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## **The Technologocal Frontier**

*an International Inter-industrial Empirical Investigation of Efficiency, Technological Change and Cenvergence*

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## The Technological Frontier

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Thomas Fredholm\* and Stefano Zambelli†

### ABSTRACT

The approach taken in this paper, to capture the international state of technological progress, goes through the so-called *technological frontier*. The technological frontier shows, for a given distribution of the net national product, the combination of production activities that would yield the highest wage income; equivalently the cost minimising choice of techniques. Different versions of the technological frontier are computed for a selection of OECD countries using input–output data 1970–2005. From these frontiers a set of indices is extracted to provide global as well as country-specific condensed measures of technological progress. Among the results are evidence of a global development that shows a more moderate rate of productivity growth compared with the conventional stylized facts of a 1.5–2.0 percent year-to-year increase and evidence of an economic development in the US driven at least as much by an increased work effort as by an increased productivity. The technological frontiers are intrinsically difficult to compute, but by applying two theoretical properties associated with the switch points between techniques of production, an algorithm is developed and invoked to efficiently compute the frontiers.

*Keywords:* Technological Change, Convergence, Input–output analysis, Technological Frontier, Computational Techniques

*JEL classifications:* C61, C63, C67, O47

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# 1 Introduction

The concept of technological progress is not a simple or straightforward one. Intratemporal and intertemporal comparison of technological possibilities is problematic, because also in its ideal type conceptualization the intrinsic nature of most commodities changes according to time and space. New commodities are introduced and either substitute old ones or coexist; new methods of production and new markets emerge so as to influence the prices and trade; exhaustible resources are depleted cutting off access to old production techniques; and so on and so forth. Normally the process of technological innovation is studied focusing on the creation of new products or on new ways to produce the same product. But for the whole system the detection of technological progress is problematic.

The problem of measuring technological progress is related to aggregation and hence to some form of indexation. Normally comparison between bundles of different types of commodities is in the literature made following two methods or a combination of them: the *value method* and the *index number method*. In the value method the different commodities are assigned a value in terms of the value of a *numéraire* (which is either one single commodity or a bundle of commodities), while in the index number method the heterogeneous physical commodities are transformed into an index number. Both methods are problematic and have been widely discussed in the literature. The value method has been studied since Smith and Ricardo's labour theory of value and the index number problem is still centred around Irving Fisher's 1922 study on 'The Making of Index Numbers'.

The literature on index number is enormous and here we will not make a review. What are sufficient to point out are two things. Firstly, that the function of an index number is to transform something which is intrinsically heterogeneous into a homogeneous (scalar) magnitude.

Second, that there is broad consensus that the ideal index number, in the sense of Fisher (1922), does not exist and hence cannot be constructed. See among many Leontief (1936), Afriat (1977), Samuelson and Swamy (1974), and Velupillai and Zambelli (1993). Samuelson and Swamy (1974, p. 568) in their survey on invariant economic index numbers summarize and declare at the outset that

we cannot hope for one ideal formula for the index number: if it works for the tastes of Jack Spratt, it won't work for his wife's tastes; if, say, a Cobb-Douglas function can be found that works for him with one set of parameters and for her with another set, their daughter will in general require a non-Cobb-Douglas formula! Just as there is an uncountable infinity of different indifference contours—there is no counting tastes—there is an uncountable infinity of different index number formulas, which dooms Fisher's search for the ideal one. It does not exist even in Plato's heaven.

The approach taken in this paper, to capture the state of technological progress, goes through the so-called *technological frontier*. The technological frontier shows, for a given uniform rate of profit, what combination of production activities that would yield the highest wage income, i.e., the envelope of all the possible wage-profit frontiers that can be constructed from a given set of production techniques; equivalent to the cost minimising choice of production activities.<sup>1</sup> One advantage of using wage-profit frontiers is that it takes into account the fact that new techniques generate a different demand vector of the factors of production and eventually different prices and hence different costs and revenues.

From this approach we will construct indices that in several ways differ from the orthodox indices referred to by Samuelson and Swamy in the quotation above. The orthodox indices take as given (homothetic) preference and use these as weights in the process of aggregation. As pointed out, Jack Spratt might possess a particular Cobb-Douglas utility function, but his family might not.<sup>2</sup>

The method we propose use on the other hand production costs, measured in prices of production, as weights in the process of aggregation. The prices of production have the advantage of taking into account both the production techniques and the demand for means of production. Hence, where neoclassical indices are generated using axiomatic preferences, we use uniquely determined (endogenous) prices of production.<sup>3</sup> The prices of production do not however take into account the demand for final consumption, but following the *Non-substitution theorem*, the production prices are independent of the composition of the vector of final consumption.<sup>4</sup> It is a major advantage that the wage-profit frontiers are independent of simple differences in the scale of production, even if the changes are asymmetric across industries.

