Nonseparable Walsh-type functions on $\mathbb{R}^d$

We study wavelet packets in the setting of a multiresolution analysis of $L^2(\mathbb{R}^d)$ generated by an arbitrary dilation matrix A satisfying $|\det A| = 2$. In particular, we consider the wavelet packets associated with a multiresolution analysis with a scaling function given by the characteristic function of some set (called a tile) in $\mathbb{R}^d$. The functions in this class of wavelet packets are called generalized Walsh functions, and it is proved that the new functions share two major convergence properties with the Walsh system defined on [0,1). The functions constitute a Schauder basis for $L^p(\mathbb{R}^d)$, $l < p < \infty$, and the expansion of L^p-functions converge pointwise almost everywhere. Finally, we introduce a family of compactly supported wavelet packets in $\mathbb{R}^2$ of class $C^r(\mathbb{R}^2)$, $1 \leq r \leq \infty$, modeled after the generalized Walsh function. It is proved that this class of smooth wavelet packets has the same convergence properties as the generalized Walsh functions.