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Numerical methods for optimizing the performance of buildings

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SUMMARY:
The many different parties that influence design and control decisions for buildings, such as building owners, users, architects, consulting engineers, contractors, etc, may have different and to some extent contradicting requirements to buildings. Furthermore, national building regulations specify requirements to (among others) the energy performance and quality of the indoor climate, which also must be satisfied. This paper describes numerical methods intended for estimating design decisions that satisfy the given requirements, and that at the same time are optimal in some sense, for instance with respect to the economy, energy performance, or indoor climate. This is addressed by combining building simulation methods with numerical optimization methods. The paper describes a problem formulation that represents optimal design decisions, and the numerical simulation and optimization methods used for solving the problem. The paper furthermore provides a case study regarding a small office building.

1. Introduction

The parties who influence design and control decisions for buildings (referred to as *decision makers*), such as building owners, users, architects, consulting engineers, contractors, etc., often have different and to some extent conflicting requirements to buildings. For instance, the building owner may be more concerned about the budget for the building, rather than the indoor climate, which is more likely to be a concern of the building user.

Furthermore, it is a well-established fact that it is easier and less costly to make design changes in the early stages of the design process for buildings rather than later. See for instance Poel (2005) and Nielsen (2003) for a more detailed description and discussion of the design process for buildings. Decision-makers may therefore benefit from software-based building optimization methods, developed for the early stages of the design process.

Simulation and optimization methods have been combined in many different ways for supporting building-related decisions. For instance, the studies by Peippo et al. (1999), Bouchlaghem et al. (2000) and Wright et al. (2002) all use this approach for estimating efficient design decisions related to, among others, the shape and orientation of the building, the amount of insulation, and the shape and area of the windows. The problem formulations used in the studies are single- or multi-criteria optimization problems, involving either energy performance, construction or operational costs, or measures for thermal discomfort.

In general, there are many combinations of decision variables and performance calculations that are relevant to include in the problem formulation. This motivates the development of building optimization methods that give the end-user full control over the problem formulation.

Furthermore, the numerical optimization methods must address the following issues: (1) partial derivatives of the functions can not be expected to be available, and (2) the optimization method must provide valid input to the simulation methods; otherwise they may not provide valid output.

The purpose of this paper is to describe a combination of numerical simulation and optimization methods that can be used for estimating efficient design decisions at the early stages of the design process for buildings. The problem formulation and the involved numerical methods are described. The paper furthermore provides a case study involving a small office building.

This paper is based on the thesis by Pedersen (2006), where further details can be found.

2. General aspects of the method

Decision-making is supported by calculating optimal design decisions for a conceptual building model. The problem formulation, the model and the elements included in the problem formulation are described in the following.

2.1 Problem formulation

Figure 1 shows an illustration of the elements involved in decision-making. The figure illustrates the simple fact that decisions made under given circumstances result in a number of consequences. The figure furthermore illustrates the requirements made by decision-makers to the decisions as well as to the resulting consequences.

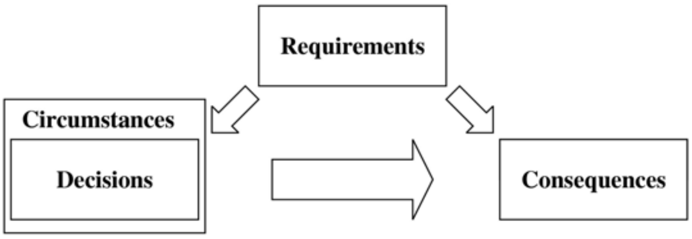

FIG. 1: Elements involved in decision-making.

The term *decision* refers to the aspects of the building that decision-maker has control over, and *circumstances* refers to the aspects that the decision-maker has no control over, or do not wish to control.

