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# An Analytical Equation for Cogging Torque Calculation in Permanent Magnet Motors

Kaiyuan Lu, Peter Omand Rasmussen, Ewen Ritchie

**Abstract**—This paper proposes a new approach to the prediction of the cogging torque in motors with parallel or radial magnetized permanent magnets. The magnetic field energy associated with the permanent magnets is used for cogging torque calculation, similar to the flux-magnetomotive force (MMF) method. Mathematical development from the energy variation in the magnets to a simple, compact equation using the flux inside the magnets for cogging torque prediction is given. Validation of the cogging torque equation is performed by comparing results obtained using the proposed method with those obtained using the Maxwell stress method and with laboratory test results.

**Index Terms**—Analytical; Cogging torque; permanent magnet motors.

## I. INTRODUCTION

PERMANENT magnet (PM) motors have been widely used in high performance applications. Cogging torque is an important problem with PM motors as it causes speed pulsations and induces vibration and acoustic noise. Cogging torque is produced by the interaction of the permanent magnets and the soft magnetic parts of the magnetic circuit. Cogging torque may be calculated accurately using the Finite Element Method (FEM). However, the FEM is a numerical method that does not easily allow parametric study [1]. An analytical method would give greater insight into the mechanism of cogging torque production, and are to be preferred in the initial design stage and for cogging torque minimization studies.

Many analytical cogging torque calculation methods have been proposed, e.g. [2]–[8]. Cogging torque is most commonly estimated from the air gap magnetic flux density distribution calculated from the virtual work principle using the energy stored in the air gap [2]–[7]. Cogging torque may also be estimated by summing the net lateral magnetic force acting along the sides of the stator teeth [8].

In [9], Deodhar *et al.* proposed applying the flux-magneto-motive force (MMF) method to estimate the cogging torque of permanent magnet motors. The flux-MMF method uses only the magnetic field energy associated with the magnets. This method, is claimed by the authors to be ‘a true universal technique of cogging torque prediction and gives greater insight into many of the methods used for torque minimization’. Watterson, [10]

provided the theoretical foundation for the flux-MMF method. Discussion by Lovatt and Watterson in [11], and by Campbell in [12] is helpful to increase understanding of the magnetic field energy represented by the B-H curve of the permanent magnets.

[9], presents the flux-MMF method with an emphasis on applications requiring cogging torque minimization. It lacks a detailed explanation of the theoretical background for the flux-MMF method, which has caused confusion among some readers in understanding this method [11], [12]. A systematic review of the flux-MMF method from theory to application is the first objective of this paper. Based on the flux-MMF method and using mathematical manipulation, a new cogging torque equation with a simple, compact expression is then proposed. Using this equation, graphical construction of the flux-MMF diagram is avoided, which simplifies the cogging torque calculation. This equation also has potential value for application in cogging torque minimization studies.

In section II of this paper, a brief review of the flux-MMF method, and the work of Watterson [10] is given. Mathematical development starting from the virtual work principle and the energy stored in the magnets proceeding to an analytical equation for cogging torque calculation is found in section III. In section IV, this equation is applied to a Surface Mounted Permanent Magnet, Transverse Flux Motor (SMPMTFM) to calculate the cogging torque. The result is compared with the cogging torque calculated by the Maxwell stress method and to test results. Finally, a conclusion is given in section V.

## II. A BREIF REVIEW OF THE THEORY BEHIND THE FLUX-MMF METHOD

The cogging torque may be easily calculated using (1) if the stored energy, denoted  $W$  is known:

$$T_{cog}(\theta) = -\frac{dW}{d\theta} \quad (1)$$

where  $\theta$  is the mechanical rotor position.

Often, the energy stored in the air gap is used for cogging torque calculation. In [10], Watterson has theoretically proved that if an electromagnetic system consists of linear soft magnetic materials and linear permanent magnets only, and has no current flowing in coils, the integral of the shaded area shown in Fig. 1 over all the magnet material alone is equal to the energy of the entire system.

In Fig. 1, the line connecting the working point of the permanent magnet and the origin is known as the load line.

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The load line is mainly determined by the air gap reluctance. In PM motors with salient or slotted stators, when the rotor rotates, the working point of the magnets changes. For each rotor position, if the allocation of the working point on the B-H curve of the permanent magnet could be determined, e.g. by using the reluctance network model or by using FEM, the total energy stored in the system could be calculated by summing the volume integral of the shaded area over all the magnets, and the cogging torque could then be calculated by (1).

