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Čizmar, Dean; Sørensen, John Dalsgaard; Kirkegaard, Poul Henning; Rajčić, Vlatka

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Dean Čizmar
Department of Structural Engineering, Faculty of Civil Engineering, University of Zagreb, Croatia.

John Dalsgaard Sørensen and Poul Henning Kirkegaard
Department of Civil Engineering, Aalborg University, Denmark.

Vlatka Rajčić
Department of Structural Engineering, Faculty of Civil Engineering, University of Zagreb, Croatia.

Abstract

This paper presents a probabilistic approach for structural robustness assessment for a timber structure built a few years ago. The robustness analysis is based on a structural reliability based framework for robustness assessment. The complex timber structure with a large number of failure modes is modelled by only a few dominant failure modes. A component based robustness analysis is performed based on reliability indices of remaining elements after assumed failure of selected critical elements. Two different approaches are used; first failure of elements is assumed to be brittle and second where material ductility of timber is taken into account. The robustness is expressed and evaluated by a robustness index.

Keywords

Robustness of structures; Probabilistic Methods; Timber truss structure, Ductility

1. Introduction

1.1 Robustness of structures in the codes

Robustness of structural systems has attracted a renewed interest due to a much more frequent use of advanced types of structures with limited redundancy and serious consequences in case of failure. The interest has also been stimulated due to severe structural failures such as that at Ronan Point in 1968 (Pearson & Delatte, 2005) and at the World Trade Centre towers in 2001. In order to minimize the risk of such disproportionate structural failures many modern building codes consider the need for robustness in structures and provide requirements, strategies and methods to obtain robustness, see e.g. (CEN, 2006; CEN, 2004a). The requirement for robustness is specified in most buildings codes in a way like the general requirements in the two Eurocodes, EN 1990 Eurocode 0: Basis of Structural Design (CEN, 2006) and EN 1991-1-7 Eurocode 1: Part 1-7 Accidental Actions (CEN, 2004a). The first provides the basic requirements, e.g. it is stated that a structure shall be "designed in such a way that it will not be damaged by events like fire, explosions, impact or consequences of human errors, to an extent disproportionate to the original cause." The second provides strategies and methods to obtain robustness though actions and design situations to consider.

1.1.1 Definitions of robustness

During the last decades a variety of researchers have attempted to quantify aspects of robustness such as redundancy and to identify design principles that can improve robustness. All the proposed attempts for quantification of robustness can be divided into three main categories of measures: deterministic, probabilistic and risk based.

1.1.2 Deterministic robustness measures

A simple and 'easy-to-use' deterministic measure is given in (ISO19902, 2007). In this robustness measure the ratio of the base shear capacity of the platform and the design load are compared. The base shear capacity is estimated using non-linear structural models with and without failed elements. In (Starossek & Haberland, 2008) a measure of robustness is proposed where the stiffness matrix of the intact structure and the stiffness matrix after removal of a structural element are compared and a robustness index is derived. The same authors also proposed energy and damaged based definitions of robustness. Quite recently, a multi-level framework for progressive collapse assessment of building structures subject to sudden column losses is presented by (Izzuddin, Vlassis, Elghazouli, & Nethercot, 2008). The proposed assessment framework employs three stages, first determination of the nonlinear static response, then a simplified dynamic assessment and finally a ductility assessment. In (Vlassis, Izzuddin, Elghazouli, & Nethercot, 2008) is presented an application of the proposed design-oriented method for progressive collapse assessment of multi-storey buildings.

