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## BUOYANCY-AFFECTED FLOWS IN VENTILATED ROOMS

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*Calculations of velocity and temperature distributions in rooms with ventilation arrangements are reported. The method involves the solution, in finite-difference form, of two-dimensional equations for the conservation of mass, momentum, energy, turbulence energy, and dissipation rate, with algebraic expressions for the turbulent viscosity and heat diffusivity. The results are shown to be in reasonable agreement with available experimental data and the method is then applied to provide additional information useful for design purposes.*

### INTRODUCTION

The main purpose of air-conditioning systems is to maintain a suitable distribution of air velocity, temperature, and humidity in some closed space, typically a room. The designer of ventilated rooms is required, therefore, to know in advance the flow characteristics of a given air-conditioning arrangement in a particular room geometry.

In the past, calculation methods have made use of extensive assumptions to estimate the velocity levels without actually solving the flow equations. At present, however, direct solution of the equations of motion, in time-averaged form and with modeled turbulent stresses, lies within reasonable limits of computer usage and cost, and the interest in this approach has increased rapidly. In a previous paper [1] the velocity characteristics of room and ventilation arrangements were calculated; the present contribution extends the method to allow the calculation of two-dimensional temperature distributions in ventilated rooms with buoyancy effects.

Temperature gradients are usually present as a result of heat transfer through walls and windows, heat generation by the room occupiers or heaters, or air at a different temperature being blown through the ventilation system. Energy is transported throughout the room as a passive property and interacts with the velocity field, inducing nonuniform buoyancy forces that affect both the mean flow and the turbulence. The effect of buoyancy is commonly expressed in air-conditioning literature by a global Archimedes number

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## NOMENCLATURE

$a, b$	boundaries surrounding inlet opening	$x, y$	coordinates
$Ar$	Archimedes number ( $= \beta g h \Delta T_0 / U_0^2$ )	$x_R$	abscissa of reattachment point
$B$	buoyancy parameter [ $= -\beta g (K/\epsilon)^2 \partial \theta / \partial y$ ]	$\beta$	coefficient of thermal expansion
$C_1, C_2, C_\mu$	constants in turbulence model	$\gamma_{h,t}$	turbulent heat diffusivity
$f$	wall parameter in expression for $\sigma_t$	$\Gamma_{h,eff}$	effective exchange coefficient for heat
$g$	gravitational acceleration	$\Gamma_\phi$	exchange coefficient in $\phi$ equation
$G, G_B$	generation of $K$ by shear and buoyancy	$\Delta T_0$	temperature difference between outlet and inlet
$h$	height of inlet slot	$\epsilon$	dissipation of turbulence kinetic energy
$H, L$	dimensions of room or model	$\theta$	excess temperature ( $= T - T_0$ )
$K$	turbulence kinetic energy	$\kappa$	von Kármán constant
$P$	pressure	$\mu_{eff}, \mu_t$	effective and turbulent viscosities
$Re$	Reynolds number ( $= h U_0 / \nu$ )	$\nu$	fluid laminar viscosity
$S_\phi$	source term in $\phi$ equation	$\nu_t$	turbulent momentum diffusivity
$T_0$	inlet fluid temperature	$\rho$	fluid density
$U, V$	components of mean velocity	$\sigma$	fluid Prandtl number
$U_m$	peak velocity across jet	$\sigma_t$	turbulent Prandtl number
$U_{rm}$	maximum velocity in reverse flow	$\sigma_k, \sigma_\epsilon$	constants in turbulence model
$U_0$	bulk inlet fluid velocity	$\phi$	general dependent variable

$$Ar = \frac{\beta g h \Delta T_0}{U_0^2}$$

with  $\Delta T_0$  the temperature difference between outlet and inlet, and  $h$  and  $U_0$  a characteristic length and velocity of the diffuser employed.

Previous numerical studies of buoyancy-affected flows of this type include the works of Nielsen [2] and Larson [3] with a stream function-vorticity formulation. Experimental investigations have been reported, for example, by Linke [4], Jackman [5], Schwenke [6], Hestad [7], Nagasawa [8], and Nielsen [2], who provided qualitative information by flow visualization and quantitative information by the measurement of local values of temperature and velocity. In spite of numerous data, experiments with simple and known boundary conditions—for example, with one wall uniformly heated and the others adiabatic—are in short supply. Nevertheless, sufficient data exist to allow an appraisal of the present method.

The following section introduces the equations solved and the calculation method. Calculated results are presented and discussed and conclusions are drawn in the remaining sections.