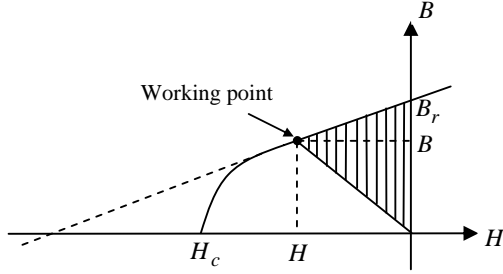


Fig. 1 B-H diagram and the energy density used for energy calculation of the entire system

To perform the volume integral over the magnets, it is pointed out in [10] that for parallel and radial magnets, the magnet flux on surfaces perpendicular to the magnetization direction should be calculated first. The flux should then be integrated along the magnetization direction to obtain the total system energy. In the practice of FEM, this is not easily done. The proposed integral method may be simplified by assuming a uniformly distributed field inside the magnets.

If it is assumed that the flux, over a surface perpendicular to the magnetization direction, does not change when this surface is moved along the magnetization direction, and the whole magnet has a uniform remanent flux density value throughout the magnet volume, according to the equation presented in [10], the calculation of the system energy related to cogging torque production could be simplified to:

$$W = -\frac{1}{2}(Hl_m)(B_r S_m) = -\frac{1}{2}F_m \Phi_r \quad (2)$$

Where  $H$  and  $B_r$  are as illustrated in Fig. 1.  $S_m$  is the magnet area perpendicular to the magnetization direction and  $l_m$  is the magnet length along the magnetization direction.  $F_m$  is the magnet MMF and  $\Phi_r$  is the component of remanent flux inside the magnet that is perpendicular to the magnetization direction. It should be noted that  $H$  and  $F_m$  in (2) are negative.

Equation (2) suggests that instead of using the B-H curve, and using a volume integral for energy calculation, the energy may directly be obtained from the magnet MMF and flux. The volume integral over the magnets could be avoided. This leads to the so-called flux-MMF diagram method. The flux-MMF diagram proposed by Deodhar *et al.* in [9] is illustrated in Fig. 2.

Using the flux-MMF diagram of the permanent magnet to calculate the cogging torque, as proposed in [9], the motor FE model may need to be solved for successive rotor positions with an equal angular shift of  $\Delta\theta$ . For each rotor

position, the magnet flux is first calculated and the corresponding MMF value is found. The flux-MMF diagram as shown in Fig. 2 could then be constructed and the shaded area corresponding to the energy change, denoted  $\Delta W$ , may be obtained from the flux-MMF diagram. The cogging torque may then be calculated by:

$$T_{cog}(\theta) = -\frac{1}{\Delta\theta} \Delta W(\theta) \quad (3)$$

where  $\Delta\theta$  is specified and is known. Often,  $\Delta\theta$  is a constant.

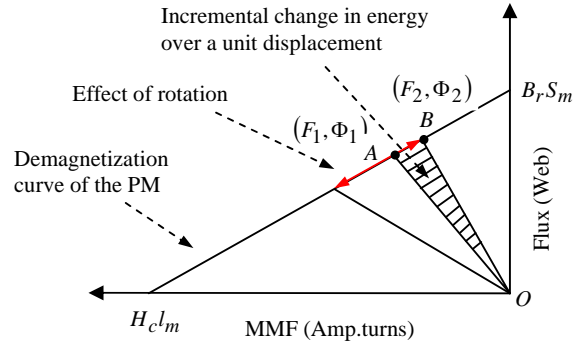


Fig. 2 Sketch of the flux-MMF diagram of a permanent magnet [9]

### III. CALCULATION OF COGGING TORQUE

The previous discussed flux-MMF method may be further simplified.

Suppose that at an arbitrary rotor position  $\theta$ , the magnet operates at point A, as illustrated in Fig. 2. The corresponding MMF and flux are  $F_1$  and  $\Phi_1$ . Then, the rotor position is changed through an infinitesimally angle,  $\Delta\theta$ . The operating point of the magnet becomes B and the magnet MMF and flux become  $F_2$  and  $\Phi_2$ . During this movement, the change of the energy, which is the shaded area shown in Fig. 2, could be calculated by:

$$\Delta W = \frac{1}{2} \Phi_r (F_1 - F_2) \quad (4)$$

For example, when the operating point moves from A to B (e.g. when the magnets are moving towards the direction where they are aligned with the stator teeth), the absolute value of MMF decreases and the magnet flux increases. Because both  $F_1$  and  $F_2$  are negative, so  $\Delta W$  given by (4) is less than zero and the calculated cogging torque by (3) would be positive.

