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Abstract—**Joint processing of multiple communication flows in wireless systems has given rise to a number of novel transmission techniques, notably the two-way relaying, but also more general traffic scenarios, such as coordinated direct and relay (CDR) transmissions. In a CDR scheme the relay has a central role in managing the interference and boosting the overall system performance. In this letter we consider the case in which an amplify-and-forward relay has multiple antennas and can use beamforming to support the coordinated transmissions. We focus on one representative traffic type with one uplink user and one downlink user. Two different criteria for relay beamforming are analyzed: maximal weighted sum-rate and maximization of the worst-case weighted SNR. We propose iterative optimal solutions, as well as low-complexity near-optimal solutions.**

Index Terms—**Analog network coding, relay beamforming.**

I. INTRODUCTION

Joint processing of multiple flows is beneficial and has recently been a focus of extensive research, most notably in the area of two-way relaying (TWR), where throughput gains have been demonstrated by utilizing the ideas of wireless network coding. Various transmission designs have been proposed for multi-antenna relay beamforming in TWR systems [1] and the transceivers in multi-user TWR systems [2].

Leveraging on principles from wireless network coding, we have proposed *coordinated direct/relay* (CDR) transmission schemes with amplify-and-forward (AF) relaying in [3]. The CDR transmission considers wireless cellular scenarios with one direct user (UE) which is directly served by the base station (BS) and one relayed UE which requires a relay to facilitate the communication. The value of CDR lies in the fact that it extends the gains of the analog network coding introduced in the TWR scenarios: while the TWR scenarios feature a very specific two-way pattern, CDR shows how similar gain can be introduced for other traffic patterns.

The relayed UE is assumed to have no direct link to the BS due to large path loss and relies only on the ampli-fied/forwarded signal from the relay. Each user might have a downlink or uplink traffic and thus there are different traffic types. We focus on one representative type with one relayed uplink UE and one direct downlink UE. In this case, we are using the principles of analog network coding for traffic flows that are not related to the TWR traffic in an obvious way. Furthermore it showcases the principle of overheard information where a node overhears a signal that is not intended to itself and uses it as side information to cancel interference in a previous transmission phase.
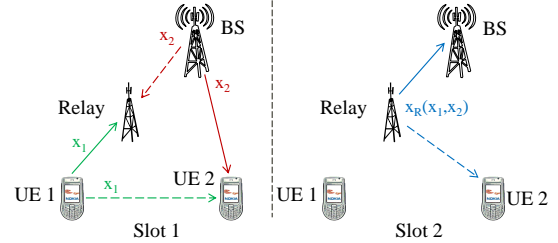
Figure 1. System Model.

In the scheme on Fig. 1 with one BS, one relay, and two UEs, a relayed UE has one signal to deliver to the BS, while a direct user wants to receive a signal from the BS. Notice in a conventional wireless cellular system, these signals are sent over two orthogonal uplink and downlink phases for the two separate information flows, respectively. Instead in the CDR system, the BS first sends the signal to the direct UE and simultaneously the relayed UE transmits the signal to the relay station in slot 1. The relay receives two signals: the desired signal from the UE and an interfering signal from the BS. It forwards them in slot 2 using the principle of network coding. The simultaneous two-flow transmissions improve the *spectral efficiency* compared to the conventional method.

In the CDR transmission, there is interference from the relay to the direct UE. The relay has a central role in managing the two information flows and it is therefore interesting to investigate how multiple antennas at the relay can improve the coordinated transmission scheme. A natural step for fu-ture work is to equip also the BS with multiple antennas; however, this is a highly non-trivial extension, with multiple possibilities for novel transmission schemes. Previously in [3], we have only considered a suboptimal algorithm for relay beamforming, assuming that the relay and the reception nodes have a perfect channel state information (CSI). In this work with the same CSI assumption, we target two general prob-lems: weighted sum-rate (WSR) maximization and worst-case weighted signal-to-noise ratio (SNR) maximization. The main difference is that in WSR maximization, we are simultaneously focusing on both flows; while in the worst-case problem we are still working with two-flows, but only optimizing the worse one. We propose iterative algorithms to obtain respective global optimal solution, based on the optimal result from a quality of service (QoS)-aware relay power minimization problem. In general, the weights are useful for prioritizing different flows and thus find various practical applications. We also propose one low-complexity suboptimal design, linear space spanning to approach the optimal solutions. The optimal solutions provide the performance upper bound for the low-complexity proposal, as well as other practical schemes.

