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an Experimental Approach

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On the Constitutional Choice of a Democratic Voting Rule: An experimental approach

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Abstract. *In this work we attempt to evaluate the performance of different voting rules subject to different sets of voters and parties. The method of investigation is experimental: Different sets of agents have been artificially generated and the performance of different voting rules have been evaluated relative to a simple additive social evaluation function. Our experiments suggest that "winner take all" voting schemes outperform voting schemes that are based on proportional representation.*

Keywords: Constitutional voting rules, direct vs. representative democracy, multiple issues, multiple candidates, binary choice functions.

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1. INTRODUCTION

Even though modern democracies have existed for more than 200 there is still little agreement on which particular voting rule should be regarded the superior foundation for collective decision making. Early in the second half of this century the search for one universal voting rule was hampered by emergence of the impossibility theorems presented by Arrow (1951) and followers.[1] Regardless of the difficulties raised within social choice theory there is general consensus that democratic societies function better and are ethically superior to non-democratic ones. But to survive through time a democratic society has to be based on agreement concerning principles which guarantee the implementation of fixed rules for changes of political leadership. If there exist no implicit or explicit contract within society which allows the chosen voting rule to function regardless of the specific historical determined distribution of preferences, the survival of democracy is endangered because some citizens might believe that changes in voting rules might be made in order to favour the maintenance of power by the current leadership. Such a situation would be perceived as undemocratic by (some) citizens and would most likely lead to social unrest. It is therefore important that the choice of the voting rules to select the body of representatives is made at the constitutional level. But in a second best world with absence of perfect aggregation mechanisms and no theoretical consensus on which voting rule society should adopt to make collective decisions, it seems important to ask on a pragmatic basis which type of mechanism democracies should use to guide their collective actions. Our contribution in this direction is made following an experimental approach with the use of computer simulations.

Studies on (optimal) collective decision procedures can be grouped into several heavily related categories. A first categorisation can be made in terms of the axiomatic foundation.[2] In these studies citizens' preferences are described by a limited set of axioms that are used to define *rational optimal* decisions. Given the individual choice functions the conditions under which a unique optimal social welfare function exists are studied and the results turn out bo be rather restrictive.

A second categorisation can be made for those studies in which focus is put on the specifics of

[1]Most of the literature in the wake of Arrows impossibility theorem has concentrated on relaxation and reinterpretation of the assumptions in Arrow (1951). Sen (1970) constitutes another excellent analysis of the basic social choice problem. A collection of important work in the area can be found in Rowley (1993).

[2]This is the strand of literature normally referred to as social choice theory, see also footnote 1.

the decision (voting) mechanism. In these studies different voting procedures are evaluated against specific criteria which generally are regarded as good measures of democratic performance, fairness or social welfare.[3] These studies typically restrict themselves to look at direct decision procedures in the sense that focus is placed on the search for characteristics of different voting rules, given some specific assumptions regarding voters and alternatives of choice. In this respect the literature on voting rules concentrate on voting within the committee (or society) thereby largely ignoring the issues brought up by the two-layer structure of representative democracy.[4]

Finally, another categorisation can be made in terms of that strand of literature, mainly within the public choice tradition, which starts out by assuming that democracy is representative; i.e.,collective decisions are made by an elected subset of the agents, often organised in political parties, who through their decisions affect the welfare of all agents in society. Although some branches within this strand of literature concentrates on the workings of different voting rules; Austen-Smith and Banks (1988), Ortuno-Ortin (1997), Persson, Roland and Tabellini (1997), Osborne and Slivinski (1996) and Hamlin and Hjortlund (1998) all constitute recent examples, most studies concentrate on the particular institutional and behavioural effects induced by majority voting.[5]

In the present work, given that we want to provide a contribution to the question of which voting rule the society should adopt for collective decision making, we assume that a modern democracy must be representative in nature, i.e. that direct democracy is the ideal democratic institution but that costs of operating a direct democracy are too high for it to be practically implementable given the number of agents within each collective cohort and the number of questions to be resolved by collective decisions. By taking this view we implicitly follow the long tradition within political theory which regards direct decision procedures as the first best democratic rule in an ideal

[3]See for example Black (1958), Fishburn (1973,1974), Chamberlin and Cohen (1978), Merrill (1984,1985), Bordley (1983,1985,1986), Nurmi (1992) and Kollman et al. (1997).

[4]Exceptions are Black (1958) and Chamberlin and Courant (1983).

[5]Hotelling and Downs (1957) are the main intellectual pioneers in the tradition of spatial politico-economic equilibrium models. Subsequently a vast amount of literature have emerged in this tradition. Recent developments which include substantial differences from the original set up include work on the threat of entry (Palfrey (1984), equilibrium with abstention (Ledyard (1984), Brennan and Hamlin (1998b)) and the Citizen-Candidate model (Osborne and Slivinski (1996), Besley and Coate (1997).

world.[6] But at the same time we attempt to evaluate different voting schemes in relation to some normative criterion just as it has been done within the empirical social choice literature.

In our study we have created artificial societies through computer simulations in which political interaction is carried out. The artificial environments consist of a number of voters and a number of political candidates who each have binary and separable preferences over a k-dimensional issue space. We have carried out a high number of elections using the following procedure: Before each election an exogenously determined number of parties announce their preferred policy (other policy announcements are not credible). A set of voters, identified by their preferences, are artificially created. Voters vote honestly for the party with whom they identify the most. Given a set of voters and a set of candidates elections are simulated according to three different voting schemes. The voting schemes used are plurality rule (PL), run-off election (RO) and proportional representation (PR). While the two "winner take all schemes" are defined in a traditional Downsian manner, we have chosen a formulation of the political process under proportional representation rule which can be seen as somewhat unusual because policy is the outcome of separate issue majority voting in a parliament with perfect transformation of votes into voting power: Each candidate gets a vote-share in the legislative body which is perfectly proportional to her vote-share in the election and then policy is decided by majority voting on each issue separately.

