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Published in:
Proceedings of the Eighth International Conference on Sound and Vibration, Hong Kong, China, July 2-6, 2001

Publication date:
2001

Document Version
Publisher's PDF, also known as Version of record

Link to publication from Aalborg University

Citation for published version (APA):
Andersen, L., & Nielsen, S. R. K. (2001). Vehicle Moving along a Beam on a Random Modified Kelvin Foundation. In *Proceedings of the Eighth International Conference on Sound and Vibration, Hong Kong, China, July 2-6, 2001* (pp. 2811-2818)

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The 8th International Congress on Sound and Vibration
2-6 July 2001, Hong Kong, China

VEHICLE MOVING ALONG A BEAM ON A RANDOM MODIFIED KELVIN FOUNDATION

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Abstract. The paper deals with the stochastic analysis of a single-degree-of-freedom vehicle moving at a constant velocity along an infinite Bernoulli-Euler beam. The beam is supported by a Kelvin foundation which has been modified by the introduction of a shear layer. The spring stiffness in the support is assumed to be a stochastic homogeneous field consisting of a small random variation around a deterministic mean value. First, the equations of motion for the vehicle and beam are formulated in a moving frame of reference following the vehicle. Next, a perturbation analysis is performed to establish the relationship between the variation of the spring stiffness and the response of the mass of the vehicle as well as the displacement variance of the beam under the oscillator.

INTRODUCTION

A road, runway or railway track is often modelled as a beam structure on a Kelvin foundation. In itself this model only weakly describes the real situation of a track resting on a subsoil. However, a reasonable model may be obtained by adjusting the stiffness of the Kelvin foundation as proposed by Dieterman and Metrikine [1] and Metrikine and Popp [2], respectively, for representation of a visco-elastic half-space or a layer over a bedrock. Alternatively Vallabhan and Das [3] showed that an elastic layer under static load may be approximated by a Winkler foundation modified by the inclusion of a shear layer. Intuitively this is a better model since the modified support is at least capable of transporting energy in the along-beam direction. Analytic solutions for shear and translational spring stiffnesses were derived by Krenk [4].

Recently the vertical stiffness of the ballast and sleepers that are used to support railway tracks has been found to vary significantly along the track [5] with a correlation length much smaller and a variation coefficient much larger than the sub-soil. This variation leads to vibrations of a moving vehicle and the track itself, even when no external excitation is applied. In the literature similar problems have been treated numerically using finite elements in a number of papers. Yoshimura *et al.* [6] analysed a vehicle moving along a simply supported beam with random surface irregularities and varying cross-section. Frýba *et al.* [7] examined the behaviour of an infinitely long Euler beam on a Kelvin foundation with randomly varying parameters along the beam.

In the present paper a novel numerical method will be presented for the analysis of a single-degree-of-freedom (SDOF) vehicle moving uniformly along a beam on a random modified Kelvin foundation. The vertical support stiffness is described by a weakly homogeneous random process.

The randomness is primary due to the sleeper and ballast stiffness variation, as mentioned above. Furthermore it should be noticed that the parameters of a sub-soil under e.g. a railway track is usually known beforehand from field observations and/or laboratory tests. The problem is formulated in a local coordinate frame, which follows the vehicle, and the interaction between the vehicle and the beam is taken into account. Based on a perturbation analysis, the stationary response of the vehicle and beam due to the variation of the spring stiffness is calculated. No resort to Monte Carlo simulation is necessary since a closed form solution feasible for numerical treatment is derived.

THEORY

A vehicle modelled as an SDOF system with deterministic mass m_0, spring stiffness k_0 and viscous damping c_0 is moving uniformly in permanent contact along the smooth surface of a Bernoulli-Euler beam at the velocity v, thus having the along beam position $x = vt$ at time t. The beam has the deterministic bending stiffness EI and mass m per unit length and the beam axis forms a straight line in the state of static equilibrium. The beam rests on a modified Kelvin foundation with deterministic shear stiffness G and viscous damping γ per unit length of the beam, see Fig. 1.