Decisions are represented by a set of *decision variables* $x \in \mathbb{R}^n$, and the circumstances are represented by a set constant parameters $y \in \mathbb{R}^{n_y}$. The consequences are represented by a set of *utility functions* $q : D \times \mathbb{R}^{n_y} \rightarrow \mathbb{R}^{n_q}$, that depend on x and y. The consequences of a set of decisions x made under the circumstances y can therefore be evaluated by calculating the function values $q(x, y)$.

The domain $D \subseteq \mathbb{R}^n$ for the utility functions is defined in the following way:

$$D = \left\{ x \in \mathbb{R}^n : d(x) \geq 0 \right\}, \tag{1}$$

where the functions $d : \mathbb{R}^n \rightarrow \mathbb{R}^{n_D}$ are referred to as *domain constraint functions*.

The following optimization problem is used for estimating efficient decisions:

$$\begin{aligned} &\min_{x \in D} && r_O^T q(x, y) \\ &\text{subject to} && A_I \begin{bmatrix} q(x, y) \\ x \end{bmatrix} \geq b_I \quad \text{and} \quad A_E \begin{bmatrix} q(x, y) \\ x \end{bmatrix} = b_E \end{aligned} \tag{2}$$

This formulation allow the user to specify which utility function to minimize or maximize, as well as linear inequality and equality relations, involving decision variables and utility functions. The user can thus choose between, for instance, energy optimal or economical optimal design decisions. The inequality constraints can be used for specifying upper and lower bounds on utility functions and decision variables. The equality constraints can be used for specifying required values for utility functions and decision variables.

2.2 A conceptual building model

The required utility functions are based on performance calculations for a building with a simple geometry, representing general features, such as volume, surface area, mass of constructions, window area, etc. This so-called *conceptual building model* is shown in Figure 2.

All floors are identical, and each floor has window “bands” on two of the four external walls. The staircase tower is omitted, and only a single internal wall is included, which divide the building into two thermal zones. The performance of each of the two thermal zones is calculated separately.

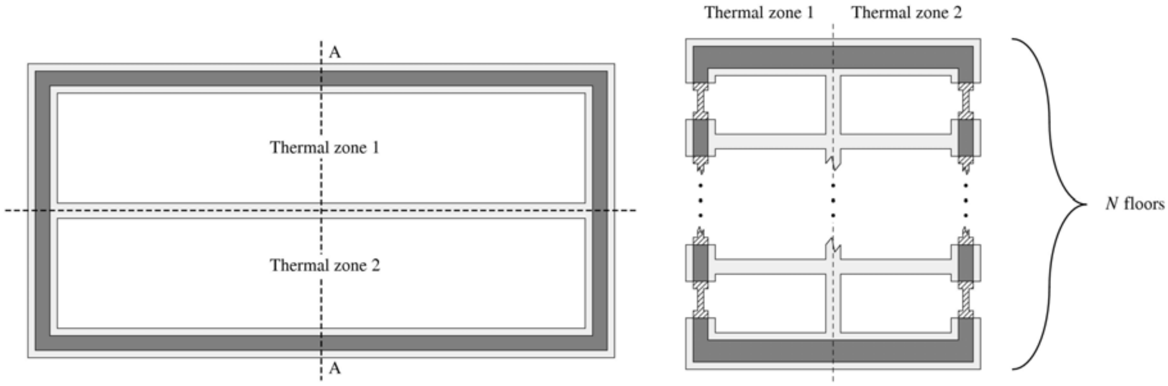

FIG. 2: Left: Layout of the conceptual building. Right: The cross-section A-A.

2.3 Decision elements

The following decision variables are included in the problem formulation:

- The shape of the building (represented by the width to length ration and the number of floors)
- The window fraction of the façade areas
- Discrete selections of windows from a product database
- Amount of insulation used in ground slab, roof construction and external walls

The following main groups of utility functions are included in the problem formulation:

- The energy performance (energy for heating, cooling, ventilating, producing DHW, U-values for constructions, among others)
- The indoor environment (overheating and daylight utilization)
- Economy (construction and operational costs)

3. Utility functions

The utility functions involved in the problem formulation are described in the following, as well as the domain constraint functions, that specify the domain of the utility functions.

