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Sun, Fan; Popovski, Petar; Thai, Chan T.D.; Carvalho, Elisabeth De

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# Sum-Rate Maximization of Coordinated Direct and Relay Systems

(Invited Paper)

Fan Sun, Petar Popovski, Chan Dai Truyen Thai, and Elisabeth de Carvalho

Department of Electronic Systems, Faculty of Engineering and Science

Aalborg University, DK-9220 Aalborg, Denmark

Email: {fs,petarp,ttc,edc}@es.aau.dk

**Abstract**—Joint processing of multiple communication flows in wireless systems has given rise to a number of novel transmission techniques, notably the two-way relaying based on wireless network coding. Recently, a related set of techniques has emerged, termed coordinated direct and relay (CDR) transmissions, where the constellation of traffic flows is more general than the two-way. Regardless of the actual traffic flows, in a CDR scheme the relay has a central role in managing the interference and boosting the overall system performance. In this paper we investigate the novel transmission modes, based on amplify-and-forward, that arise when the relay is equipped with multiple antennas and can use beamforming. We focus on one representative traffic type, with one uplink and one downlink users and consider the achievable sum-rate maximization relay beamforming. The beamforming criterion leads to a non-convex problem and we introduce a low-complexity iterative solution, as well as derive a sum-rate upper performance bound. Numerical results demonstrate an obvious benefit from the usage of multiple antennas at the relay node.

**Index Terms**—Analog network coding, amplify-and-forward, beamforming, a priori information.

## I. INTRODUCTION

Recently there have been extensive studies on cooperative, relay-based transmission schemes for extending cellular coverage or increasing diversity. Several basic relaying transmission techniques have been introduced, such as amplify-and-forward (AF) [1], decode-and-forward (DF) [2] and compress-and-forward (CF) [3].

These transmission techniques have been applied in one-way, two-way or multi-way relaying scenarios. There has been a particularly high interest in two-way relaying scenarios [4], [5], [6], where throughput gains have been demonstrated by utilizing the ideas of wireless network coding. The two underlying principles used in designing throughput-efficient schemes with wireless network coding:

- 1) Aggregation of multiple communication flows: instead of transmitting each flow independently, the principle of network coding is used in which flows are sent/processed jointly;
- 2) Intentional cancellable interference: network coding allows interference and simultaneous usage of the shared wireless medium, leaving to the receivers to remove the adverse impact of interference by using any side information.

Leveraging on these principles, we have proposed schemes with non-regenerative AF relaying in [7], [8] that feature more

general traffic patterns compared to the two-way relaying. These schemes are termed *coordinated direct/relay* (CDR) transmissions. The CDR transmission considers scenarios where one direct user (UE) and one relayed UE are served in uplink/downlink. The relayed UE is assumed to have no direct link to the base station (BS) due to large path loss and relies only on the amplified/forwarded signal from the relay in order to decode the signal from the BS. Transmission schemes that are related to some of the schemes have appeared before in [9], [10], [11].

Each user might have a downlink or uplink traffic. Hence, there are different traffic configurations. We focus on one representative traffic type with one relayed uplink UE and one direct downlink UE. This case displays the merits of analog network coding in a setting that is more general than the usual two-way relay scenario. Furthermore it showcases the principle of overheard information where a node overhears a signal that is not intended to itself and uses it as *a priori* information to cancel interference in an ulterior transmission phase.

In the scheme on Fig. 1, we assume that a relayed UE has one signal to deliver to the BS through the assistance of the relay station, while a direct user wants to receive a signal from the BS. Notice in a conventional wireless cellular system, these signals are sent over two orthogonal uplink and downlink phases for the two separate information flows, respectively. Instead in the CDR system, the BS first sends the signal to the direct UE and simultaneously the relayed UE transmits the signal to the relay station in phase 1. The relay receives two signals: the desired signal from the UE and an interfering signal from the BS. It does not decode the signals but instead forward them in phase 2 using the principle of analog network coding. The simultaneous two-flow transmissions improve the *spectral efficiency* compared to the conventional method. The key points are the BS can use the *a priori* information to perform self-interference cancellation and enable interference-free reception and decoding; the direct UE can use the overheard information in phase 2 to help decoding the desired signal.

In the works that deal with the CDR transmission, the relay has a central role in managing the interference. Therefore, in this work we investigate the qualitative changes and the performance improvements that arise when the relay node in the CDR schemes is equipped with multiple antennas.

Differently from the previous works, the usage of multiple antennas at the relay permits to manage the interference and boost the overall system performance through beamforming. This is a significant conceptual difference compared to the original CDR schemes, while the usage of multiple antennas at the BS and the UEs is a clear future work. We consider AF operation at the relay, assuming that the relay and the reception nodes have a perfect channel state information (CSI). Our objective is to maximize the achievable sum-rate of the system. Our design shows that the overall system performance is improved by allowing the relay beamformer to deliver the interfered signal to both the BS and the direct UE in phase 2. Meanwhile,

- the BS completely cancels the self-interference;
- the direct UE applies linear interference minimization receiver to decode the desired signal.

The sum-rate maximization is achieved by a low-complexity iterative algorithm. Simulation results show a clear tradeoff between the rate maximization of individual information flows. The gain via possessing multiple relay antennas is also shown compared to the original CDR transmission.

*Notation:* We use uppercase and lowercase boldface letters to represent matrices and vectors, respectively.  $\otimes$  refers to the Kronecker product and  $\|\cdot\|_F^2$  denotes the Frobenius norm of a matrix.  $\mathbb{I}$  is the identity matrix.

## II. SYSTEM OVERVIEW

The basic setup is the scenario in Fig. 1 with one BS, one relay, and two UEs. The relay is equipped with  $M$  antennas. The BS and the UEs are equipped with one antenna only. The transmission from the relayed UE to the relay has the same duration as the transmission from the relay to the BS. The relay is deployed to help the relayed UE which has no direct link to the BS due to large path loss.

