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QUATERNION FEEDBACK CONTROL FOR RIGID-BODY SPACECRAFT

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Abstract: This paper addresses three-axis attitude control for a Danish spacecraft, Rømer. The algorithm proposed is based on an approximation of the exact feedback linearisation for quaternionic attitude representation. The proposed attitude controller is tested in a simulation study. The environmental disturbances correspond to those expected for the Rømer mission. The pros and cons of the algorithm are discussed. The results of the study show that the controller is a successful candidate for on-board implementation.

Keywords: Attitude control, Nonlinear control, Feedback linearisation

Acronyms

{P}	Spacecraft principal axis coordinate frame.
{I}	Earth centred celestial coordinate frame.
${}^{x}\boldsymbol{v}$	A vector in a certain coordinate system {X}, where {X} is {P} or {I}.
a_1	Constant: $\frac{I_{2,p}-I_{3,p}}{I_{1,p}}$
a_2	Constant: $\frac{I_{3,p}-I_{1,p}}{I_{2,p}}$
a_3	Constant: $\frac{I_{1,p}-I_{2,p}}{I_{3,p}}$
${}^{p}_{i}\boldsymbol{A}$	Transformation matrix from {I} to {P}.
$L_{\boldsymbol{f}}\boldsymbol{h}$	Lie derivative of a function $\boldsymbol{h}$ along vector field $\boldsymbol{f}$.
$\boldsymbol{I}_p$	Inertia tensor for the satellite about the principal axes.
${}^{x}\boldsymbol{L}$	Total angular momentum in {X}.
${}^{p}\boldsymbol{N}_{control}$	Control torque in {P}.
${}^{p}\boldsymbol{N}_{dist}$	Disturbance torque in {P}.
${}^{p}\boldsymbol{N}_{ext}$	External torque in {P}.
${}^{p}\boldsymbol{\omega}_{p,i}$	Angular velocity of {P} in relation to {I} observed in {P}.
${}^{p}_{i}\mathbf{q}$	Attitude quaternion representing the transformation from {I} to {P}.
${}^{p}_{i}\boldsymbol{q}$	Vector part of the quaternion ${}^{p}_{i}\mathbf{q}$.
$\boldsymbol{Q}({}^{p}_{i}\mathbf{q})$	Orthogonal matrix used for kinematics.
$\boldsymbol{u}$	Input vector.
$\boldsymbol{U}$	Feedback linearising input vector.
$\boldsymbol{x}$	State vector.

1. INTRODUCTION

Over the last three decades nonlinear controllers have proved to be of increasing interest. This is due to the fact that most systems are inherently nonlinear, and nonlinear controllers might be designed to guarantee global stability, improved efficiency and increased control performance for such systems. This is especially interesting for spacecrafts, since they are nonlinear and efficiency is an important factor as the energy used on board the satellite is self-obtained.

The attitude controller presented in this paper was proposed as an attitude controller for the next Danish satellite, Rømer. The purpose of the Rømer mission is to measure oscillations of 25 nearby stars and is planned to be launched in

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2002. The objectives of the attitude controller is to make attitude corrections and perform slew manoeuvre.

This paper uses quaternion feedback linearisation for control synthesis. Feedback linearisation is a well-known type of nonlinear control, that is particularly well-suited for implementation on simple and well-defined systems, with precisely known constants.

The unit quaternion has been successfully used in several spacecraft for attitude representation, due to its advantages over other attitude representations such as the Gibbs vector, rotation matrices etc. The unit quaternion provides a simple equation for kinematics where the composition of successive rotations corresponds to the product of corresponding quaternions.

Nonlinear attitude control has been based on passivity, sliding mode and feedback linearisation. Previously the exact feedback linearisation was conducted locally, when the attitude was parameterised by Euler angles (Byrnes and Isidori, 1991), or globally when feedback linearisation was applied only to the spacecraft dynamics (Wen and Kreutz-Delgado, 1991).

