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*Published in:*  
8th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes

*DOI (link to publication from Publisher):*  
[10.3182/20120829-3-MX-2028.00117](https://doi.org/10.3182/20120829-3-MX-2028.00117)

*Publication date:*  
2012

*Document Version*  
Early version, also known as pre-print

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*  
Tabatabaeipour, S. M., Stoustrup, J., & Bak, T. (2012). Control Reconfiguration of LPV Systems Using Virtual Sensor and Virtual Actuator. In *8th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes* (Vol. 8, pp. 818-823). Elsevier. <https://doi.org/10.3182/20120829-3-MX-2028.00117>

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# Control reconfiguration of LPV systems using virtual sensor and actuator

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**Abstract:** In this paper, a fault tolerant control method for linear parameter varying (LPV) systems using a virtual actuator and a virtual sensor is proposed. The basic idea of the method is to insert a reconfiguration block, which consists of an LPV virtual actuator and an LPV virtual sensor, between the plant and the nominal controller such that the fault tolerant goal is achieved without re-designing the nominal controller. The role of the reconfiguration block is to transform the signals from the faulty system such that its behavior is similar to the nominal system from the point of view of the controller and to transform the output of the controller for the faulty system such that the stability and performance goals are preserved. In this paper, we consider the weak fault-hiding goal and stability of the closed loop system. Input to state stabilizing LPV gains of the virtual actuator and sensor are found by solving linear matrix inequalities (LMIs). We show that separate design of these gains guarantees the closed loop input to state stability of the closed loop reconfigured system.

**Keywords:** Fault-tolerant control, virtual actuator, virtual sensor, reconfigurable control, linear parameter varying systems

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## 1. INTRODUCTION

Due to increasing demands on the safety and reliability in modern technological systems, it is important to design control systems that can automatically detect and diagnose the occurrence of a fault, maintain the overall functionality of the system, and ensure an acceptable performance for the faulty system. A control system with these properties is called a Fault Tolerant Control (FTC) system. The area of fault tolerant control has attracted a lot of attention during the past 15 years; see review papers Patton (1997), Blanke et al. (2000), Jiang (2005).

FTC systems are either passive or active. In a passive FTC (PFTC), the structure of the controller and its parameters are designed and fixed such that it can tolerate occurrence of a set of faults without any change during the operation. A PFTC solution is a common solution for a set of control problems where online reconfiguration cannot be allowed. A PFTC solution in general results in a conservative solution with a possibly low performance. In active FTC (AFTC), a fault is detected and diagnosed by a fault detection and diagnosis (FDD) scheme. Then, the controller is redesigned or reconfigured in the case of severe faults. Control reconfiguration considers the problem of changing the control law or the controller structure by selecting a new set of inputs and outputs. After choosing the new configuration, new control parameters should be found such that the new controller can maintain the original system performance, if possible, or at least ensure a tolerable performance degradation in the faulty process, see Blanke et al. (2006).

In this paper, the idea of control reconfiguration using a virtual sensor and actuator proposed in Niemann and Stoustrup (2003) and Steffen (2005) for linear systems is extended to LPV systems. The basic idea is to place a reconfiguration block between the faulty system and the nominal controller instead of replacing the nominal controller with a new one designed for

the faulty system. The goal of the reconfiguration block is to transform the output of the faulty plant to an appropriate signal such that from the nominal controller's viewpoint its behavior is similar to that of the nominal plant.

A control reconfiguration method using an observer for sensor faults and its dual for actuator faults based on loop transfer recovery design is proposed in Niemann and Stoustrup (2003). In Steffen (2005) virtual actuators and virtual sensors for linear systems are investigated. In Lunze and Steffen (2006) it is shown that control reconfiguration of a linear system after an actuator fault is equivalent to disturbance decoupling. Control reconfiguration using virtual actuators and sensors for piecewise affine systems and Hammerstein-Wiener systems are proposed in Richter et al. (2008), Richter and Lunze (2008), Richter et al. (2011) and Richter (2011).

de Oca and Puig (2010) propose using virtual sensors for fault tolerant control of polytopic LPV systems. It is assumed that the nominal controller consists of a state feedback controller combined with an LPV observer. Then, when a sensor fault occurs, a virtual sensor is used to mask the fault. In this paper, we consider both sensor and actuator faults, and propose a control reconfiguration method that uses an LPV virtual sensor and an LPV virtual actuator to achieve the reconfiguration goal. We consider LPV systems with general dependence on the varying parameter. We do not assume any specific structure for the nominal controller. It is only assumed that the nominal controller is designed such that the nominal closed loop system is input to state stable (ISS). Then, we show that if we design the virtual actuator and the virtual sensor separately such that each of them is ISS, we can guarantee that the closed loop reconfigured system is also ISS. We derive sufficient conditions for designing input to state stabilizing virtual actuator and sensors in terms of LMIs.

This paper is organized as follows. In Section II, preliminaries and some basic definitions are given. In Section 3 LPV systems and faults are introduced and reconfiguration problems for LPV systems that are considered in this paper are defined. Control reconfiguration of LPV systems using a virtual actuator and a virtual sensor is discussed in Section 4. In Section 5, the method is demonstrated on a numerical example. Finally conclusions and directions for future works are given in Section 6.

## 2. PRELIMINARIES

The function  $\alpha: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is called a class  $\mathcal{K}$  function denoted by  $\alpha \in \mathcal{K}$  if it is continuous, strictly increasing, unbounded and satisfies  $\alpha(0) = 0$ . The function  $\beta: \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is called a class  $\mathcal{KL}$  function denoted by  $\beta \in \mathcal{KL}$  if  $\beta(\cdot, t) \in \mathcal{K}$  and  $\beta(r, t) \rightarrow 0$  as  $t \rightarrow \infty$ .