Notation: $\otimes$ refers to the Kronecker product and $||\cdot||_F^2$ denotes the Frobenius norm. $\mathbb{I}$ is the identity matrix.

II. SYSTEM OVERVIEW

We consider a relay with M antennas and the BS and the UEs are equipped with one antenna. The transmission from the relayed UE to the relay has the same duration as the transmission from the relay to the BS. There are two information flows: the relayed UE (UE 1) delivers x_1 to the BS and the BS transmits x_2 to the direct UE (UE 2).
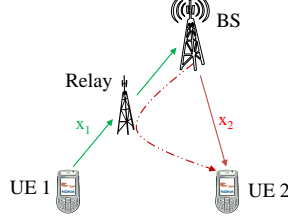

Figure 2. Two Information Flows.

Fig. 2 describes the two flows and the network coding principle essentially different from the conventional TWR scenario. The first phase of TWR network coding and CDR scheme is the same when UE 1 and the BS simultaneously transmit. However, in the second phase, instead of forwarding the signal to UE 1, the relay "re-directs" it to UE 2. Here UE 2 somehow plays the role of UE 1 in the TWR because it also overhears a signal from UE 1 as side information. Each channel element is assumed to be an independent complex Gaussian random variable with zero mean and unit variance. All links are assumed to be static within the two slots. Assume P to be the transmit power of the BS and each UE, the received signals at the relay and UE 2 in the first slot are

$$\begin{aligned} \mathbf{y}_R[1] &= \sqrt{P}\mathbf{h}_{R1}x_1 + \sqrt{P}\mathbf{h}_{RB}x_2 + \mathbf{n}_R \\ y_2[1] &= \sqrt{P}h_{21}x_1 + \sqrt{P}h_{2B}x_2 + n_2[1] \end{aligned} \quad (1)$$

where $\mathbf{n}_R$ is the complex white Gaussian noise vector at the relay with the covariance matrix $\mathbb{E}[\mathbf{n}_R\mathbf{n}_R^{\mathrm{H}}] = \mathbb{I}$ and $n_2[1]$ is the complex white Gaussian noise variable at UE 2 in the first slot with unit variance[1]. The received signals at the BS and UE 2 in the second slot are $y_B[2] = \mathbf{h}_{BR}\mathbf{x}_R + n_B$, $y_2[2] = \mathbf{h}_{2R}\mathbf{x}_R + n_2[2]$ where the signal vectors transmitted from the relay is in the form $\mathbf{x}_R = \mathbf{W}\mathbf{y}_R[1]$ with $\mathbf{W}$ being the $M \times M$ relay beamforming matrix. At the relay, $\mathbf{W}$ is used here to linearly process $M \times 1$ received signal vector and form the $M \times 1$ transmit signal vector. n_B and $n_2[2]$ are the complex white Gaussian noise variables with unit variance each at the BS and UE 2 respectively. The relay beamforming matrix is calculated based on the CSI of all links and will be broadcasted to both the BS and the direct UE before the actual relay beamforming phase. The relay transmit power is constrained not to exceed a power budget $\mathbb{E}[\mathbf{x}_R^{\mathrm{H}}\mathbf{x}_R] = P(\mathbf{h}_{RB}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{W}\mathbf{h}_{RB} + \mathbf{h}_{R1}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{W}\mathbf{h}_{R1}) + ||\mathbf{W}||_F^2 \le P_R$.