1.1.3 Probabilistic robustness measures

In the late 80's (Frangopol & Curley, 1987) proposed reliability-based indices as measures of structural redundancy through the residual strength of a damaged system. The same authors also proposed a redundancy factor where the reliability indexes of the both intact and damaged systems are used to determine this factor. (Lind, 1995) proposed a generic measure of system damage tolerance, where a vulnerability parameter is used as indicator of the loss of system reliability due to damage. As progressive collapse is characterised by the disproportion between the magnitude of a triggering event and the resulting collapse of large part or the entire structure (Ellingwood & Leyendecker, 1978) defined the probability of such collapse as a chain of partial probabilities: the probability of an abnormal event that threatens the structure (generally a hazard), the probability of local damage as a result of this event and the probability of failure of the structure as a result of the local damage. The term hazard refers to abnormal loads or load effects (Ellingwood, Smilowitz, Dusenberry, Duthinh, & Carino, 2007). Abnormal loads can be grouped as pressure loads (e.g., explosions, detonations, tornado wind pressures), impact (e.g., vehicular collision, aircraft or missile impact, debris, swinging objects during construction or demolition), deformation-related (softening of steel in fire, foundation subsidence), or as faulty design and construction (human errors). These loads usually act over a relatively short period of time in comparison with ordinary design loads. The loads generally are time-varying, but may be static or dynamic in their structural action (Ellingwood et al., 2007). Recently (Starossek & Haberland, 2010) proposed a definition of both the progressive collapse and the robustness. Similar to the approach described above the probability of disproportionate collapse is calculated as a product of probabilities: the probability of an abnormal event that threatens the structure, the probability of initial damage as a result of event and the conditional probability of a disproportionate spreading of structural failure due to the initial damage. Based on this, there are the three main strategies to limit the probability of a disproportional collapse, first is to prevent the occurrence of abnormal events, the second is to prevent the occurrence of an initial damage in consequence of the occurrence of abnormal events. A third strategy is to prevent disproportionate spreading of failure of the initial damage. This part relates to the internal properties of the structure though its robustness. As such the robustness is a property that depends on the structure itself and the amount of initial damage (Starossek & Haberland, 2010). Vulnerability is defined as the susceptibility of a structure to suffer initial damage, when affected by abnormal events. Vulnerability is related to local conditions while robustness is related to global system behaviour (Starossek & Haberland, 2010). An example of a robustness assessment is presented in (Kirkegaard & Sørensen, 2008) where the robustness analysis is based on the framework for robustness analysis introduced in the Danish Code of Practice for the Safety of Structures and a probabilistic modelling of the timber material proposed in the Probabilistic Model Code (PMC) of the Joint Committee on Structural Safety (JCSS, 2001). The framework mentioned above considers the structural robustness

at system-level and has the potential to take into account uncertainties inherent in description of unintentional loads and defects, static layout and structural composition. (Cizmar, Rajcic, Kirkegaard, & Sorensen, 2010) generalised this approach and used a robustness index defined as a ratio of the reliability indices of the damaged and intact structure with values between 0 (non robust structure) and 1 (ideally robust structure).

1.1.4 Risk based robustness measure

Few years ago, an index of robustness has been proposed taking basis in decision theory (a risk based definition) following (Baker, Schubert, & Faber, 2008) which states that a decision theory framework can be used to assess robustness in a general manner. The index of robustness is obtained by computing both the direct risks, which are associated with the direct consequences of potential damages to the system, and the indirect risks, which correspond to the increased risk of a damaged system. Indirect risks can be interpreted as risks from consequences disproportionate to the cause of the damage, and so the robustness of a system is indicated by the contribution of these indirect risks to the total risk. This framework was then as an example applied to assess the robustness of an externally and internally post-tensioned highway bridge designed according to present best practice (Radowitz, Matthias, & Faber, 2008).

1.2 Robustness of timber structures

In the last few decades research in assessment of reliability of timber structures has been quite intensive, but robustness of timber structures has not been shown much attention. One of the reasons for the lacking interest about robustness of timber structures is that a unified approach for assessment of robustness in general has not been available. Since timber is a rather complex building material, assessment of robustness of timber structures is difficult to conduct. In the frame of the COST E55 Action (Dietsch & Winter, 2010) have made a deterministic robustness analysis of the collapses of both the Siemens Arena and the Bad Reichenhall Ice Arena. The Siemens Arena which was build in 2001 as a large span timber truss system, two of the trusses collapsed without warning at a time with almost no wind and only a few millimetres of snow. The partial collapse happened just a few months after the inauguration of the arena. An investigation showed that the cause of the failure could be localised to one critical cross-section in the tension arch near the support, where the load-bearing capacity was found to be between 25% and 30% of the required capacity. It is noted that the collapse did not occur due to an unknown phenomenon. The design of the trusses was not checked by the engineer responsible for the entire structure due to unclear specification of the responsibility and duties of that engineer. The Bad Reichenhall Ice-Arena built in 1971/1972 is a large span roof structure was supported by 2.87 m high main girders produced as timber box-girders. The box-girders featured upper and lower laminated timber members and lateral web boards. On January 2nd 2006, the entire roof collapsed without warning during a period of

