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Calculation of the Wave Conditions in Nissum Bredning

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Contents

Chapter 1

Wave Energy Flux in Nissum Bredning

For the purpose of determining the optimal position in Nissum Bredning for placement of Wave Dragon, the wave energy flux in Nissum Bredning has been calculated. It has not been posible to retrieve satisfactory measured wavedata for Nissum Bredning, therfore the calculations are based on the SPM-method[1]. By this method the significant wave height, H_S, and the peak period, T_P, are calculated from the fetch, F, and wind stress factor, U_A.

1.1 Fetch

Wave heights in Nissum Bredning are fetch limited. Due to the limited water depth, the SPM-method for wave generation in shallow waters has been used. The SPM-method makes it possible to calculate H_S and T_P for areas with constant water depth, therefore the sea bed has been approximated with areas of constant water depth as shown in figure 1.1

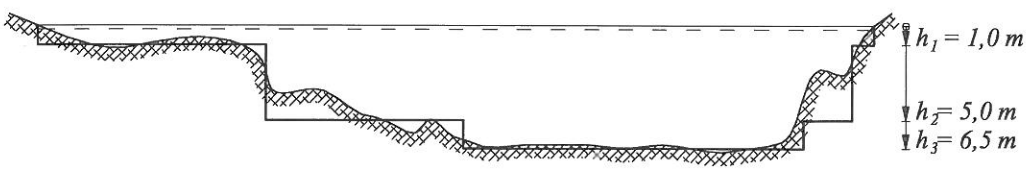

Figure 1.1
Cross section of Nissum Bredning with real sea bed topography and the approximation used in the SPM-calculation. The cross section matches the dassed line shown in figure 1.2.

The contour lines at water depths of 2 and 6 m are chosen as boundery for the areas with constant depth, see figure 1.2.

[1] "Shore Protection Manual"

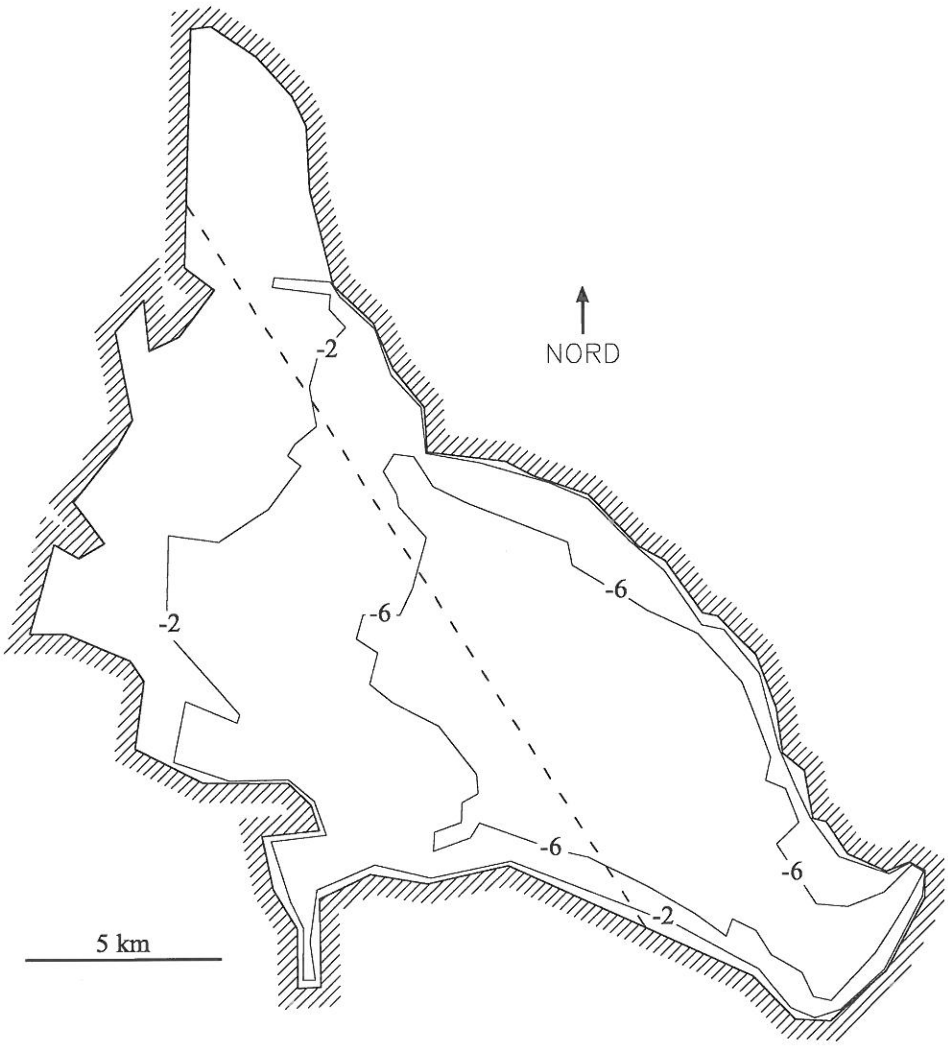

Figure 1.2
Depth contour used for determining the fetch. The dassed line corresponds to the section shown in figure 1.1.