A. SNR Expressions

From the network coding principle, x_2 is known at the BS and the related interference is mitigated via self-interference cancellation. Therefore, there is no interference when the BS wants to decode x_1. Notice we then use linear receivers in the CDR system to decode the desired signals. For the BS, after self-interference cancellation, we will have $\hat{y}_B[2] =$

$\sqrt{P}\mathbf{h}_{BR}\mathbf{W}\mathbf{h}_{R1}x_1 + \mathbf{h}_{BR}\mathbf{W}\mathbf{n}_R + n_B$. Then the SNR at the BS to decode x_1 is expressed as $\mathrm{SNR}_1 = \frac{P\mathbf{h}_{BR}\mathbf{W}\mathbf{h}_{R1}\mathbf{h}_{R1}^{\mathrm{H}}\mathbf{W}^{\mathrm{H}}\mathbf{h}_{BR}^{\mathrm{H}}}{\mathbf{h}_{BR}\mathbf{W}\mathbf{W}^{\mathrm{H}}\mathbf{h}_{BR}^{\mathrm{H}}+1}$.

Meanwhile, the direct UE uses $y_2[1]$ from the first slot and $y_2[2]$ from the second slot to form a virtual 2-antenna received signal vector $\mathbf{y}_2 = \begin{bmatrix} y_2[1] & y_2[2] \end{bmatrix}^{\mathrm{T}}$

$$\mathbf{y}_2 = \begin{bmatrix} \sqrt{P}h_{2B} \\ \sqrt{P}\mathbf{h}_{2R}\mathbf{W}\mathbf{h}_{RB} \end{bmatrix} x_2 + \begin{bmatrix} \sqrt{P}h_{21} \\ \sqrt{P}\mathbf{h}_{2R}\mathbf{W}\mathbf{h}_{R1} \end{bmatrix} x_1 + \begin{bmatrix} n_2[1] \\ \mathbf{h}_{2R}\mathbf{W}\mathbf{n}_R + n_2[2] \end{bmatrix}.$$

The direct UE wants to estimate the desired signal x_2 while x_1 is the interference from the other information flow. We cancel the interference linearly using a zero-forcing (ZF) receiver at UE 2 for analytical convenience and $\mathrm{SNR}_2 = \frac{P\|h_{2B}\mathbf{h}_{2R}\mathbf{W}\mathbf{h}_{R1} - h_{21}\mathbf{h}_{2R}\mathbf{W}\mathbf{h}_{RB}\|_2^2}{|h_{21}|^2(\mathbf{h}_{2R}\mathbf{W}\mathbf{W}^{\mathrm{H}}\mathbf{h}_{2R}^{\mathrm{H}}+1) + \|\mathbf{h}_{2R}\mathbf{W}\mathbf{h}_{R1}\|_2^2}$. Comparing CDR to the TWR in [1], the main difference is that in TWR both flows have to traverse the relay, while in this CDR scenario UE 2 has a direct-link signal component. Moreover, using ZF at UE 2 results in a different SNR form compared to TWR.

To rewrite the expressions in a simple way, the beamforming matrix $\mathbf{W}$ is converted into a vector form using the vectorization operation, $\mathbf{w} = \mathrm{vec}(\mathbf{W})$ where $\mathbf{w}$ is a $M^2 \times 1$ complex vector. With $\mathrm{vec}(\mathbf{MWN}) = (\mathbf{N}^{\mathrm{T}} \otimes \mathbf{M})\mathrm{vec}(\mathbf{W})$, we have:

$$\mathrm{SNR}_1(\mathbf{w}) = \frac{\mathbf{w}^{\mathrm{H}}\mathbf{e}\mathbf{e}^{\mathrm{H}}\mathbf{w}}{\mathbf{w}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{w} + 1}$$

$$\mathrm{SNR}_2(\mathbf{w}) = \frac{\mathbf{w}^{\mathrm{H}}\mathbf{f}\mathbf{f}^{\mathrm{H}}\mathbf{w}}{\mathbf{w}^{\mathrm{H}}(\mathbf{C}^{\mathrm{H}}\mathbf{C} + \mathbf{b}\mathbf{b}^{\mathrm{H}})\mathbf{w} + |h_{21}|^2}$$