This paper will present the problem of feedback linearising the attitude dynamics including quaternion kinematics. The limitations of the presented attitude controller for three orthogonally placed thrusters will be discussed. The paper shows whether the controller presented in this paper is a successful candidate for on-board implementation.

The rest of the paper is organised as follows: Section 2 reviews the mathematical models of rigid-body dynamics and kinematics, which uses the quaternion for attitude representation. Section 3 contains the basic principle of feedback linearisation, that were used for designing the nonlinear controller in section 4. Section 5 presents the results on functionality and stability tests. The concluding remarks comprises section 6.

2. MATHEMATICAL MODELS

The model of a satellite can be divided into a model of the rigid-body dynamics (the Euler equations), and a model of the attitude kinematics. The kinematics are here represented using the unit quaternion.

2.1 *Model of the Rigid-body Dynamics*

The model of a rigid body is as follows (J.R. Wertz, 1997):

$$\frac{d}{dt}\,{}^{p}\boldsymbol{L} = -\,{}^{p}\boldsymbol{\omega}_{p,i} \times\ {}^{p}\boldsymbol{L} + {}^{p}\boldsymbol{N}_{ext} \tag{1}$$

The term ${}^{p}\boldsymbol{N}_{ext}$ represents the external torques applied to the satellite along its principal axes. The input signals to the rigid body is angular momentum about the principal axes, and the term ${}^{p}\boldsymbol{N}_{ext}$ is therefore:

$${}^{p}\boldsymbol{N}_{ext} = {}^{p}\boldsymbol{N}_{dist} + {}^{p}\boldsymbol{N}_{control} \tag{2}$$

The term ${}^{p}\boldsymbol{N}_{dist}$ represents the disturbance torques applied to the satellite. The attitude controller presented in this paper was designed without taking the disturbance torques into account, however they were incorporated in the simulation facility used in section 5.

When ${}^{p}\boldsymbol{\omega}_{p,i}$ is isolated on the left side of equation 1, the following equation for the rigid-body dynamics, is derived:

$$\frac{d}{dt}\begin{bmatrix} {}^{p}\omega_{1,p,i} \\ {}^{p}\omega_{2,p,i} \\ {}^{p}\omega_{3,p,i} \end{bmatrix} = \begin{bmatrix} a_1\,{}^{p}\omega_{2,p,i}\,{}^{p}\omega_{3,p,i} + \dfrac{{}^{p}N_{1,control}}{I_{1,p}} \\ a_2\,{}^{p}\omega_{1,p,i}\,{}^{p}\omega_{3,p,i} + \dfrac{{}^{p}N_{2,control}}{I_{2,p}} \\ a_3\,{}^{p}\omega_{1,p,i}\,{}^{p}\omega_{2,p,i} + \dfrac{{}^{p}N_{3,control}}{I_{3,p}} \end{bmatrix} \tag{3}$$

2.2 *The Unit Quaternion*

The unit quaternion gives a minimal global representation of the attitude using only four parameters, and geometrically it corresponds to a point on a 3-sphere ($\mathbb{S}^3$) in $\mathbb{R}^4$.

A quaternion is defined as:

$$\mathbf{q} \equiv \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \equiv \begin{bmatrix} \boldsymbol{q} \\ q_4 \end{bmatrix} \tag{4}$$

where q_4 is the scalar part of the quaternion and $\boldsymbol{q}$ is the vector part. The vector part is described by the components $iq_1 + jq_2 + kq_3$ in which i, j and k are hyper-imaginary numbers.

