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LMI-based Gain Scheduled Robust Flux Observer for Induction Motors

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Abstract

In this paper a robust flux observer for an induction motor is developed through an LMI approach. The observer is robust to changes in rotational speed and in rotor and stator resistances. The problem is formulated as complex-valued rank constrained LMIs and solved through alternating projections. The method achieves good performance with very little tuning needed.

1 Introduction

Induction motors are widely used in industry, due to their relatively low cost and high reliability. One way to obtain a speed or torque control with a dynamic performance similar to that of a more expensive DC-motor is to use Field Oriented Control (FOC) [11]. Many other methods have been suggested [16] [17] [14], but in general an estimate of the rotor flux is needed in most of these control schemes. Therefore a rotor flux observer must be employed.

The dynamic behaviour of the induction motor is affected by time variations, mainly in the rotational speed and in the rotor and stator resistances. The rotor flux observer must be robust with regard to these variations.

The simplest flux estimation method is an open loop observer based on stator current measurements [10]. This method suffers from poor robustness and a slow convergence rate. Several methods have been suggested to overcome this [15] [12], but most of these are hard to tune or difficult to implement. For industrial purposes the ideal observer scheme is easy to implement in hardware and does not require tuning.

In this paper a robust flux observer is developed using structured singular value (μ) and Linear Matrix Inequalities (LMI). The method used makes it very simple to include on-line measurements of the rotational speed for gain scheduling, but the main objective is to achieve an observer suitable for speed sensorless control. The observer requires very little tuning and is robust to variations in rotor and stator resistances.

In the nineties, several control design problems have been formulated in terms of LMIs. Efficient methods exist for

solving these convex optimisation problems [2]. In [5] Gahinet and Apkarian provided solutions to $\mathcal{H}_\infty$ control problems in terms of LMIs. In 1993 Packard suggested using LMIs for gain scheduling synthesis of systems on linear fractional form [13]. In [8] Helmersson suggested a controller synthesis method including both robustness to uncertain parameters and gain scheduling. The synthesis problem is in the form of a rank constrained LMI problem. This approach will be taken here.

Section 2 describes the model of the induction motor and the uncertainties considered. Section 4 deals with the applied method and how the rank constrained LMI problem is solved. The performance of the observer will be demonstrated through simulations in Section 5.

1.1 Redheffer star product

The Redheffer star product, $\star$, represents the interconnection in Figure 1, i.e.

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = (A \star B) \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

Note that $A \star B$ depends on a partitioning of A and B . This partitioning will always be clear from the context. See [18] on how to compute $A \star B$.

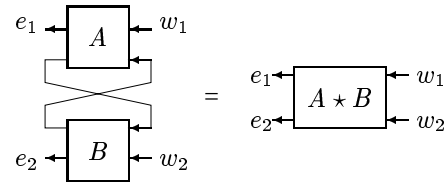


Figure 1: Redheffer star product.

2 Induction motor model

With widely used simplifying assumptions on symmetry the following state space model can be derived for a VSI-controlled squirrel cage induction motor [11]:

$$m_e = \frac{3Z_p L_m^2}{2L_r} \Im\{\bar{i}_s(\bar{i}_m - \bar{i}_s)^*\} \quad (3)$$

$$\dot{x}_{sr} = A_{sr}x_{sr} + B_{sr}u_{sr}$$

$$x_{sr} = \begin{bmatrix} i_{sD} \\ i_{sQ} \\ i_{mD} \\ i_{mQ} \end{bmatrix}, u_{sr} = \begin{bmatrix} u_{sD} \\ u_{sQ} \end{bmatrix}$$

