On topological routing on degree 3 chordal rings

Gutierrez Lopez, Jose Manuel; Riaz, M. Tahir; Pedersen, Jens Myrup; Cecilio, Sergio Labeaga; Madsen, Ole Brun

Published in:
Proceedings on the First International Conference on Image Processing & Communications

Publication date:
2009

Document Version
Early version, also known as pre-print

On topological routing on degree 3 chordal rings

J. Gutierrez, T. Riaz, J. Pedersen, S. Labeaga and O. Madsen

Department of Electronic Systems, NetSec, Aalborg University, Denmark
jgl@es.aau.dk, tahir@es.aau.dk, jens@es.aau.dk, sl@es.aau.dk

Summary. Chordal Rings are degree three regular topologies and broadly studied since they are one of the organized upgrades of Single Rings. In addition, topological routing has been successfully proposed for several regular topologies. This paper proposes two topological routing algorithms for degree three choral rings, analyzing their efficiency in terms of path distances and routing delay.

1 Introduction

Topological routing is an alternative to traditional routing methods, based on tables. It allows for very fast restoration, and is particularly well suited for large-scale communication where table updates can be time consuming and introduces significant overheads [1]. Topological routing is defined as follows:

At a given address scheme, from any node any packet can be routed given only knowledge of the addresses of the current node and the destination node, with no routing tables involved [2].

This type of routing has been proposed for several topologies such as degree 4 Grid [3], Honeycombs [4] and N2R [5], including failure support mechanism and dynamic approaches. The complexity of topological routing algorithms should be independent of the number of elements in the network and only local information should be required, mostly neighboring information. This kind of algorithms can be classified as "*forwarding algorithms*" since what they do is to determine the best outgoing link for any incoming packet at a node.

In relation to Chordal Rings routing theoretical approaches have demonstrated the possibility of optimal distance routing and effective failure supporting for certain configurations [6]. Authors use Triple-loop graphs to formulate a routing methodology and they prove that it is possible to calculate optimal paths for each of the optimal configuration for a given diameter. This type of routing is not the goal of this study since the complexity of the methods increases with the size of the network and it is only valid for certain configurations. However, it inspires the work for the use of honeycomb structures.

The goal of this study is to test the feasibility and performance of two topological routing algorithms, varying their complexity and comparing the path distances and routing delay on the transmissions. Unified algorithms, valid for all the possible configurations of degree three Chordal Rings are implemented. Obviously it is possible to define a specific algorithm for a specific configuration, but in order to analyze the properties, compare potential algorithms and, for further research, compare topological routing on different topologies, a generic approach will provide more documented conclusions.

The implementation of failure supporting mechanisms is not covered yet by this work due to the extension of the topic. But as a proposal for further work, restoration schemes known as "*Lake Algorithms*" have achieved good results for mesh network topologies and therefore might be adjusted to CR due to the similarity to Honeycombs (degree 3 mesh networks) [3].

The rest of the document is as follows. Section 2 treats the background, definitions and proper notation to understand the network structure under study and algorithms. Section 3 introduces algorithms under study and the restoration mechanism proposal. In Section 4 the results are exposed and compared. Section 5 exposes the conclusions extracted from this paper.

2 Preliminary concepts

2.1 Degree Three Chordal Rings (CR)

Let w be an even integer such that $w \geq 6$, and let q be an odd integer, such that $3 \leq q \leq w/2$. w and q then define *CR(w, q)* with w nodes labeled $N_0, \ldots, N_{w-1}$. For $0 \leq i \leq w-1$ there exists a line between each of the following pairs of nodes $(N_i, N_{i+1(mod(w))})$ and $(N_i, N_{i+1(mod(w))})$ for i even [7]. Fig. 1(a) illustrates this scheme.

Shortest Optimal Chord (SOC): This term is used along the document and it is related to the optimal Chordal Rings configurations. For the same Chordal Ring size, same w, there might be more than one configuration, several q, which gives the shortest average and maximum path distances. In real networks it is more likely to implement short chord lengths, therefore, SOC is defined as the shortest of the optimal q.

Link notation: The link notation must be mentioned for the proper understanding of the studied algorithms. Each node is connected to its neighbors by three links. Ring links L (Left) connects N_i to $N_i + 1$ and R (Right) connects N_i to $N_i - 1$ and Chordal link C (Center) connects N_i to $N_i + w$ if N_i is even or N_i and $N_i - w$ if odd. Considering $mod(w)$ for all the nodes addresses. Fig. 1(a) presents the idea for a selected node 0.