significant snowfall (Winter & Kreuzinger, 2008). The review of the structural calculation revealed severe human errors in design and heavy misuse of building codes. These errors, humidity exposure and general lack of maintenance lead to the collapse of a structure. Based on the robustness framework described above (Kirkegaard & Sørensen, 2008) presented a reliability-based robustness analysis of a glued laminated frame structure supporting the roof over the main court in a Norwegian sports centre. Progressive collapse analyses are carried out by removing potential critical elements, and then assessing the reliabilities of the remaining structural elements. The results show that the timber structure of Norwegian sports centre can be characterized as robust with respect to the robustness framework used for the evaluation. The robustness analysis in this paper is based on the general framework mentioned above (Kirkegaard & Sørensen, 2008) and a probabilistic modelling of the timber material proposed in the Probabilistic Model Code (JCSS, 2001) of the Joint Committee on Structural Safety (JCSS). The main difference with respect to the work by (Kirkegaard & Sørensen, 2008) is that in this paper the material ductility in compression is taken into account. The robustness assessment is made on componential level where reliabilities of the remaining components (after failure of one critical element) are compared with the reliability of the intact elements.

2. Overview of a structure

Many recent structures in Croatia, especially sports halls, swimming pools, tourist objects, passages and pedestrian bridges were built using timber (mainly glued laminated timber). A sport centre in Samobor (small town near Zagreb, Croatia) is considered in this paper. The total area of the sport centre is 5910 m^2. It consists of three main parts:

- main hall with dimensions 36,5x45, 9 (m) height for 600 visitors,
- swimming pool with dimensions 12,5x25, 10 (m) and depth from 1,8 to 2,4 (m),
- two smaller halls with dimensions 20x15 (m).

This paper focus is on the main hall. The main hall of this sport centre was erected in 2005. It is a plane frame truss equally spaced at 5 meters each. The structure was calculated according to Eurocode 5 (CEN, 2004b). The design was performed by the Chair for the Timber Structures at the Structural Department at the Faculty of Civil Engineering, University of Zagreb. Figure 1 shows a computer rendering of the whole sport hall. For the design of the structure characteristic values of permanent load (g= 6.38 kN/m), snow load (s=7.5 kN/m) and wind load (w=0.9 kN/m) are used. The material chosen is timber GL32c. The following cross section dimensions were chosen: upper chord 20/52 cm, lower chord 20/69 cm and diagonal elements 20/24 cm. Figure 2 shows the built structure and figure 3 shows the static system of the timber structure.

Figure 1: Computer rendering of the sport hall.

Figure 2: Sport hall in Samobor.

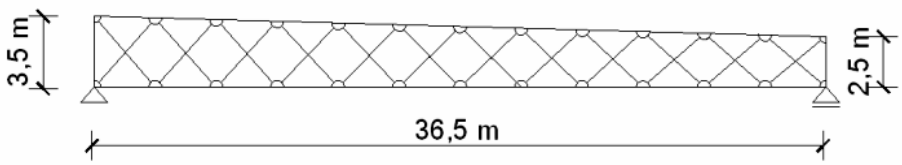

Figure 3: Static system of the timber truss structure.

3. Probabilistic model

Reliability assessments were done by First Order Reliability Methods (FORM) where a reliability index is estimated based on limit state functions for each of the considered failure modes. The probabilistic analysis is performed with a stochastic model for the strength parameters for structural elements, and not to the strength for the single laminates and the glue. Second order effects are neglected for beams subjected to compression and combined compression and bending, respectively. For the structural analysis a linear Finite Element analysis has been performed where the glued laminated truss has been modelled by beam and truss elements. Identification of the significant failure modes of the structure is difficult to perform since there are numerous possible failure elements. Based on a deterministic structural analysis only four significant different failure modes are considered: 1) combination of bending and compression (M+N) in the upper chord, 2) combination of bending and tension (M+N) in the lower chord, 3) compression (N) and 4) tension in diagonal elements (N). M and N denote bending moment and normal force, respectively. The ultimate limit state failures are assumed to be brittle (i.e. when an element fails there is no bearing capacity left). Thus the following failure elements are considered for these failure modes:

1. Failure in bottom cord (N+M)
2. Failure due to tension in diagonal element (N)
3. Failure due to compression in diagonal element (N)
4. Failure in top chord (N+M)

