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Upscaling Wind Turbines

theoretical and practical aspects and their impact on the cost of energy

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SPECIAL ISSUE PAPER

Upscaling wind turbines: theoretical and practical aspects and their impact on the cost of energy

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ABSTRACT

Wind turbines with a rated power of 5 to 6 MW are now being designed and installed, mostly for offshore operation. Within the EU supported UpWind research project, the barriers for a further increase of size, up to 20 MW, are considered. These wind turbines are expected to have a rotor diameter up to 250 m and a hub height of more than 150 m. Initially, the theoretical implications of upscaling to such sizes on the weight and loads of the wind turbines are examined, where it is shown that unfavourable increases in weight and load will have to be addressed. Following that, empirical models of the increase in weight cost and loads as a function of scale are derived, based on historical trends. These include the effects of both scale and technology advancements, resulting in more favourable scaling laws, indicating that technology breakthroughs are prerequisites for further upscaling in a cost-efficient way. Finally, a theoretical framework for optimal design of large wind turbines is developed. This is based on a life cycle cost approach, with the introduction of generic models for the costs, as functions of the design parameters and using basic upscaling laws adjusted for technology improvement effects. The optimal concept or concepts is obtained as the one that minimizes the total expected costs per megawatt hour (levelized production costs). Copyright © 2011 John Wiley & Sons, Ltd.

1. INTRODUCTION

In an effort to increase the penetration of wind power to the energy mix in Europe, a large number of onshore and (mostly) offshore wind farms are scheduled for the next years. Typical sizes of such farms are in the 500 MW range, using the recently developed 5 to 6 MW wind turbines. In order to achieve the ambitious goals for wind power utilization, however, it is envisaged that a new generation of even larger machines will be required, for better addressing of issues related to operation of farms of very large sizes (of the order of 500–1000 MW), leading also to deeper water exploitation in both northern and southern Europe. While, for onshore applications, transport and installation difficulties may prevent a further increase in size, for offshore applications, the difficulties are fewer and the benefits from the reduction of operational and other costs could justify the size increase.

It is not, however, *certain* that upscaling will continue to result in a reduction of the cost of energy, without investigating the associated costs and benefits. In order to do this, a series of actions related to evaluating the upscaling process have been undertaken, in the course of the UpWind project, and are summarized in the present paper. These include the following:

- Theoretical work in upscaling laws
- Evaluation of current trends
- Development of cost models
- Integrated procedures for optimum design (focusing on larger sizes)

The final goal of the work is to present a framework for evaluating different technological improvements developed in the UpWind project and to determine areas where development work is needed, in order to increase the cost-effectiveness of the next generation wind turbines.

2. THEORETICAL UPSCALING

The first approach that has been followed for determining the characteristics of the proposed large wind turbines was based on standard, self-similar geometric upscaling, assuming geometric and aerodynamic similarity (fixed tip speed). This pathway provides a first approximation of critical operational and structural properties and helps in identifying possible technical barriers associated with upscaling. If geometrical similarity is enforced, the weight and power, which are the main criteria we are using, scale according to $m \sim s^3$ and $P \sim s^2$, respectively, where s is the scaling factor. In Table I, the main characteristics (rotor diameter, tip speed and hub height) for power outputs from 5 to 20 MW are given, when simple scaling laws are used. The IEA 5 MW wind turbine is used as reference[1] for the 5 MW size.

Scaling in such a way, the aerodynamic forces for any linear scale (denoted by s) follow an s^2 rule (as the area increases in this manner), while the corresponding moments follow a cubic law (s^3). The section bending stiffness (EI) follows an s^4 rule, resulting in scale-invariant bending stresses due to aerodynamic forces (as they follow $M \cdot y / EI$). The same applies for tension stresses, with centrifugal forces scaling as $m \cdot \omega^2 \cdot R$, so that, for constant tip speed, the overall force scales following the square law. The cross section will also follow the square law resulting in constant tension stresses. On the other hand, tension/compression and bending loads due to self-weight scale as s^3 and s^4, respectively, resulting in a linear scaling of the corresponding stresses.

In short, neglecting second-order aerodynamic effects and assuming linear structural behaviour, we end up with what we call 'classical similarity laws'. A key finding of classical similarity, as we described it, is that stresses due to aerodynamic loading appear to be invariant during upscaling, whereas those due to weight are linearly increasing with the geometric scaling factor.

A typical example of a beam-like structure subject to such loadings is a tubular support structure with linearly varying diameter (D) and wall thickness (t). Because of the simple geometry of this component, we will use it to illustrate the results of geometrical upscaling. In this context, upscaling will be considered under the prism of the following combined design loads

- Tower bending due to rotor thrust
- Tower bending due to wind loads on the tower
- Tower axial and bending loads from weight, including tower self-weight and tower-top components (nacelle and rotor)

The design criterion is buckling under ultimate compression, considering a quasi-steady load condition. The design work is carried out according to the guidelines of DIN 18800 on Structural Steelwork.[2] The maximum normal stress in an arbitrary section (z) of a beam-like structure, subjected to a normal force F and a bending moment M, is given by

$$\sigma_{\max} = \frac{F}{A} + \frac{M}{W_m} \rightarrow \sigma_{\max}(s) = \frac{F(s)}{A(1) \cdot s^2} + \frac{M(s)}{W_m(1) \cdot s^3} \tag{1}$$

where $A(z)$ is the area of the section and $W_m(z)$ is its resistance in bending. The overall design stress is then calculated from the superposition of the individual maximum normal stresses that correspond to the different load components ($F_{1,2}, M_{3,4,5}$), as $\sigma_{\text{ALL,max}} = \sum \sigma_{\max,i}$. For the case of geometrical upscaling, the result of the superposition is shown in Figure 1, with the calculation performed for the base section. The buckling limit is also shown, calculated according to the guidelines of DIN 18800 on Structural Steelwork.[2] Note that the buckling limit is completely insensitive to scale *only* in the strict case of geometrical upscaling, where its deriving formula—based on non-dimensional geometric properties such as the t/D ratio—is scale independent. It is obvious that, as discussed, stresses due to aerodynamic forces are scale invariant, whereas stresses due to weight are linearly increasing with the scale factor. As a consequence of this, classical upscaling is not possible beyond a certain s value, as the stress, increasing linearly, will exceed at that point the design

Table I. Design parameters and 'base' values.