Consider the following nonlinear system:

$$\Sigma: \begin{cases} \dot{x} = f(x(t), u(t)) \\ y(t) = h(x(t)) \end{cases} \quad (1)$$

$$x(0) = x_0, \quad (2)$$

where  $x(t) \in \mathbb{R}^n$  is the state,  $u(t) \in \mathbb{R}^m$  is the input,  $y(t) \in \mathbb{R}^q$  is the output. We use the following stability definitions.

**Definition 1. 0-global exponential stability** (see Sontag (2008)) The system (1) with  $u(t) = 0, \forall t \in \mathbb{R}_+$  is called 0-globally exponentially stable (0-GES) if for all  $t_0$  and  $x(t_0)$ , the solution of the system satisfies:

$$\|x(t)\| \leq c\|x(t_0)\|e^{-\lambda(t-t_0)} \forall t \geq t_0,$$

for some  $c, \lambda \in \mathbb{R}_+$ .

**Definition 2. Input-to-state stability** (see Sontag (2008)) The system (1) is called input-to-state stable (ISS) with respect to the input  $u$  if there exist some  $\beta \in \mathcal{KL}$  and some  $\gamma \in \mathcal{K}$  such that for all  $t_0$  and  $x(t_0)$  and all inputs  $u$ , all solutions of the system satisfy:

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t) + \gamma(\|u\|_\infty).$$

**Definition 3. Input-to-output stability** (see Sontag (2008)) The system (1) is called input-to-output stable (IOS) with respect to the input  $u$  and the output  $y$  if there exist some  $\beta \in \mathcal{KL}$  and some  $\gamma \in \mathcal{K}$  such that for all  $t_0$  and  $x(t_0)$  and all inputs  $u$ , the output of the system satisfies:

$$\|y(t)\| \leq \beta(\|x(t_0)\|, t) + \gamma(\|u\|_\infty).$$

**Theorem 1. IOS of interconnected systems** (see Jiang et al. (1994)) Consider the following interconnected systems:

$$\begin{cases} \dot{x}_2 = f_2(x_2(t), y_1(t), u(t)), \\ y_2 = h_2(x_2(t), y_1(t), u(t)), \\ \dot{x}_1 = f_1(x_1(t), u(t)), \\ y_1(t) = h_1(x_1(t), u(t)). \end{cases} \quad (3)$$

Assume that the first system is IOS w.r.t the input  $u$  and the output  $y_1$  and the second system is IOS w.r.t the input  $(y_1, u)$  and output  $y_2$ . Then the interconnected system is IOS w.r.t the input  $u$  and outputs  $(y_1, y_2)$ .

## 3. LINEAR PARAMETER VARYING SYSTEMS

We consider the following LPV systems:

$$\Sigma_P: \begin{cases} \dot{x}(t) = A(\theta)x(t) + B(\theta)u_c(t) + B_d(\theta)d(t), \\ y(t) = C(\theta)x(t), \\ z(k) = C_z(\theta)x(t), \end{cases} \quad (4)$$

$$x(0) = x_0$$

where  $x(t) \in \mathbb{R}^n$  is the state,  $u(t) \in \mathbb{R}^m$  is the control input,  $y(t) \in \mathbb{R}^p$  is the output, and  $z(t) \in \mathbb{R}^q$  is the performance output vector, and  $d(t) \in \mathbb{R}^l$  is the disturbance. The matrices  $A(\theta), B(\theta), E(\theta), C(\theta), D(\theta), F(\theta)$  are continuous functions of the time varying parameter vector  $\theta \in \mathbb{R}^{n_\theta}$ . It is assumed that the parameter  $\theta(t)$  is bounded in a given compact set  $\Theta$  i.e  $\theta(t) \in \Theta \forall t \geq 0$ . We also assume that the rate of variation of  $\theta$  satisfies  $|\dot{\theta}_i(t)| < v_i, i = 1, \dots, n_\theta$  i.e  $\dot{\theta}(t)$  is bounded in a given hyperrectangle:

$$\mathcal{V} = \{\dot{\theta}(t) : |\dot{\theta}_i(t)| \leq v_i, i = 1, \dots, n_\theta, \forall t \geq 0\}. \quad (5)$$

Then, the set of vertices of  $\mathcal{V}$  is given by:

$$\mathcal{V}_v = \{[\theta_{d_1}, \dots, \theta_{d_{n_\theta}}]^T : \theta_{d_i} \in \{-v_i, v_i\}, i = 1, \dots, n_\theta, \forall t \geq 0\}. \quad (6)$$

The set of  $\theta$  satisfying the above assumption are denoted by  $\mathcal{F}_\Theta^v$ . The dynamic operator  $\Omega_p: \mathcal{L}_1^{loc}(\mathbb{R}^n) \times \mathcal{L}_1^{loc}(\mathbb{R}^l) \times \mathcal{L}_1^{loc}(\mathbb{R}^n) \rightarrow \mathcal{L}_1^{loc}(\mathbb{R}^p) \times \mathcal{L}_1^{loc}(\mathbb{R}^q)$  is associated with the LPV system  $\Sigma_p$  such that:

$$(y, z) = \Omega_p(u_c, d, x_0), \quad (7)$$

where  $\mathcal{L}_1^{loc}$  represents the space of locally integrable functions. In the case that matrices of the LPV systems have affine dependence on the parameter  $\theta$ , we have

$$M(\theta) = \begin{bmatrix} A(\theta) & B(\theta) & B_d(\theta) \\ C(\theta) & 0 & 0 \\ C_z(\theta) & 0 & 0 \end{bmatrix} = \begin{bmatrix} A_0 & B_0 & B_{d0} \\ C_0 & 0 & 0 \\ C_{z0} & 0 & 0 \end{bmatrix} + \sum_{i=1}^{n_\theta} \begin{bmatrix} A_i & B_i & B_{di} \\ C_i & 0 & 0 \\ C_{zi} & 0 & 0 \end{bmatrix} \theta_i. \quad (8)$$