$$A_{sr} = \begin{bmatrix} \frac{L_m^2 R_r + R_s L_r^2}{L_r(L_m^2 - L_s L_r)} & 0 & \frac{-L_m^2 R_r}{L_r(L_m^2 - L_s L_r)} & \frac{-L_m^2 \omega_r}{L_r(L_m^2 - L_s L_r)} \\ 0 & \frac{L_m^2 R_r + R_s L_r^2}{L_r(L_m^2 - L_s L_r)} & \frac{-L_m^2 \omega_r}{L_r(L_m^2 - L_s L_r)} & \frac{-L_m^2 R_r}{L_r(L_m^2 - L_s L_r)} \\ \frac{R_r}{L_r} & 0 & -\frac{R_r}{L_r} & -\omega_r \\ 0 & \frac{R_r}{L_r} & \omega_r & -\frac{R_r}{L_r} \end{bmatrix}$$

$$B_{sr} = \begin{bmatrix} \frac{L_r}{L_s L_r - L_m^2} & 0 \\ 0 & \frac{L_r}{L_s L_r - L_m^2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$m_e = \frac{3Z_p L_m^2}{2L_r} (i_{sQ} i_{mD} - i_{sD} i_{mQ}) \quad (1)$$

$$\dot{\omega}_r = \frac{Z_p}{J} (m_e - m_L) \quad (2)$$

i_{mD} and i_{mQ} are the magnetising currents which we wish to estimate based on measurements of the stator currents i_{sD} , i_{sQ} and the stator voltages u_{sD} , u_{sQ} . ω_r is the rotational speed.

The model has five states, the four currents and the rotational speed. The latter has been left out of the state space equations for two reasons. Firstly, without knowledge of the shaft load m_L , very little information about the magnetising current can be extracted from the equation. Secondly, excluding it allows the model to be written as a linear parameter varying model with ω_r as the varying parameter.

As described in Section 3 the special structure of the state space equations governing the currents allows them to be rewritten in a complex form:

$$\dot{x}_{im} = A_{im}x_{im} + B_{im}\bar{u}_s \quad x_{im} = \begin{bmatrix} \bar{i}_s \\ \bar{i}_m \end{bmatrix}$$

$$y = C_{im}x_{im} + D_{im}\bar{u}_s$$

$$A_{im} = \begin{bmatrix} \frac{L_m^2 R_r + R_s L_r^2}{L_r(L_m^2 - L_s L_r)} & \frac{L_m^2 (jL_r \omega_r - R_r)}{L_r(L_m^2 - L_s L_r)} \\ \frac{R_r}{L_r} & j\omega_r - \frac{R_r}{L_r} \end{bmatrix}$$

$$B_{im} = \begin{bmatrix} \frac{L_r}{L_s L_r - L_m^2} \\ 0 \end{bmatrix}$$

$$C_{im} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad D_{im} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\dot{\omega}_r = \frac{Z_p}{J} (m_e - m_L) \quad (4)$$

where $\bar{i}_m = i_{mD} + j i_{mQ}$, $\bar{i}_s = i_{sD} + j i_{sQ}$, $\bar{u}_s = u_{sD} + j u_{sQ}$, and $\Im\{\cdot\}$ means imaginary part. As discussed in Section 3 this is the model which will be used further on. The matrices C_{im} and D_{im} have been added to model the measurement of stator current and voltage.

2.1 LFT form

The method presented in Section 4 requires the model to be in the form of a linear fractional transformation (LFT) as shown in Figure 2.

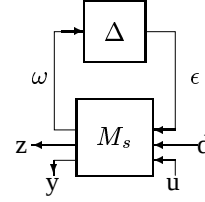


Figure 2: Linear fractional transformation form.

M_s is a constant matrix, and Δ is a block diagonal matrix of bounded uncertainties. In Section 4 it is described how to design a controller using measurements y and control input u to minimise the $\mathcal{H}_\infty$ -norm of the transfer function from disturbance input d to performance output z . When designing an observer, the control input u will be the state estimate, and the performance output z will be the estimation error. In order to fit the model presented above into the LFT framework M and Δ can be chosen as

$$M_s = \begin{bmatrix} A_{im} & B_{im} & 0 \\ [0 & 1] & 0 & -1 \\ C_{im} & D_{im} & 0 \end{bmatrix} \quad \Delta = s^{-1} I_2$$

This form cannot be used directly, since s^{-1} is not bounded. The following section describes how to overcome this.

