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# PROBABILISTIC DESIGN OF OFFSHORE STRUCTURAL SYSTEMS

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# Probabilistic Design of Offshore Structural Systems

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Probabilistic design of structural systems is considered in this paper. The reliability is estimated using first-order reliability methods (FORM). The design problem is formulated as the optimization problem to minimize a given cost function such that the reliability of the single elements satisfies given requirements or such that the systems reliability satisfies a given requirement. Based on a sensitivity analysis optimization procedures to solve the optimization problems are presented. Two of these procedures solve the system reliability-based optimization problem sequentially using quasi-analytical derivatives. Finally an example of probabilistic design of an offshore structure is considered.

# Introduction

The reliability analysis in this paper is based on first-order reliability methods (FORM), see e.g. Madsen, Krenk & Lind [2] and Thoft-Christensen & Murotsu [6]. Reliability-based optimization problems are formulated with elements and systems reliability constraints, see Sørensen & Thoft-Christensen [4]. Next, a sensitivity analysis of the elements and systems reliability-based constraints are performed. This analysis assumes that the reliability is estimated by a first-order reliability method. Based on a general optimization algorithm, e.g. the NLPQL algorithm, Schittkowski [3], and on the results of the sensitivity analysis some new optimization procedures to solve the reliability-based optimization problems are described next.

Finally, in an example the proposed optimization procedures are compared. The example considered is a plane model of an offshore steel jacket platform, where the loads and the yield strengths of the tubular elements are modelled as random variables. The design variables are the thicknesses of the tubular elements and the shape of the structure.

# Reliability-Based Structural Optimization

In reliability-based structural analysis and design some of the quantities describing the load and/or the strength of the elements are modelled as random variables. The random variables are denoted $\overline{X} = (X_1, X_2, \ldots, X_n)$. A reliability model of the structural system is then formulated. The elements in this model are failure elements modelling potential failure modes of the elements of the structural system, e.g. yielding of a cross-section or fatigue failure of a tubular joint. Each failure element is described by a failure function $g(\overline{x}) = 0$. Realizations $\overline{x}$ of $\overline{X}$ where $g(\overline{x}) \leq 0$ correspond to failure states while $g(\overline{x}) > 0$ correspond to safe states.

In first-order reliability methods (FORM) the reliabilty index $\beta$ is determined as described in [2]. Let $\overline{U}$ be standardized and normally distributed variables and let $\overline{u}^*$ denote the design point. Linearization of the safety margin M in the design point gives

$$M = g(\overline{T}(\overline{U})) \approx \frac{\nabla_u g^T}{|\nabla_u g|}\overline{U} + \beta = -\overline{\alpha}^T\overline{U} + \beta \tag{1}$$

where $\nabla_u g$ is the gradient of $g$ with respect to $\overline{u}$ in the design point $\overline{u}^*$.

Each of the failure elements represents a potential failure mode. Therefore, the reliability model of the whole structural system can be modelled as a series system. Let the number of failure modes be m. If the linearized linear safety margins are used then the systems reliability index $\beta^s$ can be estimated from

$$\beta^s = -\Phi^{-1}\Big(1 - \Phi_m(\overline{\beta}; \overline{\overline{\rho}})\Big) \tag{2}$$

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where $\Phi_m(\cdot)$ is the m-dimensional normal distribution function. $\beta_1, \ldots, \beta_m$ are the reliability indices of the failure elements determined by the FORM analysis. The elements in the correlation coefficient matrix $\overline{\overline{\rho}}$ are determined by $\rho_{ij} = \overline{\alpha}_i{}^T \overline{\alpha}_j \quad i,j = 1,2,\cdots,m$

An element reliability-based optimization problem can now formulated as

$$\min \quad W(\overline{z}) \tag{3}$$

$$s.t. \quad \beta_i(\overline{z}) \geq \beta_i^{min} \qquad i = 1,2,\ldots,m \tag{4}$$

$$z_i^l \leq z_i \leq z_i^u \qquad i = 1,2,\ldots,N \tag{5}$$

$z_1, z_2, \ldots, z_N$ are the optimization variables and $\beta_1, \beta_2, \ldots, \beta_m$ are the reliability indices of the m failure elemnts. $W(\overline{z})$ is the objective function, e.g. the structural weight and $\beta^{min}$ is the minimum acceptable reliability index. $\overline{z}^l$ and $\overline{z}^u$ are lower and upper bounds of $\overline{z}$. Alternatively a system reliability index-based optimization problem can be formulated. Instead of (4) the following constraint is used

$$\beta^s(\overline{z}) \geq \beta^{min} \tag{6}$$

where $\beta^s$ is determined using (2). This optimization problem is in general a non-linear non-convex constrained optimization problem.