The stochastic model is shown in table 1 and is mainly based on recommendations in (Koehler, Sørensen, & Faber, 2007). For the calculations permanent load, *G* due to self weight and a variable snow load, Q are taken into account. The permanent load of the roof structure, is assumed Normal distributed with an expected value μ_G = 6.38 kN/m and a coefficient of variation COV = 0.1. For the region in Croatia where the structure is located the annual maximum snow load at the ground is Gumbel distributed with a characteristic value S_{gc}= 1.5 kN/m^2 (7.5 kN/m as the distance between the trusses is 5 meters) corresponding to a 98% quantile in the distribution function of the annual maximum snow load. Based on this the snow load *Q* on the roof can be modelled by:

$$Q = S_g \cdot C \tag{1}$$

where S_g refers to snow on ground and *C* (modelled as a deterministic variable) is the roof snow load shape factor. It is assumed (based on the available data) that the coefficient of variation for the region near Zagreb is COV = 0.58 (Androic, Cizmar, & Rajcic, 2008). The following equations show how to calculate the mean value. If COV for ground snow load is assumed to be V_Q, then the expected value μ_Q can be determined from the Gumbel cumulative distribution function $F_Q(\cdot)$ as:

$$F_Q(Q_c) = \exp(-\exp(-\alpha(Q_c - \beta))) \quad (2)$$

$$\mu_Q \approx \beta + \frac{0.577216}{\alpha}, \quad \sigma_Q = \frac{\pi}{\alpha \cdot \sqrt{6}} \quad (3)$$

$$V_Q = \frac{\sigma_Q}{\mu_Q} \quad (4)$$

The strength variables f_c, f_m and f_t (compression strength parallel to grain, bending strength and tensile strength, respectively) are modelled based on the reference properties given in table 1 (Koehler et al., 2007) where MOE is the modulus of elasticity. Table 2 shows all stochastic variables used. The correlation coefficients between the stochastic variables are taken as proposed in (Koehler et al., 2007).

Table 1: Reference properties. N: Normal; LN: LogNormal.

Variable	Distribution	COV
Bending strength f_m	LN	15%
Bending MOE E_s	LN	13%
Density	N	10%

Table 2: Stochastic variables (dimensions in mm, strengths in N/mm^2 and loads in N/mm). N: Normal; LN: LogNormal; G: Gumbel.

Label	Variable	Distribution	Mean value	COV
Es	Bending MOE	LN	11700	13%
X	Model uncertain.	LN	1.00	10%
A	Joint distance	N	3041	1%
b_d	Width of diagonals	N	200	4%
h_d	Height of diagonals	N	240	4%
b_{dp}	Width bottom chord	N	200	4%
h_{dp}	Height bottom chord	N	690	4%
b_{gp}	Width top chord	N	200	4%
h_{gp}	Height top chord	N	520	4%
f_c	Compression strength	LN	26.6	12%
f_m	Bending strength	LN	41.4	15%
f_t	Tension strength	LN	24.8	18%
G	Permanent load	N	6.38	10%
S	Snow load	G	3.00	58%

For the failure elements the following limit state functions are used:

$$g_1 = X - \frac{N_{iE}(G,S)}{0.8 \cdot f_t \cdot b_{dp} \cdot h_{dp} \cdot k_{\text{mod}}} - 6 \cdot \frac{M_{iE}(G,S)}{0.8 \cdot f_m \cdot b_{dp} \cdot (h_{dp})^2 \cdot k_{\text{mod}}} \tag{5}$$

$$g_2 = X - \frac{N_{iE}(G,S)}{0.8 \cdot f_t \cdot b_d \cdot h_d \cdot k_{\text{mod}}} \tag{6}$$

$$g_3 = X - \frac{N_{iE}(G,S)}{f_c \cdot b_d \cdot h_d \cdot k_{\text{mod}}} \tag{7}$$

$$g_4 = X - \frac{N_{iE}(G,S)}{k_c \cdot f_c \cdot b_{dp} \cdot h_{dp} \cdot k_{\text{mod}}} - 6 \cdot \frac{M_{iE}(G,S)}{k_{crit} \cdot f_m \cdot b_{gp} \cdot (h_{gp})^2 \cdot k_{\text{mod}}} \tag{8}$$

where N_{iE} and M_{iE} are the load effects of the permanent load, G and snow load, S for the corresponding element number *i*. Details can be found in (Kirkegaard & Sørensen, 2008). The load duration factor, k_{mod} is considered deterministic with a value of 0.9. k_{crit} and k_c (coefficients taking into account lateral torsion buckling and buckling, respectively) are calculated as required in (CEN, 2004b) and assumed to be deterministic. Other variables used in (5) - (8) are defined in table 2.

