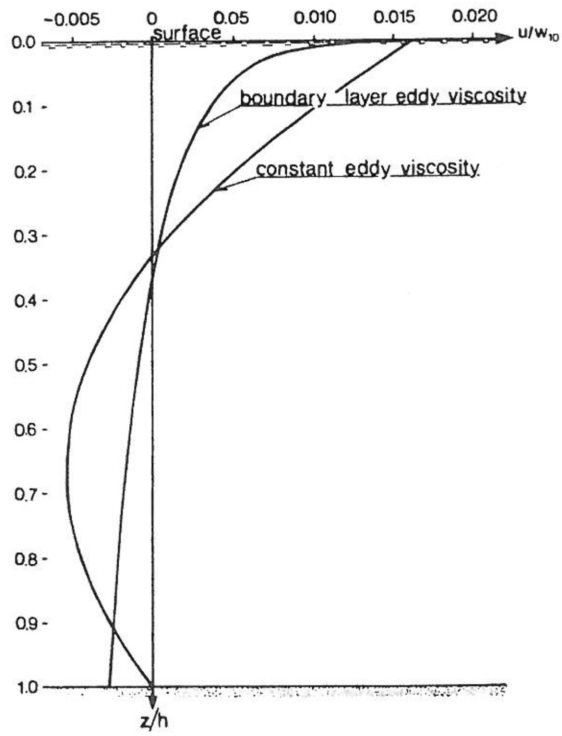

Fig. 4 Vertical velocity profile induced by wind in the case of a boundary layer eddy viscosity and constant eddy viscosity

The diffusion coefficient is assumed equal to the eddy viscosity and given by

$$D_x = D_y = D_z = 0.41\ u_f\ z\ (1 - z/h) \tag{11}$$

In order to avoid a resulting depth-averaged wind drift going in the onshore direction slightly different equations for u_w and v_w have been applied.

For numerical reasons it is furthermore assumed that the vertical position z of a particle never becomes less than h/1000.

Bacteriological mortality and sedimentation

The bacteriological disappearance due to mortality and sedimentation is assumed to follow an exponential decay

$$m/m_o = 10^{-(t/T_{90})} \qquad (12)$$

where m is the actual number of bacteria in a particle to the time t after release, m_o is the initial number and T_{90} is a decay factor.

The structure of the computer-program

The advantage of the random walk principle is that each particle can be treated independently. This means that only the final position of the particle has to be kept in the memory. This final position is kept in a matrix of the surface concentration as seen on figure no. 3. The grid size in this matrix has to be chosen as a compromise between the wish of resolution and accuracy. Before the mass of the particle is added to the respective element in the matrix, the mass is corrected for disappearance.

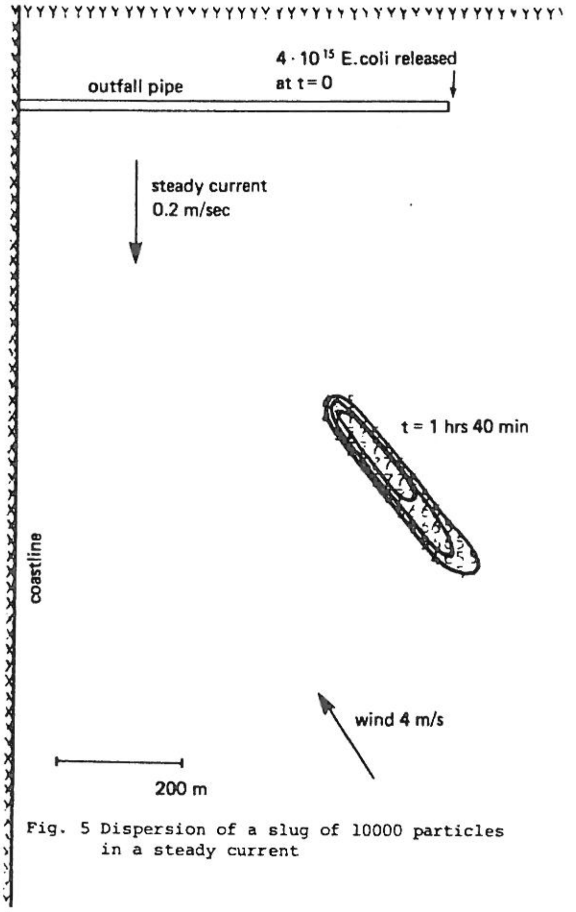

Fig. 5 Dispersion of a slug of 10000 particles in a steady current

RESULTS

The result of the model described in this paper is the computer program. It is the point of view of the author that computer simulations should not be used to set up general graphs and formulas but to present specific solutions to specific problems.

Three examples of results are shown in figure 5, 6 and 7. Figure no. 5 shows the dilution of a slug of sewage in a steady longshore current and a crossing wind drift. Figure no. 6 gives the dilution of a continuous source of sewage under the same situation as in figure 5. Finally figure 7 shows a complex situation of a continuous plume 1 hour after the tide has turned.