# Optimization Procedures

Let the failure functions be written as

$$g_i(\overline{u}, \overline{z}, \overline{\overline{b}}(\overline{z})) = 0 \qquad i = 1,2,\ldots,m \tag{7}$$

where $\overline{\overline{b}}$ is a matrix of coefficients of influence corresponding to unit loads on the structure. The coefficients of influence can e.g. be cross-sectional forces such as the axial force and the end moments of a beam element. Then it follows that

$$\frac{\partial \beta_i}{\partial z_j} = \frac{1}{|\nabla_u g_i|} \left[ \frac{\partial g_i}{\partial z_j} + \sum_{l=1}^{L} \sum_{k=1}^{K} \frac{\partial g_i}{\partial b_{lk}} \frac{\partial b_{lk}}{\partial z_j} \right] \tag{8}$$

L is the number of not fully correlated stochastic variables modelling the loads on the structure and K is the number of coefficients of influence for each load case. Usually the terms $\frac{\partial g}{\partial z_j}$ and $\frac{\partial g}{\partial b_{lk}}$ can easily be estimated numerically. The last term $\frac{\partial b_{lk}}{\partial z_j}$ is estimated using pseudo load vectors.

The sensitivity coefficients of the system reliability index $\beta^s$ can be estimated from

$$\frac{\partial \beta^s}{\partial z_j} = \sum_{i=1}^{m} \frac{\partial \beta^s}{\partial \beta_i} \frac{\partial \beta_i}{\partial z_j} + 2 \sum_{i<k}^{m} \frac{\partial \beta^s}{\partial \rho_{ik}} \frac{\partial \rho_{ik}}{\partial z_j} \tag{9}$$

where

$$\frac{\partial \beta^s}{\partial \beta_i} = \alpha_i^s = \frac{\varphi(\beta_i)}{\varphi(\beta^s)} \Phi_{m-1}(\overline{\beta}_i^a; \overline{\overline{\rho}}_i^a) \tag{10}$$

$$\frac{\partial \beta^s}{\partial \rho_{ik}} = \frac{\varphi_2(\beta_i, \beta_k; \rho_{ik})}{\varphi(\beta^s)} \Phi_{m-2}(\overline{\beta}_{ik}^b; \overline{\overline{\rho}}_{ik}^b) \tag{11}$$

$\overline{\beta}_i^a$ ,$\overline{\overline{\rho}}_i^a$ , $\overline{\beta}_{ik}^b$ and $\overline{\overline{\rho}}_{ik}^b$ are conditional reliability indices and correlation coefficients. $\frac{\partial \beta_i}{\partial z_j}$ is given by (8). $\frac{\partial \rho_{ik}}{\partial z_j}$ can be determined as described in Bjerager & Krenk [1]. Usually $\frac{\partial \beta^s}{\partial z_j}$ can be approximated with a sufficient degree of accuracy by neglecting the correlation terms.

In the optimization procedures proposed in the following the general non-linear programming algorithm NLPQL is used. The element reliability index-based optimization problem (3-5) can be solved directly using NLPQL. Gradients of the reliability constraints are determined using (8).

The system reliability index-based optimization problem (3),(5-6) can also be solved directly using NLPQL when gradients of the reliability constraint are determined by (9). However, such a procedure can be expected to be rather costly. Especially the terms concerning the correlation coefficients are computer time consuming. If the correlation coefficient terms are neglected convergence problems can be expected using a mathematical programming algorithm as NLPQL.

The system reliability index-based optimization problem can alternatively be solved by solving a sequence of element reliability index based problems, each of which usually converges quickly. In this sequence the reliability constraints are

$$\beta_i(\overline{z}) \geq \beta_i^k \qquad i = 1, 2, \ldots, m \tag{12}$$

where the sequence of lower bounds $\beta_i^k \quad, k = 0, 1, 2, \ldots$ is determined using

$$\overline{\beta}^{k+1} = \overline{\beta}^k + \Delta\overline{\beta}^k = \overline{\beta}^k + c\overline{d}^k \tag{13}$$

Initially for $k = 0$ : $\quad \overline{\beta}_i^0 = \beta^{min} \quad , i = 1, 2, \ldots, m$. The search direction $\overline{d}^k$ is determined from the following optimization problem

$$\min \quad -\beta^{sk} - \overline{\alpha}^{sk^T}\overline{d} \tag{14}$$

$$s.t. \quad \overline{\alpha}^{wk^T}\overline{d} = 0 \tag{15}$$

$$\overline{d}^T\overline{d} = 1 \tag{16}$$

where $\alpha_i^{wk} = \sum_{j=1}^{N} \frac{\partial W(\overline{z}^k)}{\partial z_j}\frac{\partial z_j}{\partial \beta_i}$ The constant c in (13) is determined such that

$$\beta^s \approx \beta^{sk} + \overline{\alpha}^{sk^T}\Delta\overline{\beta}^k = \beta^{min} \tag{17}$$

$\beta^{sk}$ is the systems reliability index corresponding to the solution of (3-5) and $\overline{\alpha}^{sk}$ is defined in (10).

