Rudder-Roll Damping Controller Design using Mu Synthesis

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Abstract: The effectiveness of rudder roll damping control is very sensitive to uncertainty in ship dynamic parameters. In this paper, an H_∞ controller is designed using μ synthesis and an uncertainty model for roll and yaw that was identified earlier from experiments at sea. The properties of the resulting controller are discussed and seaway simulations with linear and non-linear models of a container ship illustrate the controller performance. The simulation results show that the μ synthesis controller is able to obtain robust performance over an envelope of operational conditions.

Keywords: Ship control, multivariable control, structured singular value, uncertainty, robust control

1. INTRODUCTION

Roll damping systems for ships are employed to increase comfort for passengers, maintain full working capabilities for members of the crew, and prevent cargo damage. Dedicated fins for roll damping is traditionally installed to provide damping of roll motion, but utilisation of the rudder can be a much cheaper, yet also effective solution. Several researchers have analysed roll damping by means of rudders. Investigations have shown that the roll reduction obtained by the rudder could be 50%~70% for a specific vessel, however it highly depends on the dynamics of the ship. Experiments have been carried out in which a rudder-roll damping (RRD) controller performed satisfactorily for one ship but unsatisfactorily for another, although the two were sister ships with the same hull geometry but small differences in the form of bilge keels, rudder shape and loading (Blanke and Christensen, 1993). The different results were referred to ship model perturbation, indicating that RRD is highly sensitive to model uncertainty. Hence, it is crucial that design of rudder-roll damping control is done with appropriate robust performance in mind.

The aim is to enable the control system to cope with model uncertainties. Earlier results have shown that a controller can obtain good robust stability and robust performance when designed by H_∞ control theory. A multi-objective design was proposed by Stoustrup, et al., (1995), but that design did not consider model uncertainty. Some work was done using mixed sensitivity approach with unstructured perturbation (Yang and Blanke, 1997). This paper will use μ synthesis for design the controller.

The mathematical framework for the ship model is first outlined. Modelling uncertainty is then addressed, and a μ synthesis is conducted. Properties of the resulting controller are issued and simulations used to illustrate time responses in a seaway.

2. SHIP MODEL AND DISTURBANCE

2.1 Non-linear model

Six degrees of freedom are needed to describe ship kinematics and basic ship hydrodynamics. Pitch and heave can generally be neglected if the study concentrate on course keeping and roll damping. Ship motion modelling is thus considered in surge, sway, yaw and roll. The ship motion equations of these four-degrees of freedom are given based on the Newtonian equation (Blanke and Jensen 1997):

surge: $m(\dot{u} - vr - x_G r^2 + z_G pr) = X$ (1)

sway: $m(\dot{v} + ur + x_G \dot{r} - z_G \dot{p}) = Y$ (2)

yaw: $I_z \dot{r} + m x_G(\dot{v} + ur) = N$; (3)

roll: $I_x \dot{p} - m z_G(\dot{v} + ur) = K - \rho g \nabla G_z(\varphi)$ (4)

Where ∇ denotes the ship displacement, g the gravity constant, ρ the mass density of water and $G_z(\varphi)$ the righting arm which is a known function of roll angle φ. The centre of gravity is assumed at position (x_G, 0, z_G), the ship mass is m while I_x and I_z are the inertias with respect to the x and z axes. The linear surge and sway velocities are denoted by u and v and angular yaw and roll velocities by r and p. The corresponding angles are ψ and φ in an inertial frame.

The terms X, Y, N and K denote the hydrodynamic forces and moments. They can be calculated by expanding to a 3rd-order Taylor series at $u'=u'_0$, $v'=0$, $p'=0$ and $'r=0$. The ' indicate non-dimensional quantities in the "prime" system.

$$X' = X'_u\Delta u' + X'_{\dot{u}}\dot{u}' + X'_v v' + X'_{\dot{v}}\dot{v}' + X'_\delta\delta' + X'_\varphi\varphi' + X'_r r' + X'_{\dot{r}}\dot{r}' + X'_p p' + X'_{\dot{p}}\dot{p}' + X'_{vv}v'^2 + X'_{\delta\delta}\delta'^2 + X'_{\varphi\varphi}\varphi'^2 + X'_{rr}r'^2 + X'_{pp}p'^2 + X'_{vu}v'\Delta u' + \ldots \quad (5)$$