**Theorem 2.** (see Wu (1995)) The LPV system in (4) with the given compact set  $\Theta$  and the hyperrectangle  $\mathcal{V}$  is parameterically -dependent quadratically stable (PDQ-stable) if there exist a parameter dependent continuously differentiable symmetric function  $X(\theta) > 0$  such that

$$A^T(\theta)X(\theta) + X(\theta)A(\theta) + \sum_{i=1}^{n_\theta} \pm(v_i \frac{\partial X(\theta)}{\partial \theta_i}) < 0 \quad (9)$$

or if there exist a parameter dependent continuously differentiable symmetric function  $Y(\theta) > 0$  such that:

$$Y(\theta)A^T(\theta) + A(\theta)Y(\theta) - \sum_{i=1}^{n_\theta} \pm(v_i \frac{\partial Y(\theta)}{\partial \theta_i}) < 0 \quad (10)$$

Moreover, the system (4) is 0-GES. The notation  $\sum_{i=1}^s \pm(\cdot)$  means that all possible combinations of  $+(\cdot)$  and  $-(\cdot)$  must be considered in the inequality.

### 3.1 Control Design

We assume that a nominal controller  $\Sigma_C$  is designed for the nominal system with the internal state  $x_c \in \mathbb{R}^{n_c}$  and the reference input  $r(t) \in \mathbb{R}^p$  which generates the control input  $u_c$ . The controller is associated with the operator  $\Omega_C(r, y, x_{c0}): \mathcal{L}_1^{loc}(\mathbb{R}^p) \times \mathcal{L}_1^{loc}(\mathbb{R}^p) \times \mathcal{L}_1^{loc}(\mathbb{R}^{n_c}) \rightarrow \mathcal{L}_1^{loc}(\mathbb{R}^m)$ ,

$$u_c = \Omega_C(r, y, x_{c0}). \quad (11)$$

It is assumed that the nominal closed loop system  $(\Sigma_P, \Sigma_C)$  is stable. We do not make any assumption about the structure of the controller. It could be for example a dynamic or static output feedback controller.

**Assumption 1. IOS of the nominal closed loop system.** The nominal closed loop system  $(\Sigma_P, \Sigma_C)$  is IOS w.r.t the inputs  $(r, d)$  and the output  $(u_c, x)$ .

### 3.2 Faults

We consider actuator and sensor faults. Actuator faults are events that changes the input matrix of the LPV system such

that as a result the input matrix changes from  $B$  to  $B_f$ . Similarly sensor faults are events that changes the measurement matrix from  $C$  to  $C_f$ . Therefore, the model of the faulty plant is given by:

$$\Sigma_{P_f} : \begin{cases} \dot{x}_f(t) = A(\theta)x_f(t) + B_f(\theta)u_f(t) + B_d(\theta)d(t), \\ y_f(t) = C_f(\theta)x_f(t), \\ z_f(k) = C_z(\theta)x_f(t), \end{cases} \quad (12)$$

$$x_f(0) = x_0.$$

The faulty plant is associated with the dynamic operator  $\Omega_{P_f} : \mathcal{L}_1^{loc}(\mathbb{R}^n) \times \mathcal{L}_1^{loc}(\mathbb{R}^l) \times \mathcal{L}_1^{loc}(\mathbb{R}^m) \rightarrow \mathcal{L}_1^{loc}(\mathbb{R}^p) \times \mathcal{L}_1^{loc}(\mathbb{R}^q)$ ,  $(y_f, z_f) = \Omega_{P_f}(u_f, d, x_0)$ .

### 3.3 Reconfiguration problems

In most of the approaches for AFTC, when a fault occurs a new controller  $\Sigma_{C_r}$  is designed and replaces  $\Sigma_C$  such that the new closed loop system  $(\Sigma_{P_f}, \Sigma_{C_r})$  is stable and provides an acceptable performance. In this paper, we use the proposed approach in Richter (2011) in which instead of replacing the nominal controller with a new controller designed for the faulty system, a reconfiguration block is inserted between the nominal controller and the fault system. The reconfiguration block receives the output of the nominal controller  $u_c$  and the output of the faulty system  $y_f$  as input and generates  $u_f$  and  $y_c$  as shown in figure 1. The LPV reconfiguration block is an LPV system with the internal state  $\xi$ :

$$\Sigma_R : \begin{cases} \dot{\xi} = A_r(\theta)\xi(t) + B_r(\theta)u_c(t) + E_r(\theta)y_f(t), \\ y_c(t) = C_r(\theta)\xi(t) + F_r(\theta)y_f(t), \\ u_f(k) = G_r(\theta)\xi(t) + H_r(\theta)u_c(t), \\ \xi(0) = \xi_0, \end{cases} \quad (13)$$

and is associated by the operator  $\Omega_R$ :

$$(u_f, y_c) = \Omega_R(u_c, y_f, \xi_0). \quad (14)$$

$\Sigma_R$  should be designed such that the overall closed loop system  $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$  is stable and satisfies some closed loop performance requirement. The series connection of the plant with the reconfiguration block  $(\Sigma_{P_f}, \Sigma_R)$  is called the reconfigured plant and the series connection of the nominal controller and the reconfiguration block  $(\Sigma_{P_f}, \Sigma_R)$  is called the reconfigured controller. Different goals in the design of the reconfiguration block may be considered and based on them different reconfiguration problems are defined. Here we state two of them that are addressed in this paper, see Richter (2011).

**Problem 3. Stability recovery for LPV systems.** Consider the nominal LPV systems  $\Sigma_P(4)$  and the faulty LPV system  $\Sigma_{P_f}$  (12), find, if possible, a reconfiguration block  $\Sigma_R$  such that for all  $\Sigma_C$  that  $(\Sigma_C, \Sigma_P)$  is ISS w.r.t the input  $(r, d)$ , we have  $(\Sigma_{P_f}, \Sigma_R, \Sigma_C)$  is ISS w.r.t the input  $(r, d)$ .