The water depths in the chosen areas are $h_1 = 1,0\ m$, $h_2 = 5,0\ m$ og $h_3 = 6,5\ m$. The depth contours and shore lines are described with straight lines for determining the fetch. The fetch is calculated at a web of points in the directions N, NE, E, SE, S, SW, W and NW.

1.2 Wind Data and Wind Stress Factor

Measurement locations

Wind data given in "Danmarks Klima i Vind, Standardnormaler 1931-60" is used. The mean wind forces over a 30 year period from the 8 wind directions are shown in table 1.1. Wind data from four locations around Nissum Bredning is available: Thyborøn, Vestervig, Bovbjerg Lighthouse and Humlum, see figure 1.3.

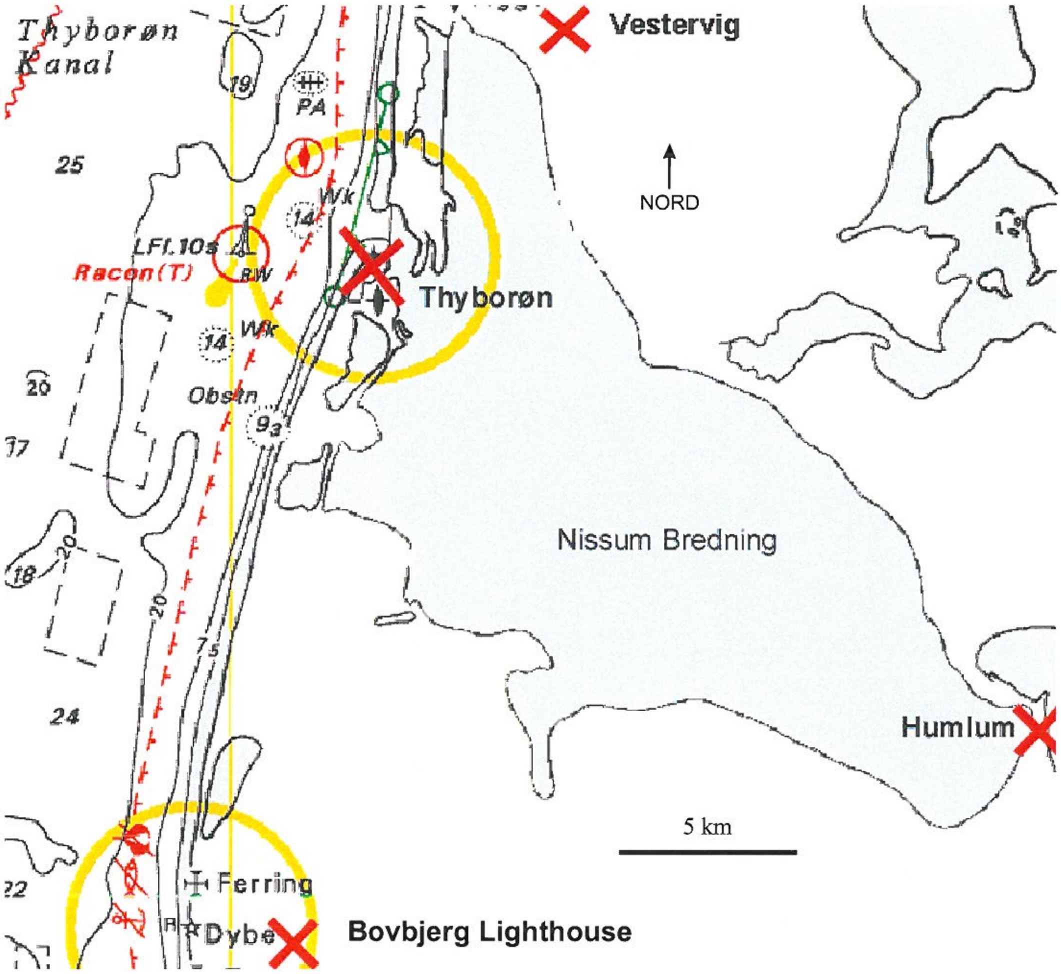

Figure 1.3
The four wind measurement stations situated around Nissum Bredning.

For calculations of waves wind data from Thyborøn is used. The data from Vestervig and Bovbjerg Lighthouse stations are used to verify these data, see section 1.3. The registrations from Humlum just give the wind directions.