### EQUATIONS, BOUNDARY CONDITIONS, AND NUMERICAL PROCEDURE

The equations solved are the continuity, momentum (two components), and energy equations, written in time-averaged form and expressing the turbulent stresses and heat fluxes in terms of turbulent diffusivities for momentum  $\nu_t$  and heat  $\gamma_{h,t}$ ,

respectively. The two diffusivities are connected by the turbulent Prandtl number  $\sigma_t = \nu_t / \gamma_{h,t}$  and the former is calculated in terms of the turbulent kinetic energy  $K$  and its dissipation rate  $\epsilon$ ,

$$\nu_t = \frac{C_\mu K^2}{\epsilon}$$

$K$  and  $\epsilon$  are derived from two additional transport equations. All differential equations are reducible to the general form

$$\frac{\partial}{\partial x} (\rho U \phi) + \frac{\partial}{\partial y} (\rho V \phi) = \frac{\partial}{\partial x} \left( \Gamma_\phi \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( \Gamma_\phi \frac{\partial \phi}{\partial y} \right) + S_\phi$$

where  $\phi$ ,  $\Gamma_\phi$ , and  $S_\phi$  assume the values listed in Table 1, with

$$G = \mu_t \left\{ 2 \left[ \left( \frac{\partial U}{\partial x} \right)^2 + \left( \frac{\partial V}{\partial y} \right)^2 \right] + \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right)^2 \right\}$$

$$G_B = \rho \beta g \frac{\nu_t}{\sigma_t} \frac{\partial \theta}{\partial y}$$

$\theta$  is the excess temperature  $T - T_0$ , where  $T_0$  is some reference value; the effective viscosity  $\mu_{\text{eff}}$  is defined by

$$\mu_{\text{eff}} = \rho(\nu + \nu_t)$$

and the effective exchange coefficient for heat  $\Gamma_{h,\text{eff}}$  by

$$\Gamma_{h,\text{eff}} = \rho \left( \frac{\nu}{\sigma} + \frac{\nu_t}{\sigma_t} \right)$$

Table 1 Values of  $\phi$ ,  $\Gamma_\phi$ ,  $S_\phi$  and Buoyancy Terms

$\phi$	$\Gamma_\phi$	$S_\phi$	Buoyancy
1	0	0 (continuity)	
$U$	$\mu_{\text{eff}}$	$-\frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left( \mu_{\text{eff}} \frac{\partial U}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_{\text{eff}} \frac{\partial V}{\partial x} \right)$	
$V$	$\mu_{\text{eff}}$	$-\frac{\partial P}{\partial y} + \frac{\partial}{\partial x} \left( \mu_{\text{eff}} \frac{\partial U}{\partial y} \right) + \frac{\partial}{\partial y} \left( \mu_{\text{eff}} \frac{\partial V}{\partial y} \right)$	$-\rho \beta g \theta$
$h$	$\Gamma_{h,\text{eff}}$	0	
$K$	$\frac{\mu_{\text{eff}}}{\sigma_K}$	$G - \rho \epsilon$	$+ G_B$
$\epsilon$	$\frac{\mu_{\text{eff}}}{\sigma_\epsilon}$	$\frac{\epsilon}{K} (C_1 G - C_2 \rho \epsilon)$	$+ C_1 \frac{\epsilon}{K} G_B$

Since temperature differences are small in air-conditioning applications (typically less than  $10^\circ\text{C}$ ), the coefficient of volumetric expansion  $\beta$  was assumed constant, and the influence of density variations was neglected in terms that do not contribute to buoyancy effects. Buoyancy terms are listed in the last column of Table 1, for the vertical momentum equation ( $y$  axis directed vertically downward) and the  $K$  and  $\epsilon$  equations. The terms in the momentum equations are exact and those in the  $K$  and  $\epsilon$  equations have been modeled in the way proposed by Launder [9]. The turbulent heat fluxes have been expressed in terms of a turbulent diffusivity for heat, as in the energy equation. The turbulent Prandtl number  $\sigma_t$  has been made dependent on buoyancy, making use of an expression proposed by Gibson and Launder [10] for horizontal flows:

$$\sigma_t = 0.67 \frac{1 + 0.83f + 0.112f^2 + (0.267 + 0.242f)B}{1 + 0.417f + 0.093B}$$

where  $f = (K^{3/2}/\epsilon)/c_\mu^{-3/4}kd$ , with  $d$  equal to the distance to the nearest boundary, accounts for the vicinity of walls and  $B$  is a parameter characteristic of the buoyancy effects, defined by

$$B = -\beta g \frac{K^2}{\epsilon^2} \frac{\partial \theta}{\partial y}$$

The five "constants"  $C_1$ ,  $C_2$ ,  $C_\mu$ ,  $\sigma_k$ , and  $\sigma_\epsilon$  were assigned the values used in the earlier isothermal calculations, listed in [1].

The boundary conditions for the velocity and turbulence properties were similar to those in [1]; i.e., zero gradients in the exit plane and in plane  $b$  of Fig. 1, prescribed values in plane  $a$  of Fig. 1, and turbulent wall functions to describe the near-wall regions close to all solid surfaces. In addition, identical conditions were imposed on the temperature in the exit plane and planes  $a$  and  $b$ , and all surfaces were assumed adiabatic except the bottom one, where a uniform heat flux distribution was prescribed. The wall functions, similar to those in [1], need not be extended to the temperature for this type of boundary condition.

The equations were solved in finite-difference form with the computer program TEACH, first described by Gosman and Pun [11] and extensively used by several workers at Imperial College. Calculations were performed in a CDC 6600 computer,

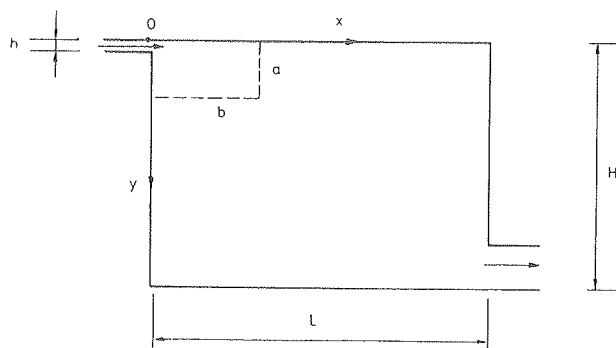


Fig. 1 Boundaries of calculated flows and coordinate system.