3.1 Energy and indoor climate

The performance with respect to energy and indoor climate is calculated using the method described by Nielsen (2005). One simulation is conducted for each of the two thermal zones of the building. The method gives, among others, hourly values for the internal air temperature, and hourly values for energy required for heating, cooling and ventilating the building.

These results are used for calculating the utility functions related to the energy performance of the building. The internal air temperatures are furthermore used for calculating the annual number of hours, where overheating occurs, which is used as a measure for thermal discomfort. The ratio between the depth of the room and the window height is used as a (primitive) measure for the level of daylight utilization. The calculation of this measure is based on the geometry of the building. Notice that high values represent low daylight utilization, and low values represent high daylight utilization.

3.2 Economy

Two utility functions are used for representing economical consequences of design decisions: The cost of constructing the building, and the annual cost of operating the building. The cost of constructing the building is estimated by interpolating values found in price catalogues, such as the V&S price catalogue (2005), which concerns unit prices for construction jobs in Denmark.

Some of the prices only depend on the number of purchased units, where other prices also depend on secondary parameters. For instance, the unit price of pouring concrete depends on the amount of concrete, and the required strength of the concrete.

The prices are interpolated using the following three models:

$$p_1(u,\beta) = \beta_1 \exp(\beta_2 u) + \beta_3 \tag{3}$$

$$p_2(u,s,\beta) = \beta_1 s + \beta_2 \exp(\beta_3 u) + \beta_4 \tag{4}$$

$$p_3(u,s,\beta) = \beta_1 \exp(\beta_2 s) + \beta_3 \exp(\beta_4 u) + \beta_5, \tag{5}$$

where u is the number of purchased units, β is a vector of model parameters, and s is the secondary parameter. Note that the length of β depends on the model.

The model (3) is used for representing unit prices that do not involve a secondary parameter, and the models (4) and (5) are used for representing unit prices involving a secondary parameter.

The model parameters β are calculated as a least squares solution to the over-determined system of non-linear equations that can be formed using the unit prices from the price catalogue.

The cost of operating the building is calculated using the annual consumption of electrical energy and energy for heating the building, together with the energy prices for electricity and district heating.

3.3 Domain constraints

The purpose of the domain constraints is to ensure that the input to the simulation methods is valid. The domain constraints ensure (among others) that the input satisfies the following requirements:

- The width to length ratio of the building is positive
- The number of floors is larger than or equal to 1
- The window fraction of the façade area is between 0 and 1
- The amount of insulation used in the ground slab, roof construction and external walls is positive.

4. A gradient-free SQP filter algorithm

The method described in this section is based on the SLP filter method by Fletcher (1998), but with a number of modifications, in order to make it suitable for solving (2). First of all, the method must not require gradient information of the functions used for defining (2), secondly, it may only evaluate these functions for iterates x_k, that belong to the domain D.

The method is intended for finding solutions to constrained optimization problems on the following form:

$$\begin{aligned} &\min_{x\in D} \quad f(x) \\ &\text{subject to} \quad c_I(x) \geq 0 \quad \text{and} \quad c_E(x) = 0, \end{aligned} \tag{6}$$

which includes the problem (2). The method needs to distinguish between the following three situations:

- The current iterate x_k belongs to the domain D
- The current iterate has provided an unsolvable (or *incompatible*) subproblem
- The current iterate does not belong to the domain D.