	Power, P (MW)			
	5	10	15	20
Rotor diameter, D (m)	126	178	218	252
Maximum Tip speed, U_t (m s^{-1})	80	80	80	80
Hub height, H (m)	90	116	136	153

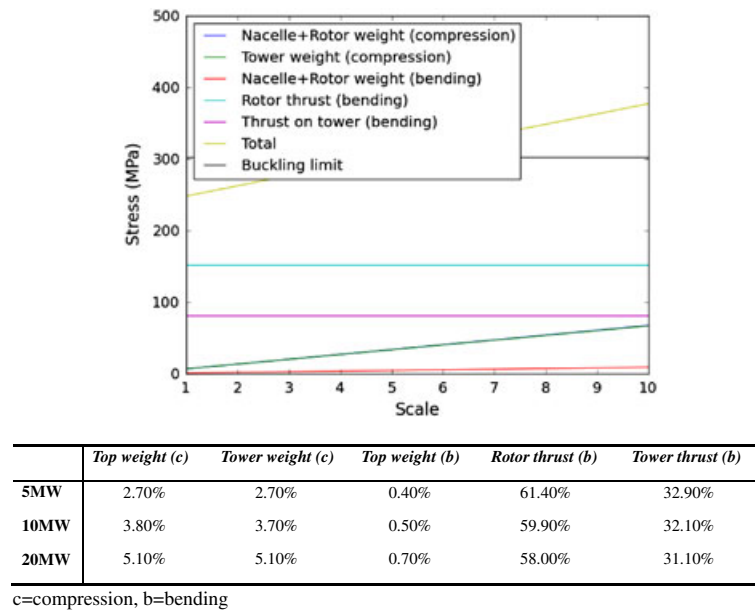|	Top weight (c)	Tower weight (c)	Top weight (b)	Rotor thrust (b)	Tower thrust (b)
5MW	2.70%	2.70%	0.40%	61.40%	32.90%
10MW	3.80%	3.70%	0.50%	59.90%	32.10%
20MW	5.10%	5.10%	0.70%	58.00%	31.10%

c=compression, b=bending

Figure 1. Stress distribution for different scales from 1 to 10. The percentile contribution of each source to the overall stress is given in the table for different sizes.

stress limit. The contribution of the various stress components for three turbine sizes (5, 10 and 20 MW) is also shown in Figure 1, illustrating the relative magnitude of each stress component, based on the same reference 5 MW wind turbine.[1]

The calculation of the various loads is not performed for the same operating point, as the extreme aerodynamic thrust on the rotor (based on rated conditions) and the tower (based on extreme gust conditions) are not fully consistent. Assuming that, for a pitch controlled turbine, the rotor thrust under extreme gust is less or equally severe with the thrust at rated conditions, we apply the above-mentioned combination for simplicity, having also in mind that we are not mainly interested in the exact loads of the reference turbine but in establishing the scaling trends of these loads.

Similar conclusions can be derived for the other parts of the wind turbine. The rotor blades, being cantilevered beams with the design driven by aerodynamic bending moments, will also scale cubically in mass, unless the cyclic weight fatigue loading becomes the design-driving load instead. The scaling rules for the drive train are not as straightforward and depend on its actual configuration. The gearbox input torque will increase cubically, but if a constant output speed is required, the increased gear ratio required will result in a further mass increase. This could be offset by the fact that the constant-speed generator size will then increase following power (s^2). For a direct-drive generator, the situation is simpler and a cubic law is expected (following torque).

A full set of scaling laws for all the components has been developed during this project, including loads, eigenfrequencies, masses, etc., and is reported in detail in Chaviaropoulos.[3]

However, these geometric similarity rules are approximations. Some of the effects that have not been included in the 'theoretical' upscaling laws are as follows:

- Boundary layer effects related to the blades—change in the Reynolds number implies change in loads. As the Reynolds number is already large for the reference wind turbine, it is not expected that the effect will be substantial.
- Wind shear effect. The power P from a wind turbine rotor of radius R in a steady wind field of velocity V is proportional to $V^3 R^2$, and assuming geometric similarity, the hub height h is proportional to R. In the presence of wind shear characterized by a wind shear exponent α, V is proportional to h^α. Hence, power becomes proportional to $R^{2+3\alpha}$. The bending moment on a blade (M) is proportional to $V^2 R^3$, and a similar analysis establishes that it will be proportional to $R^{3+2\alpha}$. Hence, both power and loading will be somewhat influenced by the difference in shear expected in large sizes.
- Size effects related to the fracture mechanics of materials—implying an upscaling exponent larger than 3
- Size effects related to increased risk of buckling failure modes by upscaling
- Increased risk by geometric upscaling of low-cycle fatigue failure from weight-induced loads
- Non-linearities due to large deflections (e.g. see Riziotis *et al.*[4] for findings concerning the 5 MW machine considered here)

- Effects of the inflow turbulence on the dynamic behaviour. As the rotor size increases, the spatial coherency of the incoming wind decreases, resulting in lower loads for the rotor and tower, better power quality and lower rotor speed fluctuations. On the other hand, the energy of the wind concentrates mainly on multiples of the rotational frequency that indicates that the wake-induced effects will have a strong variation with the azimuth.[5]
- Changes in design choices, especially for offshore applications where the technology is still evolving and different configurations are being considered for both fixed-bottom and floating structures. These can result in large variations in the structure weight that are not a direct result of the scaling.

It is these shortcomings of the simplified method that led to the next step, where the actual upscaling trends of wind turbines are studied.

3. REAL UPSCALING

Purely geometrical upscaling offers useful insight but fails to cover all aspects of the operation of large size wind turbines. In order to quantify the *actual* upscaling process that has been used in wind turbine design and manufacturing, we employ a different method, based on the study of observed industrial trends. The main questions that need to be answered by such a study are as follows:

- How do *actual* loads scale in real wind turbine designs?
- What is the weight increase law for upscaling?
- What is the cost increase law for upscaling?

In order to answer the first question, we use the load calculations that have been performed by GH (Garrad Hassan) for wind turbine manufacturers as a typical set.[6] These calculations are typically performed in conformance with GL or IEC standards.[7] Because of design configuration differences and the influence of wind class, the load cases that create design-driving extreme or fatigue loads vary considerably. Nevertheless, such data can be viewed collectively to see what information can be derived in terms of scaling trends.

While the disadvantage of this data source is in the diversity of designs and external design conditions considered, a major advantage is that the data are totally 'real world' and in no way simplistic, with loads derived by rigorous calculation procedures, approved by certification bodies and used by manufacturers in the design of their turbines. Only pitch regulated wind turbines have been considered for this study, and trend lines for the various examined loads have been derived. All kinds of extreme and fatigue loads have been examined, and typical results (for moments at the tower base and at the blade root) are shown in Figure 2 (the load component names conform to the GL coordinate system). In scaling with similarity, moments would follow a cubic law or greater where self weight loading is involved. Each of the curves of Figure 2 are based on approximately 50 data points from detailed load calculations performed to certification standards. A wide variety of turbine design styles (concepts and operational characteristics) and design site conditions (IEC Class 1, Class 2 and others) are grouped together. This limits the statistical significance in determination of characteristic exponents, although some variations are striking and appear to be consistent.

The high scaling exponent of tower base M_z (yaw moment) is interesting (the same happens for the tower top that is not shown). The yaw torque arises as a consequence of differential blade loading. It may be that the effect of turbulence in creating differential loading across the rotor disc, which is becoming more severe at large scale, is being registered here.