It should be noted that the MMF function in (4) could assume any arbitrary shape, and  $F_2$  is a representation of  $F_1(\theta + \Delta\theta)$ . According to the theory of Taylor's series,  $F_2$  may then be represented by  $F_1$  and its derivatives as:

$$F_2 = F_1(\theta + \Delta\theta) = F_1 + \frac{dF_1}{d\theta} \cdot \Delta\theta + \frac{1}{2!} \cdot \frac{d^2 F_1}{d^2 \theta} \cdot (\Delta\theta)^2 + \dots \quad (5)$$

Substituting (5) into (4) yields:

$$\Delta W = \frac{1}{2} \Phi_r \left( \frac{dF_1}{d\theta} \cdot \Delta\theta + \frac{1}{2!} \cdot \frac{d^2 F_1}{d^2 \theta} \cdot (\Delta\theta)^2 + \dots \right) \quad (6)$$

In the limit, as  $\theta \rightarrow 0$ , (1) may be rewritten as:-

$$T_{cog}(\theta) = - \frac{dW}{d\theta} = - \lim_{\Delta\theta \rightarrow 0} \frac{\Delta W}{\Delta\theta} \quad (7)$$

By observing (6) and (7), it may be found that any expression in the right side of (6) having a coefficient of  $(\Delta\theta)^k$  and  $k \geq 2$  will become zero when it is substituted into (7). Therefore the differential of the system energy  $W$  with respect to rotor position  $\theta$  may be simplified to:

$$T_{cog}(\theta) = - \lim_{\Delta\theta \rightarrow 0} \frac{\Delta W}{\Delta\theta} = \frac{1}{2} \Phi_r \frac{dF_1}{d\theta} \quad (8)$$

Noting that  $F_1$  represents the magnet MMF at an arbitrary rotor position, the subscript 1 may be removed from  $F_1$  in (8).

For a permanent magnet, if  $B$  and  $H$  are aligned everywhere throughout the magnet volume, and assuming the magnet has a uniform value of  $B$  everywhere inside the magnet at a certain rotor position, then the relationship between magnet MMF and flux may be expressed as:

$$F = \frac{l_m}{\mu_0 \mu_r} \left( \frac{\Phi}{S_m} - B_r \right) \quad (9)$$

where  $\mu_0$  is the permeability of free-space and  $\mu_r$  is the relative permeability of the magnet material.  $\Phi$  is the magnet flux calculated over a surface perpendicular to its direction of magnetization.

Substituting (9) into (8), gives:

$$T_{cog}(\theta) = \frac{1}{2} \Phi_r \frac{dF}{d\theta} = \frac{1}{2} B_r \frac{l_m}{\mu_0 \mu_r} \frac{d\Phi}{d\theta} \quad (10)$$

If the electrical angular position is used instead of the mechanical angular position, the cogging torque may be finally expressed as:

$$T_{cog}(\theta_e) = \frac{1}{2} p B_r \frac{l_m}{\mu_0 \mu_r} \frac{d\Phi}{d\theta_e} \quad (11)$$

where  $p$  is the number of magnet pole pairs, and  $\theta_e$  is the electrical rotor angular position. Often, flux  $\Phi$  is calculated over a surface allocated in the middle of the magnet length along its magnetization direction.

Equation (11) shows that the amplitude of the cogging torque is proportional to the number of magnet pole pairs, the remanent flux density value, the magnet length in the direction of magnetization and the variation of the magnet flux with respect to the rotor electrical position. Compared to the flux-MMF diagram technique, if (11) is used and when the magnet flux is obtained, the corresponding MMF need not be calculated and the construction of the flux-MMF diagram is avoided.

Besides cogging torque calculation, this equation gives a convenient analysis of some of the cogging torque

minimization techniques. The application of this equation for cogging torque minimization studies will be presented in a future paper.

#### IV. VALIDATION OF THE COGGING TORQUE EQUATION

Validation of the proposed equation was performed using a surface mounted, permanent magnet, transverse flux motor. The SMPMTFM has a complicated magnetic flux path. Analytical prediction of the magnet flux variation is difficult. The magnet flux here was calculated from a 3D FE model.

Some details about the prototype SMPMTFM used are given in table 1. A 3D FE model of the prototype motor is shown in Fig. 3.

Table 1 Dimensions of the SMPMTFM prototype

Outer diameter (mm)	240
Stack length (mm)	47
Ferrite permanent magnets (L x W x H) (mm)	15 x 12 x 7
Number of rotor poles	32

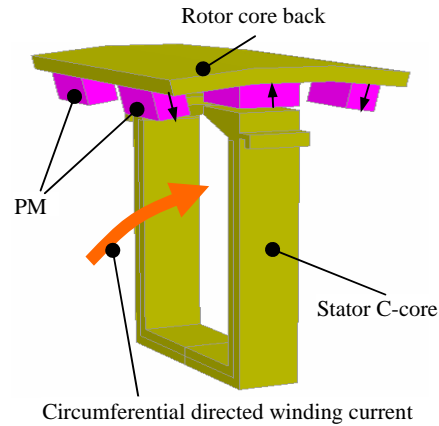


Fig. 3 A 3D single-pole-pair model of the prototype SMPMTFM

The number of rotor poles on the prototype is 32. There are 32 magnets on each side and the total number of magnets is 64.