2.2 Finite frequency method

In order to describe a dynamic system by a constant matrix the frequency must be included in the uncertainties. For the observer synthesis it is necessary to have bounded uncertainties. Therefore we cannot directly use the LFT realisation presented above. Two ways to overcome this are suggested in [7]. The first way is to use a mapping from the unit disc to the right half plane:

$$\frac{1}{s} = \frac{1 + \delta}{1 - \delta} \quad (5)$$

This mapping can be realised via an LFT:

$$s^{-1}I = \delta I \star N_s \quad N_s = \begin{bmatrix} I & \sqrt{2}I \\ \sqrt{2}I & I \end{bmatrix} \quad (6)$$

The system can then be written as an LFT of the bounded δ :

$$(s^{-1}I) \star M_s = ((\delta I) \star N_s) \star M_s = (\delta I) \star (N_s \star M_s) \quad (7)$$

Defining the new matrix $M \equiv N_s \star M_s$ the frequency can then be treated as a complex uncertainty $|\delta| < 1$.

Alternatively a mapping from the interval $[-1, 1]$ to the negative imaginary axis can be performed with a similar LFT. The frequency can then be treated as a real uncertainty $\delta \in [-1, 1]$.

In [7] the method is only proposed for μ -analysis, but here it will be used for synthesis. The second mapping can therefore not be used as it will not guarantee stable observers. In addition it only covers the negative half of the axis, but for a complex-valued system the frequency response will not necessarily be symmetric about the real axis.

2.3 Uncertain and time-varying parameters

The dynamical behaviour of the induction motor is affected by time variations in the parameters. Parametric uncertainties can easily be formulated as LFTs, see for instance [18].

The rotor resistance R_r can change as much as 50 % due to heating. Since the change is slow the uncertainty is modelled as real.

The stator resistance R_s can also change, but is usually better ventilated than R_r , so the variations will not be quite as large. The stator resistance uncertainty is also modelled as real.

The rotational speed ω_r can change due to load disturbances or as a result of a command change to the controller. As the changes can be fast the uncertainty is modelled as complex. Sometimes ω_r is measured, but avoiding the use of a speed sensor is often desirable, due to the relatively high cost and high sensitivity to the environment of speed sensors. Since the framework described in Section 4 easily allows for both known and unknown uncertainties, both approaches are tried here.

3 Complex system

Certain real valued systems can be written on a complex form with half the number of states, inputs, and outputs. Define the following convex matrix function set

$$\mathcal{C}^s = \{M(s) : M(s) = \begin{bmatrix} M_r(s) & -M_i(s) \\ M_i(s) & M_r(s) \end{bmatrix}\} \quad (8)$$

and the matrix

$$J = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} \quad (9)$$

where the dimension will be apparent from the context.

Lemma 1 Let $G, H, F = F^* \in \mathcal{C}^s$ and

$$JG + G^*J + JHH^*J + F < 0 \quad \forall s \quad (10)$$

Then

$$F < 0 \quad \forall s \quad (11)$$

Proof: Obviously $N = JG + G^*J + F < 0$. Then also

$$\begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} N \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} + N = 2F < 0 \quad (12)$$

□

Lemma 2 Let $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & 0 \end{bmatrix}$, where $M_{11}, M_{12}, M_{21} \in \mathcal{C}^s$. If a controller K exists yielding closed-loop $\mathcal{H}_\infty$ performance $\|M \star K\|_\infty < \gamma$ then a controller $K_1 \in \mathcal{C}^s$ exists with equal or better closed loop performance, i. e. $\|M \star K_1\|_\infty < \gamma$.