In the first situation, the method calculates a step Δx_k towards a stable point for (6), by forming an approximated subproblem $\mathrm{QP}(x_k, \mu_k)$, using first order Taylor expansions of the functions involved in (6). The step length is restricted by adding a quadratic damping term μ_k to the objective function. This approach provides the following quadratic program:

$$\mathrm{QP}(x_k, \mu_k) = \begin{cases} \min\limits_{\Delta x \in \mathbb{R}^n} & B_{f,k} \Delta x + \frac{1}{2} \mu_k \Delta x^T \Delta x \\ \text{subject to} & c_I(x_k) + B_{c_I,k} \Delta x \geq 0 \\ & d(x_k) + B_{d,k} \Delta x \geq 0 \\ & c_E(x_k) + B_{c_E,k} \Delta x = 0 \end{cases}, \tag{7}$$

The following gradient approximations are used in the definition of (7):

$$B_{f,k} \approx \nabla f(x_k),\ B_{c_I,k} \approx J_{c_I}(x_k),\ B_{c_E,k} \approx J_{c_E}(x_k) \text{ and } B_{d,k} \approx J_d(x_k). \tag{8}$$

These approximations are initialized using finite difference calculations, and subsequently updated using the rank one updating formula described by Broyden (1965).

In the second situation, where $\mathrm{QP}(x_k, \mu_k)$ is incompatible, the method calculates a step towards the feasible region of (6). This is done by calculating a so-called *regular restoration step*, which is a step in a direction that minimizes the maximum violation of the constraints in (6). The step length is restricted by adding a quadratic damping term to the objective function. This approach provides the following subproblem:

$$\mathrm{RRQP}(x_k, \mu_k) = \begin{cases} \min\limits_{z \in \mathbb{R}, \Delta x \in \mathbb{R}^n} & z + \frac{1}{2} \mu_k \Delta x^T \Delta x \\ \text{subject to} & z \geq v(x_k) + B_{v,k} \Delta x \end{cases}, \tag{9}$$

where $v(x_k)$ is a vector of constraint violations:

$$v(x) = \begin{bmatrix} -c_I(x) \\ c_E(x) \\ -c_E(x) \\ -d(x) \end{bmatrix}, \tag{10}$$

and where $B_{v,k} \approx J_v(x_k)$ is an approximation of the gradient of v at x_k. In (9), the parameter z is introduced in order to rearrange the problem into a QP.

The last situation, where $x_k \notin D$, is handled by calculating a so-called *domain restoration step*, which is a step in a direction that minimizes the maximum violation of the domain constraints, and can be calculated by solving the following subproblem:

$$\mathrm{DRQP}(x_k, \mu_k) = \begin{cases} \min\limits_{z \in \mathbb{R}, \Delta x \in \mathbb{R}^n} & z + \frac{1}{2} \mu_k \Delta x^T \Delta x \\ \text{subject to} & z \geq -d(x_k) - J_d(x_k) \Delta x \end{cases}, \tag{11}$$

Once a step Δx_k is calculated by solving either (7), (9) or (11), and the step is accepted, the next iterate becomes:

$$x_{k+1} = \Delta x_k + x_k, \tag{12}$$

The damping parameter μ_k is required for solving (7), (9) and (11), and is calculated by relating it to the so-called *trust region radius* ρ_k, which is an upper limit on the step length, such that $\|\Delta x_k\| \leq \rho_k$ for all iterations. The details of the relations between μ_k and ρ_k are quite lengthy, and are therefore omitted.

The filter concept, described by Fletcher (1998), is used as acceptance criteria. A filter is a set of pairs $\{(f(x_i),\ \ h(x_i))\}$, $i \in F$, that are non-dominating in the Pareto (1969) sense of the word. The function h is defined as:

$$h(x) = \max\{0, \max\{v(x)\}\}, \tag{13}$$

where $v(x)$ is given by (10). In order for a step Δx_k to be accepted, it must provide a pair $\{(f(x_{k+1}), \quad h(x_{k+1}))\}$, that is acceptable to the filter, i.e. the pair must not be dominated by any other pair in the filter. Furthermore, the iterate must be a so-called *h*-type iterate. See Fletcher (1998) for details regarding this concept.

