Discussion on the paper "Optimal Magnetic Momentum Control of Inertially Pointing Spacecraft" by Marco Loverainear

Wisniewski, Rafal

Publication date:
2001

Document Version
Også kaldet Forlagets PDF

Discussion on the paper "Optimal Magnetic Momentum Control of Inertially Pointing Spacecraft" by Marco Lovera

1 Discussion by Rafal Wisniewski

The paper by Maro Lovera proposes a novel momentum unloading algorithm for an inertially stabilized spacecraft. The spacecraft is actuated by reaction wheels working in zero momentum bias. The periodic control theory is used for synthesis of a controller compensating external disturbances by means of reaction wheels and magnetic actuators. The key issue is an observation that the magnetic field as observed from a low earth orbit is almost periodic.

1.1 Implementation Issues

The disturbances acting on the spacecraft have oscillating and secular components. Typical attitude control system compensates for the oscillating environmental torques with reaction wheels whereas the influence of the secular part is attenuated using magnetorquers. The justification of this practice is that the bandwith of the magnetic actuators is very slow in the magnitude of one orbit,while the reaction wheels are fast. The response time of the reaction wheels corresponds to one minute. Hence implementation of the control

$$\mathbf{m}_{\text{coil}}(t) = -\frac{k}{|\mathbf{b}|^2}\mathbf{b}(t) \wedge \mathbf{h}_{wheels}(t), \tag{1}$$

with a positive constant k seems to be reasonable. The desired gain can be computed using the Floquet theory [3]. The closed loop system is then parameterized by the gain k, $\dot{x}(t) = A(k,t)x$, and the root locus of the characteristic multipliers can be used for assigning the desired performance, [1].

The author has showed that the control action can be reduced if a periodic controller is implemented instead of time invariant as in (1). However, from practical point of view the complexity of the attitude control system is increased. The time dependent control gain has to be stored in the computer memory and additional information of the current spacecraft position has to be interfaced to the attitude controller. Furthermore, magnetic field of the Earth diverges from ideally periodic, therefore the control gain has to be updated once in a while during the entire mission, which adds an extra task on a ground station.

1.2 Synthesis of Periodic Control

The author has proposed two elegant schemes for synthesis of period controller, which are based on periodic solutions to periodic Riccati equations. The author has recommended the direct backward integration of the differential Riccati equation. In this perspective it is worth mentioning a result of [4] providing a hint of how to choose the final condition for backward integration.

Theorem 1 ([4]) *Let* $\mathbf{P}(t)$ *be the solution to the Riccati equation*

$$-\dot{\mathbf{P}}(t) = \mathbf{P}(t)\mathbf{A}(t) + \mathbf{A}^{\mathrm{T}}(t)\mathbf{P}(t) - \mathbf{P}(t)\mathbf{B}(t)\mathbf{R}^{-1}(t)\mathbf{B}^{\mathrm{T}}(t)\mathbf{P}(t) + \mathbf{Q}(t), \tag{2}$$

with final conditions $\mathbf{P}(\tau) = \mathbf{P}_f \geq \mathbf{0}$ *(positive semidefinite) and assume that*

1. $(\mathbf{A}(t), \mathbf{B}(t))$ *is stabilizable*
2. $(\mathbf{A}(t), \mathbf{Q}(t))$ *is detectable*
3. $\mathbf{F} \equiv \mathbf{P}(\tau) - \mathbf{P}(\tau - T)$ *is positive semidefinite.*

Then the periodic matrix function $\mathbf{P}(t)$ *is stabilizing, i.e. the control law* $\mathbf{u}(t) = -\mathbf{R}^{-1}(t)\mathbf{B}^{\mathrm{T}}(t)\mathbf{P}(t)\mathbf{x}(t)$ *is stable.*

Theorem 1 reveals that even though the stationary periodic solution is not reached, but the matrix $\mathbf{F}$ is positive definite, then the closed system proposed by the author is still stable. To keep $\mathbf{F}$ positive definite sufficiently large final condition for backward integration should be chosen.

1.3 Alternative Methods for Periodic Control Synthesis

The author has suggested to apply two methods for the control synthesis, periodic LQ and H_∞, both relying on the periodic Riccati equations. It is noteworthy that alternative methods for H_∞ and H_2 control synthesis based on the Linear Matrix Inequality technique has appeared recently in the literature [2] and [5]. To give an insight into the structure of the LMI based algorithms, a procedure for the H_2 state feedback control is presented below. This algorithm corresponds to the periodic LQ control synthesis applied by the author.