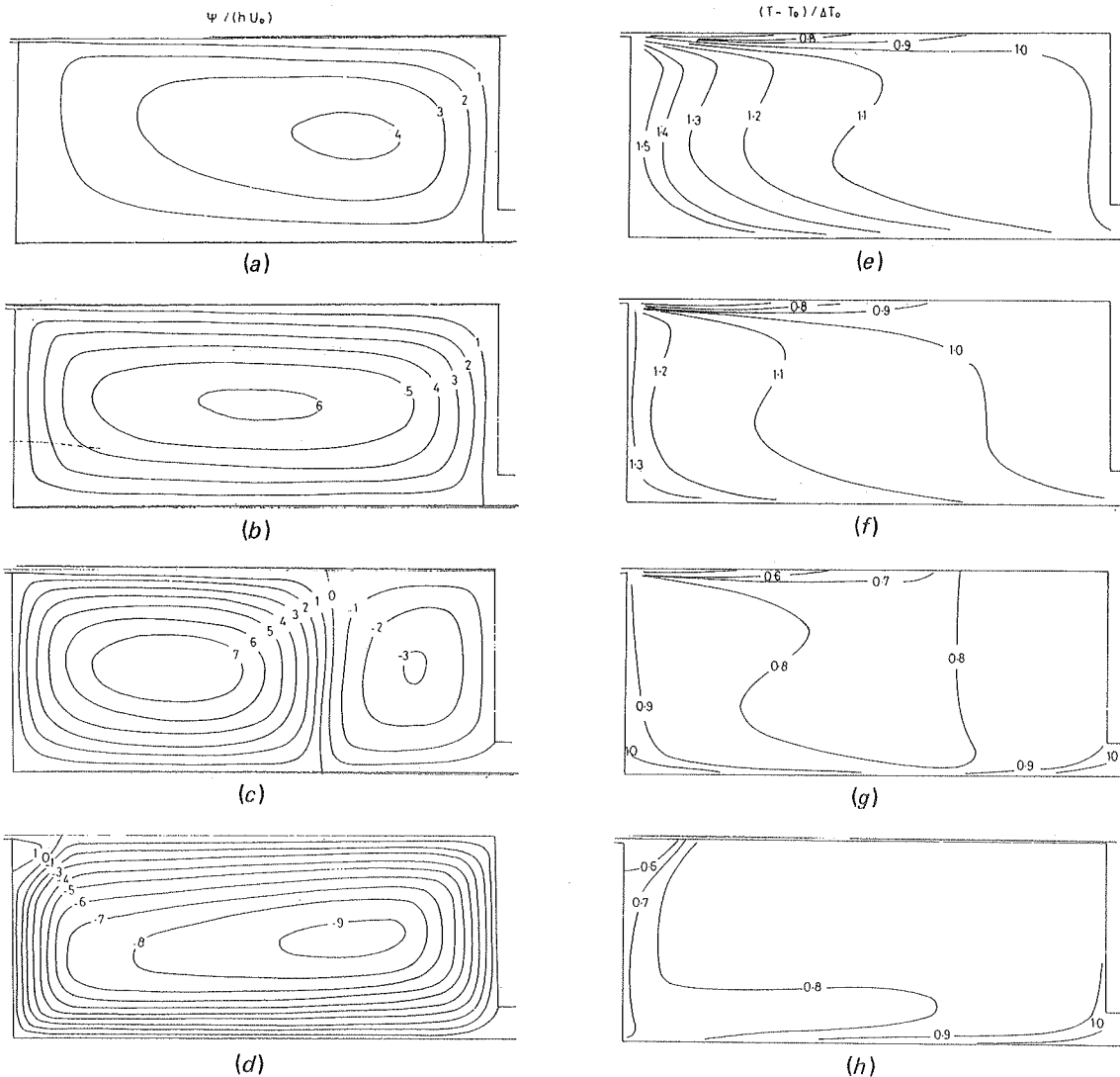


Fig. 2 Patterns of streamlines (a-d) and isotherms (e-h) at  $Re = 7350$ ,  $L/H = 3.0$ ,  $h/H = 0.017$ , and  $Ar$  as follows: (a and e) 0, (b and f)  $8 \times 10^{-3}$ , (c and g)  $2 \times 10^{-2}$ , and (d and h)  $8 \times 10^{-2}$ .

using meshes of approximately 500 grid nodes. Computing times varied from 100 s in almost isothermal flows to 300 s at high Archimedes numbers. Tests with larger numbers of grid nodes indicated that differences of the order of 2% of the maximum velocity and temperature difference in the flow may be attributed to grid dependence.

### CALCULATED RESULTS

The geometric configuration is shown in Fig. 1 and is the same as that employed in [1]. To demonstrate the effect of increasing temperature gradients on the flow, calculations were performed for the case of a uniformly heated floor at four different values of  $Ar$ .  $L/H$  and  $h/H$  were taken as 3.0 and 0.017 and the Reynolds number was fixed at  $Re = 7350$ . The results, in the form of streamlines and isotherms, are shown in Fig. 2. It is obvious that an increase in  $Ar$  is accompanied by an increase in the strength of the main vortex up to a point where the jet running along the ceiling no

longer remains attached to it. If  $Ar$  is further increased, the point of separation is accordingly moved upstream until a main vortex of opposite sign is established.