Used wind data

Wind force	1	2	3	4	5	6	7	8	9	10	11	12
North	1.4	1.5	1.1	0.7	0.4	0.3	0.1	0.1	0.0	0.0	0.0	0.0
North-east	1.7	1.8	1.7	1.2	0.6	0.3	0.1	0.1	0.0	0.0	0.0	0.0
East	1.9	2.3	2.2	2.0	1.5	0.8	0.4	0.1	0.0	0.0	0.0	0.0
South-east	2.5	3.3	2.9	2.6	1.5	0.8	0.4	0.1	0.0	0.0	0.0	0.0
South	1.9	2.2	2.0	1.5	0.8	0.4	0.2	0.0	0.0	0.0	0.0	0.0
South-west	2.0	3.2	3.3	2.5	1.4	0.6	0.2	0.1	0.0	0.0	0.0	0.0
West	2.3	3.7	4.2	3.6	2.3	1.4	0.6	0.3	0.1	0.1	0.0	0.0
North-west	2.0	2.9	3.3	3.2	2.5	1.8	1.1	0.6	0.2	0.1	0.0	0.0
Calm	4.2											

Table 1.1
Wind data from Thyborøn. The values give (in %) the frequency of a given wind force comming from a given direction. The wind force is measured in Beaufort.

The SPM-method

A "Wind stress factor" is calculated to determine the wave heights and thereby the wave energy.
After converting the wind forces to velocities they are adjusted into an equivalent velocity measured 10 m above ground by formula 1.1. The result of this is given in table 1.2.

$$U_{10} = U_z \left(\frac{10}{z} \right)^{1/7} \qquad for\ z < 20m \tag{1.1}$$

- U_{10} = Wind velocity at an elevation of 10 m $[m/s]$
- U_z = Wind velocity at an elevation of z m $[m/s]$
- z = The elevation above ground of the station. For Thyborøn the elevation is 2 m $[m]$

Correction factors

The wind velocity is measured over land. It is adjusted by multiplying a factor R_L giving the relationship between the wind velocity measured over land and sea. R_L depends on the equivalent wind velocity at an elevation of 10 m over ground and is determined by figure 1 in "Generation and analysis of random waves"[2].
Furthermore the wind velocities are adjusted for the difference in temperature between the water and the air. A factor for correction, $R_T = 1.1$[3], is used as there is no further information about these temperature differences available.

Determination of U_A

Finally the wind stress factor, U_A, is calculated by formula 1.2.

$$U_A = 0.71 \cdot U_{10}^{1.23} \tag{1.2}$$

Windforce	1	2	3	4	5	6	7	8	9	10	11	12
Windvelocity $[m/s]$	0.90	2.45	4.40	6.70	9.35	12.30	15.50	18.95	22.60	26.45	30.55	34.80
U_{10} $[m/s]$	1.13	3.08	5.53	8.43	11.77	15.48	19.51	23.85	28.44	33.29	38.45	43.80
R_L $[-]$	1.9	1.6	1.3	1.2	1.1	1.0	0.9	0.9	0.9	0.9	0.9	0.9
U_a $[m/s]$	2.05	5.69	9.05	13.75	18.62	23.20	27.09	34.69	43.08	52.27	62.41	73.26

Table 1.2
The table gives the conversion from the wind force [Beaufort] to the wind stress factor, U_A [m/s] including the calculations and the employed coefficient matching the 12 wind forces. The values are based on the wind conditions at Thyborøn.

1.3 Assessment of the Wind Data

Comparisons with wind data from the two nearest measurement stations; Vestervig and Bovbjerg Lighthouse, validates the data from Thyborøn. The distributions of the wind stress factors and the direction of the wind are shown in % in figure 1.4 and 1.5 respectively for the three measurement stations.

[2] "Generation and analysis of random waves" figure 1
[3] "Generation and analysis of random waves" s. 28