Outline of the proof: The closed loop transfer function is $C_l = M_{11} + M_{12}KM_{21}$. $\|C_l\|_\infty < \gamma$ if and only if $C_l^*C_l < \gamma^2 I$. Decompose K as $K = K_1 + K_2J$, where $K_1, K_2 \in \mathcal{C}^s$. When calculating $C_l^*C_l - \gamma^2 I$ it can be put on the form in (10), where G and H are zero when K_2 is zero and where F does not depend on K_2 . By Lemma 1 the performance will then at least be maintained by setting $K_2 = 0$. □

Note that since the performance holds at all frequencies, Lemma 2 holds equally well for $\mathcal{H}_2$ performance.

A state space system on the $\mathcal{C}^s$ -form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_r & -A_i \\ A_i & A_r \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_r & -B_i \\ B_i & B_r \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (13)$$

can be written in the complex for

$$\dot{x} = Ax + Bu \quad (14)$$

where $x = x_1 + jx_2$, $u = u_1 + ju_2$, $A = A_r + jA_i$, and $B = B_r + jB_i$. Note that the state space model of the current equations (1) can be put on this form through a state transformation.

This complex system is computationally simpler to work with, and from Lemma 2 it is known that the optimal controller is also of this form. Therefore it is chosen to work with the complex form in the following.

4 Observer synthesis

The closed loop observer system is depicted in Figure 3. Δ includes the frequency, the parameter uncertainty, and a mapping from estimation error to disturbance input. The system is scaled so that the uncertainties are bounded by $\bar{\sigma}(\Delta) \leq 1$. The system is then stable if and only if $\mu_\Delta(M_c) < 1$, where $M_c = M \star K$ [18].

Stability is not an issue in observer design as long as the observer itself is stable, but robust performance is assured by

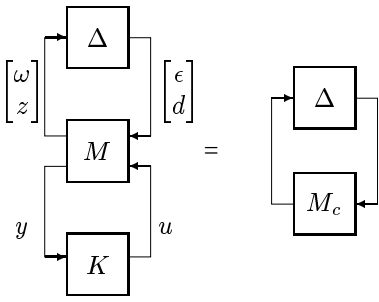


Figure 3: Observer K and uncertainty system $\Delta \star M$.

letting the uncertainty block, Δ , include a full complex block mapping from the scaled estimation error to disturbance input [18]. Robust stability of this system then implies robust performance of the original system. This section describes a method for designing an observer K making $\mu_\Delta(M_c) < 1$.

4.1 LMI synthesis

The upper bound function ν_Δ is defined as

$$\nu_\Delta(M_c) = \inf_{\substack{\nu > 0 \\ P \in \mathcal{P}_\Delta \\ G \in \mathcal{G}_\Delta}} \{ \nu : M_c^* P M_c + j(G M_c - M_c^* G) \leq \nu^2 P \} \quad (15)$$

$\mathcal{P}_\Delta$ and $\mathcal{G}_\Delta$ are block diagonal matrix sets corresponding to the uncertainty structure. If an uncertainty sub-block is full, then the corresponding sub-blocks of $\mathcal{P}_\Delta$ and $\mathcal{G}_\Delta$ are repeated scalars ($aI, a \in \mathbb{R}$). If the uncertainty sub-block is repeated scalar, then the corresponding sub-blocks of $\mathcal{P}_\Delta$ and $\mathcal{G}_\Delta$ are Hermitian full blocks. If the uncertainty sub-block is Hermitian (for repeated scalar, read real), then the corresponding sub-block of $\mathcal{G}_\Delta$ must be zero. Furthermore the sub-blocks of $\mathcal{P}_\Delta$ must be positive definite.

ν provides an upper bound for μ , i.e. $\mu_\Delta(M_c) \leq \nu_\Delta(M_c)$ [3]. Since the μ -value cannot be easily computed, the observer K will instead be designed to make $\nu_\Delta(M_c) < 1$.