The three voting schemes are evaluated with the use of a unweighted additive social evaluation function where the individual voters consensus with respect to the realised policy for each individual issue is aggregated in the same way in which one makes opinion polls. In this way we avoid the problem of comparing and aggregating individual utilities, but keep an operational measurement of the consensus of the people. From the simulations carried out we are also able to compare the performance of the voting schemes relative to the outcome that would have been prevailing under direct democracy.

Although our way of conceptualising the problem of choosing a specific voting scheme for democracy resembles some features which have been put forward in earlier work, cfr. Black

[6]Chamberlin and Courant (1983) offer some good citings dating back to Sterne on this issue. Brennan and Hamlin (1998b) give an excellent discussion on the basic difference between direct democracy and representative democracy as ideal types.

(1958), Fishburn (1973,1974), Merrill (1984,1985), Chamberlin and Cohen (1978), Chamberlin and Courant (1983), Chamberlin and Featherston (1986), Bordley (1983, 1985), Nurmi (1992) and Kollman et al (1997), the analysis differs quite sharply from these studies in several aspects. Most studies on voting rules use real valued preference orderings either in a single dimension or in multiple dimensions and a Euclidian measure of distance/utility. On the contrary we start out by assuming that political issues are binary in nature, i.e.,we assume that a binary choice has to be made on each political issue.[7] Kollman et al (1997) also use a binary formulation of politics in their computational model of Tiebout competition. While they justify their choice of binary issue spaces with the need to have simple model, we would like to make a few comments concerning the use of binary issue spaces, since we find that it have some additional advantages compared with the traditional conceptualisation of political domain in Euclidian space. Because the binary formulation of issue spaces constitutes the opposite benchmark case from the Euclidian formulation of policy spaces it could be argued that, as a starting point at least, the purely binary set up should be preferred to a combination of real valued and binary issue spaces. Moreover, in reality many political questions are clear-cut binary in nature. Prominent examples are choices such as whether to rely on nuclear power for energy or whether society should allow abortion etc. Also, from the theoretical point of view it can be claimed that any choice, as complex as it might be, can be seen as a composition of simpler basic binary choices, Cfr. Arrow (1959). The act of choosing is in itself a binary act where the actual choice is discriminated with respect to all other alternatives. Furthermore we claim that none of the actual world issues may be described by a real valued function. The encoding of any possible political decision can belong at most to the domain of the rational numbers (Cfr. Rustem and Velupillai (1990), Velupillai (1991), Zambelli (1994). This is to do with the fact that any issue has to be described in an understandable way, normally words; i.e. laws are written using an official state language. But any encoding of words has to belong to the domain of rational numbers. It is clear that a non computable irrational number cannot be communicated to others and therefore cannot be part of a specific ruling.

Apart from these theoretical underpinnings, we find anyway that binary choice functions on most issues represent a far more realistic conception of agents' preferences. It is after all rare to see political parties promising an explicit tax rate in an election campaign rather than observe them proposing higher or lower taxes. Also, weighted against the small influence any individual has

[7]Faith and Buchanan (1981) constitutes an earlier example of a model with a binary formulation of policy.

on the final outcome of mass elections it seems to be a very strong assumption that voters know all candidates' (and their own) exact positions on all issues, especially when politics is multi dimensional. Thus, contrary to most studies of electoral equilibrium, we insist that voters and candidates conceptualise collective choices as being binary and strictly separable. To make as much use of this conceptualisation as possible it is also assumed that voters use simple Boolean distance to measure how well off they are with different candidates in office, i.e.,voters compare candidates to find the candidate who agree with them on most issues. Following this line of argument we also use Boolean distance to compute the overall democratic performance of the voting schemes employed in the model. In this interpretation a good voting scheme is simply one which on average is capable of selecting a state of the world from the set of possible states of the world, in which a high fraction of voters agree with the implemented policy on many issues. Therefore, a typical social welfare function is not used to evaluate the performance of different voting schemes; instead we are measuring the social agreement with the state of the world implemented through a representative democratic process. This way of evaluating voting schemes is different from most studies which normally use either a social welfare function or, following Black (1958), the ability of a given voting rule to choose the Condorcet winner (if one exists) as the main normative criterion. Although the choice of a simple additive evaluation function is naturally connected to the binary formulation of the political issue space we could have used a traditional utility-based representation of agents' preferences and any real valued social welfare function as our normative criterion.[8] Without complicating matters greatly we could, for example, have attached different weights to each individual and to the salience of each issue in the formulations of agents preferences.[9] Such a decision might at first glance seem quite appealing but we have decided to go without it because such a weighting system is bound to be ad hoc in nature when we are dealing with artificially agents created without any underlying empirical support.[10] Thus each issue is equally important to all agents and each agent and each issue is equally important in our calculations of democratic performance. Therefore we restrict ourselves to discuss democratic performance rather than social welfare.

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[8]Following Harsanyi, Bordley (1983) also uses an additive social welfare function quite like the evaluation function used in this paper.

[9]This has been done by, for example, Bordley (1983) and Kollman et al (1997).