with $\mathbf{e} = \sqrt{P}(\mathbf{h}_{R1}^{\mathrm{T}} \otimes \mathbf{h}_{BR})^{\mathrm{H}}$, $\mathbf{G} = \mathbb{I} \otimes \mathbf{h}_{BR}$, $\mathbf{b} = (\mathbf{h}_{R1}^{\mathrm{T}} \otimes \mathbf{h}_{2R})^{\mathrm{H}}$, $\mathbf{C} = |h_{21}|(\mathbb{I} \otimes \mathbf{h}_{2R})$, $\mathbf{f}^{\mathrm{H}} = \sqrt{P}[h_{2B}(\mathbf{h}_{R1}^{\mathrm{T}} \otimes \mathbf{h}_{2R}) - h_{21}(\mathbf{h}_{RB}^{\mathrm{T}} \otimes \mathbf{h}_{2R})]$. The relay power is converted to be $\mathbf{w}^{\mathrm{H}}[\mathbf{D}^{\mathrm{H}}\mathbf{D} + \mathbf{E}^{\mathrm{H}}\mathbf{E} + \mathbb{I}]\mathbf{w}$ with $\mathbf{D} = \sqrt{P}(\mathbf{h}_{RB}^{\mathrm{T}} \otimes \mathbb{I})$, $\mathbf{E} = \sqrt{P}(\mathbf{h}_{R1}^{\mathrm{T}} \otimes \mathbb{I})$.

III. OPTIMIZING RELAY BEAMFORMING

The relay has the capability to beamform the received network-coded signal and forward the beamformed signal to both the BS and the direct UE. The central role of the relay to balance the two information flows, i.e. the transmission from the relayed UE and the transmission to the direct UE, can be observed in Fig. 2. We consider two ways for balancing the two traffic flows through two different optimization criteria: (1) weighted sum-rate maximization and (2) worst-case weighted SNR maximization.

A. Weighted Achievable Sum-Rate Maximization

The WSR maximization problem can be formulated as

$$\arg\max_{\mathbf{w}} \quad [\mu_1 C_1(\mathbf{w}) + \mu_2 C_2(\mathbf{w})] \quad (2)$$
$$\text{s. t.} \quad \mathbf{w}^{\mathrm{H}}[\mathbf{D}^{\mathrm{H}}\mathbf{D} + \mathbf{E}^{\mathrm{H}}\mathbf{E} + \mathbb{I}]\mathbf{w} \le P_R$$

where $C_1(\mathbf{w})$ and $C_2(\mathbf{w})$ denote the maximum achievable rates for the transmission of x_1 and x_2, respectively. And μ_1 and μ_2 are the corresponding rate rewards for C_1 and C_2. The rate expression for each information flow can be written as $R_1 \le C_1(\mathbf{w}) = \frac{1}{2}\log_2[1 + \mathrm{SNR}_1(\mathbf{w})]$ and $R_2 \le C_2(\mathbf{w}) = \frac{1}{2}\log_2[1 + \mathrm{SNR}_2(\mathbf{w})]$ where the factor $\frac{1}{2}$ comes from the 2 equal-duration phase transmissions. Using the monotonicity of the log function, the objective can be rewritten as $\arg\max_{\mathbf{w}}\{[1 + \mathrm{SNR}_1(\mathbf{w})]^{\mu_1}[1 + \mathrm{SNR}_2(\mathbf{w})]^{\mu_2}\}$. Because of the shapes of $\mathrm{SNR}_1(\mathbf{w})$ and $\mathrm{SNR}_2(\mathbf{w})$, the weighted

sum-rate maximization problem is a non-convex and NP-hard problem, where the global optimum solution is difficult to obtain within a reasonable computation time. This optimization problem has no general closed form solution.

Although the WSR problem in CDR differs from TWR due to the shape of $\mathrm{SNR}_2(\mathbf{w})$, we also resort to an alternative based on *rate-profile* similar to [1]. The rate-profile vector, defined as $\mathbf{a} = [\alpha_1 \, \alpha_2]^{\mathrm{T}}$, regulates the ratio between R_1, R_2 and the sum-rate $R_{\mathrm{sum}} = R_1 + R_2$ via $\alpha_1 = \frac{R_1}{R_{\mathrm{sum}}}$ and $\alpha_2 = \frac{R_2}{R_{\mathrm{sum}}}$ ($\alpha_1 + \alpha_2 = 1$). Each feasible $\mathbf{a}$ corresponds to a boundary point in the achievable rate region [1]. Given $\mathbf{a}$, we find the boundary point via (3), maximizing the sum-rate subject to the relay power and rate-profile constraints. Therefore, (2) can be solved by first discarding $\mu_i \, (i = 1, 2)$ and determining all the boundary rate-pairs obtained from (3) with respect to all different $\mathbf{a}$ values and then select the pair with highest WSR.