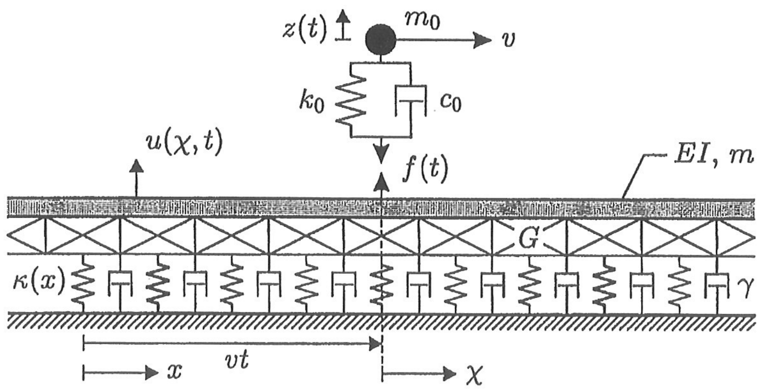

Figure 1: *SDOF vehicle on an Euler beam supported by a modified random Kelvin foundation.*

The vertical stiffness of the support $\kappa(x)$ is assumed to be described by a stochastic homogeneous field along the beam with the mean value $\bar{\kappa} = E[\kappa(x)]$. The auto-covariance function $C_{\kappa\kappa}$ and two-sided auto-spectral density $S_{\kappa\kappa}$ are defined as,

$$C_{\kappa\kappa}(\Delta x) = \sigma_\kappa^2 e^{-\left|\frac{\Delta x}{x_0}\right|}, \quad \Delta x = x_2 - x_1, \tag{1}$$

$$S_{\kappa\kappa}(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} C_{\kappa\kappa}(\Delta x) e^{-ik\Delta x} d\Delta x = \frac{x_0}{\pi} \frac{\sigma_\kappa^2}{1 + (x_0 k)^2}. \tag{2}$$

Here σ_κ^2 is the variance of $\kappa(x)$, x_0 is the correlation length and $i = \sqrt{-1}$ is the imaginary unit. k denotes a wavenumber, $k = \frac{2\pi}{L}$ where L is the corresponding wavelength. The auto-spectrum represents a first-order (so-called Ornstein-Uhlenbech) filtration of Gaussian white noise.

In a local moving coordinate system defined by the transformation $\chi = x - vt$, the equations of motion for the vehicle and beam become, respectively,

$$m_0 \frac{d^2 z}{dt^2} + c_0 \left(\frac{dz}{dt} - \dot{u}(0,t) \right) + k_0 (z - u(0,t)) = 0, \tag{3}$$

$$EI \frac{\partial^4 u}{\partial \chi^4} + m \left(\ddot{u} - 2v \frac{\partial \dot{u}}{\partial \chi} + \left(v^2 - \frac{G}{m} \right) \frac{\partial^2 u}{\partial \chi^2} \right) + \gamma \left(\dot{u} - v \frac{\partial u}{\partial \chi} \right) + \kappa u = f(t)\delta(\chi), \tag{4}$$

where the vertical displacements of the vehicle mass and beam relative to the positions in the state of static equilibrium are denoted $z = z(t)$ and $u = u(\chi, t)$, respectively. $\dot{u} = \frac{\partial u}{\partial t}\big|_\chi$ and $\ddot{u} = \frac{\partial^2 u}{\partial t^2}\big|_\chi$ are the local velocity and acceleration of the beam, respectively. The force on the beam originates from the SDOF vehicle, i.e. $f(t) = -m_0\left(g + \frac{d^2 z}{dt^2}\right)$, g being the gravitational acceleration. Analytical expressions for $\bar{\kappa}$ and G are derived in [4] for an elastic layer of finite magnitude H, defined by the mass density ρ and the Lamé constants λ, μ, and overlaying a bedrock.

Perturbation Analysis

The coefficient of variation $\frac{\sigma_\kappa}{\bar{\kappa}}$ of the spring stiffness is assumed to be sufficiently small so that a perturbation analysis may be performed for the response of the vehicle and beam. Hence, the displacements and the spring stiffness are divided up into zero- and first-order terms,

$$z(t) = \bar{z} + \hat{z}(t), \quad u(\chi, t) = \bar{u}(\chi) + \hat{u}(\chi, t), \quad \kappa(\chi + vt) = \bar{\kappa} + \hat{\kappa}(\chi + vt). \tag{5}$$

In Eq. (5) and below, the *bar* denotes mean values and the *hat* denotes stochastic deviations. The mean value of the vehicle displacement is constant and will be disregarded in the analysis. A moving vehicle on a beam and Kelvin foundation with constant stiffness causes no wave propagation in the moving frame of reference contrary to the analogous problem with a vehicle on an elastic half-space. Hence the zero-order beam displacement term $\bar{u}$ is the solution to the zero-order equation

$$EI\frac{d^4\bar{u}}{d\chi^4} + m\left(v^2 - \frac{G}{m}\right)\frac{d^2\bar{u}}{d\chi^2} - \gamma v\frac{d\bar{u}}{d\chi} + \bar{\kappa}\bar{u} = -m_0 g\delta(\chi), \tag{6}$$

which is time-independent. Four linearly independent solutions to (6) may be found on the form

$$\bar{u}_n(\chi) = -m_0 g \bar{U}_n e^{i\bar{K}_n\chi}, \quad n = 1, 2, 3, 4. \tag{7}$$

$\bar{U}_n$ are the amplitudes at $\chi = 0$ and $\bar{K}_n$ are the corresponding characteristic wavenumbers. Only solutions that are decaying in the far-field are physically valid. Thus the full solution on either side of the vehicle is the sum of two of the components, $\bar{u}_n(\chi)$. Further details are given by Andersen *et al.* [8], who also suggested a procedure for calculating $\bar{U}_n$ and $\bar{K}_n$.