[10]As we report in a later section we have used a series of increasingly egalitarian evaluation functions in order to establish whether this changes the performances of the voting rules studied. Using egalitarian evaluation functions, however, seem to generate no qualitative changes in the results.

A final point on the use of binary choice functions is that such functions are neutral in the sense that we do not need to impose any restrictions on the distribution of preferences in the model a priory. This last point has been addressed earlier by Chamberlin and Featherston (1986) in their study of four different voting rules. They note that a large number of voters combined with preference formation on the individual level drawing on a fixed parameter, multinominal probability distribution will yield very small differences in the overall preference distribution from election to election. Therefore, Chamberlin and Featherston use the Dirichlet compound multinominal distribution function to allow for changes in voters' preferences between elections.[11] In the present study, on the other hand, changes in the distribution of preferences goes through two key parameters which we have chosen to call dispersion value and preference grouping (defined in section 3). Voters' and candidates' preferences over the k-issues are generated using different probabilities for different groups of voters and by allowing these parameters to change in the simulations we achieve a large sample of different societies ranging from largely homogeneous societies (a fair coin is used to generate each voters preference on each issue) to largely heterogenous societies (voters are divided into subsets with different probabilities of preferring 0 to 1 (Yes to No) on different subsets of the issue spaces. Thus, the notion of separability of issues refers to the fact that voting on issues are done separately and that there exist no restrictions on the possible states of the world that can be implemented through the political process *a priory*. Realising this voters treat issues as separate. This does not, however, imply that there is no interdependence between issues in voters' preferences; interdependency between issues on the individual level is created precisely through the use of dispersion values in the simulations. What we are saying, though, is that the interdependency between issues are strictly personal and probabilistic such that there exist no social understanding of dependency between issues that creates a level of political dimensions which are lower than the number of issues to be decided.

A second major point on which this analysis differs from most empirical social choice literature is the fact that we are dealing with voting rules for representative democratic institutions. Due to the complexity of modern politics and to the shear number of agents in any society we start out by assuming that any democracy must be representative in nature. This has a considerable effect on the analysis because it leads us to evaluate voting schemes that are different from the voting

[11]See Chamberlin and Featherston (1986) for a discussion on this point.

rules that are typically investigated in studies where the central aim is to examine the properties of voting rules *per se*. Contrary to must studies which contrast voting rules such as majority voting and plurality voting with more information revealing schemes such as the Hare system, the Coombs system or Borda count we simply look at plurality rule, run-off election and a particular version of proportional representation; i.e. we have chosen to operate with the three types of voting schemes which are most frequently used in mass elections in real democracies. In our conceptualisation, winner take all voting schemes are used in the meaning that the winner of the election is able to implement her preferred policy unrestricted. Under proportional representation a legislative body is formed in which each candidate obtain voting power proportional to her vote share in the election. Because of the definitions we have chosen it is immediately clear that we are also comparing the performance of different electoral systems or, put another way, of different constitutional set ups. Thus the versions of plurality rule and the run-off elections that we use represent the executive, presidential system and the proportional representation scheme represents the legislative, parliamentary system. To our knowledge very little has been done in this direction. Although Black (1958), who can be seen as the primary source of intellectual inspiration for most recent work in the empirical social choice literature, explicitly makes a distinction between committee voting and representative democracy and treats the issues separately most subsequent work (with a few exceptions; most notably Chamberlin and Courant (1983)), on voting rules seems to concentrate on the specifics of voting rules within committees without adding the extra layer of analysis by investigating the choice of the committee itself. The difference between the present set up and the more traditional, direct democracy set up can be illustrated by contrasting our model with the model of Chamberlin and Cohen (1978). Chamberlin and Cohen use a spatial model with Euclidian preferences to analyse the Condorcet-efficiency of different voting rules. But while Condorcet-efficiency is coined in terms of the set of alternatives (candidates) in their study, we are interested in the actual policy outcome and the relation between this outcome and voters' policy preferences, rather than the relation between candidates and voters' preferences on candidates. Thus, in our notation, all four voting rules evaluated in Chamberlin and Cohen (1978) can be seen to constitute voting rules for what we call a presidential system, and this is exactly what we mean when we postulate that most studies fail to consider the two-layer structure of representative democracy. Thus instead of answering the question on how efficient the individual voting rule select an optimal candidate given voters' preferences, we investigate how efficient the voting rule is to select an optimal

policy, given that voters' have to choose between candidates who themselves offer policies which are non-optimal from the individual voters' point of view.[12]

Turning to the other question raised here, namely which way to conceptualise intra-parliamentary bargaining under proportional representation, we have chosen the simplest possible way to do this: Each candidate in parliament simply vote honestly for her preferred state of the world on each issue. This rule is quite different from most studies on parliamentary systems. Riker (1982b) and followers stress the importance of a zero-sum conceptualisation of politics by suggesting that coalition formation must be converging towards minimum winning coalitions, since this would allow, to the smallest subset of the relevant democratic body, to extract as much as possible from the losing coalition, but such a conceptualisation of the outcome of the parliamentary game cannot be easily introduced in the binary settings we have chosen to work with. Another possibility would be to introduce bargaining games following Baron and Ferejohn (1989), Austen-Smith and Banks (1988) but introduction of these bargaining structures would, it seems, distort the problem we are analysing. First, some particular bargaining structure would have to be chosen and this choice would have to be ad hoc in nature. Second, assuming that candidates vote honestly on each issue once elected does have some positive properties relating to the simplicity of the system and, it could be argued, represents an equally realistic way of modelling intra-parliamentary bargaining as does any particular bargaining mechanism. Finally, by using the chosen definition we can compare systems in which the actual state of the world is equal to the preferred state of the world one of the candidates (presidential systems) with systems in which the implemented policy in general will be different from the preferred state of the world of any candidate (parliamentary systems) and do so without distorting the basic problem by strategic actions of candidates.[13] In this sense the policy implementation mechanism under proportional representation which is used here should be seen as little more than a useful simplification.

