Estimation of Incident Wave Height

Frigaard, Peter; Helm-Petersen, J.

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LIP–MAST–TAW: *Caisson Investigations*

Estimation of INCIDENT WAVE HEIGHTS

1 Introduction

The following pages are the results found by Aalborg University in the calculations of the incident wave heights H_{m_0} and the reflection coefficients α from the LIP–MAST investigations in the Vinje–Basin at Delft Hydraulics during May to July 1994.

2 Presentation of the estimation methods

A short presentation of the two methods used for the estimations of the incident wave heights and reflection coefficients, namely a maximum likelihood method and the Bayesian Directional Spectrum Estimation Method, is given below.

The directional spectrum estimation methods are based on surface elevations measured by a two–dimensional wave gauge array consisting of 20 gauges. In the shown results only 10 wave gauges were used in most of the calculations.

First a **maximum likelihood method** for estimating directional spectra is presented.

For each frequency 4 parametres are estimated: incident spectral density, main direction of waves, spreading of the waves and the reflection coefficient. Reflection is assumed to occur with the reflected wave angle equal to the incident wave angle.

Generally, a directional spectrum expresses how the wave energy is distributed on both frequency and direction. The following presentation is based on Isobe, M. and

K. Kondo, 1984, Isobe, M., 1990 and Yokoki, H., M. Isobe and A. Watanabe, 1992. The directional spectrum is given in a standard form in terms of some unknown parameters which are to be estimated from measured data. In the present method only surface elevation measurements are treated.

From the measurements it is possible to identify directions and frequencies of the wave pattern. In order to perform the identification, a relation between the cross correlation matrix of the 10 elevation processes and the directional spectrum is established. The underlying assumption is that the elevation processes at one point can be considered as a sum of harmonic components having Rayleigh distributed amplitudes and uniformly distributed phases. Assuming all phases and amplitudes to be independent, the elevation processes become normally distributed. Furthermore, the theory offers the possibility of introducing reflected waves.

In Christensen and Sørensen, 1993, the likelihood function is introduced. Again, the starting point is the elevation processes. Based on an assumption of stationary processes, the time series are expressed as Fourier sums. It is emphasised, that the Fourier coefficients at a given frequency are jointly Gaussian variables. Furthermore, the mean value vector and the cross correlation matrix of the Fourier coefficients are determined, and it is shown that the elements in the cross correlation matrix are given in terms of the cross spectral density matrix of the elevation processes. The results show that the cross spectral density matrix is a function of the directional spectrum and therefore the distribution of the Fourier coefficients becomes a function of the unknown directional spectrum. A likelihood function is formulated in terms of the probability density function of the Fourier coefficients and the maximum likelihood estimates of the unknown parameters in the directional spectrum are found by maximising the likelihood function, i.e. by maximising the probability of observing the Fourier coefficients obtained.

The implementation estimates the parameters in the Mitsuyasu directional spectrum based on surface elevation time series measured simultanuosly in an arbitrary number of wave gauges. The likelihood function is maximised using the method described by Nelder, J. A. and R. Mead, 1965.

Secondly a **Bayesian approach** is presented, see Hashimoto and Kobune, 1988, and Helm–Petersen, 1993.

The method is based on the assumptions of a positive and smooth directional spreading function. *For each frequency the directional spreading function is calculated with a resolution of 5 deg.*

The Bayesian approach is based on Bayes' theorem. This approach is advantageously used where available information is limited and subjective judgments are nearby. This is generally the case when it comes to estimating directional wave spectra.

It is assumed, that the directional spreading function $H(\theta, f)$ can be expressed as a piecewise–constant function, which takes only positive values. The directional spreading function is discretized into an arbitrary number of intervals (in this case 72 intervals). Equations weighting smoothness and statistically fits to prior directional distribution are applied. Relationships between the cross–spectra and the directional spectrum are deducted and iterations on the directional spreading function based on prior estimates of the directional spreading function are performed. As the estimate of $H(\theta, f)$ becomes smoother, the weighting of the smoothness of the directional spreading function is decreased. A criterion is introduced in order to evaluate the estimates and finally the best estimate is chosen.

3 Results

The following tables show the estimated values of the incident wave height H_{m_0} and the reflection coefficient α.