In the following a uniformly heated floor is assumed. It is the simplest representation of the practical case of a room where the heat dissipated by its occupants is removed by a continuous flow of fresh air.

From a practical point of view, it is important to establish the relative magnitudes of the contributions of different terms added to the equations to describe buoyancy effects. They are shown in Fig. 3, where the maximum velocity in the return flow  $U_{rm}$  is plotted against  $Ar$  for the geometry mentioned above and also for  $L/H$ ,  $h/H$ , and  $Re$  values of 3.0, 0.056, and 7450. The continuous lines correspond to calculations with the complete equations of Table 1, and the dashed lines to parallel calculations where only the buoyancy term in the  $V$ -momentum equation was included. It is obvious that, for both values of  $h/H$ , the simpler approach produces results that are a reasonable approximation only at low  $Ar$ , where the buoyancy effect is negligible anyway. For this reason, the buoyancy terms have been included in the  $V$ ,  $K$ , and  $\epsilon$  equations employed in the remaining calculations with buoyancy.

To assess the accuracy of the present method, comparisons with experimental data are necessary. The measurements of Nielsen [2] were obtained at very low  $Ar$  ( $\sim 1.1$  to  $3.1 \times 10^{-6}$ ) and with the geometry of Fig. 1 and  $L/H = 3.0$  and  $h/H = 0.056$ . The effects of buoyancy at this value of  $Ar$  are very small, as shown by Fig. 3, and the calculated horizontal temperature profiles of Fig. 4, one at the bottom surface of the model and the other at  $Y/H = 0.75$ , are in very good agreement with the measurements. A constant heat flux distribution was assumed in the flow, in the absence of detailed experimental information, and may be the main cause of the small discrepancies. The foregoing calculations were carried out over the entire flow, instead of prescribing values along  $a$  and  $b$  of Fig. 1.

However, most of the available experimental data are related to geometries with

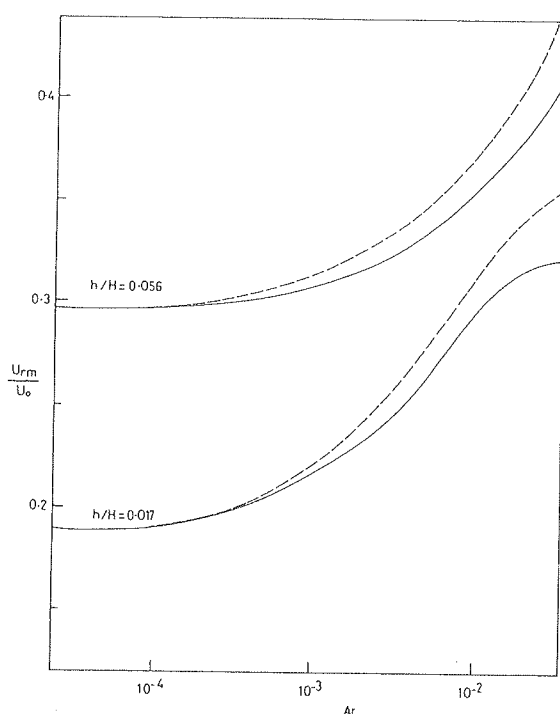


Fig. 3 Comparison of predicted  $U_{rm}/U_0$  versus  $Ar$  for  $L/H = 3.0$ ,  $h/H = 0.056$  and  $0.017$ , and  $Re = 7350$ , with (—) equations as in second section of this paper and (---) buoyancy terms ignored, except in momentum equation.

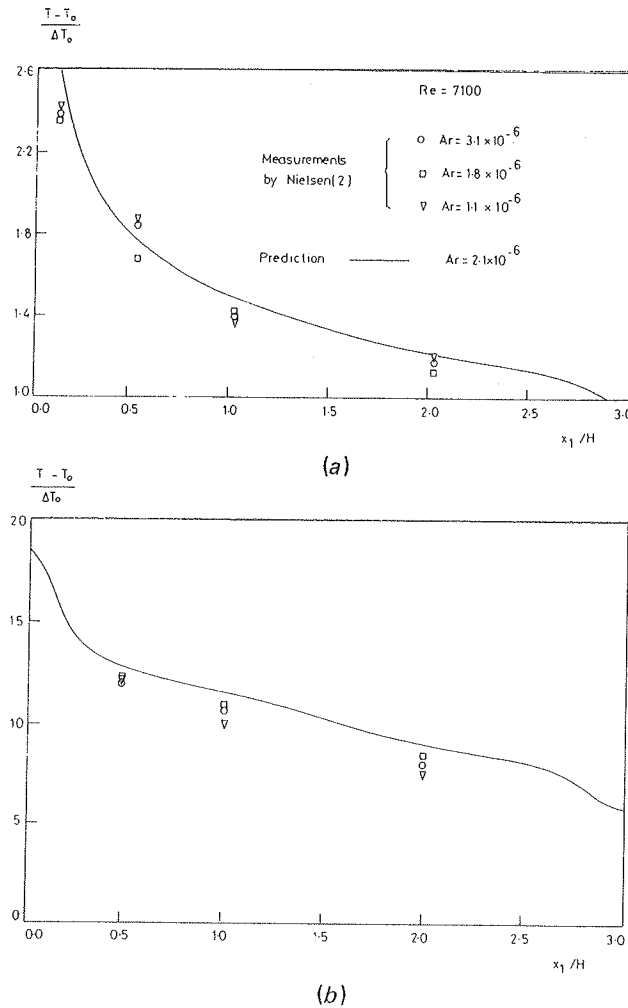


Fig. 4 Temperature profiles at (a)  $y = 0.75H$  and (b)  $y = H$  from Nielsen [2].