Fatigue load exponents also tend to exceed simple cubic scaling, given that above rated wind speed, when most fatigue damage is accumulated; the blade self weight will introduce cyclic loading in both the M_x and M_y (fore-aft and side) components (the effect will be even greater for the blades' loading). An increasing effect of turbulence with scale may also be contributory to the exponents' being greater than cubic. On the other hand, the reduction in RPM, due to the constant tip speed assumption, will reduce the fatigue loading.

A similar procedure was used to answer the second question and estimate the weight of the main components, based on actual parts instead of upscaling theory. The diameter of the wind turbine was used to indicate the scale (rated power could also be used, but a geometrical quantity was deemed preferable). Results for nacelle and blade masses are given in Figure 3. Interestingly, both seem to scale with exponents well below the cubic law that the similarity laws dictates. In the case of the nacelle weight in particular, weight seems to follow an s^2 law, indicating an almost constant power/weight ratio. For the blades, the last three points shown in the figure correspond to projections for blades larger than ones presently available, assuming that the current trend continues.

Unfortunately, the method used for the weight estimation introduces a bias that can distort the trend lines and lead to erroneous conclusions. The dataset used includes small wind turbines of relatively old design and (fewer) large wind turbines, all of which are of relatively recent design. It is therefore difficult to differentiate between the effects of upscaling and those of the technology improvements that came with the larger machines. If the same regression is performed for the nacelle weight, keeping only recent wind turbines with rotor diameter $D > 80$ m, a different picture emerges, as seen in Figure 4, with the scaling exponent increasing, in closer agreement to the upscaling theory. An indication of the bias introduced by
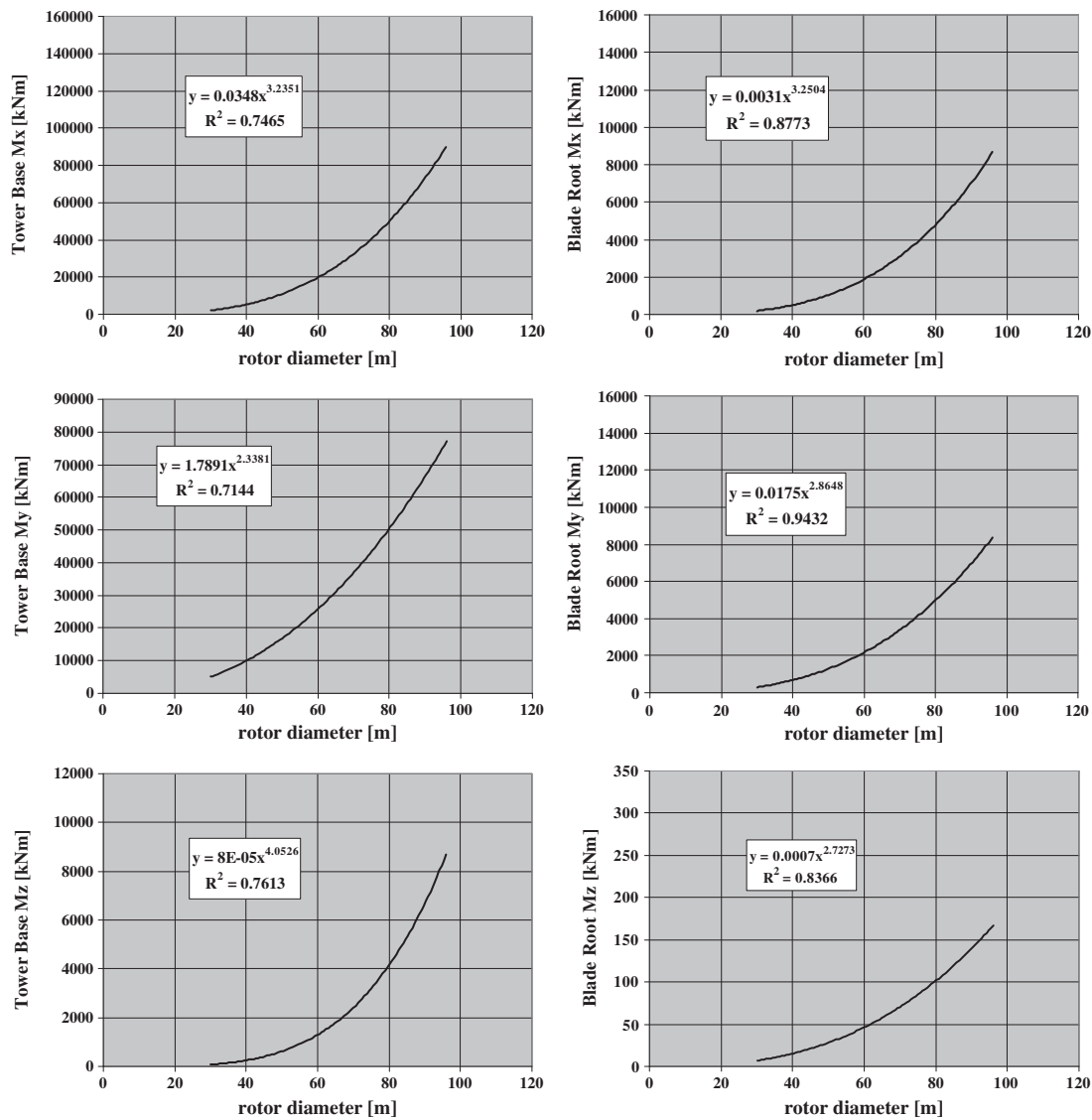
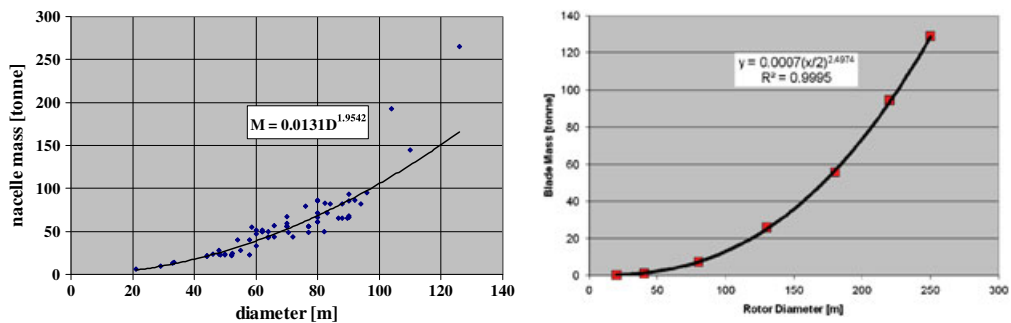**Figure 2.** Design extreme loads based on real data. Tower base (left) and blade root (right) moments.[6]

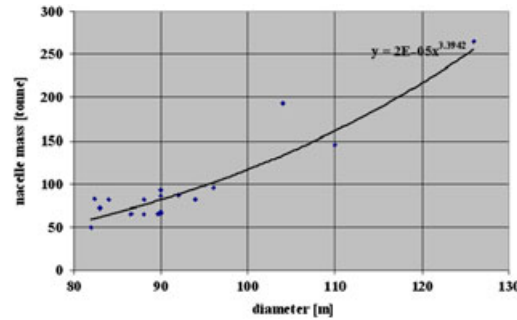**Figure 3.** Mass as a function of size for nacelle and blades.