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[12]It could, with some merit we admit, be argued that we thereby require more knowledge of voters' preferences than the direct democracy type of study. We try to overcome this by having a very large number of elections with different distributions of preferences of both voters and candidates. Furthermore, most of the one-layer studies, including the model of Chamberlin and Cohen (1978), also do specify individual preferences on policies and use these preferences to derive voters' ranking of candidates.

[13]There will, of course, be cases in which the implemented state of the world under proportional representation would be equal to the state of the world preferred by one of the candidates. This will be the case in situations where one candidate receives a majority of the votes. In addition there will be some cases where the result of majority voting in the parliament results in a state of the world which is equal to to the preferred state of the world of one of the candidates running for election, regardless of the vote share this candidate has obtained in the election.

Finally, before we present the model and the results we have derived from it we should make a brief point concerning the computational structure we rely on to derive our results. Although the approach is different from early attempts to evaluate different voting schemes such as Black (1958), a number of related studies use computational models or computer simulations. Kollman et al (1992, 1997) use computational models to investigate the effects of introducing adaptive parties into a traditional spatial model and a Tiebout model respectively. Bordley (1983, 1985, 1986) use simulations to investigate the social efficiency of different voting rules and to access the one person/one vote system. Among other prominent examples of related models that rely on computer simulations are Fishburn (1974), Chamberlin and Cohen (1978), Merrill (1985) and Nurmi (1992).

2. THE MODEL

The state of the world resulting from political interaction is determined by a binary vector of dimension k (the number of issues to be decided). The state of the world on the j'th issue after a collective decision has been carried out is given by s_j where $s_j \in (0,1)$ Consequently, after the legislative body has made decisions concerning all issues the actual state of the world will be one over 2^k possible alternatives. In our notation this state of the world is given by the vector $S^*=[s_1^*,...s_k^*]^T$.

Society is made up of a finite number of voters indexed by i=1,2,...n and a finite number of candidates indexed by h=1,2,...,m who all are identified by their preferred state of the world over multi-dimensional binary political issue spaces. In order for society to be able to make a unique decision on collective issues, voters must elect a legislative body with power to implement policy.

Before each election an exogenously determined number of candidates will announce their preferred state of the world; i.e.,candidate h announces a policy proposal in the form of a k-dimensional boolean vector, $X_h=[s_{h1},...s_{hk}]^T$. In order to avoid problems of credibility and of inconsistencies between announced policies and realized policies all elected representatives will, when elected and when in full decision capacity, seek to implement their announced policy, cfr. Alesina and Rosenthal (1995).

Voters are identified by their preferred state of the world; $X_i=[s_{i1},s_{i2},...,s_{ik}]^T$. When voting, voters

compare their preferred state of the world with the preferred state of the world of each candidate and vote for the candidate which offers the best state of the world, i.e.,voters solve;

$$\min_{h \; h=1,2..,m} \sum_{j=1}^{k} |s_i - s_h|$$

If any voter is indifferent between two or more candidates the voter abstains from voting.[14] For each configuration of voters' and candidates' preferences, elections are held using three different voting schemes:

Definition 1.a.: plurality voting

The outcome under plurality voting, denoted "Spl", is a k-dimensional Boolean vector, drawn from the total set of possible states of the world, which is equal to the proposed policy of that candidate who obtains a plurality of the votes cast in the election.[15]

Definition 1.b.: Outcome under run-off election

Run-off election rule outcome, denoted "Sro", is defined as the proposed policy vector of the candidate who has won a two-stage election. In the first stage an election is held between all candidates and the two candidates who obtains highest vote shares are elected to participate in a two-candidate, second stage election. The candidate who obtains a majority in the second run election implements her preferred policy vector.[16]

Definition 1.c. Outcome under proportional representation

Under proportional representation, denoted "Spr", voters elect a body of representatives consisting of the candidates running for office. Representatives gain voting power equal to the vote shares they obtain in the election. Issues are decided separately using majority-rule in the

[14]This assumption, implying a version of the abstention from indifference thesis, was originally chosen in order to keep the computational structure as simple as possible. In order to verify the result with the use of a more traditional tie-breaker rule, we have run simulations in which indifferent voters split their votes between all preferred candidates. These simulations show no alterations compared with the results presented below.

[15]In the simulations we ignore cases where two candidates gets the same (and winning) vote share.

[16]This definition disregards the way run-off election is implemented in reality where a candidate who obtains a majority in the first run will win the election outright. But given the assumption of honest voting, the definition here will always produce the same winner as the traditional definition, that is; a candidate who obtain a majority of the votes in the first run will always obtain a majority in the second run.

legislative body. The resulting policy on the j'th issue is the policy that gains a majority in the legislative body.

In addition to these voting rules the outcome that would prevail under separate issue, majority voting in a direct democracy, denoted "Sdd" is computed. The state of the world on the j'th issue under direct democracy voting is simply 0 if a majority of the electorate prefers 0 to 1 and 1 otherwise.