smaller values of  $h/H$  which, when applying the finite-difference scheme, require very refined grids in the vicinity of the inlet plane. The special treatment of this area was described in [1]. The calculations of Fig. 5 show the maximum velocity across the jet  $U_m$  at different values of  $Ar$  and were obtained without jet development assumptions for  $L/H = 3.0$  and  $h/H = 0.056$  and  $0.017$ . It is obvious that buoyancy forces distort the velocity field from the first few slot heights downstream of the inlet, and the procedure of [1], used for subsequent calculations, must be regarded as approximate.

The experiments of Hestad [7] are related to a geometry with  $h/H = 0.0033$  and were predicted with measured velocity and temperature values along boundary  $a$  (Fig. 1); other parameters were  $L/H = 1.867$ ,  $Re = 1400$ , and  $Ar = 5.5 \times 10^{-4}$ . Results are shown in Fig. 6, together with the isothermal prediction from [1], and also in Fig. 7, with the original tracings from Hestad's experiments. As can be seen,  $U_{rm}$  is overpredicted by approximately 10%, but the general velocity pattern is reproduced. Temperatures are slightly less well represented, but are satisfactory in view of the strong influence of buoyancy in this flow (equivalent to Fig. 2c). Also, the measured isotherms suggest that heat is transferred from the ceiling, and this has not been accounted for in the calculations, where the ceiling was treated as an adiabatic boundary.

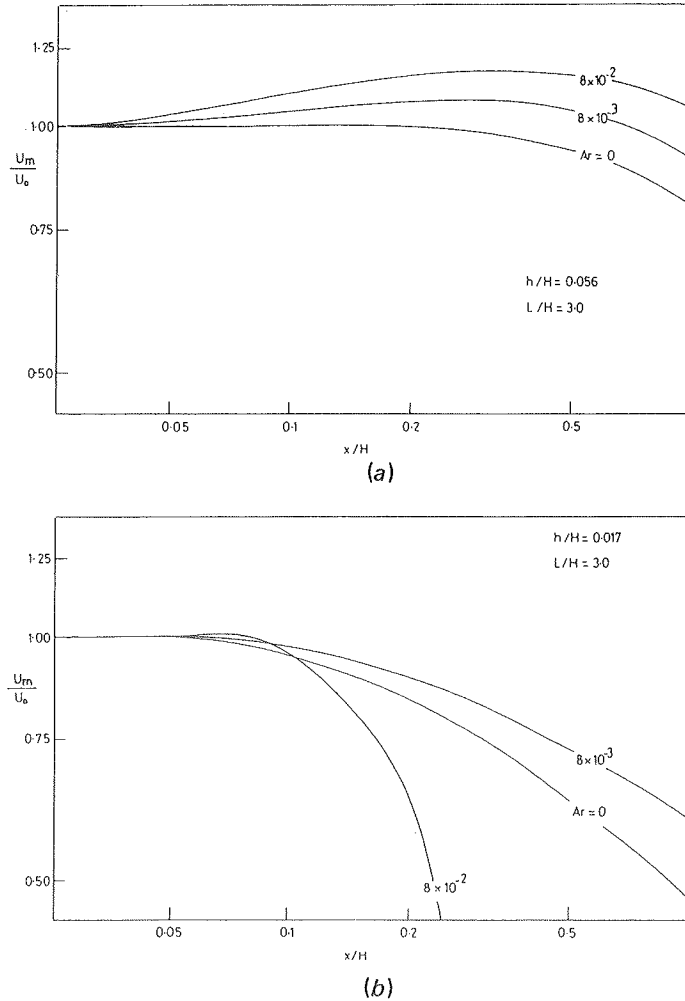


Fig. 5 Decay of maximum velocity in jet for  $L/H = 3.0$ ,  $Re = 7350$ , and  $h/H = (a) 0.056$  and  $(b) 0.017$ .