Figure 4. Mass as a function of size for nacelle considering only large size wind turbines.

technology evolution is better seen when the weight increase of blades is considered, as the technology 'steps' associated with blade materials are well defined historically.[8] The size correlations for each technology (glass epoxy, hybrid, etc.) are closer to the theoretical predictions, as seen in Figure 5. The first three curves and corresponding scatter points representing existing technologies (with hybrid having the lowest weight), whereas the last two give a projection of future technology, if the current trends continue.

In order to have a physical insight of the scaling process, independently of the technology level, we use the same simplified tower problem that was described earlier. Instead of geometrical upscaling, we require constant-stress upscaling, introducing the scaling functions $g_D(s)$ and $g_t(s)$, which quantify the departure from geometrical upscaling in diameter and thickness, respectively. The objective of the optimization problem is to minimize the product $g_D(s) \cdot g_t(s)$ (which is equivalent to minimizing the weight for each scale) while constraining the overall stress below its design limit (for local buckling in this case). Using a constrained optimization algorithm,[9] with g_D and g_t as the optimization variables, the resulting $g_D(s) \cdot g_t(s)s$ distribution, representing the extra weight needed in comparison to classical upscaling, is shown in Figure 6(a) (the optimization is performed independently for different scales). It is also seen that this extra weight term is always greater or equal ($s = 1$) to 1 and almost linearly (in this particular case) increasing with scale. Intuitively, one would expect, since $g_D(s) \cdot g_t(s) \geq 1$, that increasing the scale, the weight forces are increasing accordingly, and more material is then needed for resisting the elevated stress level.

The corresponding overall weight represented by the non-dimensional volume $s^3 \cdot g_D(s) \cdot g_t(s)$ versus s is shown in Figure 6(b) in a log–log plot. Assuming that weight scales up following an s^x law, we calculated the mean exponent (x) for this expression through a best-fit procedure. This is evidently resulting in an $x = 3^+$ exponent, as expected based on the previous discussion. It should be pointed out that the actual weight does not *really* scale following s^x (the extra weight is nearly linear with the scale). The false impression that an exponential representation of weight scaling is representative comes from the fact that the deviation of x from its prevailing component 3 is rather small. We shall nevertheless continue to use this representation to provide estimates of the deviations from geometrical similarity.

A strict proof that $g_D(s) \cdot g_t(s) \geq 1$ always holds, suggesting that we cannot upscale beam-like structures with a weight exponent less than 3, can be given. In order to do that, we further assume that the t/D ratio remains constant in upscaling
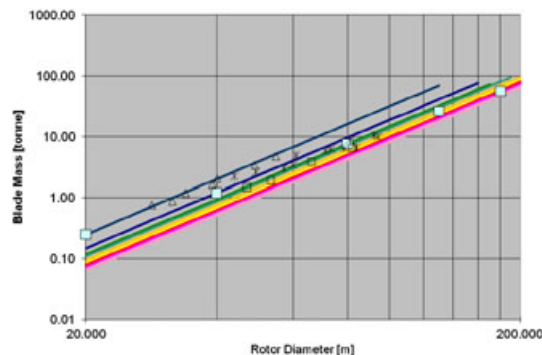

Figure 5. Blade mass as a function of size for different technologies (filled squares denote the combined size and technology improvement trend).
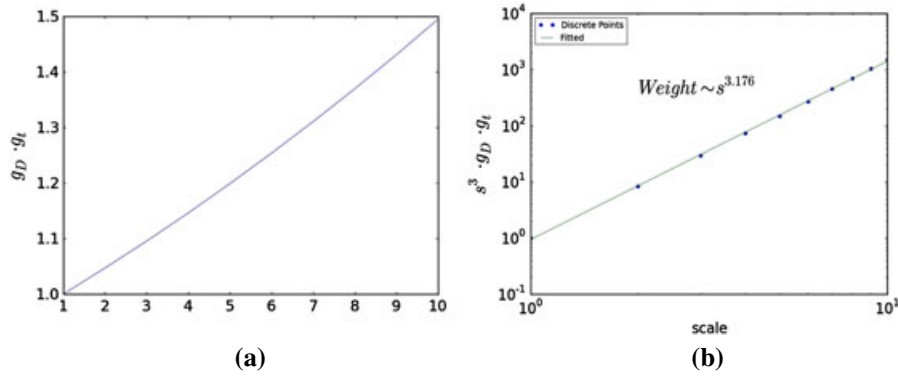
Figure 6. Scaling functions (a) and weight (b) increase with scale for a simplified tower structure.

(equivalent to setting $g_D(s) = g_t(s)$), thus eliminating one degree of freedom from the optimization procedure. The validity of the assumption is tested by comparing the results of Figure 6 (two degrees of freedom, D and t) against those of Figure 7 (one degree of freedom, D).

In this case, thickness and diameter scale in the same way, as $D(s) = D(1) \cdot s \cdot f(s)$ and $t(s) = t(1) \cdot s \cdot f(s)$, where $f(s)$ is a function of the scale with $f(1) = 1$. The area of a given section will then scale as $A(s) = A(1) \cdot s^2 \cdot f^2(s)$ and the mass as $m(s) = m(1) \cdot s^3 \cdot f^2(s)$.

Following above, the overall maximum stress reads

$$\sigma_{\text{ALL,max}}(s) = \frac{\alpha_1 \cdot s}{f^2(s)} + \alpha_2 \cdot s + \frac{\alpha_3 \cdot s}{f^3(s)} + \frac{\alpha_4 \cdot 1}{f^3(s)} + \frac{\alpha_5 \cdot 1}{f^2(s)} \tag{2}$$

where the constants $\alpha_1, \ldots, \alpha_5$ are expressed in terms of geometrical and load data of the reference ($s = 1$) turbine and represent the actual stress from each of the five load mechanisms considered (compression from top-weight, compression from tower weight, bending from top-weight offset, bending from rotor thrust and bending from thrust of air on the tower; in this order, details are in Sieros and Chaviaropoulos[10]). The last two terms, which do not have a direct dependence on scale, represent by far the largest contribution in small scales.

