Stress Analysis

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STRESS ANALYSIS

Component forces
The following types of forces contribute to the stresses in a Dolos in a pack exposed to waves:

1) Gravity forces
   Compaction forces (mainly due to settlements, gravity and flow forces)

2) Flow forces

3) Impact forces (impacts between moving concrete blocks)

Stresses created by 1 and 2 have a linear dependency on the length scale (e.g. the height of the Dolos) while stresses created by 3 have a non-linear dependency (proportional to the square root of the length scale).
Because the stresses scale differently the component stresses (forces) should be scaled correctly before being added to form the total stresses.

It is believed that the easiest way is to determine the force component M_y, M_z, N_x, T, V_y, V_z for each type of load from whatever source is available, scale them to prototype scale and then add them. The probability of simultaneous occurrence must of course be taken into account. After this the stress situation in the sections can be determined as described in the following.

The objective of the present research project is to determine stresses created by the class 1 forces only.

Cross section force components
For each of the four instrumented sections (1-4) of the Dolos the following quantities are calculated from recordings of the strain in a circular steel pipe:

M_y and M_z, bending moments
N_x, normal force
T, torsion
V_y and V_z, shear forces

The *local* coordinate systems shown in Fig. 1 are used.

As seen from Fig. 1 the x-coordinate is always normal to the section (parallel to the axes of the Dolos). The x-y coordinate plane is parallel to the shank - fluke plane and consequently it changes at the mid shank cross section. This local coordinate system is used for the analysis of the stresses in the instrumented cross sections.

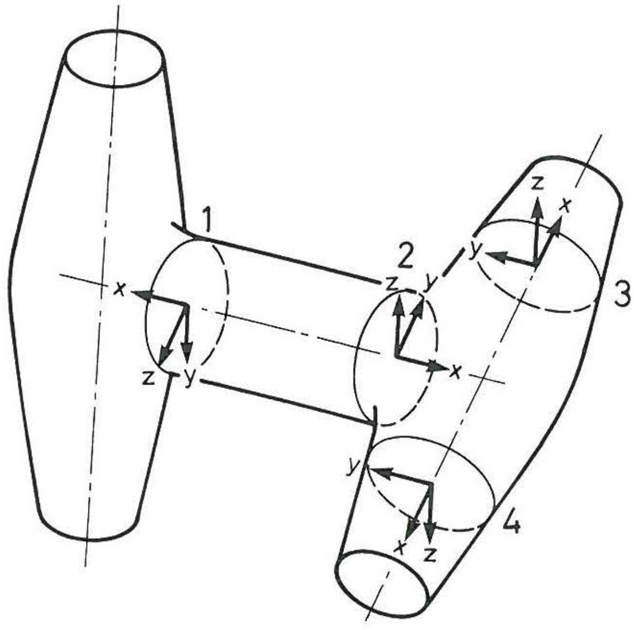


Fig. 1. Local coordinate system.

The stresses in other parts of the Dolosse might be estimated on the basis of the stresses found in the instrumented sections. However, for such an analysis the global coordinate system shown in Fig. 2 is used.

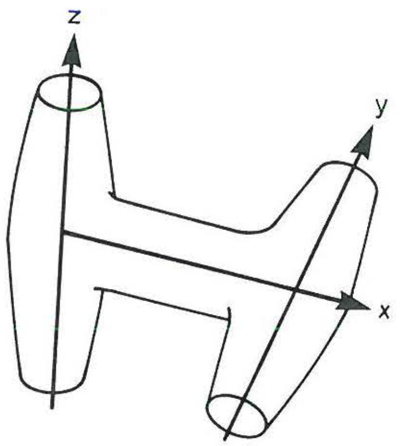


Fig. 2. Global coordinate system.

Data reduction
It should be checked if a 2-D representation of the stress field is a reasonable approximation. This can be done by comparison with a 3-D representation.

Moreover, it might be possible to leave out the stress contribution from normal and shear forces as they are expected to be of minor importance compared to bending moments and torsion. This should be checked.