2.3 *Model of the Kinematics Using the Unit Quaternion*

The kinematics are defined by using the quaternion to represent the attitude of the inertial coordinate frame in the principal coordinate frame (H.S. Morton, 1993):

$$\frac{d}{dt}\begin{bmatrix} {}^{p}_{i}q_1 \\ {}^{p}_{i}q_2 \\ {}^{p}_{i}q_3 \\ {}^{p}_{i}q_4 \end{bmatrix} = \frac{1}{2}\underbrace{\begin{bmatrix} {}^{p}_{i}q_4 & -{}^{p}_{i}q_3 & {}^{p}_{i}q_2 & {}^{p}_{i}q_1 \\ {}^{p}_{i}q_3 & {}^{p}_{i}q_4 & -{}^{p}_{i}q_1 & {}^{p}_{i}q_2 \\ -{}^{p}_{i}q_2 & {}^{p}_{i}q_1 & {}^{p}_{i}q_4 & {}^{p}_{i}q_3 \\ -{}^{p}_{i}q_1 & -{}^{p}_{i}q_2 & -{}^{p}_{i}q_3 & {}^{p}_{i}q_4 \end{bmatrix}}_{\boldsymbol{Q}({}^{p}_{i}\mathbf{q})} \begin{bmatrix} {}^{p}\boldsymbol{\omega}_{1,p,i} \\ {}^{p}\boldsymbol{\omega}_{2,p,i} \\ {}^{p}\boldsymbol{\omega}_{3,p,i} \\ 0 \end{bmatrix} \tag{5}$$

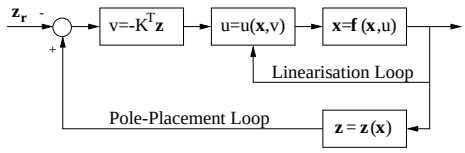

Fig. 1. Input-state feedback linearisation.

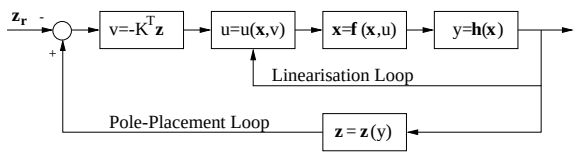

Fig. 2. Input-output feedback linearisation.

The orthogonal matrix $\boldsymbol{Q}({}^p_i\mathbf{q})$ has the following property: $\boldsymbol{Q}({}^p_i\mathbf{q})^{-1} = \frac{\boldsymbol{Q}({}^p_i\mathbf{q})^{\mathrm{T}}}{{}^p_iq_1^2+{}^p_iq_2^2+{}^p_iq_3^2+{}^p_iq_4^2} = \boldsymbol{Q}({}^p_i\mathbf{q})^{\mathrm{T}}$ since ${}^p_iq_1^2 + {}^p_iq_2^2 + {}^p_iq_3^2 + {}^p_iq_4^2 = 1$.

3. THE BASIC PRINCIPLE OF FEEDBACK LINEARISATION

The basic principle of exact linearisation is to make the nonlinear system act as a linear system and then design a simple tracking controller that stabilises the system. In other words exact linearisation is used to change the appearance or behaviour of a nonlinear system into a linear one, which is controllable, and then to design a stabilising controller, that tracks references. There exists three methods of deriving exact feedback linearisation (Marino and Tomei, 1995 *a*):

(1) State linearisation is a nonlinear change of coordinates by a diffeomorphic mapping (diffeomorphism).
(2) Input-state feedback linearisation (Sometimes called state feedback linearisation or just feedback linearisation). It consists of finding a nonlinear feedback and a diffeomorphism. It is a generalisation of the pole placement theorem for linear systems.
(3) Input-output feedback linearisation and consists of a selection of the outputs, that will make the system input-output feedback linearisable, a nonlinear feedback, and a diffeomorphism. It is a generalisation of the zero-pole cancellation technique.

The last two methods use feedback loops, which is done as in figures 1 and 2.

Nonlinear systems can be divided into those which are input-state linearisable, input-output linearisable, and those which are not feedback linearisable.