Defining the multiplier set

$$\mathcal{W}_\Delta = \{ W = P + jG : P \in \mathcal{P}_\Delta, G \in \mathcal{G}_\Delta \}$$

(15) can then be written as an LMI in W [8]:

$$\nu_\Delta(M_c) = \inf_{\substack{\nu > 0 \\ W \in \mathcal{W}_\Delta}} \{ \nu : \text{herm}((\nu I + M_c)^* W (\nu I - M_c)) \geq 0 \} \quad (16)$$

where $\text{herm}(X) = \frac{1}{2}(X + X^*)$, i.e. the Hermitian part. The structure of W is induced by the structure of the uncertainties. W is block diagonal, and each block corresponds to a block in the uncertainty. If an uncertainty is a full block, then the corresponding block of W must be repeated scalar. If an uncertainty is a repeated scalar, then the corresponding block of W can be a full block. For complex uncertainties the corresponding W -blocks must be Hermitian and positive. For Hermitian uncertainties it is only required that the W -block is positive real ($\text{herm}W_i > 0$).

Now, partition M as

$$M = \begin{bmatrix} Q & U \\ V^* & 0 \end{bmatrix} \quad (17)$$

Then

$$M_c = M \star K = Q + U K V^* \quad (18)$$

and according to [8] a controller exists making $\nu(M \star K) < 1$ if and only if a multiplier $W \in \mathcal{W}_\Delta$ exists so that

$$\text{herm}(V^\perp (I + Q^*) W (I - Q) V^{\perp*}) > 0 \quad (19)$$

and

$$\text{herm}(U^\perp (I + Q) W^{-1} (I - Q^*) U^{\perp*}) > 0 \quad (20)$$

where $X^\perp$ is a matrix satisfying $\ker X^\perp = \text{range} X$. The objective is to find a $W \in \mathcal{W}_\Delta$ satisfying (19) and (20). Once this W has been found, P and G can be found. Inserting (18) along with P and G in (15) gives a feasible LMI in K .

4.2 Gain scheduling synthesis

Some of the time varying parameters may not be known at the time of the synthesis, but can be measured on-line. To take advantage of this knowledge the controller can be made dependent on these parameters in a gain scheduling procedure. Gain scheduling is achieved by letting the controller have access to copies of some of the uncertainties. For instance the frequency which was included in the uncertainty block is known. $\bar{\Delta} = \begin{bmatrix} \Delta & 0 \\ 0 & \bar{\Delta} \end{bmatrix}$ is the uncertainty block with the known copies $\bar{\Delta}$ augmented. $\bar{M}$ is the system augmented with feed-through connections from the copies to the controller, giving a new system $\bar{M}$:

$$\bar{M} = \begin{bmatrix} \bar{Q} & \bar{U} \\ \bar{V}^* & 0 \end{bmatrix} = \begin{bmatrix} Q & 0 & U & 0 \\ 0 & 0 & 0 & I_r \\ V^* & 0 & 0 & 0 \\ 0 & I_r & 0 & 0 \end{bmatrix} \quad (21)$$

These feed-through connections allow the controller to access the copies of the uncertainties. When inserting in (19) and (20) everything but the upper left part of W corresponding to Δ vanishes. Define S and R as the upper left part of W and W^{-1} respectively. We then have

$$\text{herm}(V^\perp (I + Q^*) S (I - Q) V^{\perp*}) > 0 \quad (22)$$

and

$$\text{herm}(U^\perp (I + Q) R (I - Q^*) U^{\perp*}) > 0 \quad (23)$$

If no uncertainties are known, then $W = S = R^{-1}$. Having copies of the uncertainties gives more freedom in the choice of R and S . To guarantee the existence of a W with an upper left part equal to S and an inverse with an upper left part

equal to R certain conditions must be put on S and R . S and R must have the same block diagonal structure as the desired W . Furthermore the following rank constraint must be fulfilled for each block

$$\text{rank}(S_i - R_i^{-1}) \leq r_i \quad (24)$$

r_i is the number of known copies of the i -th uncertainty block. If the uncertainty is fully known then this is always true.