We consider the multi-antenna relay beamforming design where there are two information flows: the relayed UE (UE 1) delivers  $x_1$  to the BS and the BS transmits  $x_2$  to the direct UE (UE 2). The conventional system will create two orthogonal transmissions for separate information flows, while the CDR system enables simultaneous transmissions and thus improves the system spectral efficiency. Here we illustrate the two-phase CDR transmission in Fig. 1. In the first slot, UE 1 transmits  $x_1$  to the relay and the BS delivers  $x_2$  to UE 2 simultaneously. At the same time, UE 2 overhears the signal from UE 1 and the relay also receives the signal from the BS. Then the relay forwards the received physical layer network-coded signal to both the BS and the direct UE in the second slot. The BS has the capability to use the *a priori* information it transmits in the first slot to perform self-interference cancellation.

In summary, Fig. 2 describes the two flows and the analog network coding principle at the relay. The green lines describe the uplink traffic from the relayed UE and the red lines represent the downlink to the direct UE. This depicts a more general traffic pattern compared to the two-way relaying.

In this CDR system, each channel is assumed to be an independent complex Gaussian random variable with zero

mean and unit variance. All links are assumed to be static within the two slots. Assume  $P$  to be the transmit power of the BS and each UE, the received signals at the relay and UE 2 in the first slot are

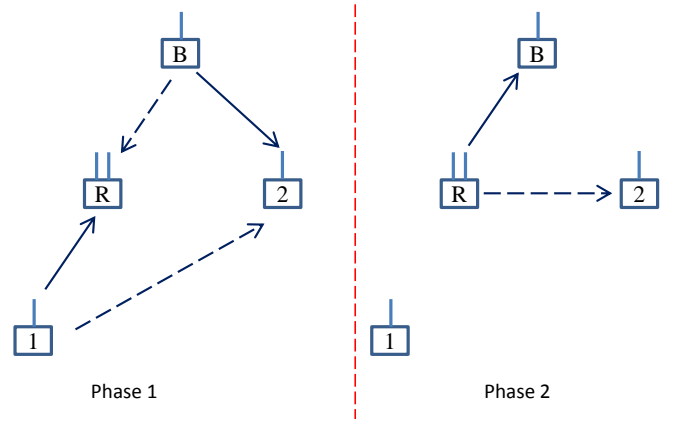


Figure 1. CDR MIMO System Model.

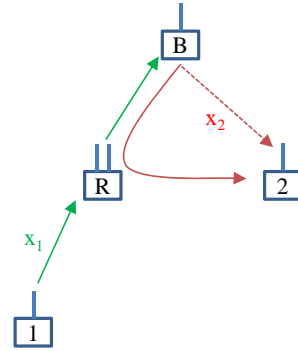


Figure 2. Two Information Flows in CDR.

$$\begin{aligned} y_R &= \sqrt{P}h_{R1}x_1 + \sqrt{P}h_{RB}x_2 + \mathbf{n}_R \\ y_2[1] &= \sqrt{P}h_{21}x_1 + \sqrt{P}h_{2B}x_2 + n_2[1] \end{aligned} \quad (1)$$

where  $\mathbf{n}_R$  is the complex white Gaussian noise vector at the relay with the covariance matrix  $\mathbb{E}[\mathbf{n}_R\mathbf{n}_R^H] = \mathbb{I}$  and  $n_2[1]$  is the complex white Gaussian noise variable at UE 2 in the first slot with unit variance<sup>1</sup>. The received signals at the BS and UE 2 in the second slot are

$$\begin{aligned} y_B &= \mathbf{h}_{BR}\mathbf{x}_R + n_B \\ y_2[2] &= \mathbf{h}_{2R}\mathbf{x}_R + n_2[2] \end{aligned} \quad (2)$$

where the signal vectors transmitted from the relay is in the form  $\mathbf{x}_R = \mathbf{W}y_R$  with  $\mathbf{W}$  being the  $M \times M$  relay beamforming matrix. At the relay,  $\mathbf{W}$  is used here to linearly process  $M \times 1$  received signal vector and form the  $M \times 1$  transmit signal vector without loss of generality.  $n_B$  and  $n_2[2]$  are the complex white Gaussian noise variables with unit

<sup>1</sup>In this work, we assume the variance of each noise component is normalized.

variance each at the BS and UE 2 respectively. The relay transmission power<sup>2</sup> is

$$\begin{aligned} & \mathbb{E}[\mathbf{x}_R^H \mathbf{x}_R] \\ &= \text{Tr}(P\mathbf{W}\mathbf{h}_{RB}\mathbf{h}_{RB}^H\mathbf{W}^H + P\mathbf{W}\mathbf{h}_{R1}\mathbf{h}_{R1}^H\mathbf{W}^H + \mathbf{W}\mathbf{W}^H) \\ &= P(\mathbf{h}_{RB}^H\mathbf{W}^H\mathbf{W}\mathbf{h}_{RB} + \mathbf{h}_{R1}^H\mathbf{W}^H\mathbf{W}\mathbf{h}_{R1}) + \|\mathbf{W}\|_F^2 = P_R. \end{aligned}$$

### III. SUM-RATE MAXIMIZATION FOR COORDINATED RELAY BEAMFORMING DESIGN

From the previous illustration, the relay has the capability to beamform the received network-coded signal and forwards the beamformed signal to both the BS and the direct UE in the second phase of the CDR transmission. Via sum-rate maximal relay beamforming design, the overall CDR system performance is enhanced by allowing the relay to balance between maximizing the rate of the transmission from the relayed UE to the BS and rate of the transmission from the BS to the direct UE. The central role of the relay in balancing the two information flows can be observed in Fig. 2. We also incorporate an iterative low-complexity algorithm to achieve the sum-rate maximization and a sum-rate upper bound.