For each of the failure elements, the element reliability index β_i is estimated using the First Order Reliability Method (FORM). The element reliability indices shown in table 3 indicate that the most significant failure modes are 1 and 4. The relative ratio between the different reliability indices corresponds very well to the results from a deterministic analysis.

Table 3: Reliability indices for failure elements (reference period: one year)

Element number	Reliability index
1	4.99
2	7.67
3	7.04
4	4.46

The requirements to the reliability of the structure can be expressed in terms of an accepted minimum reliability index, i.e. a target reliability index. The Joint Committee on Structural Safety (JCSS) has proposed target reliability values for ultimate limit states (JCSS, 2001). For the normal design situation the reliability index β_i (with a reference period equal to one year) should be larger or equal to 4.2. For the considered failure elements the reliabilities of the components are slightly larger (the lowest beta index is approximately 6% higher than target value given by JCSS).

4. Robustness analysis

4.1 Brittle behaviour of timber

Table 4: Foreseeable errors (Vrouwenvelder & Sorensen, 2010)

Accidental / Natural	Accidental / Manmade	Human influences	Normal loads	Human errors
Earthquake	Internal explosion	Vandalism	Selfweight	Design error
Landslide	External explosion	Demonstrations	Imposed load	Material error
Hurricane	Internal fire	Terrorist attack	Car park loads	Construction error
Tornado	External fire		Traffic load	Misuse
Avalanche	Impact by vehicle		Snow	Lack of maintenance
Rock fall	Mining subsidence		Wind	Miscommunication
High groundwater	Environmental attack		Hydraulic	
Volcano eruption				

For assessment of robustness the structural behavior needs to be considered with emphasis on the assessment and modeling of damage scenarios resulting from various defined or undefined exposures (Vrouwenvelder & Sørensen, 2010). Table 4 shows a list of some foreseeable exposures. The first two columns refer to the more or less extreme or accidental actions caused by nature itself or are manmade. The third column refers to human influences which are deliberate. The fourth column shows normal loads and column five ‘usual’ human errors. Robustness is considered to be related to disproportionate spreading of structural failure. As this structure is statically indeterminate, loss of one (or more) structural element(s) would not result in collapse of the whole structure, i.e. if any of the inner (truss) elements fail, force redistribution will occur and the whole system will not necessarily collapse. Since

most of the exposures are very difficult to quantify, an alternative methodology is used, based on the general framework presented above. For each of the failure elements failure is assumed (a failed element is assumed to fail in a brittle manner) and the reliability of the remaining failure elements is calculated. It is noted that only one failure element is assumed to fail at a time. The robustness index at component level $I_{rob,k,l}$ (calculated for a component *k* when component *l* is damaged or failed) is defined as:

$$I_{rob,k,l} = \min\left\{\frac{\beta_{dmg,k,l}}{\beta_{\mathrm{int},k}};1\right\} \quad \forall\beta_{dmg,k} \geq 0, \forall\beta_{\mathrm{int},k} > 0 \tag{9}$$

where $\beta_{dmg,k,l}$ denotes the reliability index of component *k* when element *l* is failed and $\beta_{\mathrm{int},k}$ denotes the reliability index of component *k* for the intact structure.

Figure 4 shows robustness indices based on element reliability indices obtained for the remaining elements after assumed failure of one element. Note that the term failure element in figure 4 refers to failure mode defined previously. Generally, after failure of one element, the reliability of the other elements is decreased (as the redistribution of the forces implies that the other elements have a higher utilization ratio). However, for assumed failure of element 4 (e.g. failure in the middle of upper chord) the reliability indices for the tensile and compressive truss elements are slightly increased. In this case, redistribution slightly decreases the load effects for elements 2 and 3, but the load effect for element 1 is highly increased and it can be concluded that the reliability for this scenario is insufficient. It is seen that with removal of the four different elements one by one, only for one failure scenario (failure in the middle of lower chord), a significant extensive failure of the entire structure or significant parts of it can be expected. This can be seen in the figure 4 where the lowest robustness index is 0.3 in case of the assumed failure of element 4. For the remaining assumed failures no significant extensive progressive failures can be expected (robustness indices are large).