An interesting problem is posed by the experiments of Schwenke [6], who measured the penetration depth  $x_R$  of the jet as a function of  $Ar$  at several values of  $h/H$  and found that, for a limited range of  $Ar$ , two different flow patterns were observed with correspondingly different values of  $x_R$ . The higher  $x_R$  (here designated solution of type I) occurred when  $Ar$  was being gradually increased and the lower  $x_R$  (type II) when  $Ar$  was gradually decreased. In principle, this situation is outside the scope of the present calculation method, since the equations ignore the time dependence of the mean flow. Two solutions may, however, be obtained at certain  $Ar$  if approximate distributions of the relevant variables are taken as initial guesses in the iterative procedure. It was found that results of type I occur when the initial guess is a converged solution of a previous calculation obtained for a slightly lower value of  $Ar$ ; as the  $Ar$  of the initial guess is increased the same type of flow exists until the flow pattern finally reverses. Thereafter, results of type II will be obtained for values of  $Ar$  higher than the critical value. Solutions of type II are obtained by progressively decreasing the value of the  $Ar$  of the initial guess from a higher value. The two types of solutions are indicated in Fig. 8 for  $h/H = 3.0$ ,  $h/H = 0.017$ ,  $Re = 7450$ , and  $Ar = 0.04$ . Figure 9 presents Schwenke's measurements for  $L/H = 1.6$  and  $h/H = 0.017$ , together with the present predictions obtained as described above. The

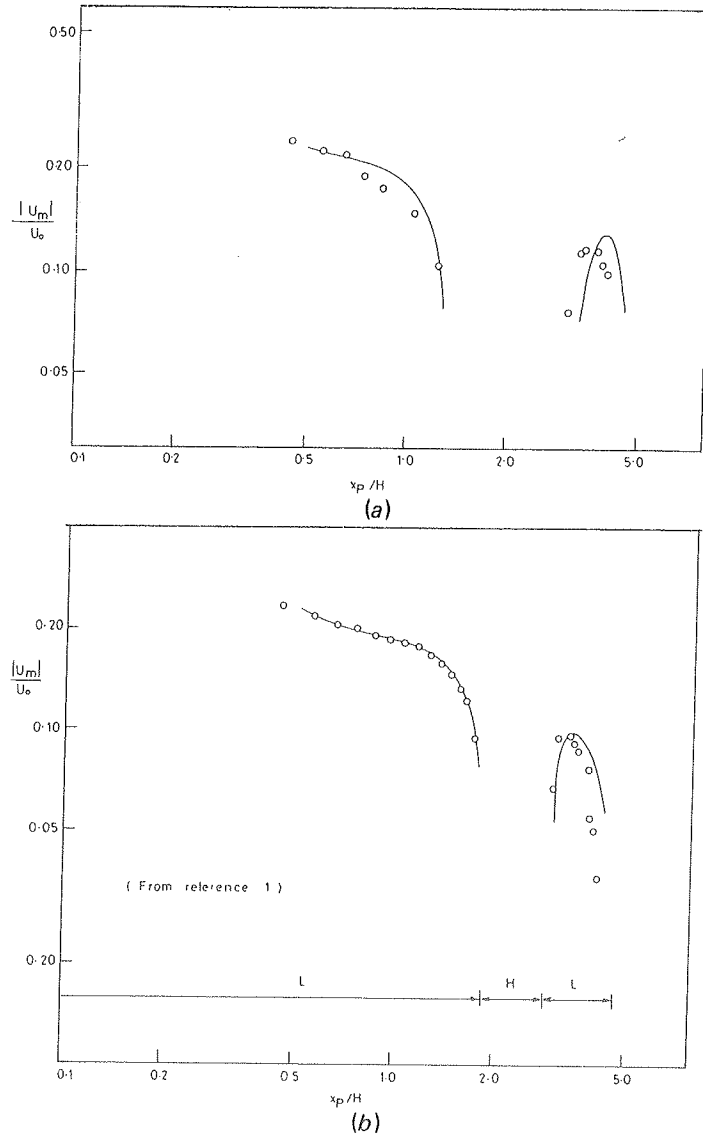


Fig. 6 Maximum velocity along periphery of room: experimental data from Hestad [7] and present prediction. (a) Buoyant and (b) isothermal.

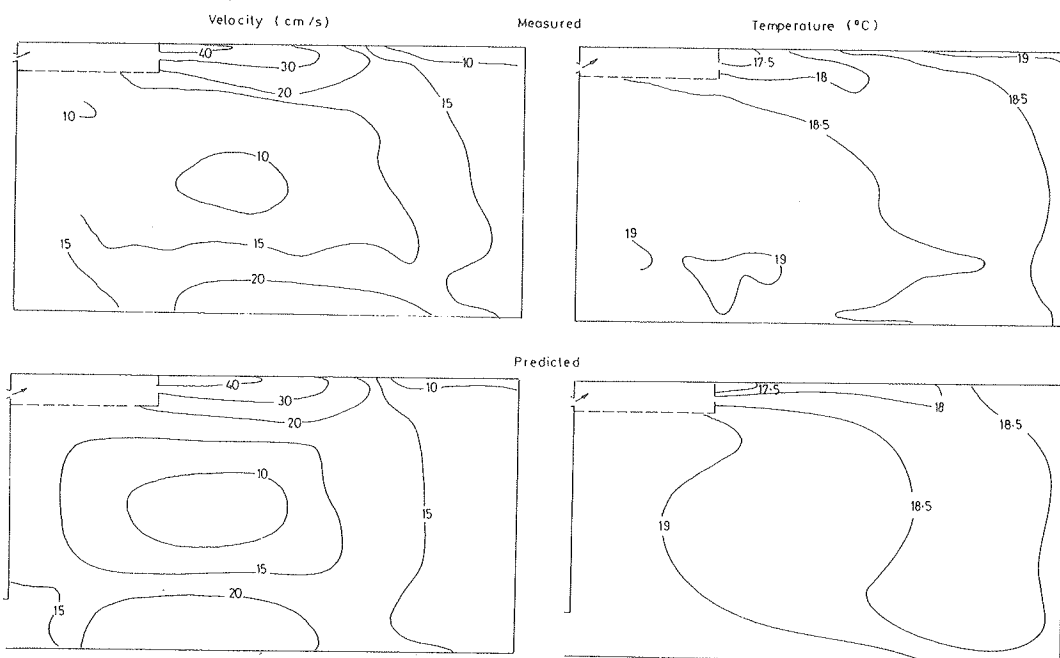


Fig. 7 Patterns of isovels and isotherms: experimental data from Hestad [7] and present prediction.