#### A. Problem Formulation

The sum-rate maximization beamforming design problem can be mathematically formulated as

$$\begin{aligned} & \arg \max_{\mathbf{W}} (R_1 + R_2) \\ & \text{s.t. } P(\mathbf{h}_{RB}^H\mathbf{W}^H\mathbf{W}\mathbf{h}_{RB} + \mathbf{h}_{R1}^H\mathbf{W}^H\mathbf{W}\mathbf{h}_{R1}) \\ & \quad + \|\mathbf{W}\|_F^2 = P_R \end{aligned}$$

where  $R_1$  and  $R_2$  denote the rate expressions for the transmission of  $x_1$  and  $x_2$ , respectively. The rate expression for each information flow can be written as

$$\begin{aligned} R_1 &= \frac{1}{2} \log_2(1 + \text{SNR}_1) \\ R_2 &= \frac{1}{2} \log_2(1 + \text{SINR}_2) \end{aligned}$$

where  $\text{SNR}_1$  is the SNR expression for the BS to decode  $x_1$  and  $\text{SINR}_2$  is the SINR expression for the direct UE to decode  $x_2$ . And the factor  $\frac{1}{2}$  is due to the two time slots transmission duration. This is because from the analog network coding principle,  $x_2$  is known *a priori* at the BS and the related interference is mitigated via the self-interference cancellation process. Therefore, there is no interference when the BS wants to decode  $x_1$ . Notice we then use linear receivers in the CDR system to decode the desirable signals at the BS and the direct UE.

Using the monotonicity of the log function and the property  $\log(ab) = \log(a) + \log(b)$ , the sum-rate maximization problem can be rewritten as

<sup>2</sup>Relay full power transmission is not necessarily the optimal strategy and relay power optimization is identified as a future task.

$$\begin{aligned} & \arg \max_{\mathbf{W}} [(1 + \text{SNR}_1)(1 + \text{SINR}_2)] \\ & \text{s.t. } P(\mathbf{h}_{RB}^H\mathbf{W}^H\mathbf{W}\mathbf{h}_{RB} + \mathbf{h}_{R1}^H\mathbf{W}^H\mathbf{W}\mathbf{h}_{R1}) \\ & \quad + \|\mathbf{W}\|_F^2 = P_R. \end{aligned} \quad (3)$$

We first take a look at the SNR and SINR expressions for both UEs. For the BS, after self-interference cancellation, we will have  $\hat{y}_B = \sqrt{P}\mathbf{h}_{BR}\mathbf{W}\mathbf{h}_{R1}x_1 + \mathbf{h}_{BR}\mathbf{W}\mathbf{n}_R + n_B$ . Then the SNR at the BS is expressed as

$$\text{SNR}_1 = \frac{P\mathbf{h}_{BR}\mathbf{W}\mathbf{h}_{R1}\mathbf{h}_{R1}^H\mathbf{W}^H\mathbf{h}_{BR}^H}{\mathbf{h}_{BR}\mathbf{W}\mathbf{W}^H\mathbf{h}_{BR}^H + 1}. \quad (4)$$

Meanwhile, the direct UE uses  $y_2[1]$  from the first slot and  $y_2[2]$  from the second slot to form a virtual 2-antenna received signal vector  $\mathbf{y}_2$

$$\begin{aligned} \mathbf{y}_2 &= \begin{bmatrix} y_2[1] \\ y_2[2] \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{P}h_{2B} \\ \sqrt{P}\mathbf{h}_{2R}\mathbf{W}\mathbf{h}_{RB} \end{bmatrix} x_2 + \begin{bmatrix} \sqrt{P}h_{21} \\ \sqrt{P}\mathbf{h}_{2R}\mathbf{W}\mathbf{h}_{R1} \end{bmatrix} x_1 \\ & \quad + \begin{bmatrix} n_2[1] \\ \mathbf{h}_{2R}\mathbf{W}\mathbf{n}_R + n_2[2] \end{bmatrix} \\ &= \mathbf{h}_2x_2 + \mathbf{n}_2 \end{aligned} \quad (5)$$

where  $\mathbf{n}_2$  denotes the effective noise vector and  $\mathbf{h}_2$  denotes the effective channel vector for  $x_2$ . Then the direct UE wants to estimate the desired signal  $x_2$  and  $x_1$  is the interference from the other information flow. Although the minimum mean squared error-successive interference cancellation (MMSE-SIC) receiver is highly desirable among many interference cancellation schemes, we use linear receiver at the direct UE to aim for a low computational complexity<sup>3</sup>. The MMSE receiver in [12] is applied to estimate  $x_2$  from  $\mathbf{y}_2$  directly without SIC. The corresponding SINR at UE 2 is derived from  $\text{SINR}_2 = \mathbf{h}_2^H \{\mathbb{E}[\mathbf{n}_2\mathbf{n}_2^H]\}^{-1} \mathbf{h}_2$  with a complete form detailed in (7) in the Appendix.

The following lemma demonstrates the main result of the problem formulation and is proved in the Appendix. We use mainly the vectorization operation to convert the beamforming matrix to an equivalent vector  $\tilde{\mathbf{w}}$ .

**Lemma 1.** *The sum-rate maximization beamforming design is equivalent to maximizing the product of two fractional quadratic functions*

$$\arg \max_{\tilde{\mathbf{w}}} G(\tilde{\mathbf{w}}) = \arg \max_{\tilde{\mathbf{w}}} \left[ \frac{\tilde{\mathbf{w}}^H \mathbf{A} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{B} \tilde{\mathbf{w}}} \times \frac{\tilde{\mathbf{w}}^H \mathbf{C} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{D} \tilde{\mathbf{w}}} \right]$$

where matrices  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$ , and  $\mathbf{D}$  are not dependent on  $\tilde{\mathbf{w}}$ .