The objective of the control design is to compute a gain $\mathbf{K}(t)$ for which the transfer function $s_c : \mathbf{w} \mapsto \mathbf{z}$ of a discrete time periodic system

$$\begin{array}{lcl} \mathbf{x}(t+1) & = & \mathbf{A}_c(t)\mathbf{x}(t) + \mathbf{B}_1(t)\mathbf{w}(t) \\ \mathbf{z}(t) & = & \mathbf{C}_c(t)\mathbf{x}(t), \end{array} \tag{3}$$

where $\mathbf{A}_c(t) = \mathbf{A}(t) + \mathbf{B}_2(t)\mathbf{K}(t)$, $\mathbf{C}_c(t) = \mathbf{C}_1(t) + \mathbf{D}_{12}(t)\mathbf{K}(t)$ satisfies

$$||s_c||_2 < \gamma. \tag{4}$$

Then the algorithm for the H_2 control synthesis is as follows.

1. Find using the LMI technique a symmetric matrix $\mathbf{Q}(t)$ and a matrix $\mathbf{Z}(t)$ for $t = 0...N-1$ (N is the period of the system) satisfying

$$\begin{array}{ll} & \left(\mathbf{W}_1(t)^{\mathrm{T}}\mathbf{A}(t) + \mathbf{W}_2(t)^{\mathrm{T}}\mathbf{C}_1(t)\right)\mathbf{Q}(t-1)\left(\mathbf{A}(t)^{\mathrm{T}}\mathbf{W}_1(t) + \mathbf{C}_1(t)^{\mathrm{T}}\mathbf{W}_2(t)\right) \\ - & \mathbf{W}_1(t)^{\mathrm{T}}\mathbf{Q}(t)\mathbf{W}_1(t) - \mathbf{W}_2(t)^{\mathrm{T}}\mathbf{W}_2(t) < \mathbf{0}, \end{array}$$

$$
\begin{aligned}
\operatorname{im}\begin{bmatrix}\mathbf{W}_1(t)\\ \mathbf{W}_2(t)\end{bmatrix} &= \ker\begin{bmatrix}\mathbf{B}_2(t)^{\mathrm{T}} & \mathbf{D}_{12}(t)^{\mathrm{T}}\end{bmatrix},\\
\begin{bmatrix}\mathbf{Q}(t) & \mathbf{B}_1(t)\\ \mathbf{B}_1(t)^{\mathrm{T}} & \mathbf{Z}(t)\end{bmatrix} &> \mathbf{0},\\
\operatorname{tr}(\sum_{\mathrm{t}=0}^{\mathrm{N}-1}\mathbf{Z}(\mathrm{t})) &< N\gamma^2,\\
\mathbf{Q}(0) &= \mathbf{Q}(N).
\end{aligned}
$$

2. For each $t = 0...N-1$ find using the LMI technique $\mathbf{K}(t)$ which satisfies

$$
\begin{aligned}
\begin{bmatrix}-\mathbf{Q}(t) & \mathbf{A}(t) & \mathbf{0}\\ \mathbf{A}(t)^{\mathrm{T}} & -\mathbf{Q}^{-1}(t-1) & \mathbf{C}_1(t)^{\mathrm{T}}\\ \mathbf{0} & \mathbf{C}_1(t) & -\mathbf{I}\end{bmatrix} &+ \begin{bmatrix}\mathbf{B}_2(t)^{\mathrm{T}} & \mathbf{0} & \mathbf{D}_{12}(t)^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\mathbf{K}(t)\begin{bmatrix}\mathbf{0} & \mathbf{I} & \mathbf{0}\end{bmatrix}\\
&+ \begin{bmatrix}\mathbf{0} & \mathbf{I} & \mathbf{0}\end{bmatrix}^{\mathrm{T}}\mathbf{K}(t)^{\mathrm{T}}\begin{bmatrix}\mathbf{B}_2(t)^{\mathrm{T}} & \mathbf{0} & \mathbf{D}_{12}(t)^{\mathrm{T}}\end{bmatrix} < \mathbf{0}
\end{aligned}
$$

In the algorithm above the burden of the direct backward integration of the differential Riccati equation is replaced by the LMI technique.

References

[1] A.Wisniewski and M. Blanke. Fully magnetic attitude control for spacecraft subject to gravity gradient. *Automatica*, 35(7):1201–1214, July 1999.

[2] G. E. Dullerud and S. Lall. A new approach for analysis and synthesis of time-varying systems. *IEEE Transactions on Automatic Control*, 44:1486–1479, 1999.

[3] R.R. Mohler. *Nonlinear Systems*, volume Dynamics and Control. Prentice Hall, 1991.

[4] G. De Nicolao. Cyclomonotonicity and stabilizability properties of solutions of the difference periodi. *IEEE Transactions on Automatic Control*, 37(9):1405–1410, September 1992.

[5] R. Wisniewski and J. Stoustrup. Generalized H_2 control synthesis for periodic systems. In *2001 American Control Conference, Arligton, Virginia, USA*, 2001.