Each implemented policy vector is evaluated in terms of a simple unweighted additive evaluation function. For each election carried out, the resulting state of the world is compared to voters' preferences, and the boolean distance between the preferences of the individual voter and the state of the world is summarized. Then this number is summarised over all n voters. Thus, the social loss from the i'th agents disagreement with the implemented policy on issue j has weight 1/kn through all n voters and all k issues. In other words we use the following definition to evaluate the performance of each voting system.

Definition two - performance levels

Let the state of the world resulting from an election that implements the electoral system g, g∈(pl,ro,pr), be denoted S_g. Then the performance level, called $W|_{Sg}$, of voting rule g in a given election resulting in the state of the world S_g is given by

$$W|_{Sg} = \frac{\sum_{i=1}^{n} \left(\sum_{j=1}^{k} s_{ij} | s_{ij} = s_j^* \right)}{kn}$$

Even though the direct democracy procedure used here for most distributions of voters' preferences will result in performance levels that are substantial less than 1, the optimal state of the world given voters' preferences is (by definition) equal to the outcome under direct democracy (S_{dd}) when definition two is used to evaluate the social performance of the system. This characteristic is a simple implication of the direct democracy decision procedure. Under direct democracy with separate voting on all issues it must be true that for all issues the policy chosen must have at least half the voters behind it given the binary formulation of the policy

space. Thus the maximum performance level that can be obtained under any voting rule cannot exceed the performance level of direct democracy for any distribution of preferences.

To see how the simulations work, we have constructed an example covering the basic features of the model. In this example there are seven voters (1,2,3,4,5,6,7) and 3 candidates (a,b,c) with preferences over six issues as presented in figure 1.

Figure 1. Voters' and candidates' preferences.

	voter							cand		
issue	1	2	3	4	5	6	7	a	b	c
1	0	0	0	1	1	1	1	0	1	0
2	1	1	1	1	0	1	1	1	0	1
3	1	0	0	1	0	1	0	1	0	1
4	1	0	1	1	0	0	0	1	0	0
5	1	0	0	0	0	1	0	0	1	0
6	1	0	1	1	1	1	0	1	1	0

In the direct democracy case with majority voting on each issue separately policy is given by (1,1,0,0,0,1). In a representative democracy each voter compares his preferred state of the world with the state of the world that is proposed by each of the three candidates and choose to vote for the candidate with whom she agrees on most issues. By doing so voter 1 find, that candidates (a,b,c) propose policies with distance (1,4,3). This imply that voter 1 votes for candidate *a*. Using the same procedure on voters' 2-7 we obtain a distribution of votes as shown in Figure 2.

Figure 2: Distribution of votes

candidate	a	b	c
number of votes	3	2	2

To keep things simple we only calculate the actual policy under plurality voting and proportional representation (in this example, run off election will result in the same policy vector as plurality voting). In the case of plurality voting, candidate *a* wins the election and implement her preferred state of the world. In the case of proportional representation policy depends on the majority on

each issue in the elected "parliament". Figure 3 shows the resulting policies under direct democracy, plurality voting and proportional representation.

Figure 3: Policy under different voting rules

	direct democracy	Plurality voting (rep.-dem)	Proportional rep.
1	1	0	0
2	1	1	1
3	0	1	1
4	0	1	0
5	0	0	0
6	1	1	1

We notice from figure 3 that both voting rules creates different policies than direct democracy. The performance levels following from different voting schemes can be computed by comparing the actual policies with table 1. Using definition 2 it immediately follows that;

$W|_{Sdd} = 28/42 \approx 0{,}667.$

$W|_{Spl} = 25/42 \approx 0{,}595$

$W|_{Spr} = 26/42 \approx 0{,}619$

In this case proportional representation beats plurality voting. It should also be noted that both schemes result in outcomes that are significantly worse than the outcome under direct democracy.

3. SIMULATIONS

In order to be able to evaluate the performance of the different voting schemes, many elections with new sets of voters and candidates are artificially generated. In order to keep things fairly simple we have had to keep some parameters fixed through all simulations. We have chosen to operate with 210 issues and 101 voters (or 101 groups of voters of equal size) in all elections.[17]

[17]The choice of these particular numbers is operational. The odd number of voters (101) allows for the unique determination of the direct democracy vector where for each individual issue majority is reached. The choice of 210 issues emerges from the need to construct homogenous subgroups of voters which are defined by different probabilities relative to different subsets of issues. The number 210 can be divided by the jprime numbers 2,3,5 and 7 which would allow for the grouping of 210, 105, 70, 42, 35, 30, 21, 15, 10, 7, 5, 3, 2, 1 issues.

The number of parties have been varied from 2 to 12 in order to capture any differences in the capabilities of the voting schemes that might be attributed to the number of alternatives, cfr. Bordley (1983) who finds that the number of alternatives do matter.

Voters' and candidates' preferences are generated randomly using two parameters to induce differences between elections. The probability distributions used to create preferences are determined by two dimensions termed preference groupings and dispersion values. We have used three different preference groupings and to illustrate how it works we briefly explain how preference grouping 1 is defined. Voters #1-50 have probability 1/2+disp (disp=dispersion value, see below) of preferring 0 on issues 1-210. Voters #52-101 have probability 1/2-disp of having preference 0 on issues 1-210. Voter #51 have probability ½ of having preference 0 on issues 1-210. Preference Groupings 2 and 3 divide voters and issues into more subsets. A summary can be found in figure 4. Dispersions values is the second parameter which is used to generate different societies. Dispersion value takes on values from 0-0.25 (in 0.05 steps) and the parameter is used to capture different distributions of preferences in societies. In general high dispersions values creates small homogenous groups of voters who are strongly opposed to each other. Thus, configurations of voters with high dispersion values can be interpreted as heterogenous societies with strong social or regional cleavages, compared to configurations with low dispersion values. Thus, the use of dispersion value has some bearing on the discussion in the literature on partial and impartial cultures, cfr. Nurmi (1992). In this notation, simulations with dispersion values equal to zero are de facto impartial cultures.