For Hermitian uncertainties we need to guarantee the existence of a positive real W_i . This is possible if the rank constraint (24) is fulfilled and in addition S_i and R_i are positive real. For complex uncertainties W_i must be Hermitian. This is possible if, in addition to the previous demands, S_i and R_i are Hermitian and positive, and if

$$\begin{bmatrix} R_i & I \\ I & S_i \end{bmatrix} \geq 0 \quad (25)$$

Once S and R have been found, W can be reconstructed from these, and K can be found as described above. Once a constant K has been found, it can be converted into a dynamic system, due to its feed-through connections to copies of the frequencies. If other uncertainties are known, the dynamics will depend on these.

4.3 Alternating projection method

The problem of finding S and R in Section 4.2 would be convex if not for the rank constraints (24). In [6] it is suggested to use alternating projections to overcome this problem. Repeated alternating orthogonal projections onto closed convex sets will converge to the intersection of these sets (if the intersection is non-empty). The basic idea is that even though the set of S s and R s fulfilling the rank constraint is not convex, it is still possible to perform a projection. Convergence to a feasible solution is only local, so the choice of initial point becomes important.

In [6] projections onto several convex sets are presented. The intersection of these forms the feasibility set of the LMIs. The projections are only given for symmetric matrices but can easily be generalised to complex matrices that are not necessarily Hermitian. In addition to these all that is needed is then a projection onto the structure of S and R . This projection is simply given by setting the elements outside the block diagonal elements to zero.

Unfortunately the computational overhead of this method is massive. In [1] it is suggested to perform the projection onto all LMI constraints in one step, by using the following lemma.

Lemma 3 [1] *The projection of Z_0 onto the convex set Γ_{convex} is the solution to the convex minimisation problem:*

$$\begin{aligned} \min & \quad \text{Trace}(X) \\ \text{Subject to} & \quad \begin{bmatrix} X & (Z - Z_0)^* \\ (Z - Z_0) & I \end{bmatrix} \geq 0 \\ & \quad Z \in \Gamma_{convex}, X = X^* \end{aligned}$$

The solution to the minimisation problem is the point in the feasibility set minimising the distance to Z_0 in the sense of the Frobenius norm. $\square$

The minimisation problem can easily be implemented in for instance the LMI Control Toolbox [4]. Complex-valued problems can be converted into real-valued problems with twice as many independent variables [4].

Projection onto the rank constraints

In addition to the projection onto the LMIs a projection onto the rank constraints (24) is needed. A block-wise projection is performed for all the sub-blocks of S and R relating to unknown uncertainties via the following lemma:

Lemma 4 [9] [6] *Define the following set:*

$$\mathcal{R} \equiv \{Z \in \mathbb{C}_{2n} : \text{rank}(Z + J) \leq k\}$$

where k is an integer fulfilling $n \leq k \leq 2n$, J is defined in (9), and $\mathbb{C}_{2n}$ is the set of all $2n \times 2n$ complex matrices.

The projection of Z_i onto $\mathcal{R}$ is given by $Z_p = U\Sigma_k V^ - J$, where $U\Sigma V^* = Z_i + J$ is a singular value decomposition, and Σ_k is obtained by replacing the $2n - k$ smallest singular values of Σ by zero. $\square$*

The rank constraint (24) is equivalent to

$$\text{rank} \begin{bmatrix} R_i & I \\ I & S_i \end{bmatrix} \leq n_i + r_i \quad (26)$$

where n_i is the size of S_i and R_i . By defining $Z_i = \begin{bmatrix} R_i & 0 \\ 0 & S_i \end{bmatrix}$ we can use Lemma 4 to project onto the rank constraint and insert the resulting Z_p as Z_0 in Lemma 3 to project back onto the convex constraints. Alternating between these two projections will in most cases converge to a feasible point fulfilling both constraints if such a point exists.

4.4 Finding the initial point

In [1] it is suggested to use the central solution to the LMIs without rank constraints as the initial point for the alternating projections. However, it was found that a better initial point was obtained by first finding the optimal $\mathcal{H}_\infty$ observer disregarding the structure of the uncertainty. This is a convex problem. Once this observer is found, the closed loop transfer matrix is found, and W is found as the central solution to (16).