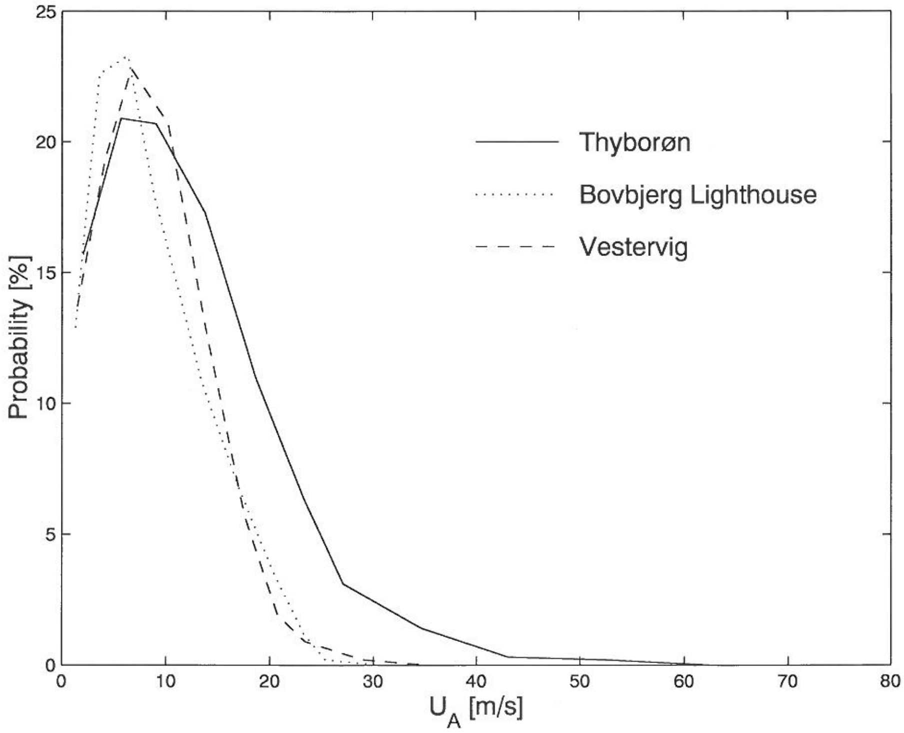

Figure 1.4
Distribution of the wind stress factor, U_A, for Thyborøn, Vestervig and Bovbjerg Lighthouse.

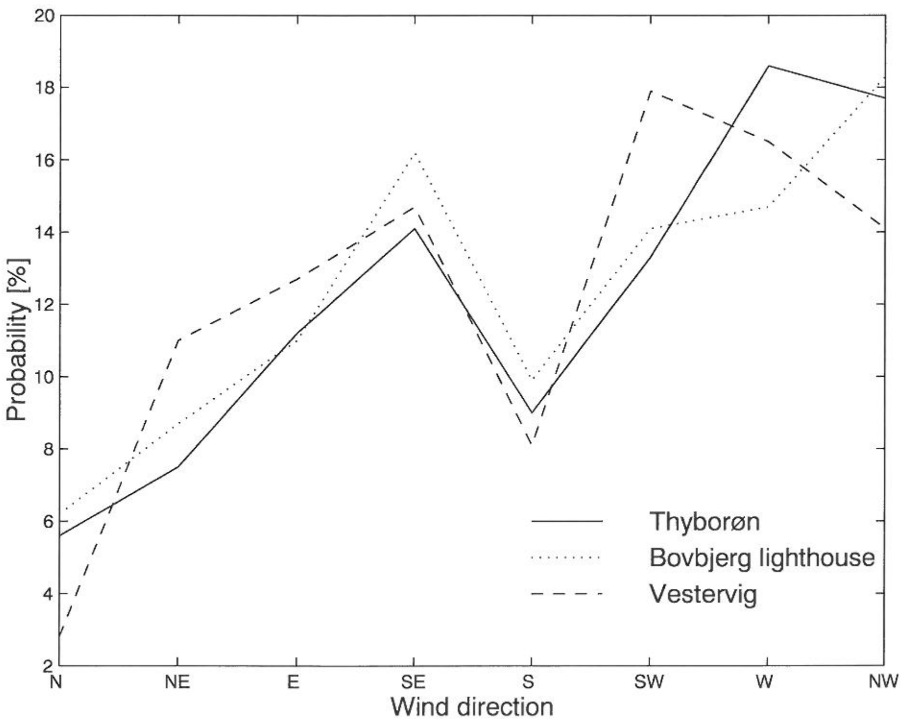

Figure 1.5
Distribution of the direction of the wind for Thyborøn, Vestervig and Bovbjerg Lighthouse.

Figure 1.5 show good consistens between the direction of the wind from the three different locations.

Figure 1.4 on the other hand shows some disparity between the maximum wind velocity at Thyborøn and the other two locations. One explanation could be that formula 1.1, used to calculate the equivalent wind velocity in 10 meters height, is unreliable for elevations over 20 m. That is exactly the cases for the measurement station at Bovbjerg Lighthouse where the elevation is 41 m, and Vestervig where the elevation is 19 m.
Wind velocities at Thyborøn are expected to be higher than wind velocities in Vestervig and Bovbjerg Lighthouse and give a better picture of the conditions at Nissum Bredning. The data from Thyborøn are therefore employed in the energy calculation.