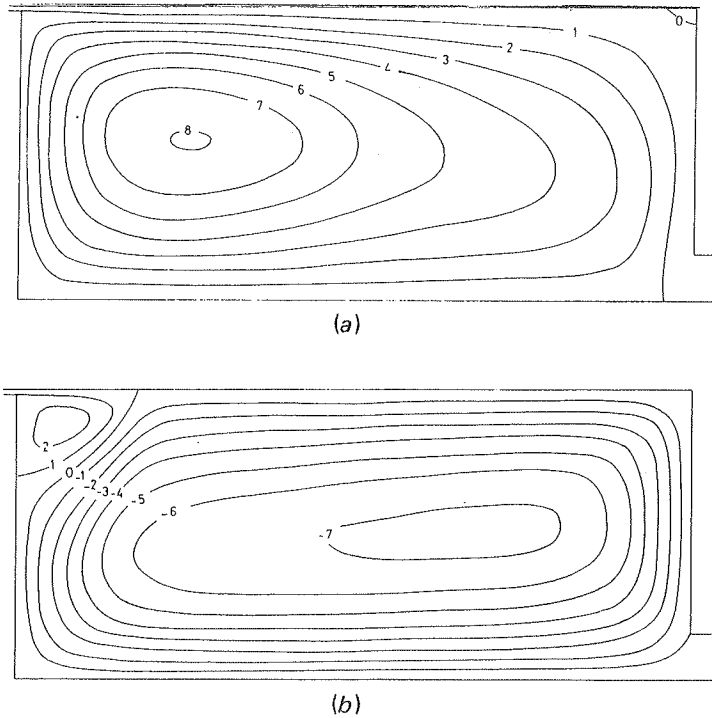


Fig. 8 Predicted flow patterns at  $Ar = 4 \times 10^{-2}$ : (a) type I and (b) type II (see text);  $L/H = 3.0$ ,  $h/H = 0.017$ , and  $Re = 7350$ .

agreement is less satisfactory than in the previous cases, but the main pattern is once more reproduced.

Comparisons of Figs. 4, 6, 7, and 9 form the basis for an evaluation of the proposed numerical approach and, with the resulting knowledge of the likely precision of the method, calculations were performed outside the range of existing measurements and are intended to provide guidelines for the designer of air-conditioning systems. The

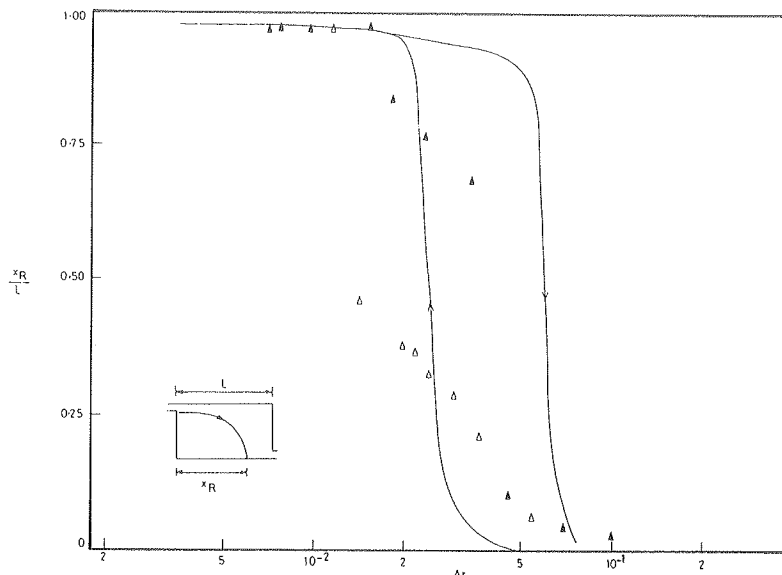


Fig. 9 Dependence of penetration depth  $x_R$  on  $Ar$ : data from Schwenke [6] and present prediction;  $L/H = 1.6$  and  $h/H = 0.017$ .