When $\bar{u}(\chi)$ has been determined, and neglecting the second-order terms, the first-order terms $\hat{z}(t)$ and $\hat{u}(\chi, t)$ can be calculated by a simultaneous solution of the equations

$$m_0\frac{d^2\hat{z}}{dt^2} + c_0\left(\frac{d\hat{z}}{dt} - \dot{\hat{u}}(0, t)\right) + k_0\left(\hat{z}(t) - \hat{u}(0, t)\right) = 0, \tag{8}$$

$$EI\frac{\partial^4\hat{u}}{\partial\chi^4} + m\left(\ddot{\hat{u}} - 2v\frac{\partial\dot{\hat{u}}}{\partial\chi} + \left(v^2 - \frac{G}{m}\right)\frac{\partial^2\hat{u}}{\partial\chi^2}\right) + \gamma\left(\dot{\hat{u}} - v\frac{\partial\hat{u}}{\partial\chi}\right) + \bar{\kappa}\hat{u}(\chi, t) = \hat{f}(\chi, t). \tag{9}$$

Here $\hat{f}(\chi, t) = -m_0\frac{d^2\hat{z}}{dt^2}\delta(\chi) - \hat{\kappa}(\chi + vt)\bar{u}(\chi)$ is the excitation of the beam. Apparently the parametric excitation due to the stiffness variation is transformed to an equivalent distributed additive excitation $\hat{\kappa}(\chi + vt)\bar{u}(\chi)$ in the first-order differential equation.

Discretisation of the First-Order Spring Stiffness Term

In order to find a solution to Eqs. (8) and (9) the influence of the distributed equivalent line load $-\hat{\kappa}(\chi + vt)\bar{u}(\chi)$ has to be evaluated. This is done numerically by a discretisation into J time-varying static equivalent point loads $\hat{f}_j(t)$ acting on the beam at the positions χ_j. The distance $\Delta\chi_j$ between each point load must be sufficiently small so that both the bending waves in the beam and the variation of the spring stiffness are described satisfactorily. In practice this requires at least 10 discrete loads per wavelength. The approximation may be written

$$-\hat{\kappa}(\chi+vt)\bar{u}(\chi) \approx \sum_{j=1}^{J} \hat{f}_j(t)\delta(\chi-\chi_j), \quad \hat{f}_j(t) = -\hat{\kappa}(\chi_j+vt)\bar{u}(\chi_j)\Delta\chi_j. \tag{10}$$

In order to obtain a good approximation the summation has to be carried out over the entire region where $\bar{u}(\chi)$ is not very close to zero. The size of this region is highly dependent on the damping, γ, in the support, and the velocity, v. Since the model is linear, the principle of superposition is applicable and the total field $\hat{u}(\chi,t)$ may be written as

$$\hat{u}(\chi,t) = \hat{u}_0(\chi,t) + \hat{u}_S(\chi,t), \quad \hat{u}_S(\chi,t) = \sum_{j=1}^{J} \hat{u}_j(\chi,t), \tag{11}$$

where $\hat{u}_0(\chi,t)$ is the contribution from the interaction force from the vehicle at $\chi=0$ and $\hat{u}_j(\chi,t)$, $j=1,2,...,J$, are the displacement fields generated by the point loads $\hat{f}_j(t)$. Equations (8) and (9) then lead to the formulation

$$m_0\frac{d^2\hat{z}}{dt^2} + c_0\left(\frac{d\hat{z}}{dt} - \dot{\hat{u}}_0(0,t) - \dot{\hat{u}}_S(0,t)\right) + k_0\left(\hat{z}(t) - \hat{u}_0(0,t) - \hat{u}_S(0,t)\right) = 0, \tag{12}$$

$$EI\frac{\partial^4\hat{u}_j}{\partial\chi^4} + m\left(\ddot{\hat{u}}_j - 2v\frac{\partial\dot{\hat{u}}_j}{\partial\chi} + \left(v^2 - \frac{G}{m}\right)\frac{\partial^2\hat{u}_j}{\partial\chi^2}\right) + \gamma\left(\dot{\hat{u}}_j - v\frac{\partial\hat{u}_j}{\partial\chi}\right) + \bar{\kappa}\hat{u}_j(\chi,t) =$$
$$\hat{f}_j(t)\delta(\chi-\chi_j), \quad j=0,1,...,J, \tag{13}$$