Introducing the non-dimensional coefficients $b_i = \alpha_i / \sigma_{\text{ALL,max}}(1)$, we come to the following equation that the unknown function $f(s)$ must satisfy for every value of s:

$$\frac{b_1 \cdot s}{f^2(s)} + b_2 \cdot s + \frac{b_3 \cdot s}{f^3(s)} + \frac{b_4 \cdot 1}{f^3(s)} + \frac{b_5 \cdot 1}{f^2(s)} = 1 \tag{3}$$

Note that, according to their defining relation, $b_1, \ldots, b_5$ express the ratio of the corresponding load component to the design stress limit of the reference wind turbine. For the given (b_i, s), equation (3) can be solved for $f(s)$. The previous equation can be written in the general form
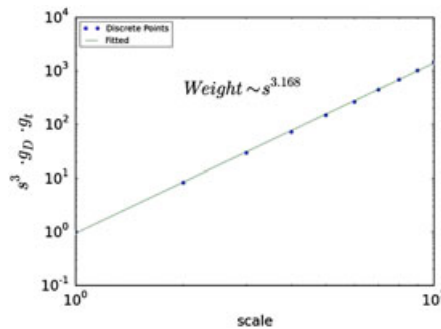
$$G(f) \equiv f^3 - af - b = 0 \tag{4}$$

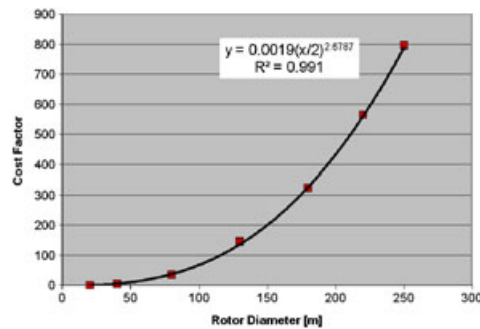**Figure 7.** Weight increase with scale-simplified model ($g_D = g_S$).

Figure 8. Blade cost as a function of scale.

The behaviour of $G(f)$ can be analysed based on its roots, and there is always one real positive solution, which is larger than 1 when $a + b > 1$ and smaller than 1 when $a + b < 1$. The above analysis shows that there is always one single positive solution of for equation (4) for $f(s)$, with $f(1) = 1$ and $f(s) > 1$ for any $s > 1$. That means that it is not possible to upscale without having a weight increase of $\geq s^3$. More details regarding this constant-stress upscaling problem can be found in Sieros and Chaviaropoulos.[10,11]

Note that the above conclusions have been derived taking account of normal stresses only and assuming that the limiting factor will always be the buckling limit. However, as long as the combined loading can be expressed as a combination of terms like the above, the conclusions will not differ greatly for other types of loading and components, although the relative magnitude of the terms will differ (e.g. weight-induced cyclic loading will have a bigger contribution in blade fatigue calculations).

The final question about upscaling concerns the cost increase associated with size. Previous studies[12] indicate that weight is the major driver for component cost, but the two do not scale in exactly the same way. For this reason, a study similar to the one for weight was performed for the cost of typical wind turbine components. Results for wind turbine blades are shown in Figure 8. Similar results can be obtained for other components, but as the variation in configurations is greater, it is harder to derive concrete conclusions.

4. OPTIMAL UPSCALING/SIZING

Up to now, we have considered the basics of upscaling for structural components, based on theoretical or empirical data. Our final goal, however, is to put together the necessary modules to calculate the cost benefits of upscaled turbines and produce optimum size configurations for very large scale wind turbines.

Based on the empirical results for cost upscaling exponents[13] and an estimation of the split-up of the cost between components (in this case, 'components' is a generic term referring to cost components so that operation and maintenance (OM) is a 'component', for example), as given in Wind Directions[14] and Fingersh *et al.*[15] and presented in Table II, a projection of the cost of upscaled wind turbines has been performed.[16] The result, as shown in Figure 9, indicates that, with the current trends, upscaling does *not* lead to a decrease of cost of energy. The reason for that is that gains in OM costs and some other 'cost components', such as installation, are offset by the increase in the cost of the rotor and nacelle.

The analysis in the previous sections has established some of the shortcomings of simplified techniques, based on empirical trends. It is therefore necessary to introduce a more generic cost model that clearly distinguishes the physics from the technology development. It is for this reason that an optimization of the whole upscaled system is proposed so that the next generation wind turbines will offer a reduced cost of energy. This also implies that the optimal size of a wind turbine will depend on the *ratio* between the capital cost and other cost (like OM), which is one reason why different optimum sizes may apply to onshore and offshore applications.

The most general approach to optimum upscaling is the adoption of a life cycle approach to describe the flow of income and costs, with the following main constituents; see also Sørensen *et al.*[17]:

- Project development
- Planning
- Investigations and tests
- Wind turbine fabrication

  - Rotor
  - Nacelle

Table II. Costs distribution.

	Cost	
	Wind Directions[14]	Fingersh *et al.*[15]
Benefits (MW h year⁻¹)	_ᵃ	_ᵃ
Benefits (MW h year^{-1})	_[a]	_[a]
Project development	_[a]	_[a]
Rotor	24.8%	17.7%
Rotor blades	22.2 %	11.8%
Rotor hub	1.4 %	2.6%
Rotor bearings	1.2 %	3.3%
Nacelle	39.0 %	52.8%
Shaft main	1.9 %	2.2%
Main frame	2.8 %	6.2%
Gearbox	12.9 %	15.1%
Generator	3.4 %	7.8%
Nacelle housing	1.4 %	1.4%
Yaw system	1.3 %	1.7%
Other	15.3%%	20.3%
Tower	26.3 %	15.5%
Miscellaneous	9.9 %	11.9%
Turbine	100.0%	100.0%
Foundation	_[a]	41.3%[b]
Electrical connection/cables, etc.	_[a]	34.3%[b]
Installation (including transportation)	_[a]	26.9%[b]
OM (per year)	_[a]	8.0%[b]
Decommissioning	_[a]	6.5%[b]

[a]Not included.
[b]In per cent of turbine cost.
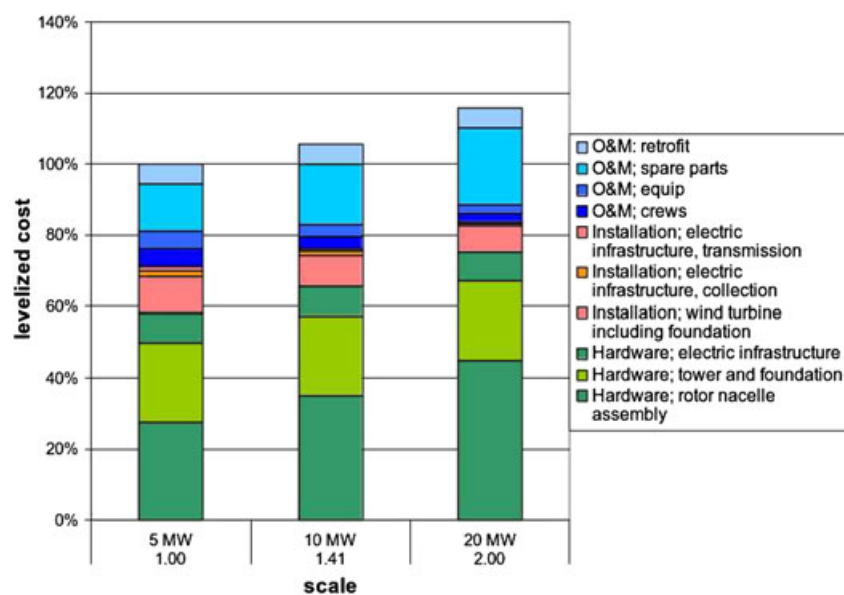
Figure 9. Cost of energy for upscaled wind turbines (estimate based on empirical data).