<sup>3</sup>The MMSE-SIC receiver changes the overall optimization problem formulation and is out of the scope of this work. It will surely bring sum-rate gain and will be considered in a future work.

### B. Sum-Rate Maximization Optimization Method

This is a non-convex problem, where global optimum solution is difficult to obtain within reasonable computation time. This optimization problem has generally no closed form solution. Well-known iterative methods can be applied such as simulated annealing and genetic algorithms which require very high computational load. We propose a low-complexity iterative algorithm that attempts to obtain a solution to the Karush-Kuhn-Tucker (KKT) conditions. The first order necessary condition  $\frac{\partial G(\tilde{\mathbf{w}})}{\partial \tilde{\mathbf{w}}} = 0$  leads to

$$\begin{aligned} & G(\tilde{\mathbf{w}}) [(\tilde{\mathbf{w}}^H \mathbf{B} \tilde{\mathbf{w}}) \mathbf{D} + (\tilde{\mathbf{w}}^H \mathbf{D} \tilde{\mathbf{w}}) \mathbf{B}] \tilde{\mathbf{w}} \\ &= [(\tilde{\mathbf{w}}^H \mathbf{C} \tilde{\mathbf{w}}) \mathbf{A} + (\tilde{\mathbf{w}}^H \mathbf{A} \tilde{\mathbf{w}}) \mathbf{C}] \tilde{\mathbf{w}} \end{aligned}$$

which can be rewritten as  $G(\tilde{\mathbf{w}}) \mathbf{V}(\tilde{\mathbf{w}}) \tilde{\mathbf{w}} = \mathbf{R}(\tilde{\mathbf{w}}) \tilde{\mathbf{w}}$ . Notice  $\mathbf{V}(\tilde{\mathbf{w}})$  and  $\mathbf{R}(\tilde{\mathbf{w}})$  depend on the unknown  $\tilde{\mathbf{w}}$ . If the dependence could be removed, then the optimizer  $\tilde{\mathbf{w}}$  is obviously the eigenvector corresponding to the largest eigenvalue of the matrix  $\mathbf{V}^{-1} \mathbf{R}$ . However, eigenvalue decomposition of the matrix  $[\mathbf{V}(\tilde{\mathbf{w}})]^{-1} \mathbf{R}(\tilde{\mathbf{w}})$  can not be accomplished in closed form. Consequently, we propose an iterative algorithm which finds the principal eigenvector corresponding to the maximum eigenvalue in  $[\mathbf{V}(\tilde{\mathbf{w}})]^{-1} \mathbf{R}(\tilde{\mathbf{w}})$  iteratively. This algorithm comes from the power iteration idea in [13], [14]. The proposed algorithm is described in Algorithm 1.

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#### Algorithm 1

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Initialization: set  $n = 0$  and  $\tilde{\mathbf{w}}^{(0)} = \tilde{\mathbf{w}}^{(\text{init})}$

iterate

update  $n = n + 1$

1)  $\mathbf{q}^{(n)} = [\mathbf{V}(\tilde{\mathbf{w}}^{(n)})]^{-1} \times [\mathbf{R}(\tilde{\mathbf{w}}^{(n)})] \tilde{\mathbf{w}}^{(n)}$

2)  $\tilde{\mathbf{w}}^{(n+1)} = \sqrt{P_R} \mathbf{q}^{(n)} / \|\mathbf{q}^{(n)}\|_2$

until  $G(\tilde{\mathbf{w}}^{(n+1)})$  or sum-rate convergence

---

Since the optimization problem is non-convex, the proposed algorithm cannot guarantee convergence. The convergence behavior of the proposed algorithm is shown numerically. For different relay antenna numbers, the sum-rate results versus the iterations are plotted in Fig. 3. The sum-rate is averaged over a sufficient number of channel realizations when SNR equals to 10 dB and  $P = P_R$ . The results show good convergence property of the proposed low-complexity algorithm: 20 iterations appear to be sufficient. Therefore, Algorithm 1 is seen to converge and provides a good sub-optimal solution to the sum-rate maximal beamforming design problem with relatively low computational complexity.

### C. Sum-Rate Upper Bound

A sum-rate upper bound for this CDR system with one information flow from the relayed UE to the BS and one information flow from the BS to the direct UE is derived in this section. The upper bound will be used to characterize the sum-rate loss resulting from Algorithm 1 which is a low-complexity sub-optimal solution.

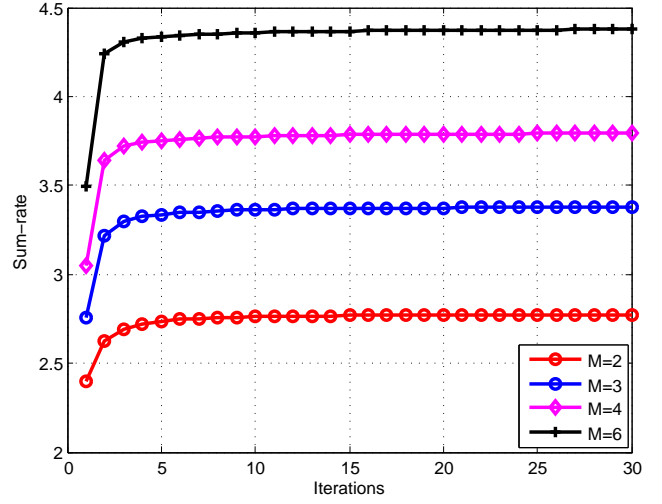


Figure 3. Convergence property according to the number of relay antennas.