1.4 H_S- and Energy Calculation

The significant wave height, H_S, is calculated for the 8 directions of wind and 12 wind stress factors in web points with a mutual distance at 250 m distributed across Nissum Bredning. For a stretch with a constant water depth, H_S is calculated by formula 1.3[4].

$$H_S = 0,283\frac{U_A^2}{g}tanh\left(0,53\left(\frac{gh}{U_A^2}\right)^{3/4}\right)tanh\left[\frac{0,00565\left(\frac{gF}{U_A^2}\right)^{1/2}}{tanh\left(0,53\left(\frac{gh}{U_A^2}\right)^{3/4}\right)}\right] \quad (1.3)$$

- F = The present fetch $[m]$
- U_A = Wind stress factor $[m/s]$
- h = Water depth $[m]$

Calculation procedure of H_S with varying water depth

To take the varying contour of the sea bed into account the calculation is divided into various parts. To determine H_S in a point A, see figure 1.6, first the wave height in point C, over the stretch with a water depth h_1, is calculated by formula 1.3 with a fetch of length $F = F_1 - F_2$. Then formula 1.3 is evaluated with respect to the fetch with the recently calculated H_S and $h = h_2$. The result is an equivalent fetch, $F_{eq,1}$, that corresponds to the stretch required to build up a H_S corresponding to the one in point C at a water depth $h = h_2$. Subsequently H_S is calculated in point B with $h = h_2$ and $F = F_{eq,1} + F_2 - F_3$. The calculation is repeated and H_S in point A is achieved.

[4]"Generation and analysis of random waves" s. 31

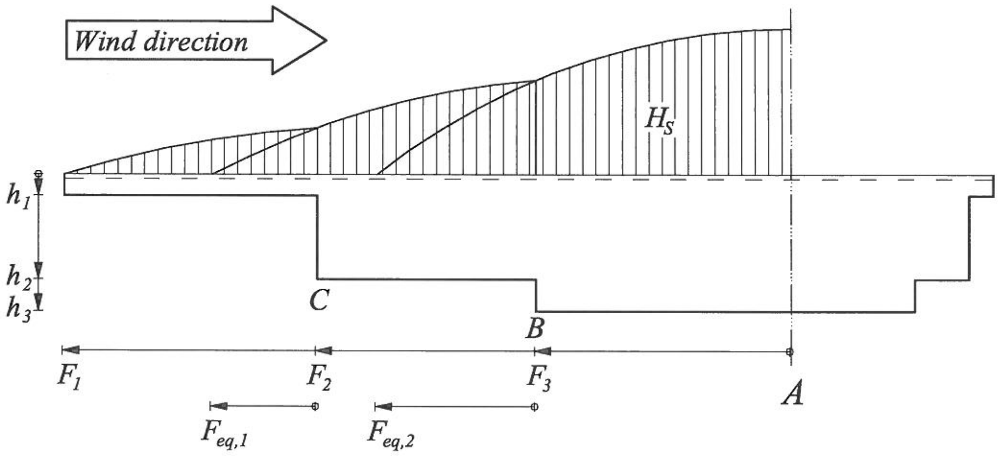

Figure 1.6
Calculation of H_S in point A.

Calculation of energy density

Under the assumption $\overline{H} = H_S/1.6$, where $\overline{H}$ is the average wave height, the total energy density per unit area, E, is calculated by formula 1.4.

$$E = \frac{1}{8}\,\varrho\, g\, \overline{H}^2 \tag{1.4}$$

Hence information about the energy density in each point of the web for the 8 directions and 12 wind stress factors are given. These energy densities are given a weight according to the percentages in table 1.1 whereupon the energies of each point is accumulated. The result is presented on figure 1.7 where a web point distance of 250 m is used.