5 Results

An observer is designed for a 1.5 kW motor with parameters $R_s = 4.5\Omega$, $L_s = 0.34H$, $L_r = 0.342H$, $L_m = 0.329H$, $Z_p = 2$, and $R_r = 4.5\Omega$ and a nominal speed $\omega_r = 280\text{rad/s}$.

When no speed sensor is available, the observer must be robust to deviations from the nominal speed for which it has been designed. A sensible design is only obtained by restricting the considered deviations to a small interval. It is assumed that a speed observer is providing an approximate value of the speed. To allow for a wider range of operation some form of gain scheduling between observers designed for a grid of operating points would have to be employed. Here we will only consider one of these observers. Since speed sensorless flux estimation is usually most difficult at low speeds, the nominal speed $\omega_r = 10\text{rad/s}$ will be considered here.

The only thing that has to be chosen is the range of parameter variations for which the observer must have robust performance. These are chosen as one tenth of the expected variations or more specifically $\omega_r \in [9.8; 10.2]\text{rad/s}$, $R_r \in [4.4; 4.6]\Omega$, and $R_s \in [4.49; 4.51]\Omega$. The intervals are chosen smaller than the expected variations in order to avoid an overly conservative observer.

5.1 Simulations

The designed observer (the ν -observer) will be compared to an observer of the type described in [10] (the JL-observer), tuned to the author's best ability. A simulation of the observers is performed on the data set illustrated in Figure 4. For the first two seconds ω_r , R_r , and R_s have their nominal values. From the time $T = 2\text{s}$ until $T = 3\text{s}$ the rotor resistance slowly increases to 110 % of the nominal value, and then slowly returns to the nominal value. In the following interval from $T = 3\text{s}$ until $T = 4\text{s}$ the stator resistance is changed in the same manner. From $T = 4\text{s}$ until $T = 5\text{s}$ the values are all nominal. Then at $T = 5\text{s}$ the rotational speed is abruptly changed to 9rad/s .

Figure 5 shows the performance of the two observers. The solid lines show the JL-observer while the dashed lines show the ν -observer. The ν -observer is considerably faster initially. The performance degradation due to changes in the resistances is approximately the same for the two observers. The period from $T = 4\text{s}$ to $T = 5\text{s}$ shows that the JL-observer once settled has slightly better nominal performance than the ν -observer. When ω_r changes, the ν -observer clearly outperforms the JL-observer. The reason for this is indicated in the steady state error plots in Figure 6. The first axis is the angular velocity of the magnetising current, and the second axis is the estimation error of the angle. The various lines show the error at different values of ω_r . As seen, the angular estimation error of the ν -observer is almost constant, whereas it is highly frequency-dependent for the JL-observer.

The Figures 7 and 8 show similar plots for variations in the resistances. The steady state errors at high frequencies

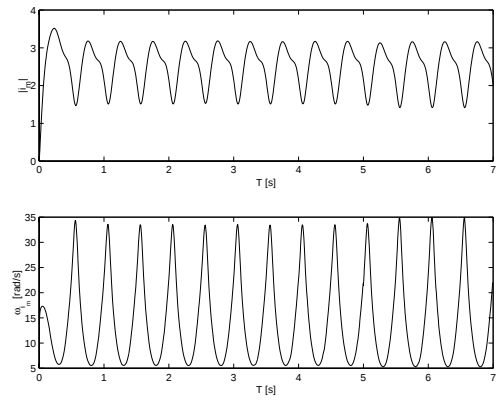


Figure 4: Simulation data. The top figure shows the magnitude of the magnetising current. The bottom figure shows the angular velocity of the magnetising current.