Take note, this does not imply that we assume *constant returns to scale*. If changes occur in the scale of production in one or more industries, without changing the proportional use of the means of production (including labour), then the wage-profit frontiers, together with the indices based hereupon, remain unaffected. If on the other hand the relative proportions of the means

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<sup>1</sup>This problem could just as well have been stated as the highest level of consumption for a given uniform growth rate. For more information on this duality see Pasinetti (1977, ch. 7), Bruno (1969), and Burmeister and Kuga (1970).

<sup>2</sup>For those, as we, who wonder whom this mysteries Jack Spratt might be. He, spelled as Jack Sprat, appears in a fitting nursery rhyme, (thanks to Wikipedia):

Jack Sprat could eat no fat.  
His wife could eat no lean.  
And so between them both, you see,  
They licked the platter clean.

<sup>3</sup>For an introduction to the neoclassical theory of index numbers and productivity measurement, see Coelli et. al. (2001).

<sup>4</sup>Explained in Section 2.

of production are affected, then it will influence the vector of production prices, which subsequently together with the new production techniques are used to assess the consequences of the technical innovations for the economy as a whole.

The technological frontier is found in both 'smooth' neoclassical economics and 'discrete' technologies models. As observed by Bruno (1969, p. 51) "any neo-classical technology could be simulated by a 'very dense' spectrum of discrete techniques", hence the approach taken in this paper is not necessarily restricted to the unorthodox setting in which it is framed.

We compute and study three different versions of the so-called technological frontier using input–output data from eight OECD countries from 1970–2005 with five year intervals. The three frontiers we shall call respectively; the *contemporary*, the *rolling*, and the *intertemporal* technological frontier—all theoretical constructs with their own set of useful applications.

The *contemporary technological frontiers* are constructed from all the production techniques extracted from the OECD input–output tables available for a given year, i.e., from a range of countries at a given point in time (one accounting period). Comparing the contemporary technological frontiers with the actual wage-profit frontiers for the individual countries can be used to study efficiency since the contemporary technological frontier, at a given point in time, provides a measure of the maximum potentials for international trade and/or gains by exchange of production techniques. Furthermore, the evolution of the contemporary technological frontiers provides a measure of global technological progress.

The *rolling technological frontiers* are the envelopes formed by the production techniques available in 1970; 1970,1975; ...; 1970,1975,...,2005, i.e., a backward looking set of production techniques, which together with the contemporary technological frontiers are used to study which countries' industry-level production techniques that are the most effective, and how this displacement of production techniques evolves over time.

The *intertemporal technological frontier*, equivalent to the last of the rolling technological frontiers, is computed from the full set of techniques available over time and across countries. The intertemporal technological frontier provides a theoretical global and intertemporal measure of the maximum economic potentials for the 'world economy' as a whole.<sup>5</sup> The intertemporal technological frontier is also useful in the study of convergence. In particular, as an alternative to the usual approach of using the US as a reference point, this is problematic because the reference point itself change over time.

The country specific wage-profit frontiers, the contemporary technolog-

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<sup>5</sup>By the world economy we here mean the eight OECD countries. Contingent on data availability, this analysis could and should of course be extended to include additional countries.

ical frontiers, and the intertemporal technological frontier are combined to construct indices for country specific technological progress and global convergence towards the maximum theoretical technical potentials. These indices provide both empirically and conceptually new insight to the well-known *catching up hypothesis*.<sup>6</sup> Among the problems confronted, is the fundamental question; is the US (the leader) catching up?

Common for the different versions of the technological frontiers is that they can be seen as an empirical proxy for what is known in the literature on economic development as the *access to technology constraint*, i.e., the situation in which a country is approaching the technological frontier and consequently *ceteris paribus* finds it more difficult to substitute currently used techniques of productions with more efficient ones.<sup>7</sup>

To actual obtain these results it has been necessary to develop an algorithm that in an effective way computes these frontiers. The mathematical notion of an envelope is conceptual straightforward, but the natural *brute force* algorithm associated with the computation of such an envelope is for every single point computational infeasible. This problem and the algorithm developed to solve it is fully described in Section 5, but can be skipped by readers more interested in the empirical results.