A sum-rate upper bound for the two-way multi-antenna AF relay system with single-antenna UEs is given in [15]. Following [15], we consider different relay beamforming matrices  $\mathbf{W}_1$  and  $\mathbf{W}_2$  are used for the link from the relayed UE and the link to the direct UE, independently. The total power of the relay is allocated to support both links of communication to maximize the total sum-rate. Then an upper bound on the sum-capacity is obtained from:

$$\begin{aligned} & \max_{\mathbf{W}_1, \mathbf{W}_2} \quad \frac{1}{2} \log_2 [1 + \text{SNR}_1(\mathbf{W}_1)] \\ & \quad + \frac{1}{2} \log_2 [1 + \text{SINR}_2(\mathbf{W}_2)] \\ & \text{s.t.} \quad P(\mathbf{h}_{R1}^H \mathbf{W}_1^H \mathbf{W}_1 \mathbf{h}_{R1} + \mathbf{h}_{RB}^H \mathbf{W}_2^H \mathbf{W}_2 \mathbf{h}_{RB}) + \\ & \quad \kappa_1 \|\mathbf{W}_1\|_F^2 + \kappa_2 \|\mathbf{W}_2\|_F^2 = P_R \end{aligned} \quad (6)$$

where  $\text{SNR}_1(\mathbf{W}_1)$  is a function of  $\mathbf{W}_1$  and  $\text{SINR}_2(\mathbf{W}_2)$  is a function of  $\mathbf{W}_2$ . And  $\kappa_1$  and  $\kappa_2$  are non-negative and fulfilling  $\kappa_1 + \kappa_2 = 1$ . The tightest upper bound based on (6) is

$$R_{\text{UB}} = \min_{\kappa_1 + \kappa_2 = 1} \max_{P_1 + P_2 = P_R} R_1(\kappa_1, P_1) + R_2(\kappa_2, P_2)$$

with

$$\begin{aligned} R_1(\kappa_1, P_1) &= \max_{\mathbf{W}_1} \frac{1}{2} \log_2 [1 + \text{SNR}_1(\mathbf{W}_1)] \\ & \text{s.t.} \quad P \mathbf{h}_{R1}^H \mathbf{W}_1^H \mathbf{W}_1 \mathbf{h}_{R1} + \kappa_1 \|\mathbf{W}_1\|_F^2 \leq P_1 \end{aligned}$$

$$\begin{aligned} R_2(\kappa_2, P_2) &= \max_{\mathbf{W}_2} \frac{1}{2} \log_2 [1 + \text{SINR}_2(\mathbf{W}_2)] \\ & \text{s.t.} \quad P \mathbf{h}_{RB}^H \mathbf{W}_2^H \mathbf{W}_2 \mathbf{h}_{RB} + \kappa_2 \|\mathbf{W}_2\|_F^2 \leq P_2. \end{aligned}$$

where  $R_1(\kappa_1, P_1)$  and  $R_2(\kappa_2, P_2)$  can be obtained via solving the above two sub-problems. The solutions to the two sub-problems can be derived via the generalized Rayleigh quotient. However, no closed form solution exists for  $R_{\text{UB}}$  and

numerical search over  $\kappa_1$ ,  $\kappa_2$ ,  $P_1$  and  $P_2$  is required. A simple bound (less tight compared to  $R_{UB}$ )  $R_{UB}^{(0)} = R_1(0.5, P_R) + R_2(0.5, P_R)$  can be used instead.

#### IV. NUMERICAL RESULTS

In this section, we present simulation results for the rates of each flow as well as the sum-rate. We assume the relay and the BS have the same transmit power, i.e.  $P_R = P$ . The simulation results are generated via the Monte Carlo technique which averages over a sufficient number of channel realizations.

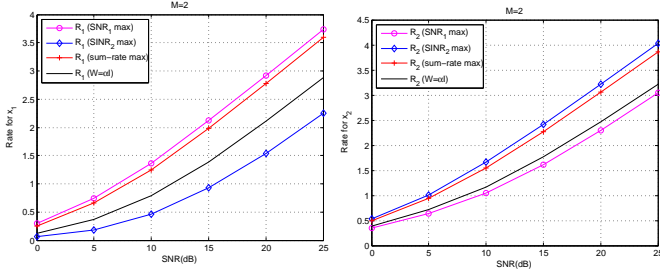


Figure 4. Rates for the two flows ( $M = 2$ ).

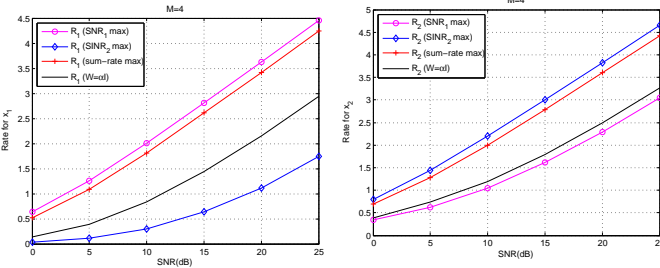


Figure 5. Rates for the two flows ( $M = 4$ ).

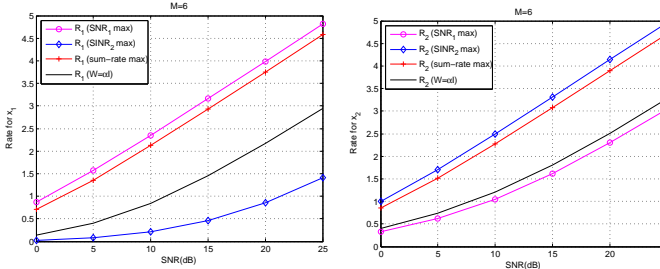


Figure 6. Rates for the two flows ( $M = 6$ ).