2.2 N2R Algorithm

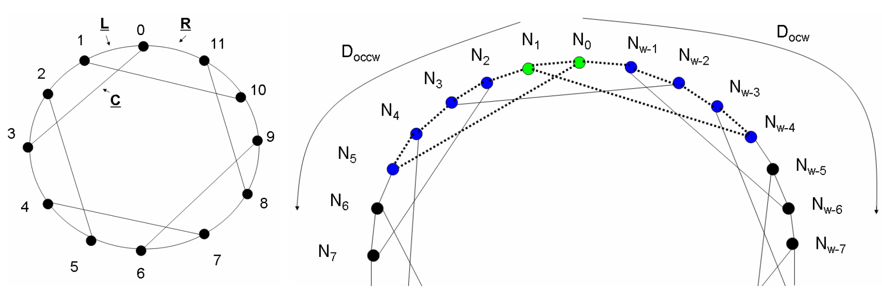
Previous work on this kind of cyclic networks can give a base for the implementation of the algorithms. N2R topological routing has been implemented [5] and the main conclusion obtained is that it is not possible to implement a generic algorithm that provides optimal paths for all the configurations and with complexity $O(1)$. The problem is when the shortest path involves "several loops" to the network at the inner ring. To be able to optimize the path distances, the complexity of the algorithm increases with the number of elements. However, simpler algorithms provide near optimal solutions and constant complexity just by not considering several loops in the inner ring.

3 Algorithms for CR topological routing

Two topological routing algorithms are proposed, for this kind of routing there is no distinction between source or middle nodes of a path, hence they will be named as current nodes when it is their time to forward a packet:

Associated Ring Algorithm (ARA): This is the simplest of the two algorithms. Let any pair of consecutive nodes be N_0 and N_1 being N_0 even and $N_0 < N_1$. Then N_0 and N_1 can be associated to two rings, AR, embedded in the CR structure. One clockwise, $N_1, N_0, N_{w-1}, N_{w-2}, N_{w-3}, N_{w-4}$, and one counterclockwise, $N_0, N_1, N_2, N_3, N_4, N_5$, Fig. 1(b). Let be N_c and N_d any current and destination nodes and D_{occw} and D_{ocw} their counterclockwise and clockwise distances using Ring links, Formula (1). If N_d belongs to any of the AR, the packet is routed using that AR. An example of AR is illustrated in Fig. 1(b) for N_0 and N_1, dotted lines. On the other hand, if N_d does not belong to any of the AR, if $D_{occw} < D_{ocw}$ the packet will be forwarded orienting the path counterclockwise, L if N_c odd or C if N_c even, and *viceversa* if $D_{occw} > D_{ocw}$, C if N_c odd or R if N_c even. Similarly to the FRA algorithm for N2R in [5].

$$D_{occ} = N_c - N_d(mod(w)) \qquad D_{oc} = N_d - N_c(mod(w)) \tag{1}$$



(a) CR(12,3) (b) Representation of a pair nodes associated rings for $q = 5$

Fig. 1. CR examples

Honeycomb Based Algorithm ($HCBA$): This algorithm deals with the comparison of two potential distances between the N_c and N_d. Let D_{qccw} and D_{qcw} be the counterclockwise and clockwise distances using the Chordal and Ring links and K_{ccw} and K_{cw} the number of Chordal hops in them. The main challenge is to calculate these distances in a simple way since no trees or precalculated paths are used in topological routing. In order to identify the patterns followed by CR, graphs similar to Honeycombs simplify the process.

The following paragraphs are focused on explaining in depth the procedure to construct these graphs. Obviously, this whole procedure is part of the pattern analysis and only the results (distance formulas) are used for the topological routing algorithm implementation, but the resulting graphs are found interesting enough to be explained for further research on CR.

The graphs represent the "*view*" from any node of the rest of the network. There are two types of honeycomb shaped graphs from a node N_c, if it is even or odd. These two graphs can be divided in two subgraphs, clockwise and counterclockwise oriented paths. All nodes can be reached using both kinds of paths, therefore all nodes can be located at both subgraphs. The use of these graphs allows to directly identify formulas for D_{qccw} and D_{qcw} based on D_{occw} and D_{ocw} and the "K" . The formulas represent the optimal D_{qccw} and D_{qcw} of any CR if several loops to the network are not considered. Once the two values are identified, the forwarding decision link is simple and direct.

Honeycomb structures can be represented as "*brick format*", similarly to grids schemes, and can be characterized by number of rows and columns, for more information it is highly recommended to read [4]. The use of this type of representation eases the analytical approach of CR.