The definition of α is $\sqrt{m_{0,reflected}/m_{0,incident}}$.

m_0 is the 0'th order moment.

θ is main direction (deg.) of the waves.

σ is the spreading (deg.).

* indicates that wave steepness is 0.02 instead of normally 0.04.

Test		Series 0				Series 2				Series 3			
θ	σ	File	H_{m_0}	α	σ	File	H_{m_0}	α	σ	File	H_{m_0}	α	σ
0	*0	001	12.4	0.90		207	12.4	0.90					
0	0	002	13.7	0.90		203	13.7	0.90		302	13.8	0.88	
0	15	003	13.6	0.90		204	12.9	0.90	22				
0	*15	004	13.3	0.90		206	13.8	0.90	25				
0	30	005	11.6	0.90	25	205	12.1	0.90	25	305	11.9	0.91	25
10	15	006	13.7	0.90		213	13.2	0.90	23				
20	15	007	12.9	0.89	24	202	12.9	0.89	23	307	13.0	0.89	22
20	*15	008	14.2	0.93	24					308	14.6	0.93	24
20	0	009	13.0	0.89						309	12.3	0.89	
20	30	010	12.9	0.89	29					310	12.6	0.89	28
30	15	011	12.6	0.90									
40	0	012	12.7	0.88									
40	15	013	13.6	0.89		201	13.2	0.89	26				
40	*15	014	13.6	0.92		212	13.6	0.92	26				
40	30	015	13.0	0.89	30					315	12.9	0.89	31
50	15	016	13.8	0.86									
60	15	017	13.6	0.61	21								
60	0					210	12.9	0.82					
60	*15					211	14.0	0.58	17	317	14.2	0.60	19

Table 1: Estimated incident waveheights (in *cm*), reflection coefficient and spreading of incident waves (in degrees).

Test		Series 4				Series 5			
θ	σ	File	H_{m_0}	α	σ	File	H_{m_0}	α	σ
0	*0								
0	0	402	14.0	0.55		502	13.3	0.35	
0	15								
0	*15								
0	30	405	12.6	0.55	22	505	11.7	0.40	19
10	15								
20	15	407	12.9	0.53	18	507	11.8	0.40	17
20	*15	408	14.2	0.72	21	508	13.3	0.42	20
20	0	409	12.6	0.61		509	12.5	0.37	
20	30	410	12.2	0.71	26	510	12.3	0.31	21
30	15								
40	0								
40	15								
40	*15								
40	30	415	12.1	0.59	27	515	12.0	0.33	26
50	15								
60	15								
60	0								
60	*15	417	13.0	0.77	23	517	12.8	0.46	18

Table 2: Estimated incident waveheights (in *cm*), reflection coefficient and spreading of incident waves (in degrees).

Test		Series 6				Series 7				Series 8			
θ	σ	File	H_{m_0}	α	σ	File	H_{m_0}	α	σ	File	H_{m_0}	α	σ
0	*0												
0	0	602	14.1	0.53		702	13.5	0.53		802	13.6	0.39	
0	15												
0	*15												
0	30	605	12.0	0.48	22	705	11.9	0.47	22	805	11.8	0.39	22
10	15												
20	15	607	13.0	0.45	19	707	13.2	0.44	19	807	13.2	0.38	19
20	*15	608	12.4	0.40	21	708	12.9	0.45	20	808	11.8	0.23	20
20	0	609	12.8	0.52		709	13.4	0.43					
20	30	610	12.2	0.45	26	710	12.5	0.42	26				
30	15												
40	0												
40	15												
40	*15												
40	30	615	12.0	0.39	27	715	12.3	0.35	27				
50	15												
60	15	618	11.6	0.30	20	718	11.5	0.32	19				
60	0												
60	*15	617	13.6	0.40		717	13.5	0.45					

Table 3: Estimated incident waveheights (in cm), reflection coefficient and spreading of incident waves (in degrees).

4 Reliability of results

Incident wave height and reflection

In all the test series the target incident wave height were 14.0 cm. This incident wave height is near the limit of the capacity of the wave generator, so therefore the measured wave heights are expected to be slightly lower.