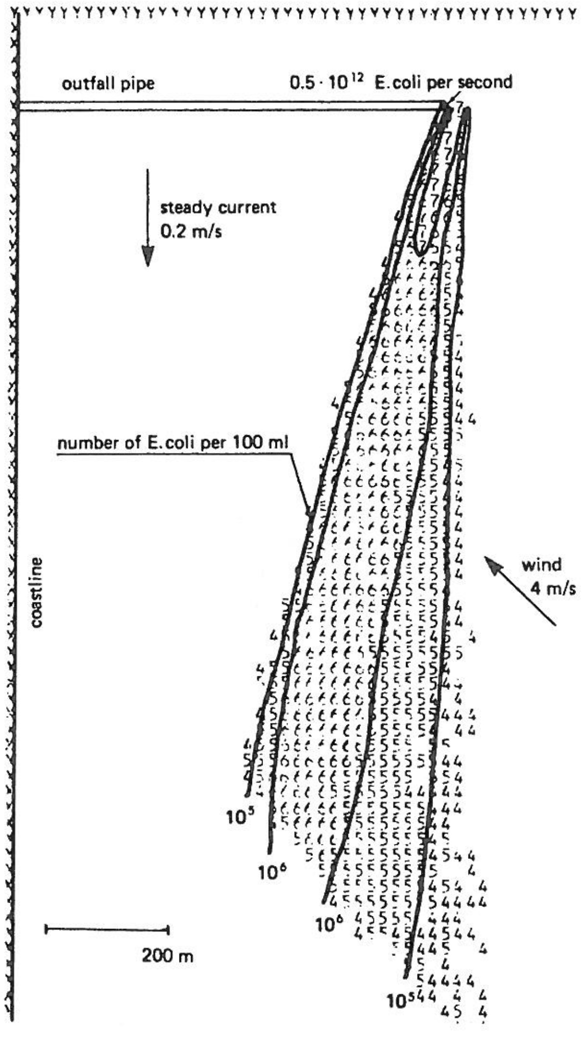

Fig. 6 Dispersion of a continuous sewage plume, slugs of 100 particles released every 30 sec., each particle represents $1.67 \cdot 10^8$ E.coli initially, surface concentration in upper 0.5 m shown.

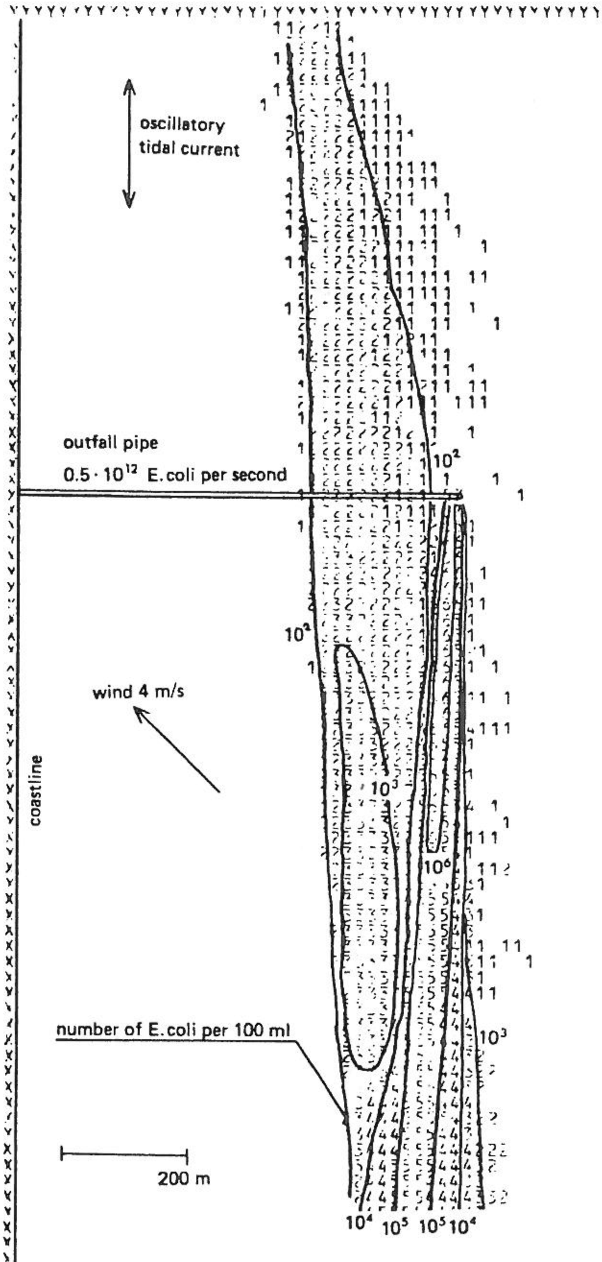

Fig. 7 As fig. 6, but with oscillatory tidal current

DISCUSSION

Wind-induced shear dispersion in relation to sea outfalls has earlier been discussed theoretically by Munro and Mollowney [10]. In their paper the 3-dimensional transport/diffusion equation was solved by a special version of finite differences. The eddy viscosity (and vertical diffusion) was assumed constant, which gave a parabolic velocity profile (see figure no. 4) according to the classic paper of Helström [11].

However, it seems obvious to utilize the experimental verification by Churchill and Csanady [12] and [13] of the logarithmic velocity profile to an improved model as done here.

Shear dispersion in steady and oscillatory currents has been the subject of a number of papers by e.g. Okubo [14] and Fisher [15] and further development was carried out by Kullenberg [16]. This paper presents an attempt to transfer their basic research to the world of practical engineers.

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