where $\hat{f}_0(t) = -m_0\frac{d^2\hat{z}}{dt^2}$, $\chi_0 = 0$, and $\hat{f}_j(t)$, $j=1,2,...,J$, are given in Eq. (10). The equations of motion (13) for $j=1,2,...,J$ are decoupled. Hence $\hat{u}_j(\chi,t)$, $j=1,2,...,J$, may be determined independently. However, subsequently Eqs. (12) and (13) for $j=0$ must be solved simultaneously.

Frequency Response Functions for First-Order Terms

The origin of the vibrations is the first-order variation of the spring stiffness, whereas the output is the displacement of the vehicle and the beam. The input-output relations are illustrated in Fig. 2, where $H_{\hat{Z}\hat{K}}(\omega)$ and $H_{\hat{U}\hat{K}}(\chi,\omega)$ are the frequency response functions for $\hat{z}(t)$ and $\hat{u}(\chi,t)$ for unit harmonic forces at $\chi_0=0$ and χ, respectively.

$$\hat{\kappa}(\chi_0+vt) \longrightarrow \boxed{H_{\hat{Z}\hat{K}}(\omega)} \longrightarrow \hat{z}(t) \qquad\qquad \hat{\kappa}(\chi+vt) \longrightarrow \boxed{H_{\hat{U}\hat{K}}(\chi,\omega)} \longrightarrow \hat{u}(\chi,t)$$

Figure 2: *Input-output relations.*

In the discretised system defined by Eqs. (12) and (13) the input is however the discrete loads $\hat{f}_j(t)$. Hence, as a first step in determining $H_{\hat{Z}\hat{K}}(\omega)$ and $H_{\hat{U}\hat{K}}(\chi,\omega)$, a relationship between the point loads and the stiffness variation must be established. Assuming $\hat{\kappa}(x)$ to vary harmonically with wavenumber k and amplitude $\tilde{K}(k)$, the variation in the moving frame of reference reads

$$\hat{\kappa}(\chi+vt) = \hat{K}(\omega)e^{i(\omega/v)\chi}e^{i\omega t}, \quad \hat{K}(\omega) = \tilde{K}(\omega/v). \tag{14}$$

Here $\omega = kv$ is the apparent circular frequency of the variation seen by an observer following the moving vehicle, and $\hat{K}(\omega)$ is the complex amplitude of $\hat{\kappa}(\chi+vt)$ at $\chi=0$. Equations (10) and (14) imply that a harmonic variation of $\hat{\kappa}(\chi+vt)$ leads to the following variation of the point loads,

$$\hat{f}_j(t) = \hat{F}_j(\omega)e^{i\omega t}, \quad \hat{F}_j(\omega) = H_{\hat{F}_j\hat{K}}(\omega)\hat{K}(\omega), \quad H_{\hat{F}_j\hat{K}}(\omega) = -\bar{u}(\chi_j)\Delta\chi_j e^{i(\omega/v)\chi_j}, \tag{15}$$

where $\hat{F}_j(\omega)$ is the amplitude of $\hat{f}_j(t)$ and $H_{\hat{F}_j\hat{K}}(\omega)$ is the frequency response function relating $\hat{f}_j(t)$ to $\hat{\kappa}(\chi+vt)$. With the point loads given by Eq. (15) four linearly independent harmonic solutions exist to the equation of motion (13) for each $j=1,2,...,J$. The solutions take the form

$$\hat{u}_{j,n}(\chi,t)=\hat{F}_j(\omega)\tilde{U}_{j,n}(\chi,\omega)e^{i\omega t},\quad j=1,2,...,J,\quad n=1,2,3,4. \tag{16}$$

Here $\tilde{U}_{j,n}(\chi,\omega)$ are the amplitudes for a harmonically varying force with unit amplitude at $\chi=\chi_j$, i.e. $f(t)=e^{i\omega t}\delta(\chi-\chi_j)$. Introducing $\tilde{U}_n(\omega)$ as the amplitudes at $\chi=\chi_j$, $\tilde{U}_{j,n}(\chi,\omega)$ may be written

$$\tilde{U}_{j,n}(\chi,\omega)=\tilde{U}_n(\omega)e^{i\hat{K}_n(\chi-\chi_j)}, \tag{17}$$

where the wavenumbers $\hat{K}_n$ are the roots to the characteristic polynomial obtained by assuming solutions of type (16) in the homogeneous version of Eq. (13), that is with $\hat{f}(t)\equiv 0$.