4. FEEDBACK LINEARISATION OF ATTITUDE DYNAMICS

Before feedback linearisation can be done, the complete system has to be defined. Using the models derived previously (Equations 3 and 5), the system takes the following form:

$$\underbrace{\frac{d}{dt}\begin{bmatrix} {}^p\omega_{1,p,i} \\ {}^p\omega_{2,p,i} \\ {}^p\omega_{3,p,i} \\ {}^p_iq_1 \\ {}^p_iq_2 \\ {}^p_iq_3 \\ {}^p_iq_4 \end{bmatrix}}_{\dot{\boldsymbol{x}}} = \underbrace{\begin{bmatrix} a_1\ {}^p\omega_{2,p,i}\ {}^p\omega_{3,p,i} \\ a_2\ {}^p\omega_{1,p,i}\ {}^p\omega_{3,p,i} \\ a_3\ {}^p\omega_{1,p,i}\ {}^p\omega_{2,p,i} \\ \frac{1}{2}\boldsymbol{Q}({}^p_i\mathbf{q}) \begin{bmatrix} {}^p\omega_{1,p,i} \\ {}^p\omega_{2,p,i} \\ {}^p\omega_{3,p,i} \\ 0 \end{bmatrix} \end{bmatrix}}_{\boldsymbol{f}(\boldsymbol{x})}$$

$$+ \underbrace{\begin{bmatrix} \frac{1}{I_{1,p}} & 0 & 0 \\ 0 & \frac{1}{I_{2,p}} & 0 \\ 0 & 0 & \frac{1}{I_{3,p}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\boldsymbol{g}} \underbrace{\begin{bmatrix} {}^pN_{1,control} \\ {}^pN_{2,control} \\ {}^pN_{3,control} \end{bmatrix}}_{\boldsymbol{u}} \tag{6}$$

4.1 *Feedback Linearising loop*

The control characteristic indices are derived using the Lie derivates for multi-variable systems:

$$\begin{aligned} L_{\boldsymbol{g}_j} L_{\boldsymbol{f}}^k \boldsymbol{h}_i(\boldsymbol{x}) &= 0 \qquad , 0 \le k \le \rho_i - 2, \forall \boldsymbol{x} \in U_0 \\ L_{\boldsymbol{g}_j} L_{\boldsymbol{f}}^{\rho_i - 1} \boldsymbol{h}_i(\boldsymbol{x}) &\neq 0 \qquad , \text{some } j, \forall \boldsymbol{x} \in U_0 \end{aligned} \tag{7}$$

where i is the number of outputs, j the number of inputs ($1 \le j \le m$) and ρ_i are the control characteristic indices (For single-variable systems these are known as the relative degree). The control characteristic indices for the system presented in equation 6 is $\{\rho_1, \rho_2, \rho_3, \rho_4\} = \{2, 2, 2, 2\}$.

The 24 lie derivates (The number of lie derivates comes from the four outputs, three inputs and control characteristic indices. $4 \times 3 \times 2 = 24$) for the model of the rigid-body were calculated, and they satisfy equations 7.

The new system with the control characteristic indices $\{2, 2, 2, 2\}$ is based upon the lie derivatives $L_{\boldsymbol{f}}^2 \boldsymbol{h}$ and $L_{\boldsymbol{g}} L_{\boldsymbol{f}} \boldsymbol{h}$, which must therefore be calculated:

$$L_{\boldsymbol{f}}^2 \boldsymbol{h} = \frac{\boldsymbol{Q}({}^p_i\mathbf{q})}{2} \begin{bmatrix} a_1\ {}^p\omega_{2,p,i}\ {}^p\omega_{3,p,i} \\ a_2\ {}^p\omega_{1,p,i}\ {}^p\omega_{3,p,i} \\ a_3\ {}^p\omega_{1,p,i}\ {}^p\omega_{2,p,i} \\ -\frac{1}{2}\left({}^p\omega_{1,p,i}{}^2 + {}^p\omega_{2,p,i}{}^2 + {}^p\omega_{3,p,i}{}^2\right) \end{bmatrix}$$

$$L_{\boldsymbol{g}} L_{\boldsymbol{f}} \boldsymbol{h} = \frac{\boldsymbol{Q}({}^p_i\mathbf{q})}{2} \begin{bmatrix} \frac{1}{I_{1,p}} & 0 & 0 \\ 0 & \frac{1}{I_{2,p}} & 0 \\ 0 & 0 & \frac{1}{I_{3,p}} \\ 0 & 0 & 0 \end{bmatrix} \tag{8}$$