To be able to make comparisons with a random chosen policy all configurations of voters have been constructed such that the expected performance level of a randomly chosen policy, denoted $EW|_{Sr}$, is ½ (where E is the expectations operator).

We have artificially generated 200 configurations of voters' and candidates' preferences with each combination of (i) the number of parties (2-12), (i) preference grouping (1-3) and (i) dispersion value (0-0.25 in 0.05 steps). This yields a total of 72.600 (200*11*3*11) different configurations of preferences. The characteristics of the simulations are summarised in table 4.

Figure 4: Summary of parameters used in the simulations

Parameters		
Number of binary issues	210	
Number of voters	101	
Number of candidates	2 to 12 (6.600 configurations for each number)	
Preference groupings	- 1: prob (s_{ij} =0): i ≤ 50: ½ + disp	½ + disp
	i = 51: ½	½
	i ≥ 52: ½ - disp	½ - disp
	- 2: i ≤ 50, j ≤ 105:	½ + disp
	j ≥ 106:	½ - disp
	i = 51:	½
	i ≥ 52, j ≤ 105:	½ - disp
	j ≥ 106:	½ + disp
	- 3 i ≤33, j ≤ 70:	½ +disp
	j ≥ 71:	½ * (1-disp)
	i = (34,68):	½
	35 ≤ i ≤ 67, j≤ 70:	½ * (1-disp)
	71 ≤ j ≤ 140:	½ +disp
	j≥ 141:	½ * (1-disp)
	i ≥ 69, j≤ 140:	½ * (1-disp)
	j≥ 141:	½ +disp
Dispersion values	varied from 0-0.25 for given m and disp, 600 configurations of voters and candidates have been created	
nb. of config. for given nb. of cand., preferrence grouping and disp.	200	
total nb. configurations	72600	

For each configuration of preferences we have computed the outcome under direct democracy (the optimal performance level (S_{dd})), and the states of the world resulting from plurality voting (S_{pl}), run-off election (S_{ro}), and proportional representation (S_{pr}) respectively. The information

obtained here is then used to create the following results:

a. Performance levels of the three voting schemes.

Using definition 2 we have computed the mean performance levels for configurations with the number of candidates (m) held constant.

b. Pair-wise comparisons of voting schemes

The performance levels of the three representative voting schemes have been compared pair-wise. These results are based on observations of the frequency by which one voting scheme outperform another.

c. Number of issues equal to the direct democracy outcome

We have used the information summarised above to calculate the average percentage of issues that is equal to the state of the world under direct democracy. These numbers are used to analyse the ability of representative democracies to implement policies that are close to the policies implemented under direct democracy. Again these results are presented for a varying number of candidates.

4. RESULTS

The result parameters defined above enable us to present a series of rather robust claims concerning the functioning of representative democracy relative to direct democracy and the social performance of different voting systems and voting schemes within representative democracy. The main result obtained from the simulations establishes the performance of different voting schemes for representative democracy:

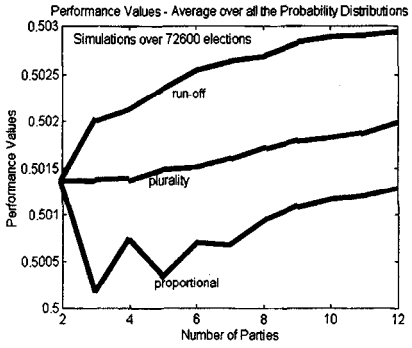
Claim one: Different voting rules for representative democracy leads to different policies and performance values. Let mean performance values be denoted av. Then;

$avW/_{Spr} < avW/_{Spl} < avW/_{Sro}$

Claim one states that - on the basis of our experiments - run-off election and plurality voting schemes lead to higher performance levels than voting based on proportional representation. This result can be seen directly from figure 5. As the figure suggests, the run-off election scheme

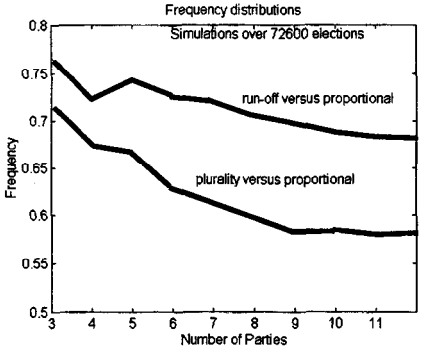
results in states of the world that are significantly better than those created by both plurality voting and proportional representation.

Figure 5



In order to analyse whether these results can be attributed to relatively few but special configurations of voters' and candidates' preferences in which proportional representation leads to very low performance levels the frequency of elections where the proportional representation scheme is beaten by run-off and plurality voting schemes has been computed. In figure 6 these results are shown.