In test series 0, test series 2 and test series 3 the models have vertical front but different types of crests, with and without noses. The reflection coefficients are expected to be in the range of $0.8 - 0.9$.

In test series 4 and test series 5 the models have a sloping berm 1:3. Test series 4 is with berm width 0 m. Test series 5 is with berm width 1 m. On top of the berm there is a vertical wall without nose. The reflection coefficients are expected to be in the range of $0.3 - 0.5$. Highest reflection is expected for test series 4.

In test series 6 and test series 7 the models have perforated fronts. The reflection coefficients are expected to be in the range of $0.3 - 0.5$.

In test series 8 the top of the caissons has been removed in order to avoid air compression inside the caissons. The reflection coefficients of these ventilated caissons are expected to be slightly lower than those obtained in test series 6.

Tests in a flume at Aalborg University with model–setup identical to the setup in test series 8 and with same wave conditions gave reflection coefficients in the range of 0.34 – 0.39.

For test series 0, test series 2 and test series 3 there are generally very good agreement between measured and target values of H_{m0} and α. The uncertainty on H_{m0} measurements is belived to be lower than ± 0.5 *cm*. The uncertainty on α measurements is belived to be lower than ± 0.05.

For test series 4 and test series 5 the estimated reflection coefficient seems to be too high. The impression is that the reflection coefficients should be reduced with app. 0.2. This means that also the prediction of the incident wave heights might be scattered. Uncertainty on α measurements is belived to be up to app. 0.2. Uncertainty on H_{m0} measurements is belived to lower than ± 1.0 *cm*. This high uncertainty is probably a result of a high amount of wave breaking in the area where the gauges are placed and a result of refraction on the slope.

Test		#	$H_{s,I}$	$\sigma_{H_{s,I}}$
θ	σ			
0	*0	2	12.4	0.00
0	0	8	13.7	0.26
0	15	2	13.3	0.50
0	*15	2	13.6	0.35
0	30	8	12.0	0.31
10	15	2	13.5	0.35
20	15	8	12.9	0.45
20	*15	7	13.3	1.04
20	0	5	12.8	0.39
20	30	5	12.5	0.27
30	15	1	12.6	0.00
40	0	1	12.7	0.00
40	15	2	13.4	0.28
40	*15	2	13.6	0.00
40	30	6	12.4	0.45
50	15	1	13.8	0.00
60	15	3	12.2	1.18
60	0	1	12.9	0.00
60	*15	6	13.5	0.55

Table 4: Average values of the estimated incident waveheights in *cm*. # indicates number of tests.

Also it is seen that estimates of the incident wave heights in case of long crested waves generally are higher than estimates of wave heights in case of short crested waves. The average of all the wave heights for the long crested seastates is 5% larger than the average of all the wave heights for the short crested waves. It is believed, that this difference is more due to differences in the wave fields than due to inaccuracy in the methods of analysis.

Peak periods of incident waves

In all tests the specified peak period of the incident wave spectrum were either $1.5s$ or $2.1s$.

Results from the analysis showed very good agreement with these target values. Only very small deviations were found. These have not been reported, because it is believed, that the deviations are due to the chosen spectral resolution rather than actual differencies in the waves. Uncertainty on T_p is believed to be lower than $0.05s$.

Incident main wave directions

Target main directions were 0, 10, 20, 30, 40, 50 and 60 degrees. Results from the analysis showed good agreement with the target values. Only small differencies were found, and they have not been reported, because it is believed that they are due to the chosen angular resolution, rather than caused by differences in the waves. Uncertainty on θ_0 is believed to be approximately 5 degrees.

Spreading of the incident waves

Target spreading of the waves were 15 and 30 degrees. Results from the analysis give an average spreading of 20 degrees in case of target spreading of 15 degrees, and an average spreading of 25 degrees in case of target spreading of 30 degrees. Uncertainty on σ is believed to be around 5 degrees.

It has not been possible to explain this difference because the trends are in conflict. Though, the differences might have two reasons. Firstly, due to re–reflection and diffraction the seastate is generally very confused, which might lead to a higher spreading than the target value. Secondly, the most oblique waves are maybe omitted in the wave generation leading to a lower spreading (especially for seastates with high spreading).

Generally most scatter and uncertainty is found in cases with wave steepness 0.02.

5 References

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