A sequential procedure similar to the above can be formulated using the Lagrange multipliers $\overline{\lambda}^k$ obtained from the solution of the element reliability index based problems. In (15) we use

$$\alpha_i^{wk} = \lambda_i^k \tag{18}$$

# Example

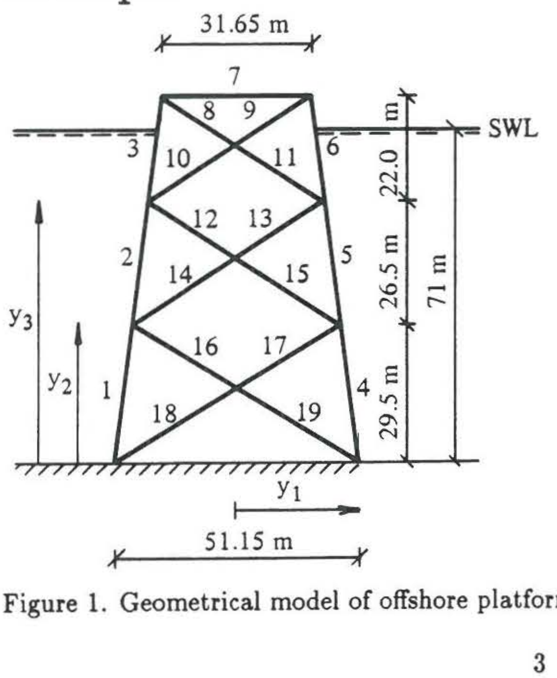


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Figure 1. Geometrical model of offshore platform.

The plane model of the tubular steel-jacket shown in figure 1 is considered. 62 failure elements (yielding, stability and punching) are used in the reliability model and 16 stochastic variables are used to model the uncertainty quantities, see Sørensen [5] for details. As design variables 6 tubular thicknesses and 3 shape variables $y_1, y_2$ and $y_3$, see fig. 1, are chosen. All structural elements are assumed to remain straight. The lower bounds on the thicknesses are 0.02 m and on ($y_1$, $y_2$, $y_3$) ($20.0m$, $20.0m$, $40.0m$) are chosen. $\beta^{min} = 4.5$ is used. Results of the element and systems reliability based problems are shown in table 1.

| variable | element | initial | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|
| $z_1$ | 1,4 | 0.040 | 0.0447 | 0.0479 | 0.0472 | 0.0469 |
| $z_2$ | 2,3,5,6 | 0.030 | 0.0352 | 0.0370 | 0.0372 | 0.0375 |
| $z_3$ | 8,9 | 0.035 | 0.0200 | 0.0200 | 0.0200 | 0.0200 |
| $z_4$ | 10,11 | 0.025 | 0.0200 | 0.0200 | 0.0200 | 0.0200 |
| $z_5$ | 12,13,14,15 | 0.030 | 0.0200 | 0.0200 | 0.0200 | 0.0200 |
| $z_6$ | 16,17,18,19 | 0.030 | 0.0200 | 0.0200 | 0.0200 | 0.0200 |
| $z_7$ | $y_1$ | 25.6 | 25.2 | 24.3 | 24.8 | 25.0 |
| $z_8$ | $y_2$ | 29.5 | 20.0 | 20.0 | 20.0 | 20.0 |
| $z_9$ | $y_3$ | 56.0 | 46.7 | 43.4 | 44.3 | 44.6 |
| $W$ | | 65.9 | 58.7 | 60.6 | 60.7 | 60.8 |
| iterations | | | 6 | 17 | 10 | 37 |
| time | | | 130 | 2958 | 216 | 833 |

Table 1. Results of optimization. 1: element reliability-based problem, 2-4: system reliability based problem ( 2: optimization problem solved using (9), 3: sequential procedure (12)-(17) used, 4: sequential procedure using (18)). Computer time is in sec. on a VAX 8700. All dimensions in m.

From the table it is seen that with element reliability constraints the objective function is reduced by about 11% and with systems reliability constraint by about 8%, while the systems reliability index is increased from 2.7 to 4.5. Further it is seen that the sequential procedures reduce the computer time drastically and give approximately the same solution as the 'exact' method.

# Conclusion

Element and system reliability-based structural design problems are formulated as optimization problems with constraints which signify that the reliability has to exceed given critical values.

Based on a sensitivity analysis of the reliability constraints sequential procedures to solve the optimization problems are presented. An example with an offshore steel platform is presented. The results show that for this example the proposed procedures to solve the design problems work well. This indicates that the procedures can be used in practical design of structural systems where the design variables are chosen as parameters of the structural elements or of the geometrical configuration.

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