$$Y' = Y'_v v' + Y'_{\dot{v}}\dot{v}' + Y'_\delta\delta' + Y'_\varphi\varphi' + Y_r'r' + Y_{\dot{r}}\dot{r} + Y'_p p' + Y_{\dot{p}}\dot{p} + Y'_{vv}v'^2 + Y'_{\delta\delta}\delta'^2 + Y'_{\varphi\varphi}\varphi'^2 + Y'_{rr}r'^2 + Y'_{pp}p'^2 + Y'_{vu}v'\Delta U' + Y'_{\delta v}\delta'v' + Y_{\varphi}'_v\varphi'v' + Y'_{\delta u}\delta'\Delta U' + \ldots \quad (6)$$

$$N' = N'_v v' + N'_{\dot{v}}\dot{v}' + N'_\delta\delta' + N'_\varphi\varphi' + N'_r r' + N'_{\dot{r}}\dot{r}' + N'_p p' + N'_{\dot{p}}\dot{p}' + N'_{vv}v'^2 + N'_{\delta\delta}\delta'^2 + N'_{\varphi\varphi}\varphi'^2 + N'_{rr}r'^2 + N'_{pp}p'^2 + N'_{vu}v'\Delta U' + N'_{\delta v}\delta'v' + N'_{\varphi v}\varphi'v' + \ldots \quad (7)$$

$$K' = K'_v v' + K_{\dot{v}}\dot{v} + K'_\delta\delta' + K'_\varphi\varphi' + K'_r r' + K'_{\dot{r}}\dot{r}' + K'_p p' + K'_{\dot{p}}\dot{p}' + K'_{vv}v'^2 + K'_{\delta\delta}\delta'^2 + K'_{\varphi\varphi}\varphi'^2 + K'_{rr}r'^2 + K'_{pp}v'^2 + K'_{vu}v'\Delta U' + K'_{\delta v}\delta v + K'_{\varphi v}\varphi'v' + \ldots \quad (8)$$

Hhere, $X_{\dot{v}}\dot{v}$, $Y_\delta\delta$ are hydrodynamic terms $\frac{\partial X}{\partial \dot{v}}\dot{v}$, $\frac{\partial Y}{\partial \delta}\delta$. Speed deviation from nominal speed is $\Delta u = u - u_0$, $\Delta U = U - U_0$ and $U = \sqrt{u^2 + v^2}$. The state vector x in Eq. (9) is obtained from (1) ~ (4). The roll and yaw angles are added to obtain

$$x = [u, v, r, p, \varphi, \psi]^T \quad (9)$$

where ψ and φ are related to p and r by $\dot{\varphi} = p$ and $\dot{\psi} = r/\cos(\varphi) \approx r$. The coefficients needed for describing the container in four degrees of freedom were given in (Blanke and Jensen, 1997)[1].

2.2 Linear model

It is difficulty to use the non-linear model directly in robust control design. The non-linear model must be replaced in calculation of controller by linear model because most of the theorems derived based on linear theory. However, it is easy to obtain a linear model if a non-linear model exist. In this paper, a linear model is obtained by linearisation of Equations (1) to (8) with $cos(\varphi) \approx 1$ because φ is assumed small.

Considering motions in roll and yaw, the surge equation has only a weak dynamic coupling. Instead of a part of the dynamics, ship speed U is a parameter, and there are only five states in the linear model, x = $[v, r, p, \varphi, \psi]^T$. The simplified linear model can be written as

$$E\dot{x} = Fx + G\delta; \quad (10)$$

where the linearized model matrices E, F and G can be seen in (Yang, 1997). The input is the rudder angle δ and ship speed U is a parameter.

The frequency characteristic of open loop transfer function from rudder to roll angle is plotted in Fig. 1. A resonant peak is observed at about 0.23 rad/s where the ship roll angle will be significant. This frequency is known as the natural roll eigenfrequency.

2.3 Model uncertainty

Model uncertainty can cause instability and poor performance of a nominally stable system, as found for sister ships mentioned in the introduction. The slight modification of ship parameters could be considered as output multiplicative model uncertainty. The relation between multiplicative model uncertainty and the nominal process $G(s)$ is

[1] *There are two sets of parameters for the container ship in the reference. One set comes from RPMM model test, the other is modified to fit full scale tests (Blanke and Jensen, 1997). The unmodified model is directionally unstable around zero turn rate, the modified is marginally stable. The coupling between steering and roll is also different for the two models.*