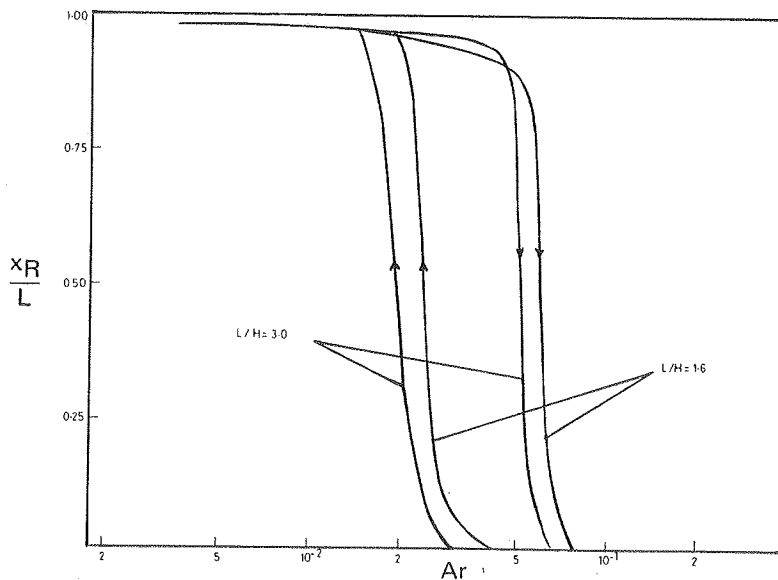


Fig. 10 Dependence of penetration depth  $x_R$  on  $L/H$  at different  $Ar$  (prediction);  $h/H = 0.017$ .

influence of the length-to-height ratio of rooms, as a function of  $Ar$ , is shown in Fig. 10. Increasing  $L/H$  from 1.6 to 3.0 is shown to have a relatively small influence in the range of  $Ar$  where the inversion of flow pattern occurs. Figure 11 is a generalization of a similar chart presented in [1] for isothermal flows. The maximum velocity in the reverse flow, normalized by the inlet velocity, is plotted against  $h/H$  for  $Ar$  of  $5 \times 10^{-4}$ ,  $10^{-3}$ , and  $5 \times 10^{-3}$ . Values of  $h/H$  lower than 0.015 are not included because of the difficulty pointed out earlier, associated with narrow slots. A uniform heat flux over the bottom surface was again assumed.

Figures 10 and 11 indicate that the maximum velocity in the reverse flow increases with the heat load in a ventilated room, and at high heat loads buoyancy leads to an inverse flow pattern. The  $Ar$  at which this inversion occurs is almost independent of the room length, and the values of  $Ar$  required to produce substantial effects on the flow patterns decrease with the relative slot size  $h/H$ .

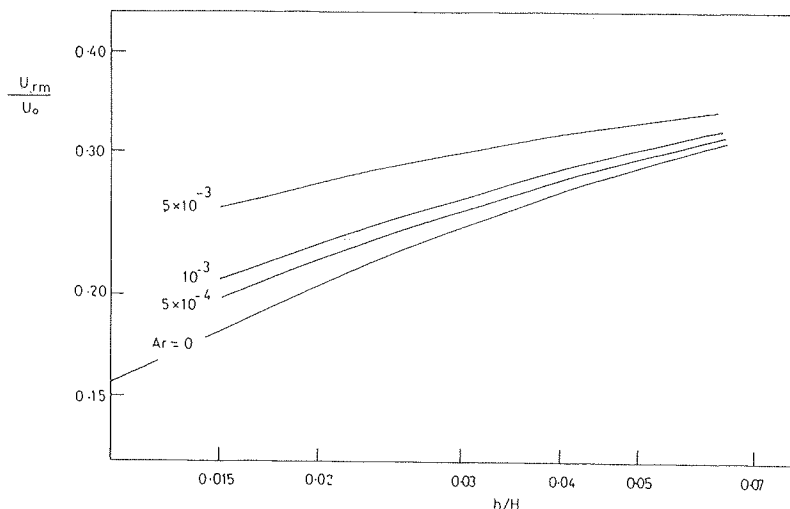


Fig. 11 Dependence of maximum velocity in reverse flow  $U_{rm}$  on slot height and  $Ar$  (prediction);  $L/H = 3.0$ .

## CONCLUSIONS

Prediction of buoyancy-affected flows in ventilated rooms has been demonstrated in a wide range of room geometries and with different heat loads, using the effective viscosity  $K - \epsilon$  model of turbulence. The main conclusions are as follows.

Up to the highest  $Ar$  investigated—i.e., 0.08 and 0.00055 in geometries with  $h/H = 0.017$  and 0.0033, respectively—which correspond to flow patterns strongly distorted by buoyancy, the degree of accuracy is shown to be adequate for design purposes. In the latter case agreement with experimental data is within 4% of the maximum velocity and 5% of the maximum temperature difference in the flow.

Part of the quoted uncertainty results from grid dependence in the calculations, estimated as 2% for both properties. The remainder may be attributed to the empirical assumptions in the turbulence model and wall functions employed, and also to the simplified treatment of the inlet region. Improvements may be expected from the solution of transport equations for the turbulent stresses and heat fluxes, particularly in strongly buoyant flows where the concepts of eddy diffusivities are less adequate, but the added computational work and cost involved are not justifiable for most practical applications. Developments in the ability to prescribe correct boundary conditions for the inlet region in more general circumstances are likely to produce more significant improvements.

The predicted results quantify the effects of buoyancy and their dependence on the geometric ratios  $h/H$  and  $L/H$ . They show, for example, that increasing  $L/H$  from 1.6 to 3.0 decreases by only 20% the range of  $Ar$  where reversal of the flow occurs, and that an  $Ar$  of  $5 \times 10^{-4}$  results in a maximum reverse velocity at  $h/H = 0.05$ , which is identical to that for isothermal flow, but in a value larger by 25% in a geometry with  $h/H = 0.015$ .

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