$$\mathbf{R}(P_R, \mathbf{a}) = \begin{cases} \max_{\mathbf{w}} & R_{\mathrm{sum}} \\ \text{s. t.} & C_i(\mathbf{w}) \geq \alpha_i R_{\mathrm{sum}} \, (i = 1, 2) \\ & \mathbf{w}^{\mathrm{H}} \left[\mathbf{D}^{\mathrm{H}}\mathbf{D} + \mathbf{E}^{\mathrm{H}}\mathbf{E} + \mathbb{I} \right] \mathbf{w} \leq P_R \end{cases} \tag{3}$$

In Algorithm 1, we use a one-dimensional bisection search to efficiently obtain the optimal sum-rate for the NP-hard problem (3). Note that R_{UB} is an upper bound on the sum-rate value (see Section III-C) and ϵ_R is a small positive constant to control the precision of the algorithm. At each bisection step, we need to verify whether it is feasible for the relay to support the rate-pair $\mathbf{a}R_0$, meaning whether the relay transmit power is sufficient. To do so, we determine the minimal transmit power at the relay subjecting to QoS constraints ($\mathrm{SNR}_i(\mathbf{w}) \geq r_i = 2^{2\alpha_i R_0} - 1$) in (4). A convergence proof of Algorithm 1 can be built based on *Appendix D* in [1].

Algorithm 1 Sum-rate max relay beamforming (given $\mathbf{a}$)

For given $\mathbf{a}$, set $R_{\min} = 0$ and $R_{\max} = R_{\mathrm{UB}}$
repeat until $R_{\max} - R_{\min} \leq \epsilon_R$
    I. set $R_0 = \frac{1}{2}(R_{\min} + R_{\max})$
    II. solve (4) and obtain P_r
        if (4) is feasible and $P_r \leq P_R$, $R_{\min} = R_0$
        else $R_{\max} = R_0$

Now we explain how to solve the relay power minimization:

$$\mathbf{P}_{\mathrm{SR}}(R_0, \mathbf{a}) = \begin{cases} \min_{\mathbf{w}} & P_r = \mathbf{w}^{\mathrm{H}} \left[\mathbf{D}^{\mathrm{H}}\mathbf{D} + \mathbf{E}^{\mathrm{H}}\mathbf{E} + \mathbb{I} \right] \mathbf{w} \\ \text{s. t.} & \frac{\mathbf{w}^{\mathrm{H}}\mathbf{e}\mathbf{e}^{\mathrm{H}}\mathbf{w}}{\mathbf{w}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{w}+1} \geq r_1 \\ & \frac{\mathbf{w}^{\mathrm{H}}\mathbf{f}\mathbf{f}^{\mathrm{H}}\mathbf{w}}{\mathbf{w}^{\mathrm{H}}\mathbf{C}^{\mathrm{H}}\mathbf{C}\mathbf{w}+|h_{21}|^2+\mathbf{w}^{\mathrm{H}}\mathbf{b}\mathbf{b}^{\mathrm{H}}\mathbf{w}} \geq r_2 \end{cases} \tag{4}$$

which is a non-convex quadratically constrained quadratic programming (QCQP) problem and hence NP-hard. Introducing $\mathbf{X} = \mathbf{w}\mathbf{w}^{\mathrm{H}}$, we can solve (4) by resorting to the widely used semidefinite programming (SDP) with rank relaxation by discarding the non-convex constraint $\mathrm{Rank}(\mathbf{X}) = 1$ [4], [5]:

$$\begin{cases} \min_{\mathbf{X}} & P_r = \mathrm{Tr}\left\{ \left(\mathbf{D}^{\mathrm{H}}\mathbf{D} + \mathbf{E}^{\mathrm{H}}\mathbf{E} + \mathbb{I} \right) \mathbf{X} \right\} \\ \text{s. t.} & \mathrm{Tr}\left\{ \left[\mathbf{e}\mathbf{e}^{\mathrm{H}} - r_1 \mathbf{G}^{\mathrm{H}}\mathbf{G} \right] \mathbf{X} \right\} \geq r_1, \, \mathbf{X} \succeq 0 \\ & \mathrm{Tr}\left\{ \left[\mathbf{f}\mathbf{f}^{\mathrm{H}} - r_2 \left(\mathbf{C}^{\mathrm{H}}\mathbf{C} + \mathbf{b}\mathbf{b}^{\mathrm{H}} \right) \right] \mathbf{X} \right\} \geq |h_{21}|^2 r_2. \end{cases}$$