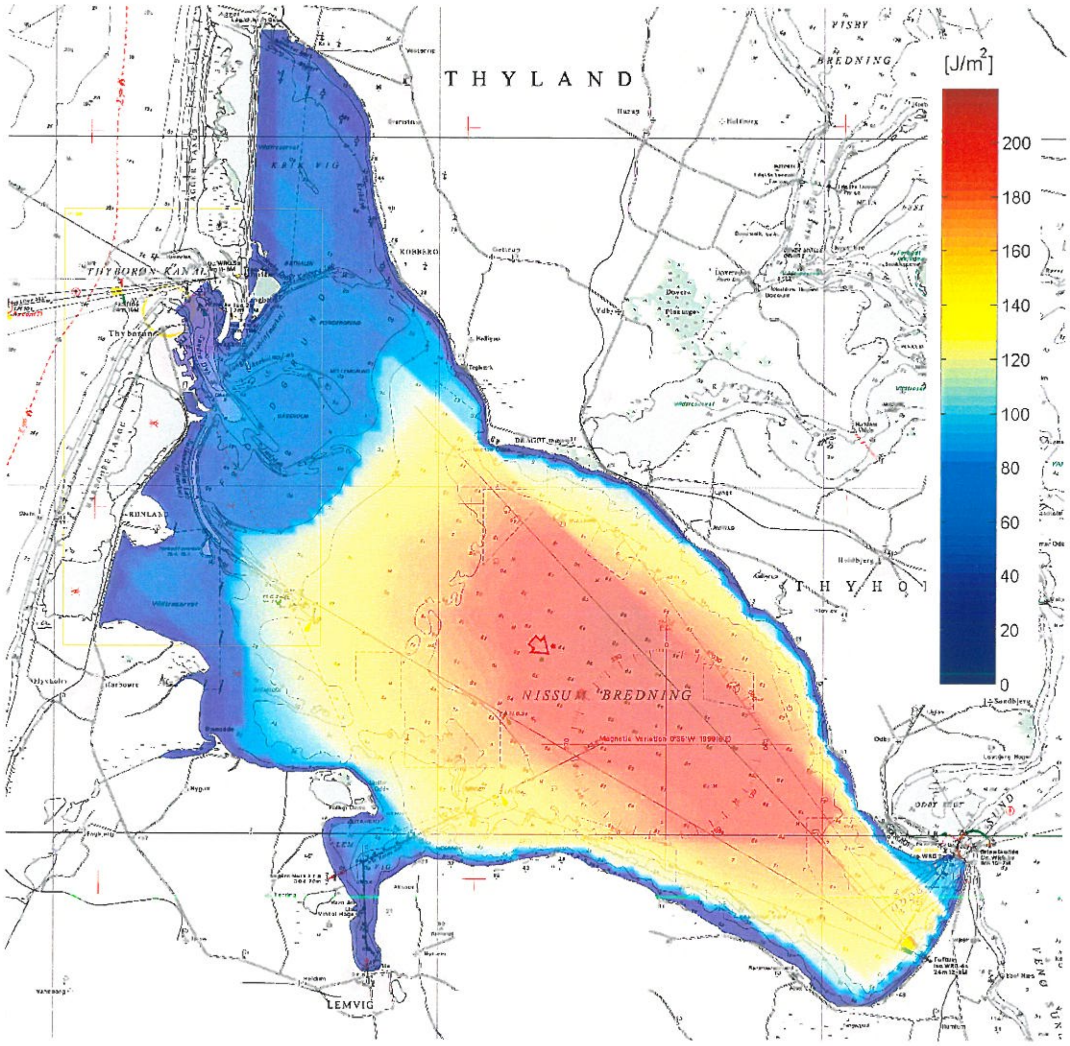

Figure 1.7
Average energy density in Nissum Bredning.

1.5 T_P- and Energy Flux Calculation

The peak period, T_P, is calculated using the same procedure as used in the calculation of the significant wave height. For a strech with constant wather depth, T_P is calculated by formula 1.5.

$$H_S = 7,54\frac{U_A}{g}tanh\left(0,833\left(\frac{gh}{U_A^2}\right)^{3/8}\right)tanh\left[\frac{0,0379\left(\frac{gF}{U_A^2}\right)^{1/3}}{tanh\left(0,833\left(\frac{gh}{U_A^2}\right)^{3/8}\right)}\right] \tag{1.5}$$

The wave length, L_P, corresponding to each peak period can be calculated by iteration using formula 1.6.

$$L_P = T_P\sqrt{\frac{gL_P}{2\pi}tanh\frac{2\pi h}{L_P}} \tag{1.6}$$

The energy propagation speed, c_g, is calculated by formula 1.7.

$$c_g = \frac{L_P}{T_P}\left(\frac{1}{2} + \frac{2\pi h}{L_P sinh\left(\frac{4\pi h}{L_P}\right)}\right) \tag{1.7}$$

Finaly the energy flux, E_f, is calculated by formula 1.8, as the product of the wave propagation speed and the energy density per unit area, E, calculated in section 1.4.

$$E_f = c_g E \tag{1.8}$$

The result is presented on figure 1.8 where a web point distance of 250 m is used.