- ○ Gearbox
- ○ Generator
- ○ Power converter
- • Substructure fabrication
  - ○ Tower

- ○ Foundation
- Installation
- Electrical connection/cables, etc.
- Operation

  - ○ OM costs
  - ○ Energy production

- Decommissioning

Based on these general constituents, upscaling becomes an optimization problem with an objective function of the form

$$\max_{z,e} \quad Z(\mathbf{z}, \mathbf{e}) = B(\mathbf{z}, \mathbf{e}) - \{C_I(\mathbf{z}, \mathbf{e}) + C_{OM}(\mathbf{z}, \mathbf{e})$$
$$+ C_F(\mathbf{z}, \mathbf{e}) + C_D(\mathbf{z}, \mathbf{e})\}$$

(5)

where B is the expected total lifetime benefits, C_I is the initial (planning, fabrication and installation) costs, C_{OM} is the expected lifetime OM costs, C_F is the expected failure costs and C_D is the demolition costs. All costs are capitalized to the time of decision (design). The optimization variables are (i) the design variables $\mathbf{z}$ (basic geometry, material selection, configuration, etc.) and (ii) the decision variables $\mathbf{e}$ defining the installation process and the OM strategy. The expected costs and the probabilities of failure in equation (5) are estimated using decision rules for OM actions given future (unknown) observations from condition monitoring using Bayesian statistical methods, for example.

In order to 'close' the design problem, we need to define the design variables, the constraints and the models for cost and benefit. Based on these, a solution strategy for the overall optimization problem can be formulated.

4.1. Design parameters

The design parameters can be defined at different levels of detail. If the optimization problem is limited to a single wind turbine, the key design drivers would include the following:

- Wind turbine configuration (number of blades, etc.)
- Rotor diameter
- Hub height
- Tip speed
- Gearbox/generator configuration
- Power regulation
- OM strategy

while in a more detailed model, where design details for the actual wind turbine are sought, the parameters would become numerous, including the following (for the blade design):

- Blade cross section parameters
- Material selection
- Internal structure selection
- Chord/twist distribution, etc.

In a full problem, an equally large number of parameters would have to be introduced for each of the components. In addition to that, in a wind farm optimization problem, further design parameters would need to be introduced, like wind turbine separation, optimum layout, local wind conditions, etc.

4.2. Design constraints

The second step in the optimization problem is the definition of the design constraints. If we formulate the problem within the framework of Bayesian statistical decision theory,[18,19] we can solve it without any explicit constraints (using the formulation in equation (5)). This, however, would require an accurate model for the failure costs in various situations. As it is difficult to define such costs, especially when hazards related to human injury due to failure need to be included, it is preferable to introduce explicit limitations in the form of the maximum annual (or accumulated) failure probability that can be accepted. The modified problem will then be formed as follows:

$$\max_{\mathbf{z}, \mathbf{e}} = Z(\mathbf{z}, \mathbf{e}) = B(\mathbf{z}, \mathbf{e}) - \{C_I(\mathbf{z}, \mathbf{e}) + C_{OM}(\mathbf{z}, \mathbf{e})$$
$$+ C_F(\mathbf{z}, \mathbf{e}) + C_D(\mathbf{z}, \mathbf{e})\}$$

(6)

$$\text{subject to } \Delta P_{F,t}(t, \mathbf{z}, \mathbf{e}) \leq \Delta P_F^{\max}, \quad t = 1, 2, \dots, T_L$$

The annual probability of failure in year t is denoted $\Delta P_{\mathrm{F},t}$, while $\Delta P_{\mathrm{F}}^{\max}$ is the maximum annual (or accumulated) failure probability that can be accepted. The annual probability of failure can be related to (electrical and mechanical) components and to structural elements, such as blades and tower. Finally, T_{L} is the design lifetime, typically 20 years.

A simpler solution is to use a deterministic formulation, based on requirements in standards, where uncertainties are taken into account through partial safety factors and appropriate characteristic values of loads and strength parameters. The optimal design is obtained as the design that maximizes the total benefits minus costs, during the lifetime, taking into account design requirements specified in standards, e.g. IEC 61400-1. In this case, the optimization problem becomes

$$\max_{\mathbf{z},\mathbf{e}} B(\mathbf{z}, \mathbf{e}, \mathbf{X_d}) - C_{\mathrm{T}}(\mathbf{z}, \mathbf{e}, \mathbf{X_d})$$
$$\text{subject to} \quad G_{\mathrm{F},i}(\mathbf{z}, \mathbf{e}, \mathbf{X_d}) \geq 0, \quad i = 1, \ldots, N_{\mathrm{D}} \tag{7}$$

where $\mathbf{X_d}$ are the design values of stochastic variables $\mathbf{X}$ obtained using characteristic values (e.g. 5% quantiles for strength parameters) and partial safety factors γ_{F} for load and strength parameters. $G_{\mathrm{F},i}(\mathbf{z}, \mathbf{e}, \mathbf{X_d})$ is the design equation for component i, e.g. the check of buckling or fatigue. $C_{\mathrm{T}}(\mathbf{z}, \mathbf{e}, \mathbf{X_d})$ is the total discounted initial (fabrication, installation, etc.), OM and demolition costs. This is closer to the standard approach to design where a typical cost/benefit problem, subject to design constraints, is solved.

4.3. Component cost models

Based on the previous results, a generalized cost model should include at least two factors, one related to the mass (weight) of the component, denoted with (M), and another indicating the cost of manufacturing this component, per unit mass, using a specific technology (T). 'Technology' is referred here in its general sense, indicating, for instance, the use of alternative materials with larger resistance, manufacturing methods with lower imperfections leading to lower safety factors, use of new devices that can lead to lightweight designs, etc. Technology improvement is anticipated as the only possible way for achieving effective cost and weight exponents down from 3^+ (based on a characteristic length such as the rotor diameter) when upscaling to 10 or 20 MW turbines, and for this reason, we believe we have to include it in our cost models.