where $\mathbf{X} \succeq 0$ means that $\mathbf{X}$ is positive semidefinite. Notice that the optimal $\mathbf{X}$ could have $\mathrm{Rank}(\mathbf{X}) > 1$, where the randomization technique [5] can be applied for near global optimal solution. However, for this special structure based on

Theorem 3.2 in [1], we can efficiently construct an optimal rank-one solution from the optimal $\mathbf{X}$. Therefore, the exact optimal solution for (4) can be efficiently obtained.

B. Worst-Case Weighted SNR Maximization

Denote $\beta_i > 0 \, (i = 1, 2)$ to be the priority factor associated with $\mathrm{SNR}_i \, (i = 1, 2)$. The max-min problem is written as

$$\mathbf{S}(P_R) = \begin{cases} \max_{\mathbf{w}} & \min_i \frac{\mathrm{SNR}_i(\mathbf{w})}{\beta_i} \\ \text{s. t.} & \mathbf{w}^{\mathrm{H}} \left[\mathbf{D}^{\mathrm{H}}\mathbf{D} + \mathbf{E}^{\mathrm{H}}\mathbf{E} + \mathbb{I} \right] \mathbf{w} \leq P_R \end{cases} \tag{5}$$

which maximizes the worst-case weighted SNR. In point-to-point MIMO systems, an algorithm to optimize the minimal SNR based on the connection with power minimization is proposed in [6]. Denote γ_0 to be the target weighted SNR value, we extend the connection to the CDR scenario via

$$\mathbf{P}_{\mathrm{SNR}}(\gamma_0) = \begin{cases} \min_{\mathbf{w}} & p_r = \mathbf{w}^{\mathrm{H}} \left[\mathbf{D}^{\mathrm{H}}\mathbf{D} + \mathbf{E}^{\mathrm{H}}\mathbf{E} + \mathbb{I} \right] \mathbf{w} \\ \text{s. t.} & \frac{1}{\beta_1} \frac{\mathbf{w}^{\mathrm{H}}\mathbf{e}\mathbf{e}^{\mathrm{H}}\mathbf{w}}{\mathbf{w}^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{w}+1} \geq \gamma_0 \\ & \frac{1}{\beta_2} \frac{\mathbf{w}^{\mathrm{H}}\mathbf{f}\mathbf{f}^{\mathrm{H}}\mathbf{w}}{\mathbf{w}^{\mathrm{H}}\mathbf{C}^{\mathrm{H}}\mathbf{C}\mathbf{w}+|h_{21}|^2+\mathbf{w}^{\mathrm{H}}\mathbf{b}\mathbf{b}^{\mathrm{H}}\mathbf{w}} \geq \gamma_0. \end{cases} \tag{6}$$

The constraints in (6) indicate that we target the same weighted SNR requirements for both flows while the constraints in (4) focus on different SNR requirements. We introduce the connection between (5) and (6) in Proposition 1 which can be proved by contradiction (similar to *Theorem 3* in [6]).

Proposition 1. *The relay power minimization problem (6) and the worst-case weighted SNR problem (5) are inverse problems. The optimal value for each problem is continuous and strictly monotonic increasing.*

To solve (5), the strict monotonicity and continuity guarantees a one-dimensional bisection search applied to (6) will obtain the optimal worst-case weighted SNR value. The algorithm is shown in Algorithm 2. Note that $\mathrm{WSNR}_{\min}$ and $\mathrm{WSNR}_{\max}$ define the range of weighted SNR values with detailed derivations in Section III-C. ϵ_r is a small positive constant. The convergence proof of Algorithm 2 can be built by contradiction and letting $\epsilon_r \to 0$, similar to *Appendix D* in [1]. To solve (6), we apply the same relaxed SDP as for (4).