To properly define these graphs it is necessary to define the "K" values K_{ccw} and K_{cw}. They represent the number of Chordal hops of a path and, also, the row in which the destination node is located in the N_c honeycomb shaped graph. The formulas of these values are presented in Table 1 for N_c even and odd. Basically, these values can be calculated for any N_d and they are required to calculate D_{qccw} and D_{qcw} to make a forwarding decision. If both K values are calculated for all the nodes from the point of view of N_c, a Honeycomb shaped graph can be formed and each of the nodes appearing twice in this graph. One representing a clockwise oriented path and another a counterclockwise path.

Fig. 2 presents the generic approach for this type of graphs for N_i even; for N_i odd it is similar, just to switch the positions in the graph of A - B and K_{ccw} - K_{cw}. A and B correspond to the values $A = \frac{w+1}{2}$ and $B = \frac{w-1}{2}$ and D_{occw} and D_{ocw} are easily calculated following Formula (1). Based on these graphs, the patterns for the distances values can be identified.

Fig. 3 presents an example of the complete graph from node "0". It can be clearly identified how each node is present twice except for 0. This kind of representation will ease future work in CR such as failure supporting schemes.

The result of this graph analysis leads to the formulas implemented in the topological routing algorithm in order to make the forwarding decision. Formula (2) for counterclockwise and Formula (3) for clockwise oriented paths; the extra three variables a ,b and c are defined in Formula (4).

Table 1. K values

N_c	$K_{ccw}(q>3)$	$K_{ccw}(q=3)$	$K_{cw}<\frac{q}{2}$	$K_{cw}>\frac{q}{2}$
Even	$\lfloor\frac{D_{occw}-A-1}{q}+1\rfloor$	$\lfloor\frac{D_{occw}+1}{q+1}\rfloor$	$\lfloor\frac{D_{ocw}-B-1}{q}+1\rfloor$	$\lfloor\frac{D_{ocw}+q-1}{q+1}\rfloor$
N_c	$K_{cw}(q>3)$	$K_{cw}(q=3)$	$K_{ccw}<\frac{q}{2}$	$K_{ccw}>\frac{q}{2}$
Odd	$\lfloor\frac{D_{ocw}-A-1}{q}+1\rfloor$	$\lfloor\frac{D_{ocw}+1}{q+1}\rfloor$	$\lfloor\frac{D_{occw}-B-1}{q}+1\rfloor$	$\lfloor\frac{D_{occw}+q-1}{q+1}\rfloor$

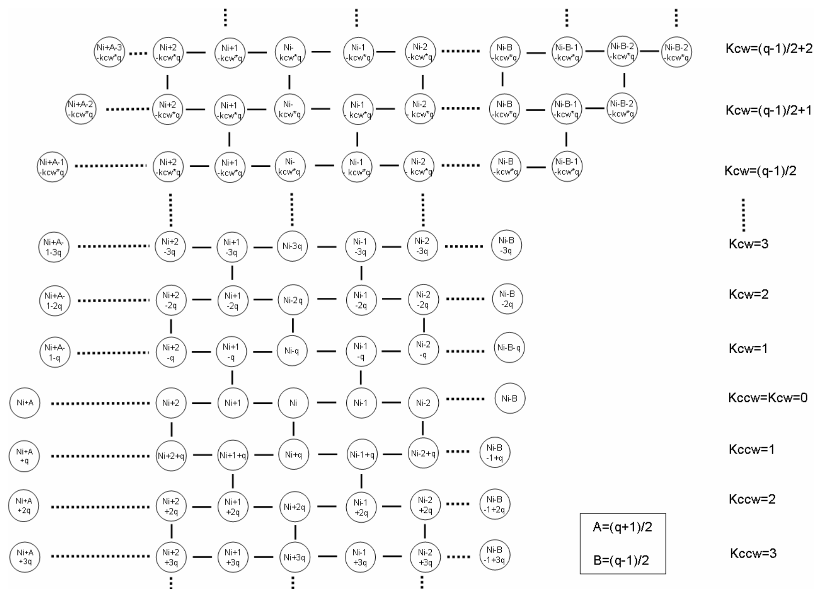


Fig. 2. Graph for a node N_i (even)