The last point is relevant also because in most ongoing experiments (prototype and small scale) only bending moments and torsion are recorded.

If only bending moments and torsion are of importance then it is clear that the maximum tensile stresses are to be found at the surface of the body.

A polar coordinate system might then be more handy especially if the cross section is approximated to a circular section, Fig. 3. If the octahedal cross section is kept the distribution of a residual stress capacity function, F, along the cross section surface should be investigated with the purpose of reducing the number of points of analysis to the eight surface corner points. However, the circular section is much easier to deal with and is a very close approximation as shown in the following.

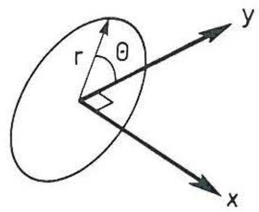


Fig. 3.

Analysis of stresses

- The concrete is assumed linear elastic.
- Failure is defined as occuring when the tensile stress of any point reaches the tensile strenght of the unreinforced concrete.
- For the evaluation of the stress conditions in a cross section the 2-dimensional principal stress failure criterion will be used

$$f = -(\sigma_1 - S)(\sigma_2 - S) \leqslant 0 \qquad \text{(no failure)} \qquad (1)$$

σ_1 and σ_2 are the principal stresses, positive as tension. S is the tensile strength.

Eq (1) is to be interpreted in such a way that the failure area is defined as depicted in Fig. 4. (σ_1 and σ_2 cannot simultaneously be larger than S without causing failure. Failure will occur if either $\sigma_1 - S \leqslant 0$ or/and $\sigma_2 - S \leqslant 0$).

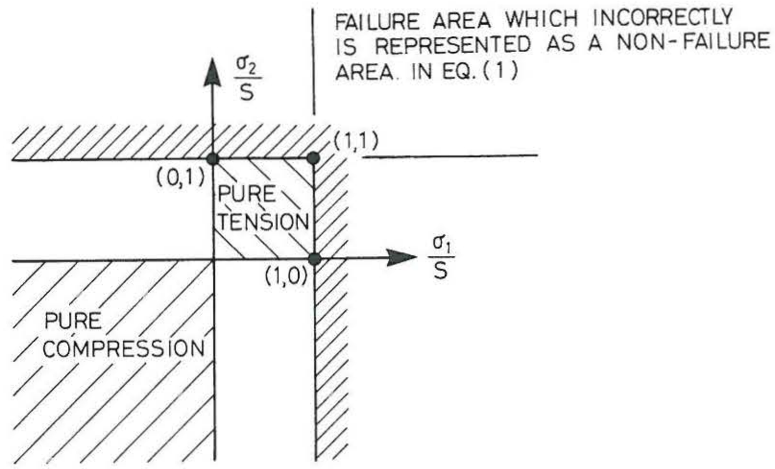


Fig. 4. 2-dimensional principal stress failure criterion.

Although the stress conditions in reality are 3-dimensional it is a resonable approcimation to use a 2-dimensional stress failure criterion because the most critical stress conditions are known to occur at the surfaces of the body.

Thus only stress conditions at points on the surface are considered in the analysis.

A local coordination system as shown in Fig. 5 is used. The x-coordinate is normal to section in question and the α - coordinate is parallel to the surface at the point of consideration. While the x - coordinate orientation is fixed relative to the cross section of the Dolos the α - coordinate changes orientation dependent on the position of the point of analysis, P, given by the angle, θ.