Algorithm 2 Minimal weighted SNR max relay beamforming

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set $r_{\min} = \mathrm{WSNR}_{\min}$ and $r_{\max} = \mathrm{WSNR}_{\max}$
repeat until $r_{\max} - r_{\min} \leq \epsilon_r$
    I. set $r_0 = \frac{1}{2}(r_{\min} + r_{\max})$
    II. solve (6) and obtain p_r
        if (6) is feasible and $p_r \leq P_R$, $r_{\min} = r_0$
        else $r_{\max} = r_0$

C. Sum-Rate and Weighted SNR Ranges

The initial values for the two optimization algorithms are now discussed. Let us first consider two benchmarks: the relay beamforming designs targeting either $\mathrm{SNR}_1(\mathbf{w})$ or $\mathrm{SNR}_2(\mathbf{w})$ maximization. It can be easily proved that an increase of the relay power will increase both SNR values. Therefore $\mathrm{SNR}_1(\mathbf{w})$ and $\mathrm{SNR}_2(\mathbf{w})$ are optimized when the relay transmits at full power. We further introduce $\mathbf{J}^{\mathrm{H}}\mathbf{J} \triangleq \mathbf{D}^{\mathrm{H}}\mathbf{D} + \mathbf{E}^{\mathrm{H}}\mathbf{E} + \mathbb{I}$ and $\tilde{\mathbf{w}} = \mathbf{J}\mathbf{w}$ to transform the relay power constraint into $\tilde{\mathbf{w}}^{\mathrm{H}}\tilde{\mathbf{w}} = P_R$. Then, $\mathrm{SNR}_1(\mathbf{w})$ and $\mathrm{SNR}_2(\mathbf{w})$ are expressed as generalized Rayleigh quotients

$$g_1(\tilde{\mathbf{w}}) = \frac{\tilde{\mathbf{w}}^{\mathrm{H}}\mathbf{J}^{-\mathrm{H}}\mathbf{e}\mathbf{e}^{\mathrm{H}}\mathbf{J}^{-1}\tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^{\mathrm{H}}\left[\mathbf{J}^{-\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{J}^{-1} + \frac{1}{P_R}\mathbb{I} \right]\tilde{\mathbf{w}}},$$

$$g_2(\tilde{\mathbf{w}}) = \frac{\tilde{\mathbf{w}}^{\mathrm{H}} \mathbf{J}^{-\mathrm{H}} \mathbf{f} \mathbf{f}^{\mathrm{H}} \mathbf{J}^{-1} \tilde{\mathbf{w}}}{\tilde{\mathbf{w}}^{\mathrm{H}} \left[\mathbf{J}^{-\mathrm{H}} \left(\mathbf{C}^{\mathrm{H}} \mathbf{C} + \mathbf{b} \mathbf{b}^{\mathrm{H}} \right) \mathbf{J}^{-1} + \frac{|h_{21}|^2}{P_R} \mathbb{I} \right] \tilde{\mathbf{w}}}.$$

The two vectors $\tilde{\mathbf{w}}_1$ and $\tilde{\mathbf{w}}_2$ maximizing g_1 and g_2 separately can be easily obtained. Thus, $R_{\mathrm{UB}} = R_1(\tilde{\mathbf{w}}_1) + R_2(\tilde{\mathbf{w}}_2)$. From [3], a suitable SNR range are $g_1(\tilde{\mathbf{w}}_2) \leq \mathrm{SNR}_1 \leq g_1(\tilde{\mathbf{w}}_1)$, $g_2(\tilde{\mathbf{w}}_1) \leq \mathrm{SNR}_2 \leq g_2(\tilde{\mathbf{w}}_2)$ because one flow is optimized when we are sacrificing the other flow. For the weighted SNR problem, $\mathrm{WSNR}_{\min} = \min \left[\frac{g_1(\tilde{\mathbf{w}}_2)}{\beta_1}, \frac{g_2(\tilde{\mathbf{w}}_1)}{\beta_2} \right]$ and $\mathrm{WSNR}_{\max} = \max \left[\frac{g_1(\tilde{\mathbf{w}}_1)}{\beta_1}, \frac{g_2(\tilde{\mathbf{w}}_2)}{\beta_2} \right]$.