The system, which is to be feedback linearised, has the following structure:

$$\begin{aligned} \frac{d^2}{dt^2} \begin{bmatrix} {}^p_i q_1 \\ {}^p_i q_2 \\ {}^p_i q_3 \\ {}^p_i q_4 \end{bmatrix} &= L_{\boldsymbol{f}}^2 \boldsymbol{h} + L_{\boldsymbol{g}} L_{\boldsymbol{f}} \boldsymbol{h} \boldsymbol{u} \\ &= \frac{1}{2} \underbrace{(2 L_{\boldsymbol{f}}^2 \boldsymbol{h})}_{\boldsymbol{f}_2(\boldsymbol{x})} + \frac{1}{2} \underbrace{(2 L_{\boldsymbol{g}} L_{\boldsymbol{f}} \boldsymbol{h} \boldsymbol{u})}_{\boldsymbol{U}(\boldsymbol{x},\boldsymbol{u})} \\ &= \frac{1}{2} \boldsymbol{f}_2(\boldsymbol{x}) + \frac{1}{2} \boldsymbol{U}(\boldsymbol{x}, \boldsymbol{u}) \end{aligned} \tag{9}$$

Using feedback linearisation theory, the new input vector $\boldsymbol{U}$ is then calculated, so that it exactly feedback linearises the system (equation 9):

$$\boldsymbol{U}(\boldsymbol{x}, \boldsymbol{u}) = -\boldsymbol{f}_2(\boldsymbol{x}) + 2\boldsymbol{v}_t \tag{10}$$

Combining the feedback linearisation loop (derived in equation 10) with the equation for the satellite model (equation 9), results in the following linear system:

$$\begin{aligned} \frac{d^2}{dt^2} \begin{bmatrix} {}^p_i q_1 \\ {}^p_i q_2 \\ {}^p_i q_3 \\ {}^p_i q_4 \end{bmatrix} &= \frac{1}{2} \boldsymbol{f}_2(\boldsymbol{x}) + \frac{1}{2}(-\boldsymbol{f}_2(\boldsymbol{x}) + 2\boldsymbol{v}_t) \\ &= \boldsymbol{v}_t \end{aligned} \tag{11}$$

The relationship between the real input vector $\boldsymbol{u}$ and the feedback linearising vector $\boldsymbol{U}$ is:

$$\begin{aligned} \boldsymbol{U} = \boldsymbol{U}(\boldsymbol{x}, \boldsymbol{u}) &= 2 L_{\boldsymbol{g}} L_{\boldsymbol{f}} \boldsymbol{h} \boldsymbol{u} \\ &= \boldsymbol{Q}({}^p_i\mathbf{q}) \underbrace{\begin{bmatrix} \frac{{}^p N_{1,control}}{I_{1,p}} \\ \frac{{}^p N_{2,control}}{I_{2,p}} \\ \frac{{}^p N_{3,control}}{I_{3,p}} \\ 0 \end{bmatrix}}_{\boldsymbol{g}_2 \boldsymbol{u}} \end{aligned} \tag{12}$$

Isolating $\boldsymbol{u}$ in equation 12 gives the following equation:

$$\underbrace{\begin{bmatrix} {}^p N_{1,control} \\ {}^p N_{2,control} \\ {}^p N_{3,control} \\ 0 \end{bmatrix}}_{\boldsymbol{u}} = \begin{bmatrix} I_{1,p} & 0 & 0 & 0 \\ 0 & I_{2,p} & 0 & 0 \\ 0 & 0 & I_{3,p} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{Q}^{\mathrm{T}}({}^p_i\mathbf{q})\ \boldsymbol{U} \tag{13}$$

The problem of feedback linearising the satellite model which incorporates the quaternion is that the number of inputs to the kinematic equation (from equation 5) is larger than the number of outputs. That is to say the feedback linearising input vector $\boldsymbol{U}$ has four components that may change from zero, whereas the real input vector $\boldsymbol{u}$ has three components, hence the system is seemingly under-actuated.