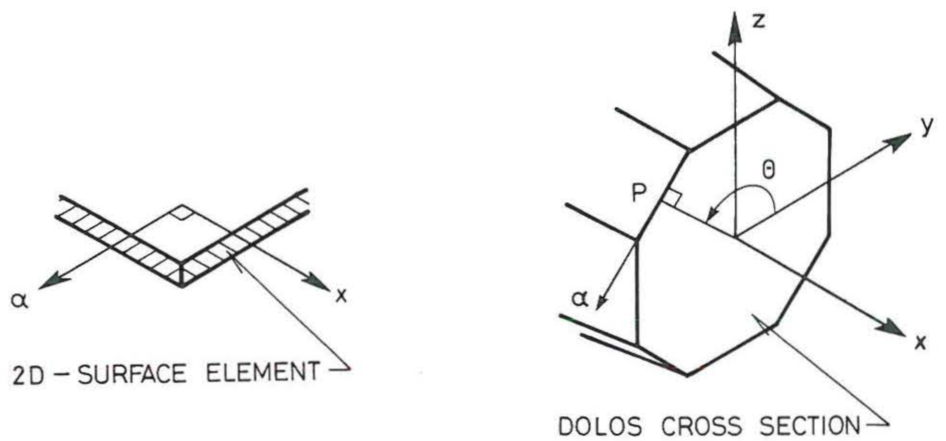


Fig. 5. Coordinate system.

Eq. (1) can be expanded to

$$f = -(\sigma_1\sigma_2 - S(\sigma_1 + \sigma_2) + S^2)$$

$$= -(\sigma_{xx}\sigma_{\alpha\alpha} - \sigma_{x\alpha}^2 - S(\sigma_{xx} + \sigma_{\alpha\alpha}) + S^2) \leqslant 0 \qquad (2)$$

or

$$F \equiv \frac{f}{S^2} = \frac{\sigma_{x\alpha}^2}{S^2} - \frac{\sigma_{xx}\sigma_{\alpha\alpha}}{S^2} + \frac{\sigma_{xx} + \sigma_{\alpha\alpha}}{S} - 1 \leqslant 0 \qquad (3)$$

F may be regarded a dimensionless measure of the residual stress capacity in the specific point.
Note the non-linear dependency of F on the stresses.

The quantity F is suitable for statistical treatment but only if representing the resulting stress conditions caused by all types of simultaneously present forces as mentioned earlier. This implies that contributions to $\sigma_{x\alpha}$, σ_{xx} and $\sigma_{\alpha\alpha}$ from each of the earlier mentioned types of forces (gravity/compaction, flow and impact forces) must be determined before a meaningful value of F can be found.

Due to the earlier mentioned different scaling with block size of the stresses created by the various types of loads one has to scale to prototype the component stresses $\sigma_{x\alpha}$, σ_{xx} and $\sigma_{\alpha\alpha}$ before F can be calculated.

Calculation of the residual stress capacity function
The two-dimensionless residual stress capacity function (failure function) F given by eq. (3) can be found from the force component.

F is related exclusive to the stress (strain) condition in a specific point at the surface of the body.

Fig. 6 shows the cross section and the local coordinate system.

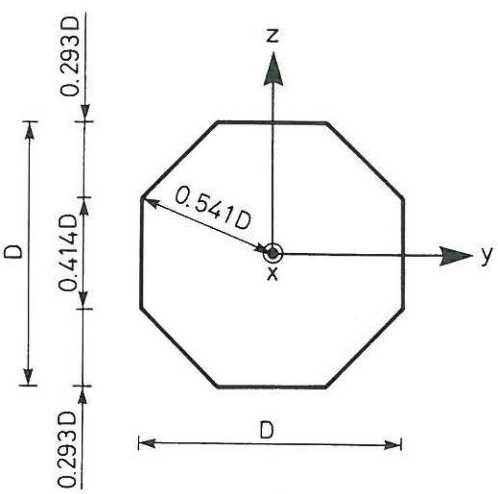


Fig. 6. Cross section with local coordinate system.

The force component M_y, M_z, N_x, T, V_y and V_z related to the local coordinate system, Fig. 5, are determined from recordings of strains in an instrumented section of a steel pipe inserted in the Dolos as shown in principle in Fig. 7.