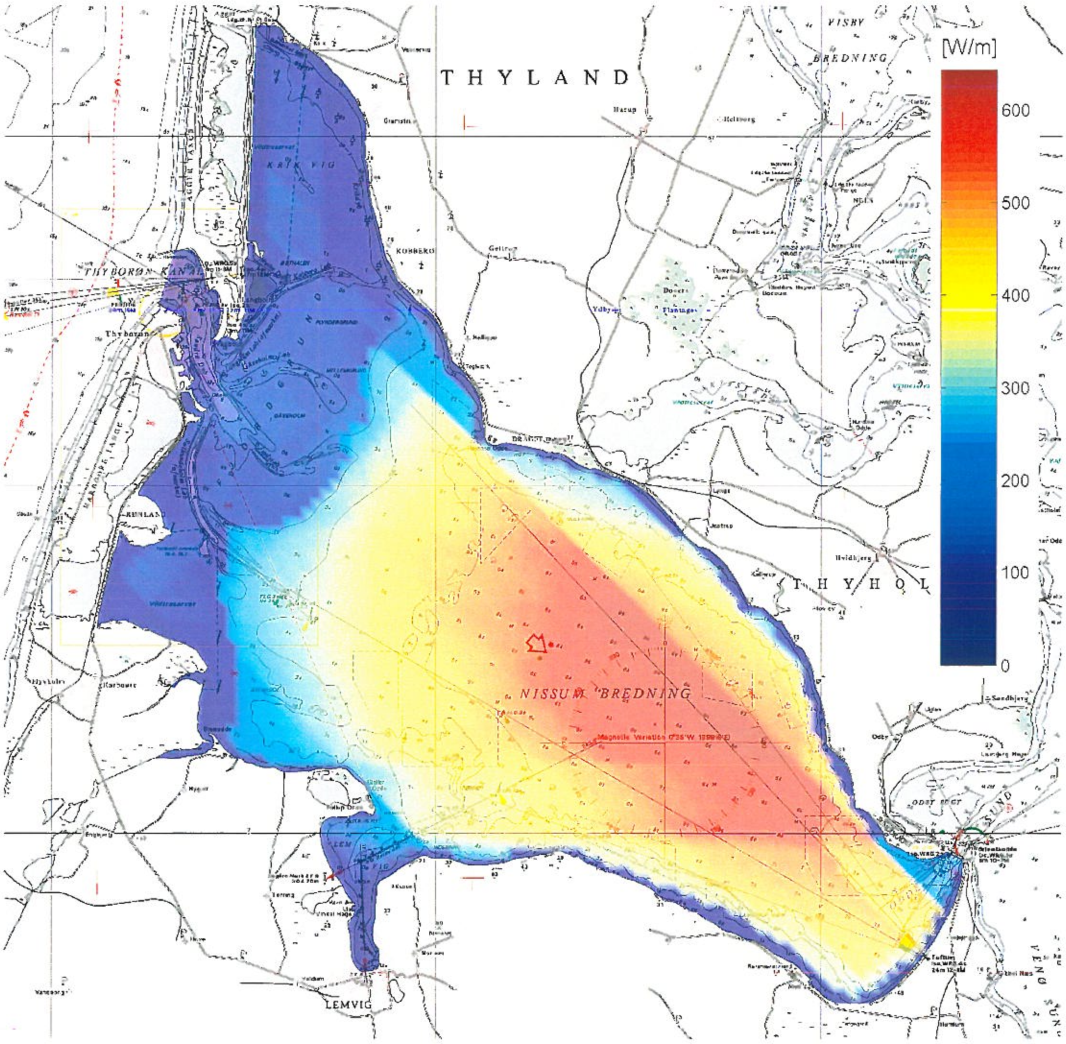

Figure 1.8
Average energy flux distribution in Nissum Bredning.

Chapter 2

Extreme value analysis

The significant wave height, H_S, is calculated as a function of the return period.

2.1 Determination af H_S

The significant wave height, H_S, at the location of Wave dragon is determined. This is done by determining an analytical expression for the relationship between H_S and the probability, P, for non-exceedens of the wave height. The used data is the percentages of wind at a given velocity listed in table 1.1. P is thereby the cumulated values of the percentages i table 1.1. The equivalent wave heights to the given wind forces i table 1.1 is calculated separately for the 8 directions of the wind using the method described in bilag 1.

Fitting the relationship between H_S and P

A Weibull distribution is applied to fit the relation between H_S and P, as this is the most applicable in practice. The analytical expression of a Weibull distribution is given by formula 2.1.

$$P = P(X < H_S) = 1 - exp\left(\left(\frac{H_S - B}{A}\right)^k\right) \tag{2.1}$$

- $P(X < H_S)$ = Probability of nonexceedens of the belonging H_S [−]
- A, B, k = Fittet distribution parameters [−]

The coefficients A, B and k are determined by rearranging formula 2.1 as given in formula 2.2 and then performing a linear regression by the method of least squares[1].

[1] "Generation and analysis of random waves" p. 45

$$
\begin{aligned}
X &= (-ln(1-P))^{\frac{1}{k}} \\
&\Downarrow \\
H_S &= AX + B
\end{aligned}
\tag{2.2}
$$

A and B in formula 2.2 are determined for k-values in a interval form 1,0 to 3,0 with a step of 0,1.
For each of the k-values the linear coefficinet of correlation is calculated and the regression with the largest coefficinet of correlation is chosen as the best fit.
The results of the performed regressions are given in table 2.1.