The vehicle displacement and the interaction terms of the beam displacement are given as

$$\hat{z}(t)=\hat{Z}(\omega)e^{i\omega t},\quad \hat{u}_{0,n}(\chi,t)=\omega^2 m_0\hat{Z}(\omega)\tilde{U}_{0,n}(\chi,\omega)e^{i\omega t},\quad n=1,2,3,4. \tag{18}$$

Here $\hat{Z}(\omega)$ is the first-order vehicle displacement amplitude and $\tilde{U}_{0,n}(\chi,\omega)=\tilde{U}_n(\omega)e^{i\hat{K}_n\chi}$ are the amplitudes for a harmonically varying force with unit amplitude at $\chi_0=0$.

At any point along the beam only two of the fundamental solutions $\hat{u}_{j,n}(\chi,t)$, $n=1,2,3,4$, are present for each $j=0,1,...,J$, as it was also the case for $\bar{u}(\chi)$. For the sake of convenience the two wave components existing *behind* the respective loads $\hat{f}_j(t)$ with reference to the direction of the velocity v will be assigned the subscripts $n=1,2$, whereas the components existing *in front of* the respective loads are assigned the subscripts $n=3,4$. From Eqs. (13) and (18) $\hat{U}_0(\chi,\omega)$, the amplitude of $\hat{u}_0(\chi,t)$, may be expressed in terms of $\hat{Z}(\omega)$. Inserting the result into Eq. (12) and assuming that J_1 of the point loads are applied behind the vehicle, the following frequency response relation is obtained by isolating $\hat{Z}(\omega)$ on the left-hand side of the resulting equation

$$\hat{Z}(\omega)=H_{\hat{Z}\hat{K}}(\omega)\hat{K}(\omega),\quad H_{\hat{Z}\hat{K}}(\omega)=N_{\hat{Z}\hat{K}}(\omega)/D_{\hat{Z}\hat{K}}(\omega), \tag{19}$$

where the numerator $N_{\hat{Z}\hat{K}}(\omega)$ and denominator $D_{\hat{Z}\hat{K}}(\omega)$ are, respectively, given as

$$N_{\hat{Z}\hat{K}}(\omega)=(i\omega c_0+k_0)H_{\hat{U}_S\hat{K}}(0,\omega), \tag{20}$$

$$D_{\hat{Z}\hat{K}}(\omega)=\left(-\omega^2 m_0+i\omega c_0+k_0\right)-\left(i\omega^3 m_0c_0+\omega^2 m_0k_0\right)\sum_{n=1}^{2}\tilde{U}_n(\omega). \tag{21}$$

$H_{\hat{U}_S\hat{K}}(0,\omega)$ is a special case of the frequency response function relating $\hat{u}_S(\chi,t)$ to $\hat{\kappa}(\chi+vt)$,

$$H_{\hat{U}_S\hat{K}}(\chi,\omega)=\sum_{j=1}^{J_1}H_{\hat{F}_j\hat{K}}(\omega)\sum_{n=3}^{4}\tilde{U}_{j,n}(\chi,\omega)+\sum_{j=J_1+1}^{J}H_{\hat{F}_j\hat{K}}(\omega)\sum_{n=1}^{2}\tilde{U}_{j,n}(\chi,\omega). \tag{22}$$

where $H_{\hat{F}_j\hat{K}}(\omega)$ and $\tilde{U}_{j,n}(\chi,\omega)$ are previously defined. Inserting Eq. (18) into (13) and making use of Eq. (19), the contribution from $\hat{f}_0(t)$ to the beam displacement can be found. The contribution from the discrete loads $\hat{f}_j(t)$, $j=1,2,...,J$, is given by Eq. (22). Adding the respective contributions, the frequency response function relating $\hat{u}(\chi,t)$ to $\hat{\kappa}(\chi+vy)$ may eventually be written

$$\hat{U}(\chi,\omega)=H_{\hat{U}\hat{K}}(\chi,\omega)\hat{K}(\omega),\quad H_{\hat{U}\hat{K}}(\chi,\omega)=H_{\hat{U}_S\hat{K}}(\chi,\omega)+H_{\hat{U}_0\hat{Z}}(\chi,\omega)H_{\hat{Z}\hat{K}}(\omega), \tag{23}$$

where

$$H_{\hat{U}_0\hat{Z}}(\chi,\omega)=m_0\omega^2\sum_{j=j_1}^{j_2}\tilde{U}_j(\omega)e^{iK_j\chi},\quad \begin{cases}\{j_1,j_2\}=\{1,2\}\text{ for }\chi\leq 0\\ \{j_1,j_2\}=\{3,4\}\text{ for }\chi>0\end{cases}. \tag{24}$$