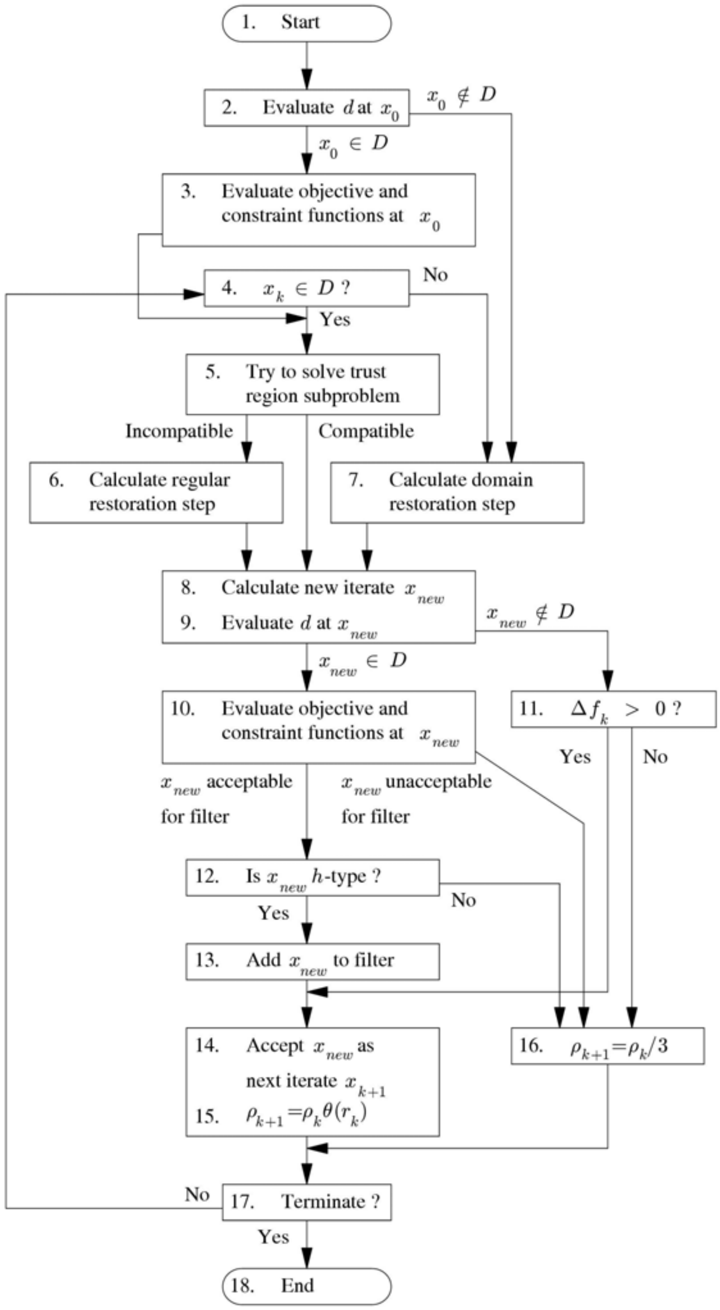

FIG. 3: Flowchart for the gradient-free SQP filter algorithm.

The trust region radius is increased if there is a good match between the expected and actual decrease in the objective function value, and decreased otherwise. This criterion is evaluated using the so-called *gain factor* r_k, given by:

$$r_k = \Delta f(x_k) / \Delta l(x_k) \, , \qquad (14)$$

where $\Delta f(x_k)$ is the decrease in the relevant objective function, and where $\Delta l(x_k)$ is the decrease in the corresponding Taylor approximation. The parameter ρ_k is updated using the expression $\rho_{k+1} = \rho_k \theta(r_k)$, where

$$\theta(r_k) = \tfrac{2}{3}\tanh\left(10\left(r_k - \tfrac{1}{2}\right)\right) + 1 \qquad (15)$$

The trust region is reduced with a factor of 3, if the iterate is unacceptable to the filter, if the iterate is not *h*-type, or if the domain constraint violation did not decrease for an iterate $x_k \notin D$.

Figure 3 shows a flowchart for the algorithm. Details regarding the stopping criteria are omitted.