Suppose, then, that starting from a reference component, built with technology (T_0), we upscale this component with a scaling factor (s) (referring to a characteristic length of the component, say the rotor diameter), changing at the same time the technology used for its design and manufacturing from (T_0) to (T). The cost C of the component can then be modelled as[13,20,21]

$$
\begin{aligned}
C(s,T)_{\mathrm{comp}} &= c(s,T) \cdot m(s,T) \\
&= c(s,T) \cdot m(1,T_0) \cdot \frac{m(s,T)}{m(1,T)} \cdot \frac{m(1,T)}{m(1,T_0)} \\
&= m(1,T_0) \cdot c(s,T) \cdot s^{\alpha_{\mathrm{comp}}(T)} \cdot r(T) \\
&= C(1,T_0) \cdot \frac{c(s,T)}{c(1,T_0)} \cdot s^{\alpha_{\mathrm{comp}}(T)} \cdot r(T)
\end{aligned}
\tag{8}
$$

where $m(1,T_0)$ is the mass of the reference component (scale 1, current level of technology T_0). $C(s,T)_{\mathrm{comp}}$ is the estimated cost of component with technology T at scale s. $m(s,T)$ is the mass of component with technology T when upscaled with scale factor s. $c(s,T)$ is the cost of component per mass unit at the new scale s and technology level T, which includes the 'technology improvement' in cost per mass unit from technology T_0 to T and upscaling with scale factor. The factor can be estimated, e.g. by evaluating changes in cost per mass unit due to technology based changes in materials, manufacturing process, etc. $r(T)$ is the technology improvement (decrease) of mass from technology T_0 to T, which can be estimated by evaluating the effect of technology improvement on mass with same size of the component(s). $\alpha_{\mathrm{comp}}(T)$ is the scaling exponent for component, based on 'similarity rules'. It is generally a function of the technology level, but as a first approximation, we can assume that it is constant for each component and that the technology effect is incorporated through the previous parameter $r(T)$. We will be using this assumption in the derivations that follow.

Each of the components in equation (8) can be assessed individually. For example, if the rotor diameter D is chosen as the main design parameter, the cost of a 'component' can thus be expressed by

$$C(D,T)_{\mathrm{comp}} = C(D_0,T_0)_{\mathrm{comp}} \cdot \frac{c(D,T)}{c(D_0,T_0)} \cdot \left(\frac{D}{D_0}\right)^{\alpha_{\mathrm{comp}}} \cdot r(T) \tag{9}$$

or equivalently

$$C(D,T)_{\mathrm{comp}} = m(D_0,T_0)_{\mathrm{comp}} \cdot c(D,T) \cdot \left(\frac{D}{D_0}\right)^{\alpha_{\mathrm{comp}}} \cdot r(T) \tag{10}$$

where the dependence on the weight of the component is clearer. The assumption that the cost is proportional to the mass could be improved, e.g. by adding a constant cost term. A simplification of the model can be achieved if scaling and technology improvements are combined in a single parameter (β) for each component, resulting in

$$C(D, T)_{\text{comp}} = m(D_0, T_0)_{\text{comp}} \cdot c(D, T) \cdot \left(\frac{D}{D_0} \right)^{\beta_{\text{comp}}(T)} \tag{11}$$

In the above model, the technology is assumed to change from T_0 to T. This change could be associated with two different times, t_0 and t, and therefore, a capitalization factor has to be included. The total costs for all components would then be written, taking t as the decision time:

$$
\begin{aligned}
C_{\text{I}}(D, T) = \sum_{i=1}^{\text{comp}} C_i(D_0, T_0(t_0)) \cdot \frac{c_i(D, T(t))}{c_i(D_0, T_0(t_0))} \\
\cdot \left(\frac{D}{D_0} \right)^{\alpha_i} \cdot r_i(T(t)) \cdot (1 + rt)^{-(t - t_0)}
\end{aligned}
\tag{12}
$$

where rt is the real rate of interest. Similar models can be written for the OM costs of the wind turbine.

4.4. Indicative optimization problems

For the general problem formulated by statistical decision theory, as represented by equation (8), significant achievements have been obtained in the development of efficient numerical techniques, during the last decades. Especially the development of FORM (first-order reliability methods), SORM (second-order reliability methods) and simulation techniques to evaluate the reliability of components and systems has been important, e.g. see Madsen.[22] In the same period, efficient methods to solve non-linear optimization problems have also been developed, e.g. the sequential quadratic optimization algorithms.[23,24] These developments have made it possible to solve problems formulated in a decision theoretical framework.

In the following, we will present some simplified cases of the general problem, which are easier to investigate. In this respect, we assume that the total expected costs during the wind turbine lifetime are obtained by summing all discounted component costs during the lifetime:

$$C_{\text{T}}(\mathbf{z}) = \sum_{j=1}^{\text{comp}} \sum_{t=1}^{T_{\text{L}}} C_{j,t}(\mathbf{z})(1 + rt)^{-t} \tag{13}$$

where rt is the real rate of interest (e.g. 3% to 8% depending on the beneficiary) and $C_{j,t}$ is the expected costs for component j in year t.

At the same time, we consider that the benefit function is represented by the total expected energy production so that the optimal design can be obtained by minimizing the levelized production cost (LPC), defined as total expected costs per megawatt hour:

$$\min_{\mathbf{z}} \text{LPC}(\mathbf{z}) = \min_{\mathbf{z}} \frac{C_{\text{T}}(\mathbf{z})}{B(\mathbf{z})} \tag{14}$$

where B is the total expected energy production during the design lifetime T_{L}.

Even with a formulation like that, the complete optimization problem is difficult to solve in a single pass. It is therefore often desirable to optimize on a component level, so as to reduce the number of design variables. For example, for the support structure (including the tower and foundation), a module has been developed that can be used to estimate the cost of the support structure (with monopile foundation for offshore applications); see de Vries.[25] Using the main design variables as input, this software module determines the support structure mass m_{support} and, thus, the support structure cost $C_{\text{support}} = c_{\text{support}} \cdot m_{\text{support}}$. For implementation in the overall cost model, the support structure cost can be approximated, e.g. by a response surface model. All models of this kind for the various components depend on the design of other components (e.g. the rotor mass is used as input in the support structure module) so that an iterative procedure has to be employed using the output of the optimization procedure for other components.

In a next step, for illustrative purposes, we introduce a simplified version of the so-called LPC of the component by dividing equation (8) with the energy produced from the hosting wind turbine, which for a given wind climate and power coefficient curve is proportional to the rated power of the turbine. Thus,

$$\text{LPC}(s, T) \approx \frac{C(s, T)}{P_{\text{rated}}(s, T)} = \frac{c(s, T) \cdot m(s, T)}{P_{\text{rated}}(s, T)} \tag{15}$$