The aim of the reconfiguration block is to transform  $y_f$  to  $y_c$  such that from the controller viewpoint the behavior of the reconfigured plant is the same as the behavior of the nominal plant. The following goal ensures that we can find a matching initial condition  $\xi_0$  for the reconfiguration block such that if there is no disturbance, the behavior of the reconfigured plant is the same as that of the nominal plant from the controller's point of view.

**Definition 4. Weak fault-hiding goal.** The reconfigured plant  $\Sigma_{P_r}$  satisfies the weak fault hiding goal if for all  $x_0$ , there exist a matching initial condition  $\xi_0$  such that :

$$\Omega_{P_r}^{y_c}(\cdot, \cdot, x_0, \xi_0) - \Omega_P^y(\cdot, \cdot, x_0) = 0. \quad (15)$$

Stronger versions of the fault-hiding goal can be defined which we do not repeat here since we are not going to address them in this work. The interested reader is referred to Richter (2011).

## 4. RECONFIGURATION BLOCK DESIGN

The reconfiguration block,  $\Sigma_R$ , consists of a virtual sensor,  $\Sigma_S$ , and a virtual actuator  $\Sigma_A$ . The virtual sensor is used to estimate the state of the faulty system  $\hat{x}_f$  based on a model of the faulty plant. The virtual actuator includes a reference model for the nominal plant with the internal state  $\tilde{x}$ . The difference between the estimate of the state of the faulty system and the state of the reference model,  $x_\Delta$ , is then fed back through the gain  $M$ . The output injection gain  $L$  and the gain  $M$  should be designed such that the estimation error goes to zero and at the same time the difference state goes to zero, hence the output of the virtual actuator approaches that of the nominal system.

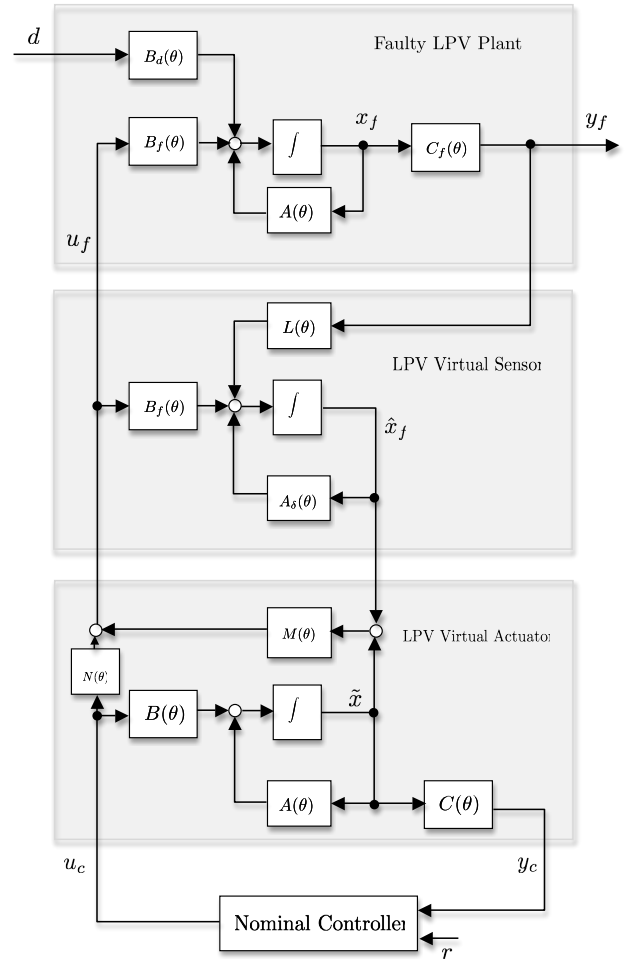


Fig. 1. LPV virtual sensor and actuator in the closed loop system

The virtual actuator block is defined as follows:

$$\Sigma_A : \begin{cases} \dot{\tilde{x}} = A(\theta)\tilde{x} + B(\theta)u_c(t), \\ u_f(t) = M(\theta)x_\Delta(t) + N(\theta)u_c(t), \\ y_c(t) = C(\theta)\tilde{x}, \\ \tilde{x} = \hat{x}_f, \end{cases} \quad (16)$$

where  $x_\Delta = \tilde{x} - \hat{x}_f$ . The virtual sensor for an LPV system is defined as:

$$\Sigma_S: \begin{cases} \dot{\hat{x}}_f(t) = A_\delta(\theta)\hat{x}_f(t) + B(\theta)u_f(t) + L(\theta)y_f(t), \\ u_f(t) = u_c(t), \end{cases} \quad (17)$$

$$\hat{x}_f(0) = \hat{x}_{f0},$$

where  $A_\delta(\theta) = A(\theta) - L(\theta)C_f(\theta)$ . To analyze the reconfigured plant we introduce the observation error defined as  $e(t) = \hat{x}_f(t) - x_f(t)$  and the difference state defined as  $x_\Delta = \tilde{x} - \hat{x}_f$  and associated with them the observation error system  $\Sigma_e$  and the difference system  $\Sigma_\Delta$  which are:

$$\Sigma_e: \dot{e} = A_\delta(\theta)e(t) + B(\theta)d(t), \quad (18)$$

$$e(0) = \hat{x}_{f0} - x_0,$$

and

$$\Sigma_\Delta: \dot{x}_\Delta = (A(\theta) - B_f(\theta)M(\theta))x_\Delta(t) + L(\theta)C_f(\theta)e(t) + (B(\theta) - B_f(\theta)N(\theta))u_c(t), \quad (19)$$