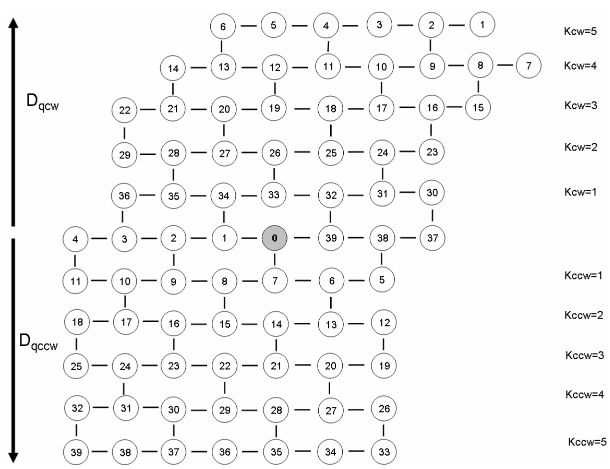


Fig. 3. Example of the graph from node 0 in CR(40,7)

$$D_{qccw} = \begin{cases} K_{ccw}(w-1) - D_{occw} + 2K_{ccw} & \text{if } K_{ccw}(w-1) - a > D_{occw} \ \& \ q > 3 \\ D_{occw} - K_{ccw}(w+1) + 2a + 2K_{ccw} & \text{if } K_{ccw}(w+1) + a < D_{occw} \ \& \ q > 3 \\ D_{occw} - K_{ccw}(w-1) & \text{if } q = 3 \ \& \ N_c \text{ even} \\ 2K_{ccw} - c & \text{rest} \end{cases} \tag{2}$$

$$D_{qcw} = \begin{cases} K_{cw}(w-1) - D_{ocw} + 2K_{cw} & \text{if } K_{cw}(w-1) - b > D_{ocw} \ \& \ q > 3 \\ D_{ocw} - K_{cw}(w+1) + 2b + 2K_{cw} & \text{if } K_{cw}(w+1) + b < D_{ocw} \ \& \ q > 3 \\ D_{ocw} - K_{cw}(w-1) & \text{if } q = 3 \ \& \ N_c \text{ odd} \\ 2K_{cw} + c & \text{rest} \end{cases} \tag{3}$$

$$a = \begin{cases} 1 & \text{if } N_c \text{ odd} \\ 0 & \text{if } N_c \text{ even} \end{cases} b = \begin{cases} 0 & \text{if } N_c \text{ odd} \\ 1 & \text{if } N_c \text{ even} \end{cases} c = \begin{cases} 1 & \text{if } N_c \text{ even } \& \ N_d \text{ odd} \\ -1 & \text{if } N_c \text{ odd } \& \ N_d \text{ even} \\ 0 & \text{rest} \end{cases} \tag{4}$$

The final step is to decide the link to forward the packet based on the shortest of D_{qccw} and D_{qcw}. The following algorithm presents part of the decision pseudo-code, when $D_{qccw} < D_{qcw}$, to illustrate the simplicity of the method:

if $D_{qccw} < D_{qcw}$ **then**
  if N_c even **then**
    if $D_{occw} < K_{ccw} * (q-1)$ **then**
      $nexthop = R$
    else if $Doccw > Kccw * (q+1)$ **then**
      $nexthop = L$
    else
      $nexthop = C$
  else
    if $D_{occw} > K_{ccw} * q$ or $K_{ccw} > q/2$ **then**
      $nexthop = L$
    else
      $nexthop = R$

4 Results

This Section presents the results when using the proposed algorithms. Transmissions for all the combinations of pairs of nodes and all the possible configurations from 6 to 100 nodes are performed. The parameters measured are the path distances and the routing time that corresponds to the execution time of the forwarding decision at every node. In transmission networks, shortest path distances are desirable since they kept as low as possible the overall traffic of the network by minimizing the routing traffic at the nodes, requiring lower capacity to support the demands.

The concept of routing delay considered in this work is similar to the table look-up time in table based schemes. Other factors may affect the final delay of packets such as queuing systems, but for the purpose of this work, the routing time is isolated to be able to compare both options. This routing delay is independent of the kind of traffic, queuing system or transmission media. Table 2 presents the difference between the shortest path distances and the algorithms results. An "error" in this case means that the distance obtained is not the shortest. The different values in Table 2 are: **Cases** is the number of different configurations tested, all the possibilities for the given w range, **Error** percentage of cases with error, 1^{st} **Error** lowest w with error and SCO **Error** Shortest Optimal Chord error percentage.