Random Stiffness Variation

In the frequency domain, the two-sided auto-spectral density for the stiffness variation becomes

$$S_{\kappa\kappa}(\omega) = \frac{x_0}{\pi} \frac{\sigma_\kappa^2}{1 + x_0^2(\omega/v)^2}, \tag{25}$$

which is obtained from Eq. (2) by the transformation $\omega = kv$. Notice that all the variation lies within the first-order terms, i.e. $S_{\kappa\kappa}(\omega) = S_{\hat{\kappa}\hat{\kappa}}(\omega)$. Next, the two-sided auto-spectral density $S_{ZZ}(\omega)$ for the SDOF vehicle displacement and the two-sided cross-spectral density $S_{UU}(\chi_1, \chi_2, \omega)$ for the beam displacement at two points χ_1 and χ_2 on the beam axis may be found. Thus, see e.g. [9],

$$S_{ZZ}(\omega) = |H_{\hat{Z}\hat{K}}(\omega)|^2 S_{\kappa\kappa}(\omega), \quad S_{UU}(\chi_1, \chi_2, \omega) = H^*_{\hat{U}\hat{K}}(\chi_1, \omega) H_{\hat{U}\hat{K}}(\chi_2, \omega) S_{\kappa\kappa}(\omega), \tag{26}$$

where $H_{\hat{Z}\hat{K}}(\omega)$ and $H_{\hat{U}\hat{K}}(\chi, \omega)$ are defined previously and $H^*_{\hat{U}\hat{K}}(\chi_1, \omega)$ is the complex conjugate of $H_{\hat{U}\hat{K}}(\chi_1, \omega)$. Again it should be noticed that the principle of superposition is valid, because the governing equations are all linear, and that all variation lies in the first-order terms. From the Wiener-Khintchine relation the auto-covariance function $C_{ZZ}(\tau)$ for the vehicle displacement and the cross-covariance function $C_{UU}(\chi_1, \chi_2, \tau)$ for the beam displacement may be expressed as

$$C_{ZZ}(\tau) = \int_{-\infty}^{\infty} \cos(\omega\tau) S_{ZZ}(\omega) d\omega, \tag{27}$$

$$C_{UU}(\chi_1, \chi_2, \tau) = 2 \int_{-\infty}^{\infty} \left(\cos(\omega\tau) S_{UU}^{\Re} - \sin(\omega\tau) S_{UU}^{\Im} \right) d\omega, \tag{28}$$

respectively, where $S_{UU}^{\Re}$ and $S_{UU}^{\Im}$ are the real and imaginary parts of $S_{UU}(\chi_1, \chi_2, \omega)$, respectively.

NUMERICAL EXAMPLES

An analysis will be carried out for the standard deviation of the SDOF mass displacement response, $\sigma_Z = \sqrt{C_{ZZ}(0)}$, and the standard deviation of the beam displacement response directly under the vehicle, $\sigma_U = \sqrt{C_{UU}(0,0,0)}$. Due to the linearity of the problem σ_Z and σ_U are proportional to σ_κ. Hence, they may conveniently be described by the *dynamic amplification factors*,

$$s_Z = \frac{\sigma_Z}{\sigma_\kappa}, \quad s_U = \frac{\sigma_U}{\sigma_\kappa}. \tag{29}$$

Notice that $[s_Z] = [s_U] = \mathrm{m}^3/\mathrm{N}$. Furthermore, the circular eigenfrequency ω_0 of the vehicle, the damping ratios ζ_0 and ζ for the vehicle and beam, and the critical velocity v_{cr} are introduced,

$$\omega_0 = \sqrt{k_0/m_0}, \quad \zeta_0 = \frac{c_0}{2\sqrt{m_0 k_0}}, \quad \zeta = \frac{\gamma}{2\sqrt{m\bar{\kappa}}}, \quad v_{cr} = \sqrt{\sqrt{\frac{EI\kappa}{m^2}} + \frac{G}{m}}. \tag{30}$$

A beam with mass per unit length $m = 1000$ kg/m and damping ratio $\zeta = 0.1$ is considered. The vehicle has the circular eigenfrequency $\omega_0 = 2\pi\ \mathrm{s}^{-1}$ and the damping ratio $\zeta_0 = 1.0$, which are assumed to be typical values. Moreover, the vehicle mass has been set to $m_0 = 1000$ kg. Various combinations of the bending stiffness EI and the correlation length x_0 are analysed. Two different supports have been examined. In Fig. 3 the results are given for the velocity range $v \in [1, 100]$ m/s and a beam resting on a foundation with $\bar{\kappa} = 10^7$ N/m^2 and $G = 5 \cdot 10^7$ Nm/m. The parameters are rather arbitrarily chosen but may to some extent represent a soft elastic layer. Figure 4 shows the results for an even softer foundation with $\bar{\kappa} = 10^6$ N/m^2 and $G = 5 \cdot 10^6$ Nm/m.