$$x_\Delta(0) = 0.$$

Dynamic of the system in terms of  $\tilde{x}$  and the new introduced variables  $e$  and  $x_\Delta$  is :

$$\begin{bmatrix} \dot{\tilde{x}}(t) \\ \dot{e}(t) \\ \dot{x}_\Delta(t) \end{bmatrix} = \begin{bmatrix} A(\theta) & 0 & 0 \\ 0 & (A(\theta) - L(\theta)C_f(\theta)) & 0 \\ 0 & L(\theta)C(\theta) & (A(\theta) - B_f(\theta)M(\theta)) \end{bmatrix} \begin{bmatrix} \tilde{x}(t) \\ e(t) \\ x_\Delta(t) \end{bmatrix} + \begin{bmatrix} B(\theta) \\ 0 \\ (B(\theta) - B_f(\theta)N(\theta)) \end{bmatrix} u_c(t) + \begin{bmatrix} 0 \\ -B_d(\theta) \\ 0 \end{bmatrix} d(t), \quad (20)$$

$$y_c(t) = [C(\theta) \ 0 \ 0] \begin{bmatrix} \tilde{x}(t) \\ e(t) \\ x_\Delta(t) \end{bmatrix}, \quad \begin{bmatrix} \tilde{x}(0) \\ e(0) \\ x_\Delta(0) \end{bmatrix} = \begin{bmatrix} \hat{x}_{f0} \\ \hat{x}_{f0} - x_0 \\ 0 \end{bmatrix} \quad (21)$$

As it can be seen from (20), the reference state  $\tilde{x}$  is decoupled from the observation error  $e$  and the difference state. Also the observation error is decoupled from the reference state, the difference state and the input  $u_c$ . The difference state is decoupled from the reference state. The weak fault-hiding goal requires finding a matching initial condition such that when  $d(t) = 0$  the output of the reconfigured plant is equal to that of the nominal plant. As it can be seen from the above equations, by choosing  $\hat{x}_{f0} = x_0$ , we have  $y_c(t) = y(t)$  because the reference model is decoupled from the rest of the system and is initialized with the same initial condition as the nominal plant. But  $x_0$  is not measurable; hence the above matching condition will not be satisfied. A remedy to that is to estimate the states of the nominal system before the occurrence of the fault denoted by  $x(t_f)$  and then set  $\hat{x}_{f0}$  to  $x(t_f)$ . Note that for this strategy, we should assume that the required time for fault detection and reconfiguration is small because otherwise it would cause a large offset between the trajectories.

In the following we find sufficient conditions in terms of LMIs for the observation error and the difference system to be ISS and then we show that these conditions also guarantees the stability of the overall closed loop system.

**Theorem 4.** Consider the faulty LPV system (12). If there exist a continuously differentiable symmetric function  $X_S(\theta)$  and a continuously differentiable function  $Y_S(\theta)$  such that:

$$A^T(\theta)X_S(\theta) + X_S(\theta)A(\theta) - Y_S(\theta)C(\theta) - C^T(\theta)Y_S^T(\theta) + \sum_{i=1}^{n_\theta} \pm(v_i \frac{\partial X_S(\theta)}{\partial \theta_i}) < 0 \quad (22)$$

for all  $(\theta, \dot{\theta}) \in \Theta \times \mathcal{V}$ , then the virtual sensor  $\Sigma_S$  with  $L(\theta) = X_S(\theta)^{-1}Y_S$  is an observer for the LPV system such that the observation error system (18) is 0-GES for  $d(t) = 0$ . Moreover the error dynamics (18) is ISS w.r.t the disturbance  $d$ .

**Proof.** To show 0-GES stability, we use Theorem 2 with  $X_S(\theta)$  and replace  $A(\theta)$  with  $A(\theta) - L(\theta)C_f(\theta)$ :

$$[A(\theta) - L(\theta)C_f(\theta)]^T X_S(\theta) + X_S(\theta)[A(\theta) - L(\theta)C_f(\theta)] + \sum_{i=1}^{n_\theta} \pm(v_i \frac{\partial X_S(\theta)}{\partial \theta_i}) < 0. \quad (23)$$

Introducing  $Y_S(\theta) = L(\theta)X_S(\theta)$  we get (22). To show ISS w.r.t the disturbance, we consider a PDLF  $V(e) = e^T X_S(\theta)e$ . Then

$$\begin{aligned} \dot{V}(e) &= \dot{e}^T X_S(\theta)e + e^T X_S(\theta)\dot{e} + e^T \dot{X}(\theta)e = \\ &= e^T [(A(\theta) - L(\theta)C_f(\theta))^T X_S(\theta) + (A(\theta) - L(\theta)C_f(\theta))X_S(\theta) \\ &\quad + \dot{X}(\theta)]e - e^T X_S(\theta)B_d(\theta)d - d^T B_d^T X_S(\theta)e. \end{aligned} \quad (24)$$

If (22) is satisfied, then for some  $a, b > 0$  and  $0 < \alpha < 1$  we have:

$$\begin{aligned} \dot{V}(e) &\leq -ae^T X_S(\theta)e - e^T X_S(\theta)B_d(\theta)d - d^T B_d(\theta)^T X_S(\theta)e \leq \\ &\leq -b\|e\|^2 + 2\|e\|\|X_S(\theta)\|\|B_d(\theta)\|\|d\| = \\ &\leq -b(1 - \alpha)\|e\|^2 - b\alpha\|e\| + 2\|e\|\|X_S(\theta)\|\|B_d(\theta)\|\|d\| \end{aligned} \quad (25)$$