Figure 6



As the figure indicate, the ratio by which run off election in terms of performance values beats

proportional representation varies between 0,77 and 0,68. The plurality/proportional representation ratio varies between 0,72 and 0,58. Thus we are able to conclude from these observations (i) that representative systems which elect a temporary dictator seem to perform better than purely representative systems and (ii) that within the "winner take all systems", the run-off election rule performs significantly better than simple plurality voting. Turning to the latter observation first, this feature is not that surprising given that voters' reveal more information under the two-step run-off election procedure than under simple plurality voting. Thus, this result is equivalent to results obtained in studies which use Condorcet efficiency as their main criterion, cfr. Merrill (1984) for a discussion on Condorcet efficiency, Nurmi (1992) for results on plurality voting versus run-off election under different preference structures and Chamberlin and Featherston (1986) for an analysis of four voting rules (plurality voting, Borda count, Hare's system and Coombs' system) in relation to Condorcet efficiency. Thus in this respect we merely underline the well known result from empirical social choice theory that apart from its good practical properties (it is very simple, calculations of election results are computational easy and voters only have to go to the poll once) plurality rule performs poorer than more information revealing voting mechanisms under a wide range of evaluation parameters. Turning to the former point that winner take all voting schemes do better than the proportional representation scheme we have chosen to operate with here, some rather interesting points should be made. Since the winner take all voting schemes can be seen to mimic presidential systems in which one candidate obtains "dictatorial" power over policy, and the proportional representation scheme can be argued to mimic parliamentary systems, the result of our simulations points towards the recommendation of a presidential system. The reason behind this however, is not that parliamentary systems create uncertainty or instability, cfr. Mueller (1989) chapters 10-12. Instead, we suggest that the reason should partly be found in the inability of a representative body to be a microcosm of the whole society since such representativeness simply cannot be said to exist in our settings, cfr. Chamberlin and Courant (1983) for a social choice analysis leading to the opposite result and Brennan and Hamlin (1998) for arguments stressing the positive properties of representation in its ideal form. Moreover, our choice of evaluation function could be acting as a mechanism which emphasises singular popular candidates, but sensitivity analyses with respect to the choice of evaluation function seem to offset this effect, see below.

Contrasting the three voting schemes with direct democracy the simulations show that the

outcome of the political process under any of the representative schemes is very different from the outcome under direct democracy and that the social performance of any representative voting mechanism is inferior to the performance of direct democracy.

Claim two: Given voters' preferences and candidates' platforms, any voting scheme used for representative democracy is likely to create results that are substantially different from the policies that would prevail under direct democracy. Assume that the configuration of voters and candidates is such that at least one candidate propose a policy with a higher performance level than the expectation of a randomly chosen policy. Then for all voting rules (g=(pl,ro,pr)) we have that; $W/_{Sr} \leq W/_{Sg} \leq W/_{Sdd}$

Claim two essentially states that any system of representative democracy leads to outcomes that are very different from outcomes generated by direct democracy and that the policy outcome of representative democracy, although better than a random chosen policy, leads to losses in social performance compared with direct democracy. These results are in sharp contrast with commonly stated arguments for representative democracy stressing that a (randomly chosen) sample of the population will create policies which do not differ much from policies chosen under direct democracy.[18] The ratio of issues that are equivalent under direct democracy and each of the three voting rules is shown in figure 7. As is evident from the figure, all three voting rules result in states of the world in general are substantially different from the outcome under direct democracy. But given our settings, it is also the case that the outcome under any of the investigated voting schemes is socially preferred to a random chosen policy. In figure 8 the estimated expected performance values (the means over the 72.600 elections) of run-off, plurality and proportional representation voting electoral schemes are computed and compared with respect to the probability distribution of the random choices of the state of the world.[19] It is clear that only a

[18]Brennan and Hamlin (1998) gives a interpretation of this view of representation.

[19]The probability distribution of the sum of a vector of binary values (with probability ½ respectively for the values 0 and 1) is determined using the pascal triangle. In the case of 2 issues the number of vectors are 2 to the power of 2, i.e.,[00,01,10,11], with the frequency for the sum values of [1,2,1]. In the case of 3 issues the number of vectores are 2 to the power of 3, i.e.,[000,001,010,100,011,110,101,111], with the frequency for the sum values of [1,3,3,1]. In the case of 4 issues we have the frequency for the sum values of [1,4,6,4,1]. Subsequently to a simple logical transformation we can assume the vector of all ones to be the direct democracy vector. In the case presented in this paper the number of issues is 210 and consequently there exist 2 to the power of 210 alternative states of the world. The probability distribution of the sum of these vectores is determined by the pascal triangle associated with the 210 issues, i.e.,[1, 210,21945, 1521520, 78738660,...., 8.3092e+61, 8.7126e+061, 8.9639e+061, 9.0492e+061, 8.9639e+061, 8.7126e+061, 8.3092e+061,....,78738660, 1521520, 21945, 210, 1]. Dividing the above vector of frequencies for 2 to the power of 210 we obtain the probability distribution for the ratio reported in the figures.

little more than two out of ten randomly picked states of the world would perform better than the run-off state of the world, and a little more than three out of ten would perform better than the state of the world under plurality voting or proportional representation. This result resembles the results reached by Bordley (1983) who finds for different parameters concerning the number of candidates, the number of voters and their preferences, that a random chosen policy is worse than any of the four voting rules he investigates.[20]

Figure 7

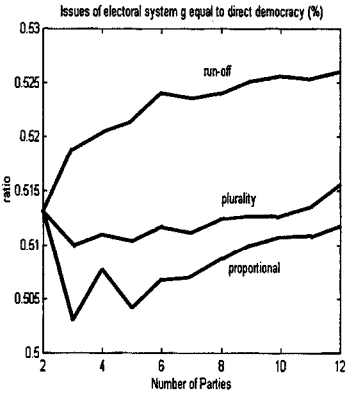
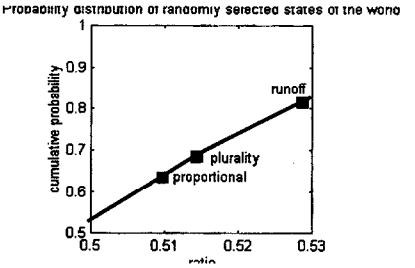


