Inequalities for integral means over symmetric sets

We prove that the integral of n functions over a symmetric set L in $\mathbb{R}^n$, with additional properties, increases when the functions are replaced by their symmetric decreasing rearrangements. The result is known when L is a centrally symmetric convex set, and our result extends it to nonconvex sets. We deduce as consequences, inequalities for the average of a function whose level sets are of the same type as L, over measurable sets in $\mathbb{R}^n$. The average of such a function on E is maximized by the average over the symmetric set E^*.