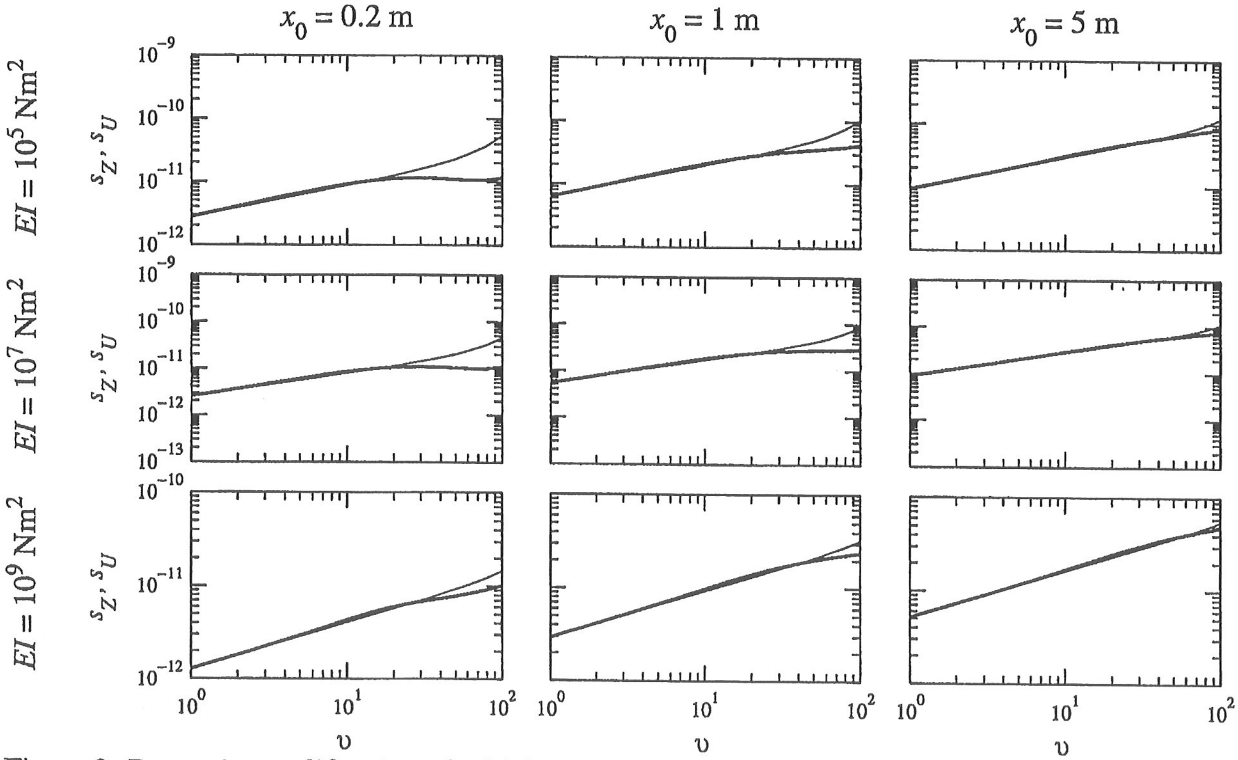

Figure 3: *Dynamic amplification of vehicle response (—) and beam response (—).* $\bar{\kappa} = 10^7$ *N/m*^{2} *and* $G = 5 \cdot 10^7$ *Nm/m.* $[s_Z] = [s_U] = m^3/N$ *and* $[v] = m/s$.

From the curves on figures 3 and 4 it may be concluded that the amplification of the responses is generally increasing with the correlation length, x_0. Also, in most cases the amplification of both the vehicle (*thick line*) and the beam response (*thin line*) grows significantly with the velocity. Two exceptions to this can be observed. First, for relatively flexible beams and low correlation lengths the vehicle response tends to stop increasing at high velocities, whereas the beam response is still growing rapidly. The phenomenon is most pronounced for the stiffer support, i.e. Fig. 3. Second, beyond a certain velocity both amplification factors for $EI = 10^5$ Nm^2 and $EI = 10^7$ Nm^2 on Fig. 4 drop off dramatically. This is due to the fact that v_{cr} in these cases lies below 100 m/s. Thus the critical velocities for $\bar{\kappa} = 10^6$ N/m^2 and $G = 5 \cdot 10^6$ Nm/m are approximately 70 m/s and 90 m/s for $EI = 10^5$ Nm^2 and $EI = 10^7$ Nm^2, respectively. However, at velocities near v_{cr}, where the sudden drop appears, a relatively strong amplification takes place.