5. Case study

The combination of simulation and optimization method are used for estimating efficient design decisions for a 3 storey, 2000 m^2 office building. The aim is to optimize the building with respect to the energy performance. The annual energy consumption is therefore required to be minimal. Table 1 shows the initial and optimal values for decision variables and utility functions, as well as the requirements to the solution. The list of decision variables and utility functions are only partial, full lists are provided by Pedersen (2006). The omitted utility functions do not influence the solution.

Notice that the optimal energy consumption is higher than the initial one. This is because the initial decisions are infeasible, since the requirements to daylight utilization are not satisfied. The building provided by the optimum decisions is the one with the lowest annual energy consumption, that at the same time satisfy all requirements. Notice also that the optimal construction cost is the one that fully exploits the allowed limits.

The solution is restricted by the upper limits on the amount of insulation used in the ground slab and roof construction, as well as the upper limits on the daylight utilization measure and the construction costs.

The requirements to the solution can be arranged as entries in the parameters r_O , A_I , b_I , A_E and b_E , in problem (2), which is solved using the gradient-free SQP filter algorithm, in order to calculate the optimum decision variables. If the decision-maker wishes to change the requirements, it can be done simply by changing these parameters. This feature enables the formulation (2) to be used for optimizing buildings in many different ways, for instance with respect to energy, economy or indoor climate.

TABLE. 1: Initial and optimum values for decision variables and utility functions.

Decision variable		Requirement	Initial value	Optimum value
Width to length ratio	[]		0.200	0.146
Number of floors	[]	= 3	3.000	3.000
Window percentage, front	[]		0.400	0.396
Window percentage, back	[]		0.400	0.396
Insulation, ground slab	[m]	≤ 0.5	0.200	0.500
Insulation, roof	[m]	≤ 0.5	0.200	0.143
Insulation, external walls	[m]	≤ 0.5	0.200	0.500
Utility function		**Requirement**	**Initial value**	**Optimum value**
Annual energy use	[kWh]	minimal	136392.96	138981.18
U-value for ground slab	[W/m^{2}K]	≤ 0.30	0.18	0.07
U-value for external walls	[W/m^{2}K]	≤ 0.40	0.17	0.22
U-value for roof	[W/m^{2}K]	≤ 0.25	0.13	0.06
Daylight utilization, front	[]	≤ 4	4.66	4.00
Daylight utilization, back	[]	≤ 4	4.66	4.00
Construction cost	[DKR]	≤ 10^7	9318393.71	10000000.00
Annual operational cost	[DKR]		67243.00	69577.85

The thermal resistance of the uninsulated parts of the ground slab, roof construction and external walls are 0.42 m^2K/W, 0.68 m^2K/W and 2.5 m^2K/W, respectively. The internal and external surface resistances are 0.13 m^2K/W and 0.04 m^2K/W, respectively. The external surface resistance is only used for the external walls. The thermal conductivity of the insulation material is 0.039 W/mK.

6. Conclusion

This paper concerns numerical methods for optimizing the performance of buildings, and describes how decision-making can be supported at early stages of the design process by combining numerical simulation and optimization methods.

A problem formulation is provided that enables decision-makers to formulate requirements to buildings in a highly flexible way. The problem formulation facilitates decision-makers to specify what aspect of the building performance to optimize, for instance energy performance, economy or indoor environment. Upper and lower bounds on decision variables and utility functions can furthermore be specified. It is believed that the proposed problem formulation is useful for developing highly flexible software systems for building-related decision support.

Efficient design decisions are estimated by optimizing decision variables for a conceptual building model. The purpose of this model is to represent general features of the building, such as volume, surface area, mass of constructions, window areas, etc.

The details of a gradient-free SQP filter algorithm are described. The method solves constrained optimization problems without requiring gradient information, and furthermore ensures that the input given to the simulation methods is valid.

A case study regarding an office building is conducted. The initial building has a low annual energy use, but is infeasible, where the optimized building has a higher annual energy use, but is feasible.

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