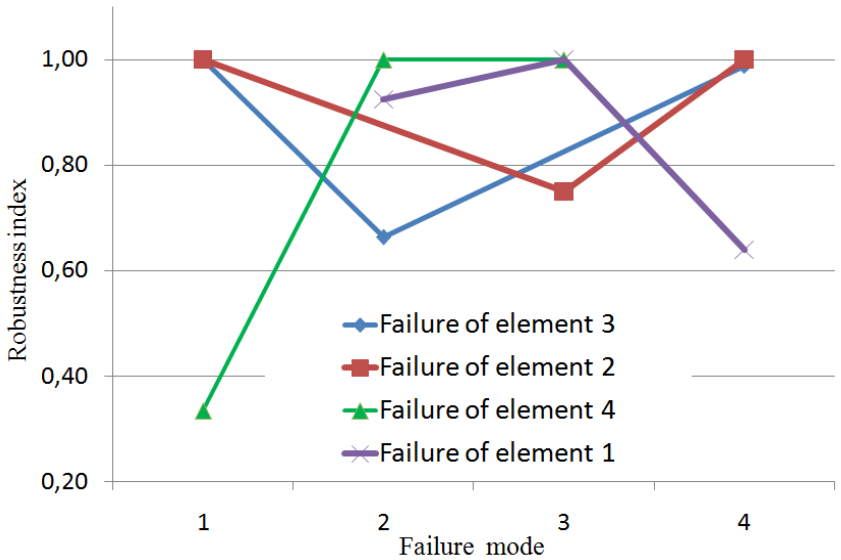

Figure 4: Robustness indices based on element reliabilities (brittle behaviour)

4.2 Ductile behaviour of timber structure

Timber is considered to be a brittle material, because failure occurs suddenly, without any warning. This can be considered as a drawback when comparing to other materials like steel. It has no or a very little ductility in the tensile area, while in compressive area linear elastic-plastic behaviour can be assumed, see figure 5. In this paper a ductile behaviour is assumed in compression and for interaction of bending and compression. The probabilistic model of stress-strain curve (figure 6 and table 5) is based on experimental data derived from compression tests. Figure 7 shows robustness indices based on ductile behaviour of elements. It can be seen, that if ductile behaviour of upper chord is assumed (element 4), the robustness index for element 1 is much higher. The same conclusion can be drawn for assumed failure of element 3 (in this case robustness index of element 2 is increased).