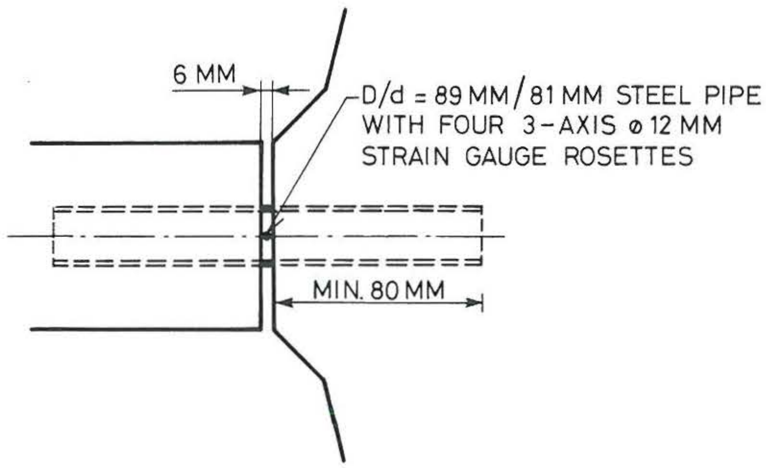


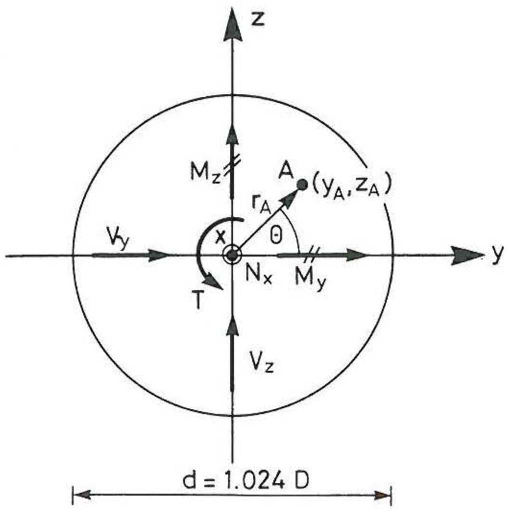
Fig. 7. Principle of instrumented sections.

The steel pipe is glued into diamant cut holes and filled with sand to maintain the mass of the Dolos.

The transformation of the above mentioned force component into the stress components σ_{xx}, σ_{yy}, σ_{xy} as given by eq. (3) and the coordinate system Fig. 3 is performed as follows:

For simplicity we shall use a circular cross section as a close approximation to the octahedal cross section. Because the maximum tensile stresses are supposed to be at the surface of the Dolos the choise of diameter of the circular section must be based on minimum deviation of the stresses at the surface.

It can be shown that if the normal forces or the bending moments are the only contributors to the stresses at the surface then the diameter d of the „equivalent" circular section should be d = 1.027 D or d = 1.023 D respectively. Because most likely the bending moments will be the main contributor to the tensile stresses a diameter d = 1.024 D is chosen for the equivalent cross section, Fig. 8.



Areas of section $F = \pi(\frac{d}{2})^2$

Modulus of section $I_y = I_z = I = \frac{\pi}{64} d^4$

Fig. 8. Equivalent circular cross section.

For a circular cross section with diameter, d, one can derive the following analytical expressions for the stress components

normal stress component in direction x

$$\sigma_{xx} = \frac{N_x}{F} + \frac{M_y}{I_y} z_A - \frac{M_z}{I_z} y_A \tag{4}$$

shear stress component in direction y

$$\sigma_{xy} = \frac{3+2\nu}{8(1+\nu)} \frac{V_y}{I_z} \left[(\frac{d}{2})^2 - y_A^2 - \frac{1-2\nu}{3+2\nu} z_A^2 \right] - \frac{1+2\nu}{4(1+\nu)} \frac{V_z}{I_y} y_A z_A - \frac{T}{I_x} z_A \tag{5}$$

shear stress component in direction z

$$\sigma_{xz} = \frac{3+2\nu}{8(1+\nu)} \frac{V_z}{I_y} \left[(\frac{d}{2})^2 - z_A^2 - \frac{1-2\nu}{3+2\nu} y_A^2 \right] - \frac{1+2\nu}{4(1+\nu)} \frac{V_y}{I_z} y_A z_A + \frac{T}{I_x} y_A \tag{6}$$

ν is Poisson's ratio.