If  $\|e\| \geq \frac{2\|X_S(\theta)\|\|B_d(\theta)\|\|d\|}{\alpha b}$ , then it follows:

$$\dot{V}(e) \leq -b(1 - \alpha)\|e\|^2, \quad (26)$$

which proves that  $\Sigma_e$  is ISS w.r.t to the disturbance  $d$ . ■

**Theorem 5.** Consider the faulty LPV system (12). If there exist a continuously differentiable symmetric function  $X_A(\theta)$  and a continuously differentiable function  $Y_A(\theta)$  such that:

$$X_A(\theta)A^T(\theta) + A(\theta)X_A(\theta) - B_f(\theta)Y_A(\theta) - Y_A^T(\theta)B_f^T(\theta) - \sum_{i=1}^{n_\theta} \pm(v_i \frac{\partial X_S(\theta)}{\partial \theta_i}) < 0 \quad (27)$$

for all  $(\theta, \dot{\theta}) \in \Theta \times \mathcal{V}$ , then the difference system (4) associated with the virtual actuator with  $M(\theta) = X^{-1}(\theta)Y_A(\theta)$  is 0-GES for  $u_c(t) = 0, e(t) = 0$ . Moreover, the difference system (4) is ISS w.r.t the inputs  $u_c, e$ .

**Proof.** To show 0-GES stability we use (10) from theorem 2 with  $X_A(\theta)$  and replace  $A(\theta)$  with  $A(\theta) - B_f(\theta)M(\theta)$ :

$$X_S(\theta)[A(\theta) - B_f(\theta)M(\theta)]^T + [A(\theta) - B_f(\theta)M(\theta)]X_S(\theta) - \sum_{i=1}^{n_\theta} \pm(v_i \frac{\partial X_S(\theta)}{\partial \theta_i}) < 0 \quad (28)$$

Introducing  $Y_A(\theta) = M(\theta)X_A(\theta)$  we get (27). To show ISS w.r.t  $(u_c, e)$  we consider a PDLF  $V(e) = x_\Delta^T Z_A(\theta)x_\Delta$ .

$$\begin{aligned} \dot{V}(e) &= \dot{x}_\Delta^T Z_A(\theta)x_\Delta + x_\Delta^T \dot{Z}_A(\theta)x_\Delta + x_\Delta^T \dot{Z}_A(\theta)x_\Delta = \\ &= x_\Delta^T [(A(\theta) - B_f(\theta)M(\theta))^T Z_A(\theta) + Z_A(\theta)(A(\theta) - B_f(\theta)M(\theta)) \\ &\quad + \dot{Z}(\theta)]x_\Delta + e^T C_f^T L(\theta)Z_A(\theta)x_\Delta + \\ &\quad + x_\Delta^T Z_A(\theta)L(\theta)C_f(\theta)e + u_c^T B_\Delta^T Z_A x_\Delta + x_\Delta^T Z_A B_\Delta u_c. \end{aligned} \quad (29)$$

If (27) is satisfied, then for some  $a, b > 0$  and  $0 < \alpha < 1$  we have:

$$\begin{aligned} \dot{V}(e) &\leq -ax_\Delta^T Z_A(\theta)x_\Delta + e^T C_f^T L(\theta)Z_A(\theta)x_\Delta + \\ &\leq x_\Delta^T Z_A(\theta)L(\theta)C_f(\theta)e + u_c^T B_\Delta^T Z_A x_\Delta + x_\Delta^T Z_A B_\Delta u_c \leq \\ &\leq -b\|x_\Delta\|^2 + 2\|x_\Delta\|\|Z_A(\theta)\|\|L(\theta)\|\|C_f\|\|e\| + \\ &\quad + 2\|x_\Delta\|\|B_\Delta(\theta)\|\|Z_A(\theta)\|\|u_c\| = \\ &\leq -b(1 - \alpha)\|x_\Delta\|^2 - b\alpha\|x_\Delta\| + 2\|x_\Delta\|\|Z_A(\theta)\|\|L(\theta)\|\|C_f\|\|e\| \\ &\quad + \|B_\Delta(\theta)\|\|Z_A(\theta)\|\|u_c\| \end{aligned} \quad (30)$$

If  $\|x_\Delta\| \geq \frac{2\|Z_A(\theta)\|\|L(\theta)\|\|C_f\|\|e\| + \|B_\Delta(\theta)\|\|Z_A(\theta)\|\|u_c\|}{\alpha b}$ , then it follows:

$$\dot{V}(x_\Delta) \leq -b(1 - \alpha)\|x_\Delta\|^2, \quad (31)$$

which proves that  $\Sigma_{x_\Delta}$  is ISS w.r.t to the inputs  $(e, u_c)$ . ■

The following theorem shows that if we design the virtual actuator and the virtual sensor independently and obtain the gains  $M, L$  based on the above theorems, then we can guarantee that the closed loop reconfigured system is ISS w.r.t to the input  $(r, d)$ .

**Theorem 6.** Consider the faulty LPV system (12). Assume that the assumption 1 holds. If there exist a continuously differentiable symmetric function  $X_S(\theta), X_A(\theta)$  and a continuously differentiable function  $Y_S(\theta), Y_A(\theta)$  such that:

$$A^T(\theta)X_S(\theta) + X_S(\theta)A(\theta) - Y_S(\theta)C_f(\theta) - C_f^T(\theta)Y_S^T(\theta) + \sum_{i=1}^{n_\theta} \pm(v_i \frac{\partial X_S(\theta)}{\partial \theta_i}) < 0, \quad (32)$$

$$X_A(\theta)A^T(\theta) + A(\theta)X_A(\theta) - B_f(\theta)Y_A(\theta) - Y_A^T(\theta)B_f^T(\theta) - \sum_{i=1}^{n_\theta} \pm(v_i \frac{\partial X_S(\theta)}{\partial \theta_i}) < 0, \quad (33)$$

for all  $(\theta, \dot{\theta}) \in \Theta \times \mathcal{V}$ , then the closed loop reconfigured system  $(\Sigma_{P_f}, \Sigma_S, \Sigma_A, \Sigma_C)$  with  $M(\theta) = Y_A(\theta)X_A(\theta)^{-1}$  and  $L(\theta) = X_S(\theta)^{-1}Y_S(\theta)$  is ISS w.r.t the input  $(r(t), d(t))$ .