Winddirection/ coefficients	k	A	B	ρ
North	1,2	0,2508	-0,0112	0,9982
North-East	1,2	0,2074	0,0008	0,9995
East	1,7	0,3543	-0,0194	0,9936
South-East	1,7	0,5125	-0,0277	0,9939
South	1,4	0,3464	-0,0103	0,9985
South-West	1,6	0,4399	-0,0104	0,9986
West	1,5	0,5441	-0,0160	0,9962
North-West	2,1	0,8472	-0,2089	0,9988

Table 2.1
Weibull- and correlation-coefficients for the 8 winddirections.

On figure 2.1 the relation between P and H_S is plotted in the case of wind form west, together with the fitted Weibull-distribution with k=1,5. Furthermore the Weibull-distributions with $k = 1,2$ and $k = 3,0$ are also plotted.

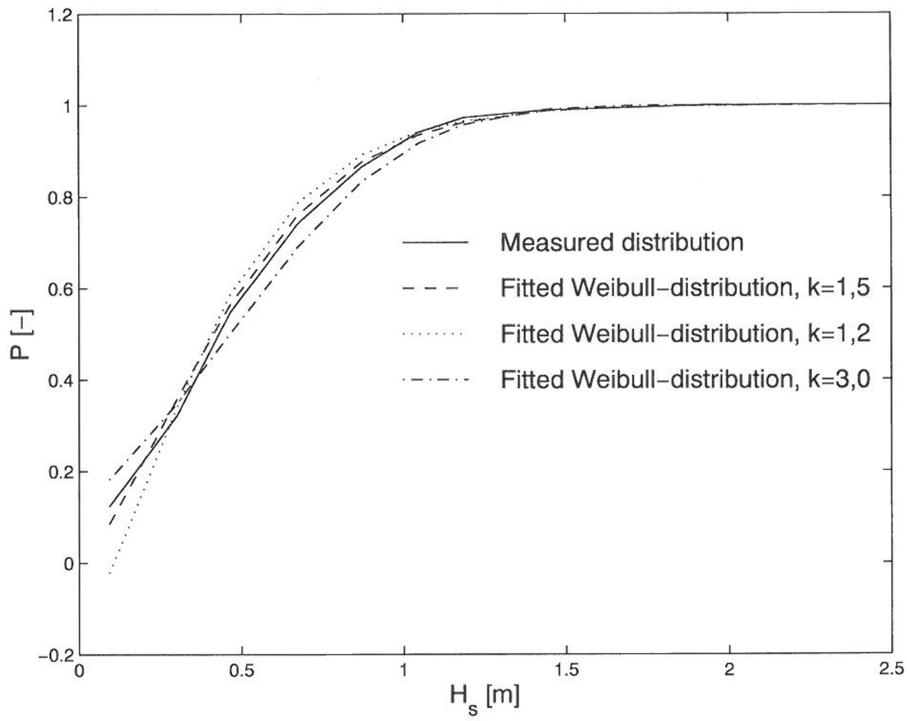

Figure 2.1
Distribution of H_S and the fitted Weibulldistribution in the case of wind form west.

2.2 H_S and Return Periods

Using formula 2.2 and the coefficients in table 2.1 the significant wave height has been calculated as a function of the return period the result of which is displayed in table 2.2. The exceedence probability is defined by a 3 hour storm within the return period.

Return period [*year*]	**1**	**2**	**5**	**10**	**15**	**20**	**25**	**50**
North	0.96	1.12	1.32	1.46	1.45	1.55	1.63	1.72
North-East	0.81	0.93	1.10	1.21	1.21	1.29	1.36	1.43
East	0.90	1.01	1.13	1.21	1.21	1.27	1.31	1.37
South-East	1.31	1.45	1.64	1.76	1.75	1.83	1.90	1.98
South	1.10	1.25	1.44	1.56	1.56	1.65	1.72	1.80
South-West	1.21	1.35	1.53	1.64	1.64	1.72	1.79	1.86
West	1.59	1.80	2.05	2.22	2.21	2.33	2.43	2.54
North-West	1.63	1.79	1.99	2.12	2.11	2.20	2.27	2.35

Table 2.2
Significant wave height, $H_S[m]$, for each wind direction as a function of the return period in years.

Chapter 3

Acknowledgement

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Refrences

Zhou Liu and Peter Frigaard. *Generation and Analysis of Random Waves.* Aalborg Universitet, 2001.

M. V. B. Jensen, M. L. Skovhus, K. B. Sørensen, J. Lynggaard, J. W. Larsen and R. Svendsen. *Wave Dragon i Nissum Bredning - placering, lastbestemmelse og fundering.* Aalborg Universitet, 2001.