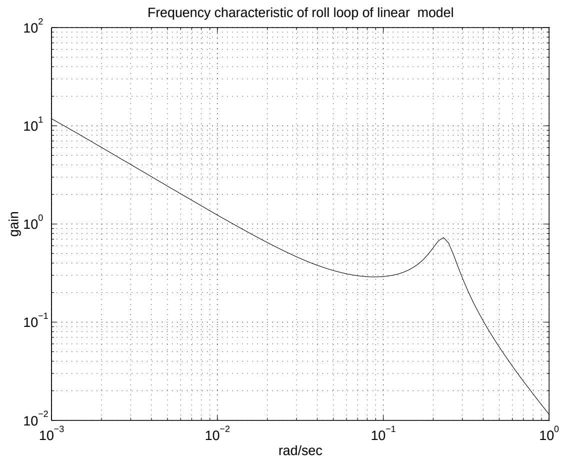

Fig. 1. Frequency characteristic of open loop transfer function from rudder to roll angle

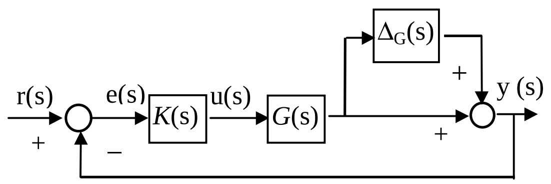

Fig. 2. Ship model multiplicative uncertainty

$$G_p(s) = (1 + \Delta_G(s))G(s) \qquad (11)$$

where $G_p(s)$ is the perturbed process, i.e. the real system. The block diagram for the closed loop is shown in Fig 2.

For a ships roll damping, the following uncertainty model was found for a multi-purpose naval vessel (Blanke, 1996). Since there are no general results for container ships, the same uncertainty model is used here,

$$\Delta_{Gr}(s) = \frac{2\zeta_0\omega_0\left(1+\alpha\frac{\nabla}{\nabla_0}-\alpha\left(\frac{\nabla}{\nabla_0}\right)^2\right)s+\omega_0^2\left(1-\left(\frac{\nabla}{\nabla_0}\right)^2\right)}{s^2+2\zeta_0\omega_0 s+\omega_0^2} \qquad (12)$$

where $\alpha = 0.5$, ∇ is displacement and ∇_0 is nominal displacement. This equation is valid only when the displacement is smaller than nominal. The terms ω_0 and ζ_0 are natural roll frequency and damping, respectively. Damping typically is in the range 0.15 - 0.25.

For yaw rate, a frequency independent uncertainty of 0.1 is suggested by the same paper (Blanke, 1996). Accordingly, the following uncertainty description is used for the rudder to yaw angle.

$$\Delta_{Gy}(s) = \frac{0.42\zeta_0\omega_0 s + 0.21\omega_0^2}{s^2 + 2\zeta_0\omega_0 s + \omega_0^2} \qquad (13)$$

2.4 Wave model

Waves, wind and current are the principal factors causing disturbances of ship on the sea. In terms of ship roll, wave is the most important disturbance. A long crested irregular sea is described by a one dimensional amplitude spectrum recommended by ISSC that can be written as

$$G_{\varsigma\varsigma}(\omega_w) = \frac{173h_{1/3}^2}{\omega_w^5 T_w^4} e^{\left(\frac{-691}{T_w^4\omega_w^4}\right)} \quad [m^2 s] \qquad (14)$$

where $h_{1/3}$ is the average height of largest third of the waves, ω_w is angular frequency, T_w is the average wave period.

The relation between ship motion response and wave height is commonly named a receptance function. It is influenced by the wave frequency and the receptance function must be transformed from wave frequency to encounter frequency before access to motion spectra can be made.

For the moving ship, the encounter frequency is associated with the wave frequency and wave energy and the speed of the ship by

$$\omega_e = \omega_w \left(1 - \omega_w \frac{U}{g} \cos\chi\right) \qquad (15)$$

where χ is the encounter angle, the direction of the wave propagation relative to the ship.

Because the wave energy in a frequency interval is unaffected by the speed of observation platform, following equation hold.

$$G_{zz}(\omega_w, \chi)d\omega_w = G_{zz}(\omega_e, \chi)d\omega_e \qquad (16)$$

An approximation of a sea spectrum by a finite sum of sinusoids with random initial phases is

$$z(t) = \sum_{i=1}^{n} a_i \sin(\omega_{e,i}t + \varphi_i + \varphi_{init}) \qquad (17)$$

The individual amplitudes of the sines are conveniently taken as median points between frequencies where the response operators are, known, tabulated (Blanke, 1981; Tiano et al. 1996). Frequencies $\omega_{e,j}$ and phase angles φ_i are tabular values from the response operator tables. The initial phase is a random number used for initialization. The amplitudes a_i are calculated from

$$a_i = \sqrt{2}\left|R_{z\varsigma}(\omega_w, \chi, U)\right|\sqrt{\sigma_w^2(\omega_{1,i}, \omega_{2,i})} \qquad (18)$$