#### A. Benchmarks

The relay beamforming targeting  $\text{SNR}_1$  maximization and the relay beamforming aiming at  $\text{SINR}_2$  maximization are also included to evaluate the individual rate performance. These two designs maximizing individual information flows are straightforwardly solved based on the generalized Rayleigh quotient and will not be detailed in this work. In addition, to assess the effect of linear relay beamforming, the trivial pure amplification relaying  $\mathbf{W} = \alpha \mathbb{I}$  with  $\alpha = \sqrt{P_R / (P \mathbf{h}_{RB}^H \mathbf{h}_{RB} + P \mathbf{h}_{R1}^H \mathbf{h}_{R1} + \|\mathbb{I}\|_F^2)}$  accounting for the total relay power constraint is also considered. The benchmark

with single antenna relay is included to evaluate the gain from deploying multiple antennas at the relay.

#### B. Individual Rate Performance

Fig. 4, Fig. 5 and Fig. 6 compare the individual rates when the number of relay antenna is  $M = 2, 4, 6$ . We use  $R_1$  and  $R_2$  to denote the rates for the transmission from the relayed UE and the transmission to the direct UE, respectively. The proposed algorithm performs close to the relay beamforming maximizing  $\text{SNR}_1$  in  $R_1$  evaluation. When evaluating  $R_2$ , our algorithm provides a lightly lower rate performance compared to the  $R_2$  optimum beamforming targeting  $\text{SINR}_2$  maximization. Therefore, the iterative sum-rate maximization algorithm performs very close to the optimal rates for individual information flows. The proposed algorithm gives a tradeoff between  $R_1$  maximization and  $R_2$  maximization. It is also observed that the pure amplification relaying  $\mathbf{W} = \alpha \mathbb{I}$  causes a significant performance loss. Hence, we neglect the pure amplification relaying in the sum-rate evaluation.

#### C. Sum-Rate Performance

Fig. 7 and Fig. 8 show the sum-rate performance comparison with respect to different relay antenna numbers. The iterative technique is better than the relay beamforming design targeting either  $\text{SNR}_1$  maximization or  $\text{SINR}_2$  maximization. And the proposed algorithm performs close to the loose sum-rate upper bound. Therefore, the proposed iterative algorithm is an efficient tool to address sum-rate maximization of the multi-antenna AF CDR system, although it is sub-optimal. Furthermore, the sum-rate gain from the multiple-antenna relay beamforming is obvious, compared to the single antenna relay transmission. With the increase of the number of antennas at relay, we can see a clear increase in the sum-rate performance.

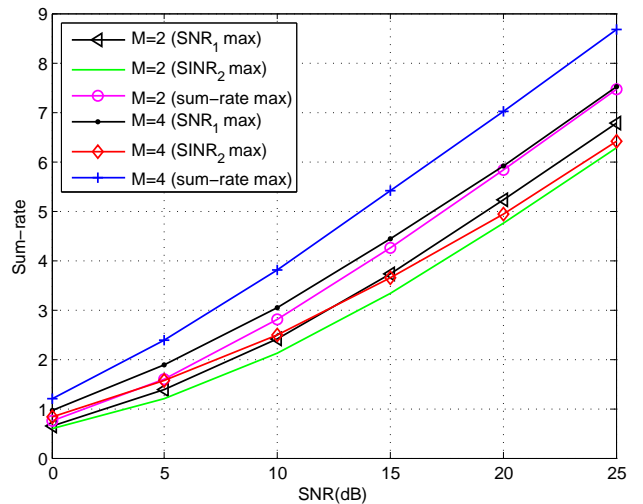


Figure 7. Sum-rate performance.

$$\begin{aligned}
\text{SINR}_2 &= \mathbf{h}_2^H \{ \mathbb{E} [\mathbf{n}_2 \mathbf{n}_2^H] \}^{-1} \mathbf{h}_2 \\
&= \begin{bmatrix} \sqrt{P} h_{2B} \\ \sqrt{P} \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{RB} \end{bmatrix}^H \begin{bmatrix} 1 & 0 \\ 0 & \mathbf{h}_{2R} \mathbf{W} \mathbf{W}^H \mathbf{h}_{2R}^H + 1 \end{bmatrix}^{-1} \begin{bmatrix} \sqrt{P} h_{2B} \\ \sqrt{P} \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{RB} \end{bmatrix} \\
&= \frac{(P^2 |h_{21}|^2 + P) \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{RB} \mathbf{h}_{RB}^H \mathbf{W}^H \mathbf{h}_{2R}^H + P |h_{2B}|^2 (\mathbf{h}_{2R} \mathbf{W} \mathbf{W}^H \mathbf{h}_{2R}^H + 1) + P^2 |h_{2B}|^2 \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{R1} \mathbf{h}_{R1}^H \mathbf{W}^H \mathbf{h}_{2R}^H}{P \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{R1} \mathbf{h}_{R1}^H \mathbf{W}^H \mathbf{h}_{2R}^H + (P |h_{21}|^2 + 1) (\mathbf{h}_{2R} \mathbf{W} \mathbf{W}^H \mathbf{h}_{2R}^H + 1)} \\
&\quad - \frac{P^2 h_{2B} h_{21}^* \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{R1} \mathbf{h}_{RB}^H \mathbf{W}^H \mathbf{h}_{2R}^H + P^2 h_{21} h_{2B}^* \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{RB} \mathbf{h}_{R1}^H \mathbf{W}^H \mathbf{h}_{2R}^H}{P \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{R1} \mathbf{h}_{R1}^H \mathbf{W}^H \mathbf{h}_{2R}^H + (P |h_{21}|^2 + 1) (\mathbf{h}_{2R} \mathbf{W} \mathbf{W}^H \mathbf{h}_{2R}^H + 1)} \tag{7}
\end{aligned}$$