We will further simplify the cost model assuming that the cost per unit mass c is only a function of T, $c(T)$. For the mass of the component, we assume, following the results of Section 3, that a scaling law of the form

$$\frac{m(s,T)}{m(1,T)} = s^3 \cdot f^2(s)\,, \ f(s) \geq 1 \tag{16}$$

applies, where $f(s)$ is considered, in a first approximation, as technology independent. Evidently, this is not completely true, as the actual form of $f(s)$ depends on the way the design stress is split to its sub-components and, in particular, on the share of the own weight. We also use the technology factor r, as defined in equation (8), as

$$r(t) = \frac{m(1,T)}{m(1,T_0)}\,, \quad r(T_0) = 1 \tag{17}$$

To work with levelized costs, we also need an upscaling rule for the rated power. Assuming classical similarity in power performance and a constant power density (P_{rated}/(rotor area)), we read

$$P_{\text{rated}}(s,T) = P_{\text{rated}}(1,T) \cdot s^2 \tag{18}$$

Defining LPCR as $\mathrm{LPC}(s,T)/\mathrm{LPC}(1,T_0)$, equation (15) will take the form

$$\mathrm{LPCR}(s,T) = Y(T) \cdot r(T) \cdot s \cdot f^2(s) \tag{19}$$
$$\text{with } f(s) \geq 1 \ \text{ and } \ f(1) = 1$$

with $Y(t) = c(T)/c(T_0)$ expressing the gain, in cost per unit mass, as a result of the transition to a new technology T. In upscaling theory, we are mostly interested in possible solutions of equation (19) with $\mathrm{LPCR}(s,T) < 1$ and, at the same time, $s > 1$. On the single component basis, this is the only reason for increasing the scale beyond 1, to $s > 1$. From equation (19), it is obvious that $\mathrm{LPCR}(s,T)$ takes its minimal value at $s = 1$ (under the constraint $s \geq 1$). That means that larger scales are always *less* cost-effective than smaller. Actually, LPCR achieves even lower (better) values when s becomes smaller than 1, suggesting that downscaling is preferable to upscaling.

In order to check for the conditions for the existence of a solution with $\mathrm{LPCR}(s,t) < 1$ for $s > 1$, we examine a modified cost model, where an additional fixed cost has been added so that the LPC function reads

$$\mathrm{LPC}(s,T) = \frac{c(T) \cdot m(s,T) + b(T)}{P_{\text{rated}}(s,t)} \tag{20}$$

If we wanted to include the influence of other factors, a more elaborate model of the form

$$\mathrm{LPC}(s,T) = \frac{c(T) \cdot m(s,T) + c_A(T) \cdot s^2 + c_L \cdot s + b(T)}{P_{\text{rated}}(s,t)} \tag{21}$$

could be used, where the additional terms correspond to costs that are proportional to area (e.g. coatings) and length (e.g. ladders), respectively, but for this analysis, we will use the simplified form of equation (20). As we are mostly interested in the effect of newly introduced technologies (that would introduce an additional fixed cost), we assume that $b(T_0) = 0$. Using the same assumptions for scaling as before, the non-dimensional LPC will then be calculated as

$$\mathrm{LPCR}(s,t) = \frac{1}{s^2}X(T) + Y(T) \cdot r(T) \cdot s \cdot f^2(s), \quad X(T) = \frac{b(T)}{C(1,T_0)} \tag{22}$$
$$\text{with } f(s) \geq 1 \ \text{ and } \ f(1) = 1$$

where $X(T)$ is the ratio of the additional fixed cost to the initial total cost. Its value will therefore drop if the fixed cost of a particular component drops with technology improvements. This means that larger scales might be optimal when a new technology, with high fixed cost, is applied to significantly reduce the weight of a component and/or its variable production cost. At the wind turbine level, this could be better understood with the following example. Suppose that a nacelle-mounted LiDAR system, or an equally expensive sophisticated pitch control system, can significantly reduce the blade loads (and therefore weight). Is it always cost-effective to use such a system? The answer from equation (22) is that cost-effectiveness is only coming at a certain scale. A parametric study was performed, in order to quantify the scale at which a certain change starts becoming cost-effective. A reference case with $X(T) = 0$ is defined for comparison. As seen in Figure 10, where solutions having the same cost at $s = 1$ are compared (see Sieros and Chaviaropoulos[11] for details), a substantial reduction in $[Y(T) \ r(T)]$ is required before the cost minimum shifts to $s > 1$ (case B). A similar trend is observed looking at overall cost as a function of size, based on historical data. The curve (Figure 11), obtained from analysis of existing designs in the context of the project, indicates a higher cost at small scales, where fixed costs are a substantial part of the total cost, an optimum at an intermediate size, and increasing costs at very large sizes. In order to shift the optimum size to larger values, new technologies that are not cost-effective at the current size will have to be introduced.
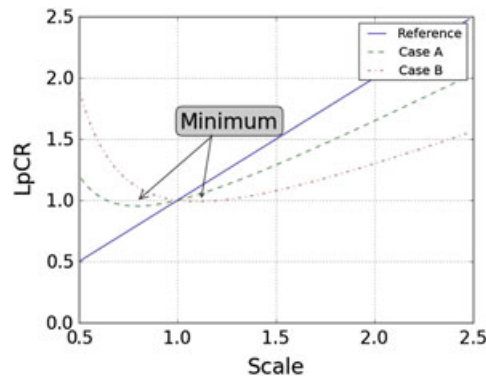
Figure 10. Levelized cost as a function of scale for different balances between fixed and variable cost (case A, fixed cost is 20% of total cost at $s = 1$; case B, fixed cost is 40% of total cost at $s = 1$; reference, only variable cost).
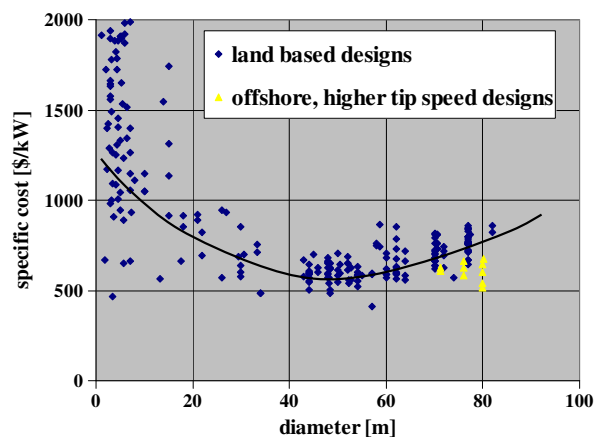

Figure 11. Specific cost trends.

5. CONCLUSIONS

We started this work demonstrating the results obtained when standard geometrical similarity is used. In this case, it was shown that the limiting loads increase with scale, imposing unfavourable conditions for the resulting wind turbine. Following that, a study of the evolving trends in size, weight and cost was performed. The results of that showed an improved scaling behaviour, which is mostly attributed to the technology improvement.

Finally, a framework for the optimization of wind turbines for large sizes was presented. Models for the cost of the various components were introduced, and a solution to simplified cases was presented. Based on the simplified problem, it was shown that, for a given technology level, upscaling always results in an unfavourable weight increase. Although these conclusions are 'exact' for the examined tower structure only, there is no obvious reason why they should not apply to other wind turbine components that can be similarly modelled as beam-like structures, e.g. blades, low speed shafts, etc.

Similarly, using a linearized weight-based cost model, it was shown that without additional technology improvements the levelized component cost increases with turbine size. If new solutions with high fixed cost are introduced, the optimum size (in terms of cost-effectiveness) for components shifts to larger values.

The overall optimization problem, including all other costs, has optimum solutions at larger scales than the component optimization problem. It is nevertheless necessary to reduce the rate at which the component levelized cost increases so that the overall cost of energy may benefit from the transition to larger scales.

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