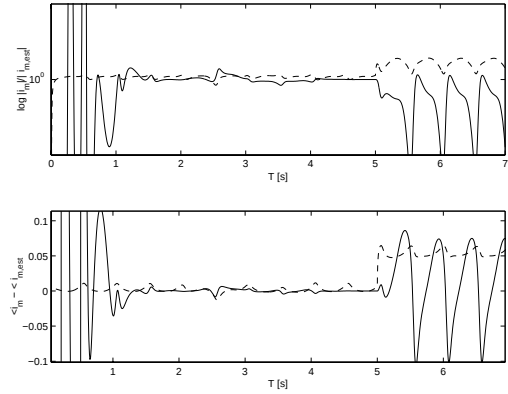


Figure 5: Estimation errors from simulation. The dashed lines show the ν -observer and the solid lines show the JL-observer. The top figure shows the magnitude estimation error. The bottom figure shows the angle estimation error.

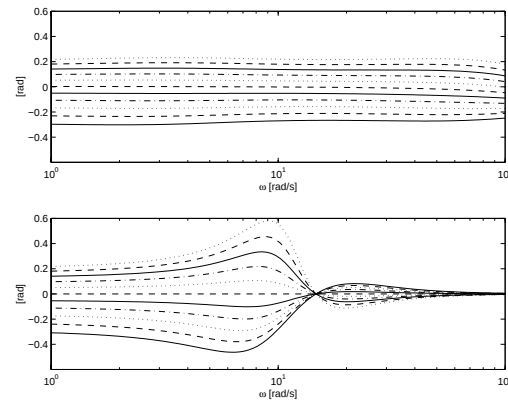


Figure 6: Steady state estimation error of the angle at different rotational speeds. The top figure shows the ν -observer, and the bottom figure shows the JL-observer.

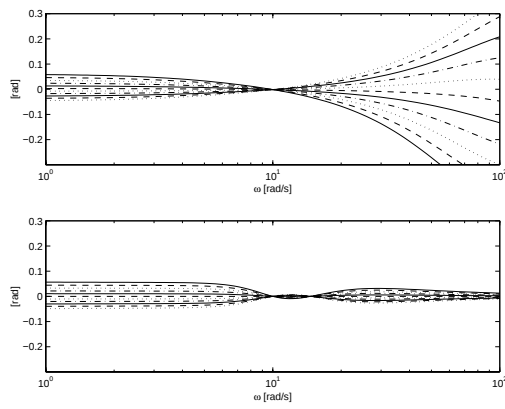


Figure 7: Steady state estimation error of the angle with various values of rotor resistance. The top figure shows the ν -observer, and the bottom figure shows the JL-observer.

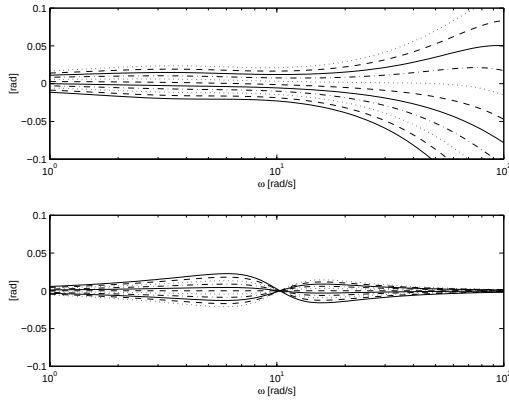


Figure 8: Steady state estimation error of the angle with various values of stator resistance. The top figure shows the ν -observer, and the bottom figure shows the JL-observer.

are very large for the ν -observer. This is fortunately a minor problem since steady state operation usually will not exceed 30rad/s in angular flux velocity.

5.2 Gain scheduling with a speed sensor

Measuring ω_r allows for gain scheduling. An observer was designed for the same system as above, including this gain scheduling, and the estimation errors due to speed variations were practically removed. The sensitivity to resistance variations was almost identical to that of the sensorless observer, so in the case of speed measurements being available no major improvements over the JL-observer were obtained.

6 Conclusion

A rotor flux observer for speed sensorless low speed operation was designed. The sensitivity of the observer to speed variations was similar in magnitude to the sensitivity obtained by the existing method to which it was compared, but

the angular estimation error is almost independent of the angular velocity of the flux, making it more suitable for control purposes.

A major advantage of the method is that very little tuning was required.

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