The formulae (5) and (6) fulfil the requirement that no stress component at right angle to the surface exists when only the above mentioned force components are present, i. e. no contact loads acting on the surface in the point of analysis.

At the surface eqs (4), (5), and (6) reduces to

$$\sigma_{xx} = \frac{N_x}{F} + (\frac{M_y}{I}\sin\theta - \frac{M_z}{I}\cos\theta)\frac{d}{2} \tag{7}$$

$$\sigma_{xy} = \frac{1+2\nu}{4(1+\nu)I}(\frac{d}{2})^2\left[V_y\sin^2\theta - V_z\sin\theta\cos\theta\right] - \frac{T}{I_x}\frac{d}{2}\sin\theta \tag{8}$$

$$\sigma_{xz} = \frac{1+2\nu}{4(1+\nu)I}(\frac{d}{2})^2\left[V_z\cos^2\theta - V_y\sin\theta\cos\theta\right] + \frac{T}{I_x}\frac{d}{2}\cos\theta \tag{9}$$

$I_x = 2I$ for the circular cross section.

With reference to the surface coordinate system given in Fig. 5 it is seen that the shear stress component $\sigma_{x\alpha}$ is given as

$$\sigma_{x\alpha} = \sigma_{xz}\cos\theta - \sigma_{xy}\sin\theta \tag{10}$$

cf. Fig. 9.

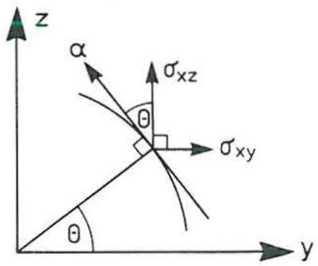


Fig. 9.

$$\sigma_{x\alpha} = \frac{1+2\nu}{4(1+\nu)I}(\frac{d}{2})^2(V_z\cos\theta - V_y\sin\theta) + \frac{T}{I_x}\frac{d}{2} \tag{11}$$

Note that the stress component $\sigma_{\alpha\alpha}$ given in eq. (2) is zero if only the cross section force components given by N_x, M_y, M_z, V_y, V_z and T are present. This is actually the case because the components are found solely from strain measurement in a section of the inserted steel pipe where no surface loads are present.

Transforming these force components to equivalent stresses at the Dolos concrete surface gives of course no possibility of determining a stress component $\sigma_{\alpha\alpha}$. Such a component of some magnitude might exist if significant contact loads from neighbour Dolosse are present at or near the section where the stresses are determined. However, because the most critical sections (i. e. sections where maximum tensile stresses are expected to occur) are thought to be near the trunk-shank corners where contact points with other blocks are less likely to occur it is assumed that $\sigma_{\alpha\alpha}$ can be ignored. This of course should be verified in some way – most probably by FEM simulations.

$\sigma_{\alpha\alpha}$ could of course be found by means of strain gauge rosettes placed directly on the Dolos concrete surface. This method was actually tried in the present case but did not work due to too small strains generated in the 200 kg Dolosse. Much larger Dolosse are needed for this method.

If $\sigma_{\alpha\alpha}$ is ignored then the residual stress capacity criterion given by eq. (3) reduces to

$$F = \frac{\sigma_{x\alpha}^2}{S^2} + \frac{\sigma_{xx}}{S} - 1 \leqslant 0 \tag{12}$$

For a Dolosse placed in a certain position in a specifik pack of blocks the stress components in eq. (12) are functions of the position of the considered cross section and the angle θ.