Figure 8



So far we have established that representative democracy in this set up produce different outcomes than direct democracy. Furthermore we have demonstrated that all types of representative voting schemes considered produce results which are socially inferior to the outcome under direct democracy but preferable to a random choice of policy. Finally we have

[20]The four voting systems used by Bordley (1983) are: Borda count, Copeland Voting, Approval Voting and One Person/One Vote (plurality voting). In addition Bordley evaluates the outcome under random policy making and under dictatorship (an agent is picked to be dictator without any voting going on).

shown that winner take all electoral systems in which a single candidate obtains absolute power outperform the proportional system in which the implemented policy is the outcome of majority voting in a parliamentary system. Before drawing too rigorous conclusions on these results, we would like to stress some points relating to partial investigations of the simulations. Going back to figure 5 an interesting point arises from the observation of the performance levels of the two winner take all electoral rules. As it can be seen from the figure the number of candidates running in the election has real effects on the social performance under winner take all systems.

Claim three: For winner take all voting rules (pl,ro), mean performance levels increase in the number of candidates running for office.

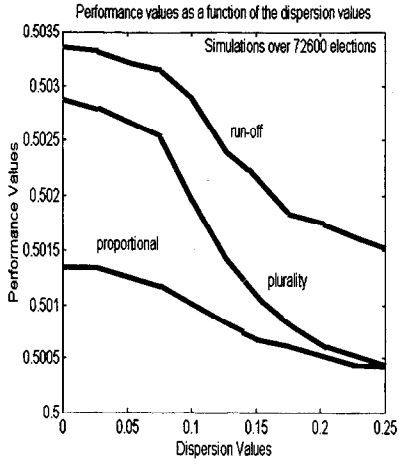
This claim simply states that for given distributions of voter preferences, a high number of political candidates should be preferred to a two-candidate system in winner take all electoral systems. In our set up candidates announce policies independently of voters' preferences. This simply means that as the number of candidates running for office goes up, the possibility of a popular (or good) candidate also increases. Although this explanation weakens the generality of the result, it does not completely destroy it. Even in highly modern democracies with frequent popularity polls, complete knowledge of the distribution of voters preferences can hardly be said to be a realistic assumption. If politics is seen as multi dimensional and the distribution of voter preferences is considered unknown, our results suggest that societies should reduce entry barriers for political candidacy.

A range of dis-aggregation of the total experiment generally does not change the overall results. There are prominent differences in the results for different dispersion values though, leading to claim 4.

Claim 4: For any voting scheme social performance is decreasing in dispersion of voters' preferences, implying that heterogenous societies with strongly opposed groups of voters are less likely to obtain high social performance values than societies with low dispersion between different groupings in the electorate.

In figure 9 performance levels are shown as a function of the dispersion value.

Figure 9



Given the overall preference grouping which determines the number of voter types, dispersion values are used to introduce differences in the distribution of preferences. When dispersion values are low it is unlikely that homogenous groups of voters exists in the electorate. In particular when dispersion is equal to 0, the probability that any voter has preference 0 (1) on any issue is ½. When dispersion values are high, on the other hand, homogenous groups of voters, which are strongly opposed to each other, are likely to exist. High dispersions value in our set up can be seen to mimic class based societies or societies with strong regional cleavages, such that each class or region constitutes a rather homogenous group of voters, who are strongly in opposition to other groups in society. The result shown in figure 9 is strongly suggesting that performance levels are lower when dispersion values are higher, thereby suggesting that societies without homogenous groups of strongly opposed voters might do "better" than societies with such groups regardless of the voting rule layed down in the constitution. Finally, figure 9 also rises questions as to whether plurality voting is a desirable voting rule when dispersion values in society are high.

The results reported are based on the use of a simple, unweighted additive evaluation function as the social welfare criterium. A natural question seems to be whether the results are depending on that type of evaluation function. To answer this question a series of elections using increasingly egalitarian evaluation functions have been carried out. Under egalitarian evaluation

functions voters who have preferences located far from the implemented policy vector get a higher weight than voters who are located close to the implemented policy vector. The simulations with egalitarian evaluation functions display the same qualitative results as reported earlier, thereby suggesting that the content of proposition three and four are rather insensitive to such changes.

5. CONCLUSION

We have used a simple model of political interaction with multiple binary issues and multiple candidates to mimic representative political systems through simulations. Our results suggest that the constitutional choice of voting rule has a substantial impact on the outcome of the political process. The state of the world resulting from direct democracy is substantially different from the state of the world resulting from representative democracy. Furthermore the outcome of representative democracy depends crucially on the voting rule society choose to elect political assemblies. Our experiment points towards a recommendation of "winner take all" electoral systems, in which the winner of the election obtains absolute power to implement policy. Finally, given the former result, a voting rule which allows voters' to reveal more information about their preferences such as run-off election comes out superior to simple plurality voting. This result seems to be stable both under an unweighted additive evaluation function and when the relevant social target is the outcome of direct democracy. Under "winner take all"electoral systems, our results suggest that a high number of political parties should be preferred to a two-party system. This result is especially interesting given the empirical observation that a high fraction of real democracies with winner take all electoral systems are two party systems or near two party systems.

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