Meanwhile, from the bisection principle, the number of iterations required for Algorithm 1 or Algorithm 2 can be obtained directly from the initial values and the threshold for the stopping criteria, i.e. ϵ_R or ϵ_r.

D. Low-Complexity Design

The optimal solutions involve SDP optimization and optimal rank-one construction via linear programming (LP) [1] inside each bisection search step. We also propose low-complexity suboptimal designs for both problems, inspired by [7]. The design is termed to be the linear space spanning (LSS), where the solution is chosen to lie in the linear space spanned by $\tilde{\mathbf{w}}_1$ and $\tilde{\mathbf{w}}_2$, $\tilde{\mathbf{w}}_L = a\tilde{\mathbf{w}}_1 + b\tilde{\mathbf{w}}_2$ (a and b are real value parameters). It is obvious that any scaling of a does not change the values for $g_1(\tilde{\mathbf{w}}_L)$ and $g_2(\tilde{\mathbf{w}}_L)$. Therefore, $\tilde{\mathbf{w}}_L$ is further simplified by letting $a = 1$ and $\tilde{\mathbf{w}}_L = \tilde{\mathbf{w}}_1 + b\tilde{\mathbf{w}}_2$. Therefore, the suboptimal designs will perform maximization of the WSR objective and the weighted SNR objective over b_{WSR} and b_{SNR} without other constraints, respectively. For either problem, a grid search using the Nelder-Mead method [7] is applied to efficiently find the solution. Then the obtained beamforming solution is scaled to satisfy the relay power constraint, which does not change the WSR and weighted SNR values.

The optimization in the suboptimal design is over one real variable. Thus the complexity reduction is huge compared to the optimal solutions. However, this advantage is obtained at the expense of a performance loss shown in Section IV.

IV. NUMERICAL RESULTS

In this section, we present simulation results with $M = 2$ antennas at the relay and all transmission nodes having the same transmit power, i.e. $P_R = P$. The thresholds for the two algorithms are $\epsilon_R = \epsilon_r = 0.01$. The two benchmarks targeting individual SNR maximization are included. Fig. 3 shows the sum-rate with $\mu_1 = \mu_2 = 1$. The proposed optimal and suboptimal solutions provide non-trivial gains compared to the two benchmarks. LSS gives performance close to the optimal solution. The conventional two orthogonal uplink and downlink transmission with three equal slots is constrained to use the same total duration as CDR and the multi-antenna relay helps only the flow from UE 1; it has a huge sum-rate loss compared to the optimal CDR transmission. We also show the worst-case SNR performance with $\beta_1 = \beta_2 = 1$ in Fig. 4 where LSS is observed to perform close to the optimal solution. The conventional transmission is not suitable to be included for SNR comparison.
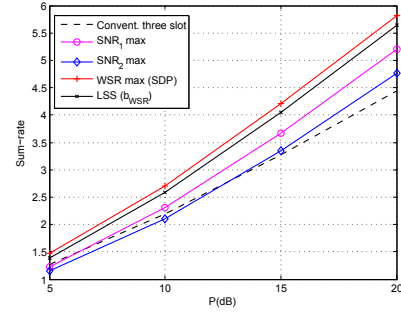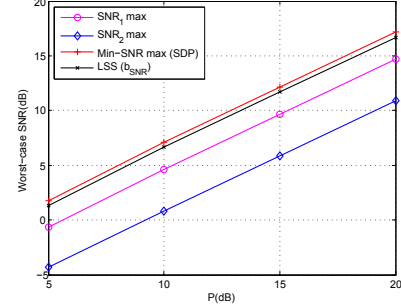

Figure 3. Sum-rate performance.

Figure 4. Worst-case SNR performance.

V. CONCLUSIONS

We have investigated the problem of beamforming optimization at a relay that participates in a coordinated direct and relay (CDR) transmission with one uplink user and one downlink user. Two different performance criteria have been considered for optimization, weighted sum-rate maximization and maximization of the worst-case weight SNR. We have devised methods to obtain the optimal solutions and proposed low-complexity alternatives with slightly suboptimal performance.

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