**Proof.** To prove ISS of the closed loop reconfigured system, we use Theorem 1 and the fact that the closed loop system is the interconnection of the  $(\Sigma_{\bar{P}}, \Sigma_C)$  and  $(\Sigma_e, \Sigma_\Delta)$  as shown in the figure 2. We also know that based on the assumption 1  $(\Sigma_{\bar{P}}, \Sigma_C)$  is ISS and based on Theorems 4 and 5  $\Sigma_e$  and  $\Sigma_\Delta$  are ISS.

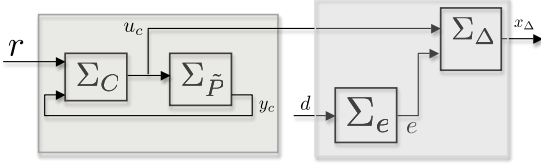


Fig. 2. Closed loop system as series connection of  $(\Sigma_C, \Sigma_{\bar{P}})$  and  $(\Sigma_e, \Sigma_\Delta)$

First, we want to show that the interconnection  $(\Sigma_\Delta, \Sigma_e)$  is ISS with respect to the inputs  $(u_c, d)$ . Based on the theorem 4  $\Sigma_S$  is ISS w.r.t  $d$  which means that there exist a  $\beta_e \in \mathcal{KL}$  and a  $\gamma_d \in \mathcal{K}$  such that:

$$\|e(t)\| \leq \beta_e(\|e(0)\|, t) + \gamma_d(\|d\|_\infty). \quad (34)$$

Based on Theorem 5  $\Sigma_\Delta$  is ISS w.r.t the inputs  $(e, u_c)$  which means that there exist a  $\beta_\Delta \in \mathcal{KL}$  and a  $\gamma_u \in \mathcal{K}$  and a  $\gamma_e \in \mathcal{K}$  such that:

$$\|x_\Delta(t)\| \leq \beta_\Delta(\|x_\Delta(0)\|, t) + \gamma_u(\|u_c\|_\infty) + \gamma_e(\|e\|_\infty). \quad (35)$$

Knowing the fact that  $\|e(t)^T, x_\Delta(t)^T\| \leq \|e(t)\| + \|x_\Delta(t)\|$ , we have:

$$\|e(t)^T, x_\Delta(t)^T\| \leq \beta_e(\|e(0)\|, t) + \beta_\Delta(\|x_\Delta(0)\|, t) + \gamma_d(\|d\|_\infty) + \gamma_u(\|u_c\|_\infty) + \gamma_e(\|e\|_\infty). \quad (36)$$

We know that  $\|e\|_\infty \leq \beta_e(\|e(0)\|, t) + \gamma_d(\|d\|_\infty)$ . Hence:

$$\begin{aligned} \|e(t)^T, x_\Delta(t)^T\| &\leq \beta_e(\|e(0)\|, t) + \beta_\Delta(\|x_\Delta(0)\|, t) + \\ &\gamma_e(\beta_e(\|e(0)\|, t) + \gamma_d(\|d\|_\infty)) + \gamma_d(\|d\|_\infty) + \gamma_u(\|u_c\|_\infty) \\ &\leq \beta_e(\|e(0)\|, t) + \beta_\Delta(\|x_\Delta(0)\|, t) + \\ &\gamma_e(\beta_e(\|e(0)\|, t)) + \gamma_e(\gamma_d(\|d\|_\infty)) + \gamma_d(\|d\|_\infty) + \gamma_u(\|u_c\|_\infty). \end{aligned} \quad (37)$$

As  $\beta_e(\|e(0)\|, t) + \beta_\Delta(\|x_\Delta(0)\|, t) + \gamma_e(\beta_e(\|e(0)\|, t))$  is a  $\mathcal{KL}$  and  $\gamma_e(\gamma_d(\|d\|_\infty)) + \gamma_d(\|d\|_\infty) + \gamma_u(\|u_c\|_\infty)$  is a  $\mathcal{K}$  function, the above inequality shows that the interconnection  $\Sigma_e, \Sigma_\Delta$  is ISS w.r.t to  $(d, u_c)$  and therefore it is IOS w.r.t the output  $(e, x_\Delta)$ . Based on Assumption 1,  $(\Sigma_{\bar{P}}, \Sigma_C)$  is IOS w.r.t to the input  $(r, d)$  and the output  $(u_c, \tilde{x})$ . Therefore, using Theorem 1 we conclude that the series connection  $(\Sigma_{\bar{P}}, \Sigma_C, \Sigma_e, \Sigma_\Delta)$  is IOS w.r.t to the input  $(r, d)$  and the output  $e, x_\Delta$ . ■

## 5. EXAMPLE

Consider an LPV system with the following parameters:

$$A(\theta) = A_0 + \theta A_1, \quad (38)$$

$$A_0 = \begin{bmatrix} -0.3839 & -10.9262 \\ 10.9262 & -0.3839 \end{bmatrix} \text{ and } A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad (39)$$

$$B(\theta) = B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } B_d = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } C(\theta) = C = [1 \ 1], \quad (40)$$

where

$$\theta = 0.5 + 0.5 \sin(0.2t). \quad (41)$$

We consider a simultaneous sensor and actuator fault that changes the input and measurement matrix to the following matrices:

$$B_f = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, C_f = [0.5 \ 0]. \quad (42)$$