The upper three components of the feedback linearising input vector $\boldsymbol{U}$ are calculated according to the formula 10, but in order for the fourth component of the $\boldsymbol{u}$ vector (u_4) in equation 13 to be zero, ${}^p_i\mathbf{q}$ has to be perpendicular to $\boldsymbol{U}$. The flexibility is in $\boldsymbol{U}$, which can be arbitrary assigned in contrast to ${}^p_i\mathbf{q}$, which is measured. In this paper U_4 is changed, but in practical applications, any of the other components of the $\boldsymbol{U}$ vector might have been chosen. The component U_4 is calculated in the following way $U_4 = -\frac{{}^p_i q_1\ U_1 + {}^p_i q_2\ U_2 + {}^p_i q_3\ U_3}{{}^p_i q_4}$.

The projection gives the following extra term to the input vector $\boldsymbol{u}$:

$$\underbrace{\begin{bmatrix} \frac{{}^p_i q_1}{2\,{}^p_i q_4} \left({}^p\omega_{1,p,i}{}^2 + {}^p\omega_{2,p,i}{}^2 + {}^p\omega_{3,p,i}{}^2 \right) \\ \frac{{}^p_i q_2}{2\,{}^p_i q_4} \left({}^p\omega_{1,p,i}{}^2 + {}^p\omega_{2,p,i}{}^2 + {}^p\omega_{3,p,i}{}^2 \right) \\ \frac{{}^p_i q_3}{2\,{}^p_i q_4} \left({}^p\omega_{1,p,i}{}^2 + {}^p\omega_{2,p,i}{}^2 + {}^p\omega_{3,p,i}{}^2 \right) \end{bmatrix}}_{\boldsymbol{v}_e} \tag{14}$$

which changes the ideal feedback linearised system presented in equation 11, into the following:

$$\frac{d^2}{dt^2} \begin{bmatrix} {}^p_i q_1 \\ {}^p_i q_2 \\ {}^p_i q_3 \end{bmatrix} = \boldsymbol{v}_t - \boldsymbol{v}_e \tag{15}$$

The extra term from the projection has the opposite sign of the term in $\boldsymbol{U}$ which feedback linearises the model of the satellite and keeps the acceleration of the quaternion (${}^p_i\ddot{q}_1$, ${}^p_i\ddot{q}_2$ and ${}^p_i\ddot{q}_3$) constant. The inclusion of the extra term contributes to damping of the system.

The proposed feedback linearising loop results in in exact feedback linearisation at the equilibrium point ${}^p_i q_4 = 1$. The attitude controller is unstable at the set $\{{}^p_i\boldsymbol{q} : {}^p_i q_4 = 0\}$, which can be geometrically interpreted as a circle in "$\mathbb{S}^3$". This circle divides the $\mathbb{S}^3$ into two stable hemispheres $\{{}^p_i\boldsymbol{q} : {}^p_i q_4 < 0\}$ and $\{{}^p_i\boldsymbol{q} : {}^p_i q_4 > 0\}$.

4.2 *Tracking Feedback*

The result of the input-output feedback linearisation loop on the system, when ${}^p_i q_4$ is disregarded, is the following linear system in the neighbourhood of the equilibrium point:

$${}^p_i\ddot{\boldsymbol{q}} = \frac{d^2}{dt^2} \begin{bmatrix} {}^p_i q_1 \\ {}^p_i q_2 \\ {}^p_i q_3 \end{bmatrix} = \boldsymbol{v}_t \tag{16}$$

The scalar part of the attitude quaternion (${}^p_i q_4$) is disregarded, since it is bounded and given

by $\sqrt{1 - {}^p_i q_1 - {}^p_i q_2 - {}^p_i q_3}$. The system is therefore stable if ${}^p_i \boldsymbol{q}$ is stable.