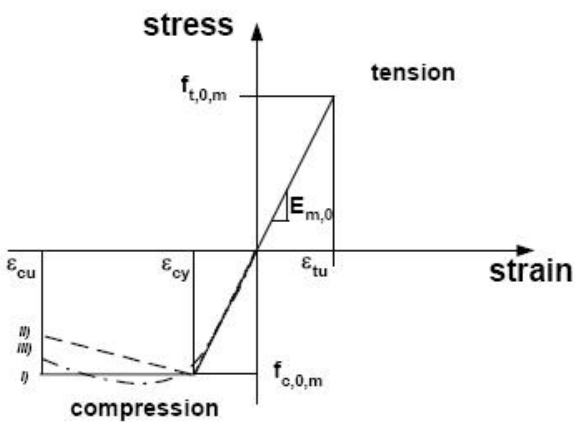

Figure 5: Typical stress strain curve of timber

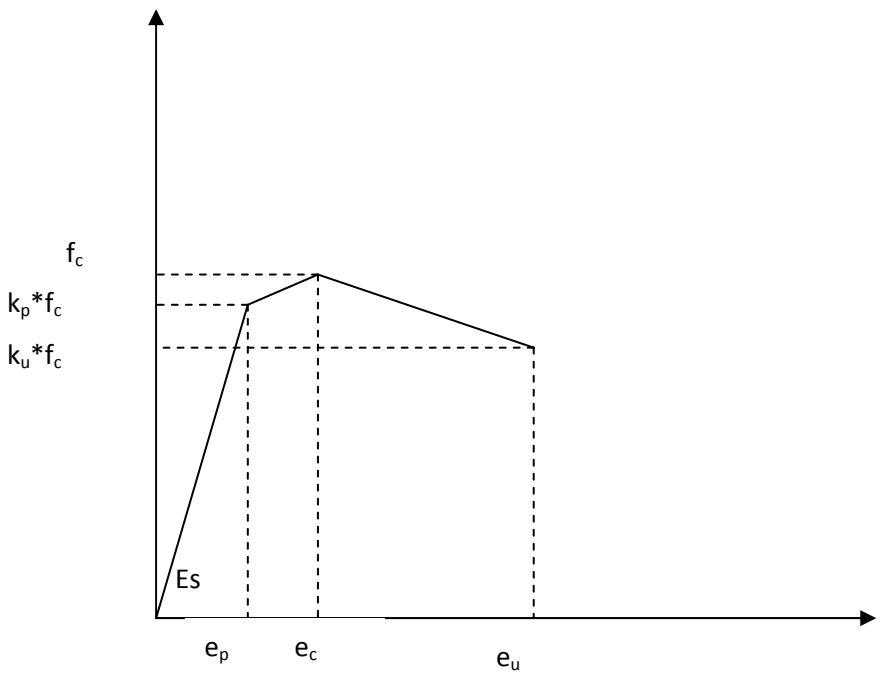

Figure 6: Idealised stress-strain behaviour in compression

Table 5: Probabilistic variables based on experimental data

Label	Variable	Distribution	Mean value	COV
k_u	Factor to modify force at ultimate strain	N	0.70	30%
k_p	Factor to modify force at proportional strain	N	0.80	5%
Es	Bending MOE [MPa]	LN	11700	11%
f_c	Compression strength [MPa]	LN	26.6	8.5%

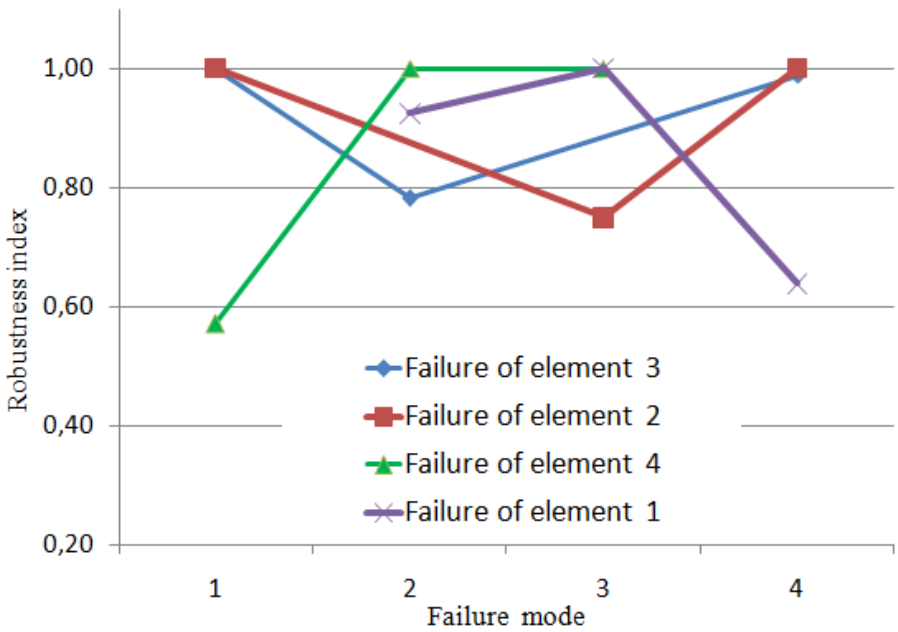

Figure 7: Robustness indices based on element reliabilities (ductile behaviour)

5. Conclusions

A general overview of different approaches for robustness assessment, including recent robustness evaluations of timber structures is presented. The robustness analysis in this paper is based on the general framework for robustness analysis introduced in the Danish Code of Practice for the Safety of Structures and a probabilistic modelling of the timber material proposed in the Probabilistic Model Code of the Joint Committee on Structural Safety (JCSS). For the purpose of robustness assessment, a reliability-based approach is used based on component reliabilities. Two different structural analysis were made, first is based on linear elastic models, i.e. non-linear effects are not taken explicitly into account and other is based on a ductile behaviour of timber in compression. Progressive collapse analyses are carried out by removing four structural elements one by one.

The results based on brittle models show that the timber structure for three of the failure scenarios can be characterized as very robust with respect to the robustness framework used for the evaluation. However, for one of the failure scenarios the robustness can be considered as significantly lower.

The results based on models with ductile behaviour of timber show that robustness indices are higher for asssumed failures of these ductile elements. Based on this model it can be concluded that for all of the failure scenarious the structure can be considered as robust. It must be noted that this concusion is only made under assumption that in interaction of bending and compression, the timber material behaves ductile in compression and brittle in bending, which may not be true.

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