As previously discussed, only the flux over a surface allocated in the middle of the magnet along its magnetization direction is needed. The two plans used to obtain the magnet flux variation with respect to rotor position are shown in Fig. 4, and the magnet flux profiles obtained for the south PM poles and the north PM poles are shown in Fig. 5.

Substituting the magnet flux profiles obtained into (11), the cogging torque may be obtained. The cogging torque produced by each magnet is independent. The total cogging torque is given by the sum of the cogging torque produced by all the magnets. The calculated cogging torque produced by all the south PM poles, north PM poles, and the resultant total cogging torque is shown in Fig. 6. A comparison of the cogging torque predicted by (11) with measured cogging torque, and cogging torque calculated using the Maxwell stress method is shown in Fig. 7.

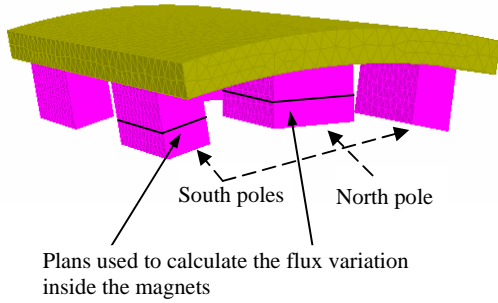


Fig. 4 The two plans used to obtain magnet flux variation in the one-pole-pair FE model

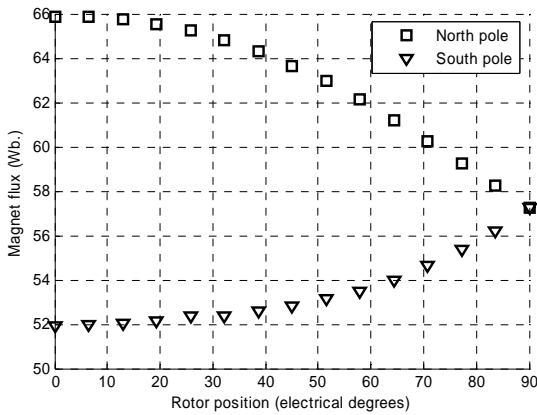


Fig. 5 Magnet flux variation obtained from the selected plans in the 3D FE model.

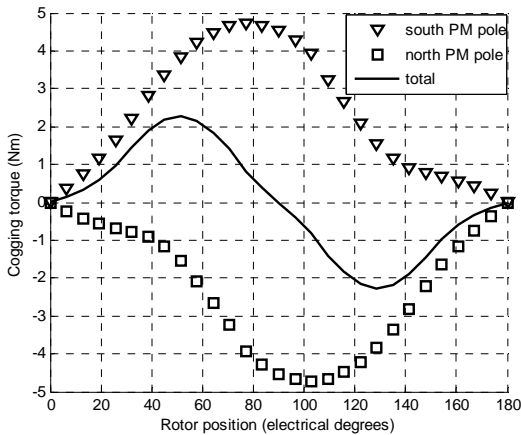


Fig. 6 Calculated cogging torque using (11) and magnet flux waveforms obtained from FE model

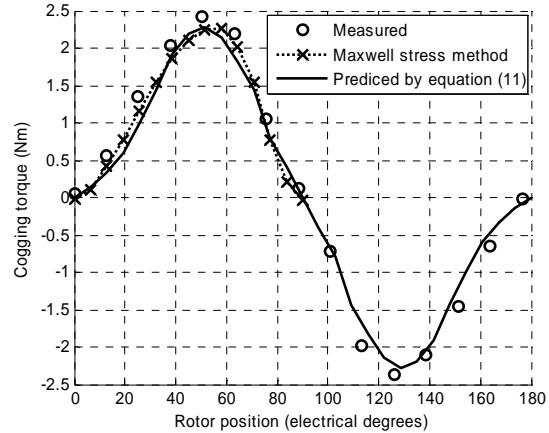


Fig. 7 Comparison of the predicted cogging torque using (11) with measurement and calculated cogging torque using Maxwell stress method

It may be observed from Fig. 6 that for this motor, the cogging torque produced by the south poles cancels some of the cogging torque produced by the north poles. The peak value of the resultant total cogging torque is only 45% of the peak cogging torque produced solely by the north poles or the south poles. Fig. 7 shows a good agreement of the predicted cogging torque with the cogging torque calculated using Maxwell stress method and measured results.

### V. CONCLUSION

In this paper, a new approach is given to prediction of the cogging torque in PM motors. It was developed based on the energy stored in the magnets, similarly to the flux-MMF method. Construction of the flux-MMF diagram may be avoided using the proposed equation. The proposed method also has potential application value for cogging torque minimization studies. The cogging torque predicted by the proposed equations shows a good agreement with the cogging torque calculated using Maxwell stress method and measured results.

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