The following feedback form has to be chosen to get asymptotic tracking (Marino and Tomei, 1995*b*):

$$\boldsymbol{v}_t = \begin{bmatrix} -k_1 \, {}^p_i q_1 - k_2 \, {}^p_i \dot{q}_1 \\ -k_1 \, {}^p_i q_2 - k_2 \, {}^p_i \dot{q}_2 \\ -k_1 \, {}^p_i q_3 - k_2 \, {}^p_i \dot{q}_3 \end{bmatrix} \tag{17}$$

The tracking loop ($\boldsymbol{v}_t$) is a simple multi-variable proportional controller, which gives asymptotic tracking on the linearised system.

5. SIMULATION TESTS OF THE ALGORITHM

The attitude controller presented in this paper is based on approximate feedback linearisation, which combines feedback linearisation theory and approximation of the feedback linearisation loop using projection described in section 4.

To thorough test the proposed controller, the performance and functionality of the feedback linearisation loop is first tested, then the stability of the attitude controller (both loops) is tested near the unstable set $\{{}^p_i \boldsymbol{q} : {}^p_i q_4 = 0\}$.

The tests are done in a Matlab simulation, which includes the realistic environmental disturbances for the stability test.

5.1 *Test of performance and functionality*

The approximate feedback linearisation loop can be tested in the following way. The rigid-body is tumbling about all three axes at the start of the simulation, and the tracking feedback loop is disconnected ($\boldsymbol{v}_t = 0$). The approximate feedback linearisation loop keeps the derivative of the quaternion (${}^p_i \dot{q}_1$, ${}^p_i \dot{q}_2$, ${}^p_i \dot{q}_3$) constant as long as the quaternion is on either of the two stable hemispheres, since the system is almost transfered into the Brunovsky controller form when it is combined together with the feedback linearisation loop. The exact feedback linearising loop would in theory keep the derivative of the quaternion constant all the time. The test is illustrated in figure 3.

The first plot in figure 3 shows the attitude quaternion. The first three components of the quaternion are moving on a straight line, except from about 20 and 65 seconds into the simulation, where ${}^p_i q_4 \approx 0$ and the system is unstable. The fourth component of the quaternion is moving so that the quaternion fulfils the unit property of the quaternion.

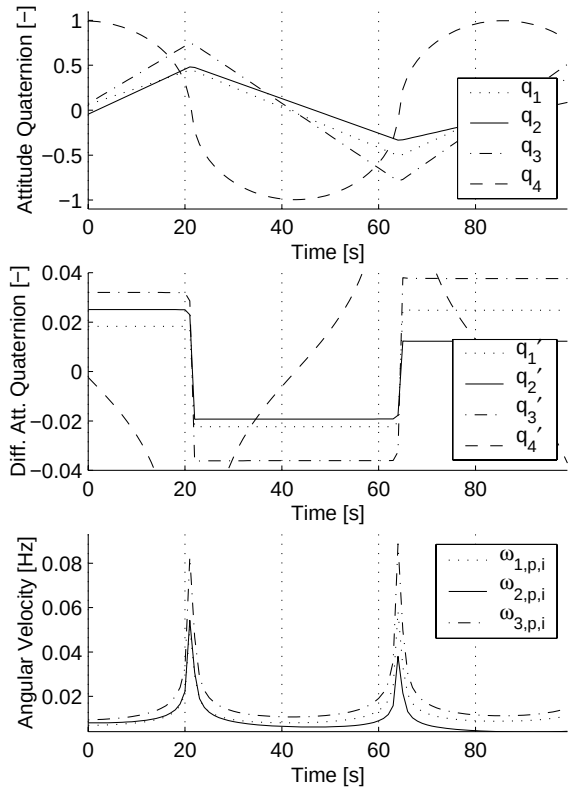

Fig. 3. Test of the feedback linearisation loop. The tracking loop is disconnected ($\boldsymbol{v}_t = 0$), so the quaternion (${}^p_i q_1$, ${}^p_i q_2$ and ${}^p_i q_3$) should be linearised.