$$\begin{aligned}
\arg \max_{\mathbf{W}} & \frac{P \mathbf{h}_{BR} \mathbf{W} \mathbf{h}_{R1} \mathbf{h}_{R1}^H \mathbf{W}^H \mathbf{h}_{BR}^H + \mathbf{h}_{BR} \mathbf{W} \mathbf{W}^H \mathbf{h}_{BR}^H + 1}{\mathbf{h}_{BR} \mathbf{W} \mathbf{W}^H \mathbf{h}_{BR}^H + 1} \times [1 + \\
& \frac{(P^2 |h_{21}|^2 + P) \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{RB} \mathbf{h}_{RB}^H \mathbf{W}^H \mathbf{h}_{2R}^H + P |h_{2B}|^2 (\mathbf{h}_{2R} \mathbf{W} \mathbf{W}^H \mathbf{h}_{2R}^H + 1) + P^2 |h_{2B}|^2 \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{R1} \mathbf{h}_{R1}^H \mathbf{W}^H \mathbf{h}_{2R}^H}{P \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{R1} \mathbf{h}_{R1}^H \mathbf{W}^H \mathbf{h}_{2R}^H + (P |h_{21}|^2 + 1) (\mathbf{h}_{2R} \mathbf{W} \mathbf{W}^H \mathbf{h}_{2R}^H + 1)} \\
& + \frac{-P^2 h_{2B} h_{21}^* \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{R1} \mathbf{h}_{RB}^H \mathbf{W}^H \mathbf{h}_{2R}^H - P^2 h_{21} h_{2B}^* \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{RB} \mathbf{h}_{R1}^H \mathbf{W}^H \mathbf{h}_{2R}^H}{P \mathbf{h}_{2R} \mathbf{W} \mathbf{h}_{R1} \mathbf{h}_{R1}^H \mathbf{W}^H \mathbf{h}_{2R}^H + (P |h_{21}|^2 + 1) (\mathbf{h}_{2R} \mathbf{W} \mathbf{W}^H \mathbf{h}_{2R}^H + 1)}] \\
\text{s.t.} & P (\mathbf{h}_{RB}^H \mathbf{W}^H \mathbf{W} \mathbf{h}_{RB} + \mathbf{h}_{R1}^H \mathbf{W}^H \mathbf{W} \mathbf{h}_{R1}) + \|\mathbf{W}\|_F^2 = P_R \tag{8}
\end{aligned}$$

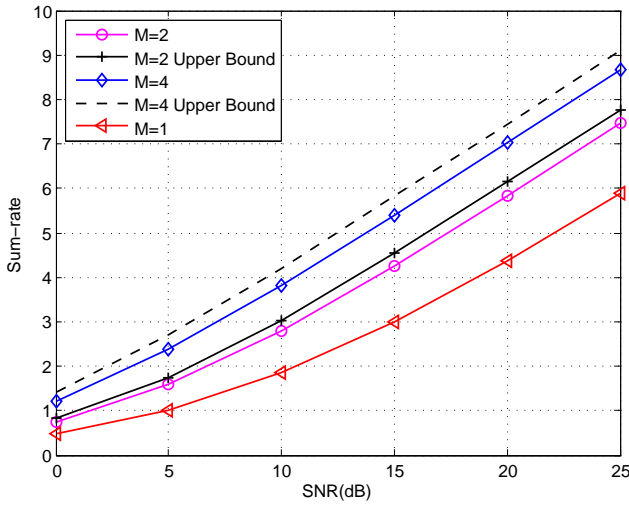


Figure 8. Sum-rate performance.

## V. CONCLUSIONS AND FUTURE WORKS

We focus on the design for the relay beamforming of the AF CDR system. Relay beamforming design for sum-rate maximization is considered. We propose a low-complexity but efficient iterative algorithm to achieve the sum-rate maximization and derive the upper bound of the achievable sum-rate. Numerical results confirm that the proposed iterative design gives comparable sum-rate and performs close to the loose upper bound as well as show a obvious sum-rate increase from the usage of multiple relay antennas.

## APPENDIX A PROOF OF LEMMA 1

*Proof:* From (4) and (7), the problem (3) can be reformulated in (8). In order to rewrite the optimization cost function in a simple way, the beamforming matrix  $\mathbf{W}$  is converted into a vector form using the vectorization operation,  $\mathbf{w} = \text{vec}(\mathbf{W})$ . With the property  $\text{vec}(\mathbf{M}\mathbf{W}\mathbf{N}) = (\mathbf{N}^T \otimes \mathbf{M})\text{vec}(\mathbf{W})$ , we can rewrite the problem in (9).

For the next step, we introduce matrix  $\mathbf{J}$  from the Cholesky decomposition

$$P(\mathbf{h}_{RB}^T \otimes \mathbb{I})^H (\mathbf{h}_{RB}^T \otimes \mathbb{I}) + P(\mathbf{h}_{R1}^T \otimes \mathbb{I})^H (\mathbf{h}_{R1}^T \otimes \mathbb{I}) + \mathbb{I} \triangleq \mathbf{J}^H \mathbf{J}.$$

We let  $\tilde{\mathbf{w}} = \mathbf{J}\mathbf{w}$ . When applying  $\mathbf{w} = \mathbf{J}^{-1}\tilde{\mathbf{w}}$ , the problem can be finally reformulated in (10). We further observe that the norm of  $\tilde{\mathbf{w}}$  does not influence the maximization at all. Hence, the constraint can be ignored. This transforms the problem (10) into an unconstrained maximization problem.