Table 2. Shortest Vs. Algorithms Distances

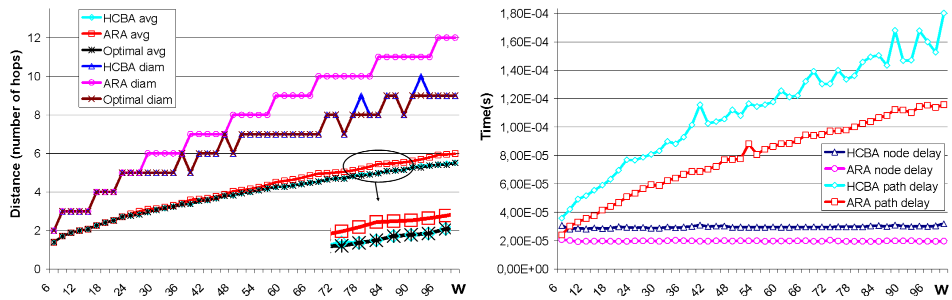
Algorthm	Cases	Error	1^{st} error	SCO error
ARA	1200	14%	20 nodes	80%
HCBA	1200	1,6%	46 nodes	6,4%

Fig. 4 illustrates the results, for SCO values, in terms of distances and routing time. Fig. 4(a) presents the average and maximum distance (diameter) of both algorithms and the optimal precalculated paths solution. The difference between ARA and $HCBA$ is clear, $HCBA$ being optimal except for few N (6,4 %). On the other hand, Fig. 4(b) presents the routing times, at each of the node and complete path, of the algorithms. The numerical values are not relevant since they vary depending on the hardware [1] used for the test, the relative difference between algorithms is the important information to look at. Even though ARA provides longer paths, the routing delay is shorter. The routing time at the nodes verifies the most important property of topological routing, independence complexity-number of elements. Both algorithms have constant node routing time, $HCBA$ obviously taking longer due to its higher complexity.

5 Conclusion

The implementation of two different topological routing schemes for degree three Chordal Ring leads to some interesting conclusions. The simplest of the algorithms, ARA, is based on associated rings to consecutive pairs of nodes and it performs perfectly with short chord length ($q = 3$) in terms of path distances. When $q > 3$ the packets are routed using non optimal paths but the routing delay is kept due to the simplicity of the algorithm.

[1] The results correspond to a AMD Athlon(tm) 64 X2 Dual Core Processor 3800+, MMX, 3DNow (2 CPUs), 2.0GHz. Memory: 990MB RAM



(a) Average and Maximum Distances (b) Topological Routing Time

Fig. 4. Performance Graphs

On the other hand, a more complex scheme, *HCBA*, based on Honeycomb graphs performs much better in terms of path distances but the delay cause at every routing node is longer. This algorithm performs optimally in terms of path distances on the 93,5% of the *SOC* cases and 98,4% of the overall tested configurations.

The decision of which algorithm is better depends on the trade off delay-path distance required by the specific kind of traffic or services of the network. But in case that some flexibility is allowed in terms of routing delay, the *HCBA* would be the best choice since the routing traffic and the necessary capacity will be reduced due to the shortest paths routing.

The most important result is that both algorithms fulfill the requirement of constant complexity. The schemes are independent of the number of elements in the network, and therefore, perfectly applicable to large scale network since the size does not affect the routing delay at the nodes.

References

1. W. D. Grover, *Mesh-Based Survivable Networks, Options and Strategies for Optical, MPLS, SONET and ATM Network*, Prentice Hall PTR, 2003.
2. Jens Myrup Pedersen, Thomas Phillip Knudsen, Ole Brun Madsen. “Topological Routing in Large-Scale Networks”. Proceedings of IEEE/ICACT 2004.
3. J. Pedersen, A. Patel, T. Knudsen , O. Madsen “Applying 4-regular Grid Structures in Large-Scale Access Networks” Comp. Com. 29 (2006) 1350-1362.
4. Ivan Stojmenavie, “Honeycomb Netwoks: Topological Properties and Communication Algorithm”, IEEE Transaction on Parallel and Distributed System, Vol8, NO. IO, October 1997.
5. Jose M. Gutierrez, Ruben Cuevas, Jens M. Pedersen and Ole B. Madsen “Improving Topological Routing in N2R Networks”. CAAN07, August 2007, Canada.
6. L. Barrierre, J. Fabrega, E. Simo, M. Zaragoza. “Fault-Tolerant Routings in chordal Ring Networks”. Networks 2000, vol. 36, pp. 180-190. ISSN 0028-3045.
7. S. Bujnowski, B. Dubalski, A. Zabludowski “The evaluation of transmission ability of 3rd degree chordal rings with the use of adjacent matrix”. 7th INFORMS 2004