CONCLUSIONS

The response of a single-degree-of-freedom vehicle moving uniformly along a Bernoulli-Euler beam resting on a modified Kelvin foundation with random spring stiffness has been investigated. The analysis was performed using the perturbation technique. Hence, it is only valid at relatively small variations of the spring stiffness. For larger variations a Monte Carlo simulation approach may be necessary as a higher-order perturbation analysis is inconvenient. Generally the analysis shows that the response of the moving vehicle and the beam underneath increases with velocity and the correlation length of the stiffness variation. However, for vehicle velocities beyond the critical velocity of the beam/support the response due to spring stiffness variation drops dramatically.

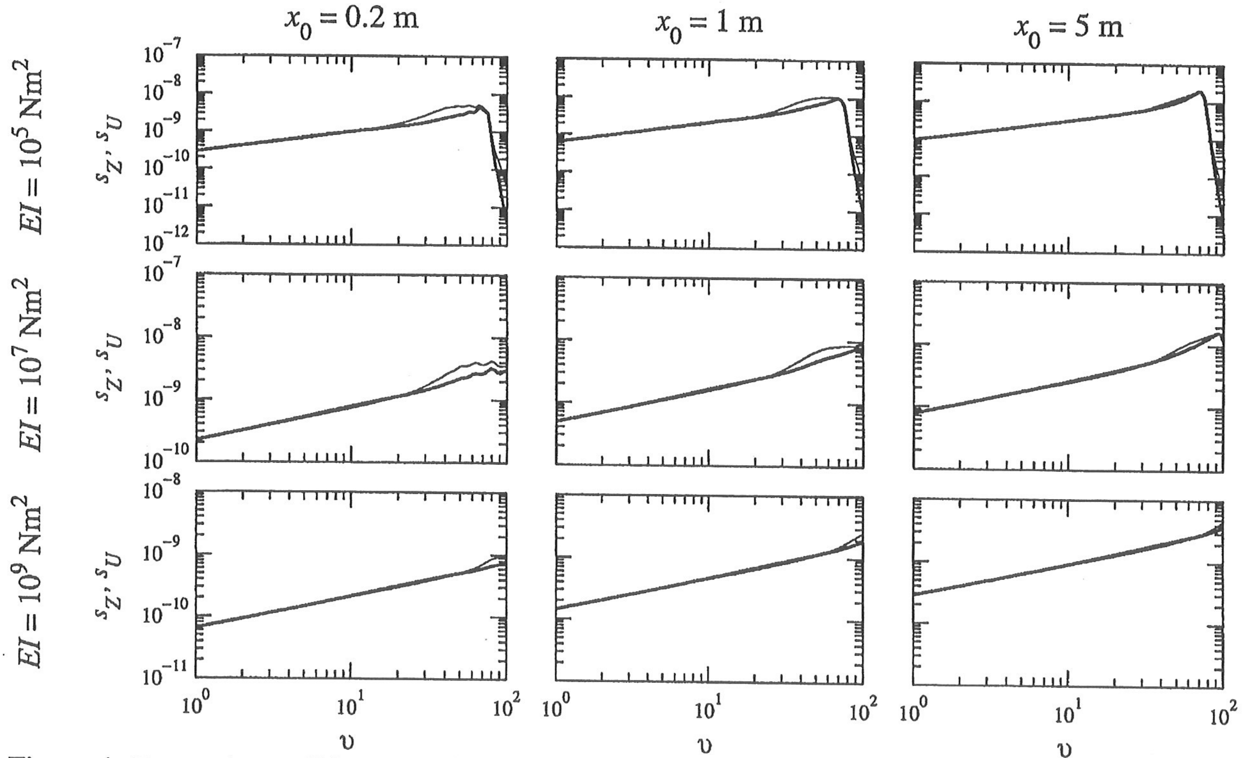

Figure 4: *Dynamic amplification of vehicle response (—) and beam response (—). $\bar{\kappa} = 10^6$ N/m^2 and $G = 5 \cdot 10^6$ Nm/m. $[s_Z] = [s_U] = m^3/N$ and $[v] = m/s$.*

ACKNOWLEDGMENT

The present research was supported by The Danish Technical Research Council within the project: "Damping Mechanisms in Dynamics of Structures and Materials".

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