The second plot shows the differentiated quaternion which is constant for the three first components of the quaternion, when the quaternion is in one of the two stable hemispheres. The extra term mentioned in equation 14 can be seen in the second plot, as it results in lower angular velocities than needed in order to keep the differentiated quaternion constant. This is best seen near the unstable point where the term is most significant. Hence the approximated feedback linearisation proposed in this paper diverges very little from the exact feedback linearisation inside the two stable hemispheres.

The third plot shows the angular velocity of the satellite model, that goes to infinity if the output from the approximate feedback linearisation loop was not bounded in this test.

5.2 *Test of stability*

In order to test stability and performance of the presented attitude controller design, both feedback loops are active and the reference is changed, so that the satellite makes a slew-manoeuvre of 160 °about one of the axes. The starting point will be the equilibrium point for the attitude controller (${}^p_i q_4 = 1$), where the extra term from equation 14 is zero. The destination point, will on the other

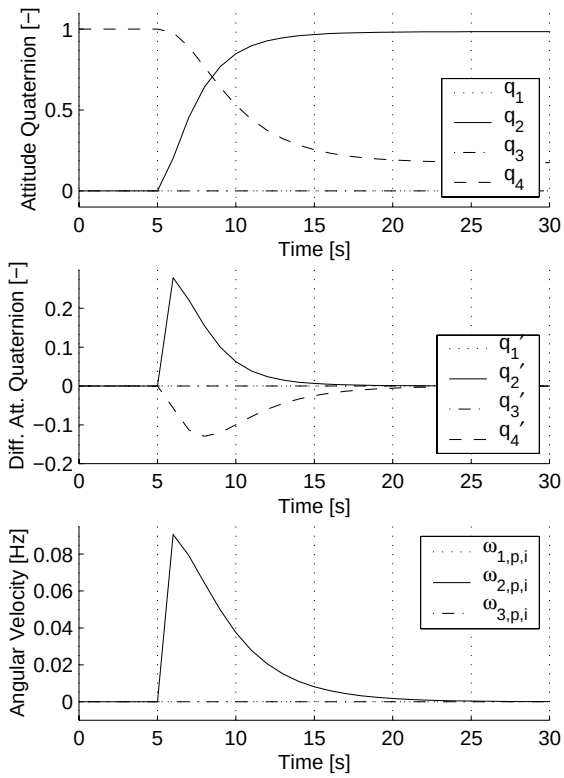

Fig. 4. Stability test of the attitude controller under slew manoeuvre of 160 °(Which is 20 °from making the system unstable).

hand be close to where the extra term has most influence. This is illustrated in figure 4.

The first plot in figure 4 shows the quaternion. The reference for the attitude controller is changed 5 seconds after start, and the second (${}^p_i q_2$) and fourth (${}^p_i q_4$) component of the quaternion changes value since the rotation is about the second axis.

The second and third plot shows the expected changes, which are a result of the reference tracking.

The result of the second test (stability test) showed, that the system is stable far away from the equilibrium point, even when the references are near the boundary of the stable hemispheres.

6. CONCLUSION

A nonlinear spacecraft attitude controller was derived using approximated feedback linearisation. The goal of this controller was to feedback linearise the attitude dynamics of a satellite model, which uses the quaternion for attitude representation.

Exact feedback linearisation was not possible due to the dependency between the four components in the attitude quaternion. The standard feedback linearisation techniques were extended through projection to approximate the feedback linearisation loop. The drawback of using projection is the loss of global stability at the circle ${}^p_i q_4 = 0$ in $\mathbb{S}^3$, due to the introduction of a division by ${}^p_i q_4$. However, the projection results in an extra term which contributes to the damping of the system.

The derived attitude controller was shown to be able to handle coarse pointing and slew-manoeuvres, thereby proving to be a candidate for on-board implementation.

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