This fault occurs at  $t_f = 10$ . The nominal plant is controlled with a static output feedback controller with  $K = -5$ . The fault is detected and the system is reconfigured at  $t_r = 15$ . To reconfigure the system we use Theorem 4 and 5. Note that these theorems are derived for LPV system with general dependence on the varying parameter  $\theta$ . The above system has affine dependence on  $\theta$ . For this class of LPV systems (22) and (27) can be relaxed into a finite set of LMIs using the method proposed in Apkarian and Tuan (2000). The virtual sensor gain  $L(t)$  is calculated as follows:

$$X_S(t) = X_{S_0} + \theta(t)X_{S_1}, Y_S(t) = Y_{S_0} + \theta(t)Y_{S_1} \quad (43)$$

$$L(t) = X_S(t)^{-1}Y_S(t). \quad (44)$$

with

$$X_{S_0} = \begin{bmatrix} 0.0228 & 0.0448 \\ 0.0448 & 0.6393 \end{bmatrix}, X_{S_1} = \begin{bmatrix} 0.3606 & -0.0224 \\ -0.0224 & 0.6394 \end{bmatrix} \quad (45)$$

$$Y_{S_0} = \begin{bmatrix} 2.307 \\ 13.4088 \end{bmatrix}, Y_{S_1} = \begin{bmatrix} 0 \\ 6.1708 \end{bmatrix}. \quad (46)$$

The same procedure is used to find  $M(t)$ :

$$X_A(t) = X_{A_0} + \theta(t)X_{A_1}, Y_A(t) = Y_{A_0} + \theta(t)Y_{A_1}, \quad (47)$$

$$M(t) = Y_A(t)X_A(t)^{-1}, \quad (48)$$

with

$$X_{A_0} = \begin{bmatrix} 1.4542 & -0.0029 \\ -0.0029 & 1.4970 \end{bmatrix}, X_{A_1} = \begin{bmatrix} 1.4008 & 0.2083 \\ 0.2083 & 1.5019 \end{bmatrix}, \quad (49)$$

$$Y_{A_0} = [-0.9844 \ 0.2016], Y_{A_1} = [-2.1155 \ 3.3985]. \quad (50)$$

The matrix  $N$  is chosen as  $I$ . Figures 3 and 4 show the simulation result. Figure 3 shows the plant output  $y$  and states  $x$  and the output of the reconfigured plant  $y_c$  which is the output seen by the nominal controller. Figure 4 shows the difference system states  $x_\Delta$ , and the output of the nominal controller  $u_c$  and the output of the reconfigured controller  $u_f$  which is the input to the plant. As it can be seen from  $y_c$  during the period  $[10, 15]$ , the controller sees the faulty plant, but for  $t > 15$  the controller sees the output of the reconfigured plant which is

equal to that of the nominal plant initialized with  $x_0 = [0, 0]^T$ . In other words, the fault is hidden from the controller's viewpoint. The difference state is bounded because the observation error is bounded which means that the difference systems is ISS. The output of the nominal controller,  $u_c$ , after reconfiguration is zero but  $M(t)x_\Delta$  is added to it to account for the fault which generates the appropriate signal  $u_f$  to keep the closed loop system ISS.

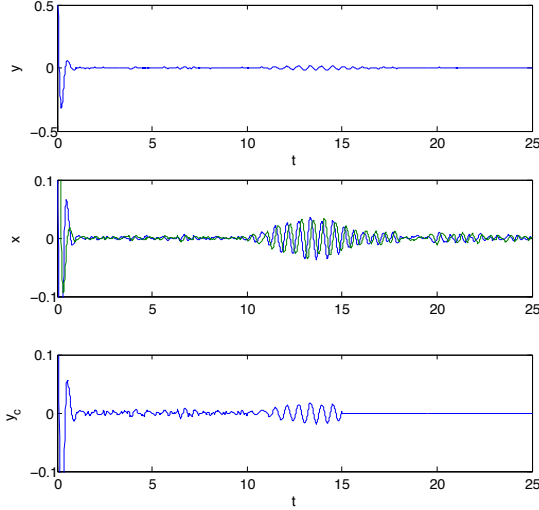


Fig. 3. Simulation results for  $y$ , the plant output,  $x$  the plant states,  $y_c$  output of the reconfigured plant

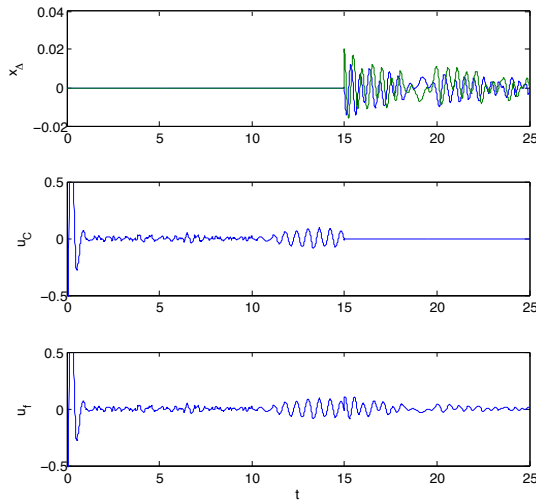


Fig. 4. Simulation results for  $x_\Delta$ , the difference system state,  $u_c$  output of the nominal system,  $u_f$  output the reconfigured controller

## 6. CONCLUSION

We presented a method for control reconfiguration of LPV systems using a reconfiguration block. We considered both actuator and sensor faults. The reconfiguration block which is realized by an LPV virtual sensor and an LPV virtual actuator is inserted between the plant and the nominal controller to guarantee the input to output stability of the overall closed loop system. We showed that if the virtual actuator and the virtual sensor are designed separately to be input to state stable and if

the nominal closed loop system is input to output stable, then the closed loop reconfigured system remains input to output stable. Sufficient conditions for input to state stability of the virtual actuator and the virtual sensor is derived in terms of LMIs. In future work, we would consider the tracking and performance properties of the reconfiguration method.

## ACKNOWLEDGEMENTS

The first author is supported by the Norwegian Research Center for Offshore Wind Energy (NORCOWE).

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