Due to the symmetry of a Dolos the probability density function (pdf) $F(\theta)$ will be identical for sections 1 and 2 and for sections 3 and 4 when the local coordinate systems are defined as depicted in Fig. 1 and the Dolosse are placed at random. Moreover, for sections 1 and 2 we have

$$\text{pdf } F(\theta) = \text{pdf } F(180^\circ - \theta) \quad \text{and pdf} F(180^\circ + \theta) = \text{pdf}(-\theta) \tag{13}$$

and for sections 3 and 4

$$\text{pdf } F(\theta) = \text{pdf } F(-\theta) \tag{14}$$

Fig. 10 shows in principle for one section the outcome of a single test.

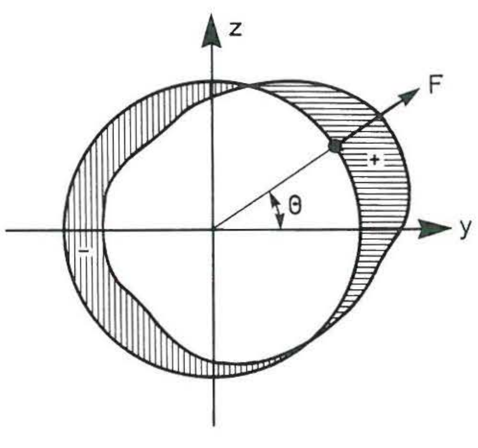


Fig. 10. Illustration of outcome of a single test.

If the tensile strength of the concrete is S then we can define the following critical quantities that produce cracking in the surface due to bending moments, torque, shear forces and normal forces respectively

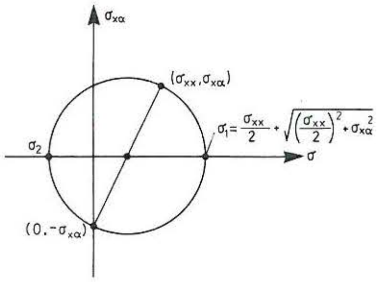
$$
\begin{aligned}
M_{cr} &= S\,\frac{2I}{d} \\
T_{cr} &= S\,\frac{I_x}{\frac{d}{2}} = S\,\frac{4I}{d} \\
V_{cr} &= S\,\frac{4I}{d^2} \\
N_{cr} &= S \cdot F
\end{aligned}
\qquad (15)
$$

Substituting eqs. (15) into eq. (12) using eqs. (7) and (11) gives the following expression for the dimensionless residual stress capacity function

$$
F = \frac{M_y \sin\theta - M_z \cos\theta}{M_{cr}} + \left(\frac{T}{T_{cr}}\right)^2 + \left(\frac{\frac{1+2\nu}{4(1+\nu)}\,(V_z\cos\theta - V_y\sin\theta)}{V_{cr}}\right)^2
$$

$$
+ \frac{2T\,\frac{1+2\nu}{4(1+\nu)}\,(V_z\cos\theta - V_y\sin\theta)}{T_{cr}\cdot V_{cr}} + \frac{N}{N_{cr}} - 1 \leqslant 0 \qquad (16)
$$

Instead of operating with the stress component σ_{xx} and $\sigma_{x\alpha}$ or with the residual stress capacity function F, one can operate with the max principal tensile stress, $\sigma_1 = \sigma_{max}$. σ_1 which is a function of θ can be found from the Mohr's diagram as follows.



$$
\sigma_1 = \frac{\sigma_{xx}}{2} + \sqrt{\left(\frac{\sigma_{xx}}{2}\right)^2 + \sigma_{x\alpha}^2} \qquad (17)
$$

The angle β between σ_1 and σ_{xx} is given by

$$
tg\,2\beta = \frac{\sigma_{x\alpha}}{\frac{1}{2}\sigma_{xx}} \qquad (18)
$$

To arrive at correct estimates of the prob. of failure it must be verified if σ_1 and σ_2 are not $> S$ simultaneously, cf. eq. (1).

Conclusion: The most convenient parameter to use is most probably $\sigma_1(\beta)$.