After some mathematical manipulations, it can be readily observed the reformulated sum-rate maximization beamforming design problem is in the form of

$$\arg \max_{\tilde{\mathbf{w}}} G(\tilde{\mathbf{w}}) = \arg \max_{\tilde{\mathbf{w}}} \left[ \frac{\tilde{\mathbf{w}}^H \mathbf{A} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{B} \tilde{\mathbf{w}}} \times \frac{\tilde{\mathbf{w}}^H \mathbf{C} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \mathbf{D} \tilde{\mathbf{w}}} \right]$$

where matrices  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{C}$ , and  $\mathbf{D}$  are not dependent on  $\tilde{\mathbf{w}}$ . This completes the proof.  $\blacksquare$

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$$\begin{aligned}
& \arg \max_{\mathbf{w}} \frac{P\mathbf{w}^H(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{BR})^H(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{BR})\mathbf{w} + \mathbf{w}^H(\mathbb{I} \otimes \mathbf{h}_{BR})^H(\mathbb{I} \otimes \mathbf{h}_{BR})\mathbf{w} + 1}{\mathbf{w}^H(\mathbb{I} \otimes \mathbf{h}_{BR})^H(\mathbb{I} \otimes \mathbf{h}_{BR})\mathbf{w} + 1} \times \{1 + \\
& \frac{\mathbf{w}^H [(P^2|h_{21}|^2 + P)(\mathbf{h}_{RB}^T \otimes \mathbf{h}_{2R})^H(\mathbf{h}_{RB}^T \otimes \mathbf{h}_{2R}) + P|h_{2B}|^2(\mathbb{I} \otimes \mathbf{h}_{2R})^H(\mathbb{I} \otimes \mathbf{h}_{2R})]\mathbf{w} + P|h_{2B}|^2}{P\mathbf{w}^H(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R})^H(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R})\mathbf{w} + (P|h_{21}|^2 + 1)[\mathbf{w}^H(\mathbb{I} \otimes \mathbf{h}_{2R})^H(\mathbb{I} \otimes \mathbf{h}_{2R})\mathbf{w} + 1]} \\
& + \frac{\mathbf{w}^H [P^2|h_{2B}|^2(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R})^H(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R})]\mathbf{w}}{P\mathbf{w}^H(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R})^H(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R})\mathbf{w} + (P|h_{21}|^2 + 1)[\mathbf{w}^H(\mathbb{I} \otimes \mathbf{h}_{2R})^H(\mathbb{I} \otimes \mathbf{h}_{2R})\mathbf{w} + 1]} + \\
& \frac{\mathbf{w}^H [-P^2h_{2B}h_{21}^*(\mathbf{h}_{RB}^T \otimes \mathbf{h}_{2R})^H(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R}) - P^2h_{21}h_{2B}^*(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R})^H(\mathbf{h}_{RB}^T \otimes \mathbf{h}_{2R})]\mathbf{w}}{P\mathbf{w}^H(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R})^H(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R})\mathbf{w} + (P|h_{21}|^2 + 1)[\mathbf{w}^H(\mathbb{I} \otimes \mathbf{h}_{2R})^H(\mathbb{I} \otimes \mathbf{h}_{2R})\mathbf{w} + 1]} \} \\
& s.t. \quad \mathbf{w}^H [P(\mathbf{h}_{RB}^T \otimes \mathbb{I})^H(\mathbf{h}_{RB}^T \otimes \mathbb{I}) + P(\mathbf{h}_{R1}^T \otimes \mathbb{I})^H(\mathbf{h}_{R1}^T \otimes \mathbb{I}) + \mathbb{I}]\mathbf{w} = P_R
\end{aligned} \tag{9}$$

$$\begin{aligned}
& \arg \max_{\tilde{\mathbf{w}}} \frac{\tilde{\mathbf{w}}^H \left\{ \mathbf{J}^{-H} [P(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{BR})^H(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{BR}) + (\mathbb{I} \otimes \mathbf{h}_{BR})^H(\mathbb{I} \otimes \mathbf{h}_{BR})] \mathbf{J}^{-1} + \frac{1}{P_R}\mathbb{I} \right\} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \left[ \mathbf{J}^{-H}(\mathbb{I} \otimes \mathbf{h}_{BR})^H(\mathbb{I} \otimes \mathbf{h}_{BR})\mathbf{J}^{-1} + \frac{1}{P_R}\mathbb{I} \right] \tilde{\mathbf{w}}} \times \{1 + \\
& \frac{\tilde{\mathbf{w}}^H \left\{ \mathbf{J}^{-H} [(P^2|h_{21}|^2 + P)\mathbf{M} + P|h_{2B}|^2\mathbf{N} + P^2|h_{2B}|^2\mathbf{K} - P^2h_{2B}h_{21}^*\mathbf{T} - P^2h_{21}h_{2B}^*\mathbf{T}^H] \mathbf{J}^{-1} + \frac{P|h_{2B}|^2}{P_R}\mathbb{I} \right\} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^H \left\{ \mathbf{J}^{-H} [(P|h_{21}|^2 + 1)(\mathbb{I} \otimes \mathbf{h}_{2R})^H(\mathbb{I} \otimes \mathbf{h}_{2R}) + P(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R})^H(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R})] \mathbf{J}^{-1} + \frac{P|h_{21}|^2 + 1}{P_R}\mathbb{I} \right\} \tilde{\mathbf{w}}} \} \\
& s.t. \quad \tilde{\mathbf{w}}^H \tilde{\mathbf{w}} = P_R \\
& \mathbf{M} = (\mathbf{h}_{RB}^T \otimes \mathbf{h}_{2R})^H(\mathbf{h}_{RB}^T \otimes \mathbf{h}_{2R}) \quad \mathbf{N} = (\mathbb{I} \otimes \mathbf{h}_{2R})^H(\mathbb{I} \otimes \mathbf{h}_{2R}) \\
& \mathbf{K} = (\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R})^H(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R}) \quad \mathbf{T} = (\mathbf{h}_{RB}^T \otimes \mathbf{h}_{2R})^H(\mathbf{h}_{R1}^T \otimes \mathbf{h}_{2R})
\end{aligned} \tag{10}$$

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