Section 2 presents the theoretical framework, Section 3 the indices used, Section 4 the data, and Section 6 the empirical results. Section 7 concludes the paper.

## 2 The Technological Frontier

The economic system consists of  $n$  industries each producing one unique commodity by means of some combination of the  $n$  commodities and labour.<sup>8</sup>

Let  $\mathbf{A}$  be a  $n \times n$  indecomposable semi-positive non-singular matrix of interindustry coefficients, where the  $ij$ th entry represents the  $i$ th industry's use of the  $j$ th commodity in the production of one unit of the industry's output. Likewise,  $\mathbf{l}$  is a  $n \times 1$  vector of labour input coefficients where the  $i$ th entry represents the  $i$ th industry's use of labour in the production of one unit of output. As usual these elements can be collected in the following long-run equilibrium relationship that captures the distribution of the total production among wages, profits, and means of production, where the wage and profit rates are assumed to be uniform:

$$\mathbf{A}\mathbf{p}(1+r) + \mathbf{l}w = \mathbf{p} \quad (2.1)$$

<sup>6</sup>See among many Abramovitz (1986).

<sup>7</sup>See Ernst et. al. (1998, p. 15–16), where the the access to technology constraint is discussed in relation to Korea and Taiwan in the 1980s and 1990s.

<sup>8</sup>This theoretical part is based on the seminal work by von Neumann (1945–46), Leontief (1941), and Sraffa (1960), and subsequent work found in Pasinetti (1977), Velupillai and Zambelli (1993), and Zambelli (2004).



Choosing a *numéraire*  $\boldsymbol{\eta}$ , for which it holds that  $\boldsymbol{\eta}'\boldsymbol{p} = 1$ , the degrees of freedom reduces from two to one, such that for a given rate of profit, the wage rate can be computed by isolating  $\boldsymbol{p}(r, \boldsymbol{A}, \boldsymbol{l}) = (\boldsymbol{I} - \boldsymbol{A}(1+r))^{-1}\boldsymbol{l}w$ , premultiplying with the *numéraire*, and rearranging, *viz.*

$$w(r, \boldsymbol{A}, \boldsymbol{l}) = \left( \boldsymbol{\eta}'(\boldsymbol{I} - \boldsymbol{A}(1+r))^{-1}\boldsymbol{l} \right)^{-1} \quad r = \{r \in \mathbb{Q} : 0 \leq r \leq R\} \quad (2.2)$$

Where  $R$ , the maximum rate of profit, can be computed as  $R = \lambda^{-1} - 1$ , where again  $\lambda$  is the maximum eigenvalue of  $\boldsymbol{A}$ .

It must be stressed that the production prices  $\boldsymbol{p}(r, \boldsymbol{A}, \boldsymbol{l})$  and the wage rate  $w(r, \boldsymbol{A}, \boldsymbol{l})$  are scale-independent, not only of the scale of the economy

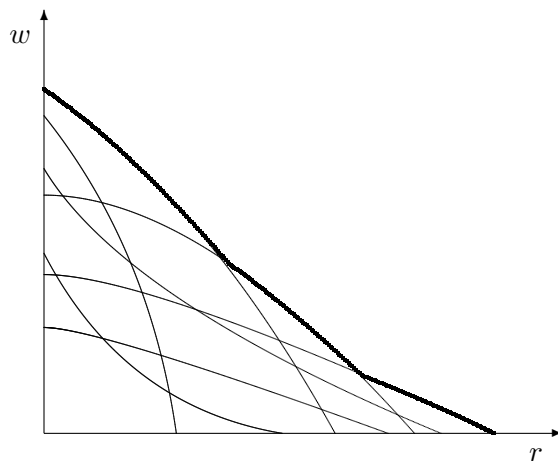


Fig. 2.1: *The technological frontier*

as a whole, but also the scale of production in the single industries. This property is known as the Non-Substitution Theorem.<sup>9</sup>

For each unique set of techniques  $\{E^{(k)}\} = \{\boldsymbol{A}^{(k)}, \boldsymbol{l}^{(k)}\}$ ,  $k = 1, 2, \dots, m$ , from the set of systems  $\boldsymbol{E} = \{E^{(1)}, E^{(2)}, \dots, E^{(m)}\}$  there is a unique wage-profit frontier. The envelope of these frontiers, illustrated in Figure 2.1, is the technological frontier, *viz.*

$$w^{\text{TF}}(r, \boldsymbol{E}) = \max \{w(r, E^{(1)}), w(r, E^{(2)}), \dots, w(r, E^{(m)})\} \quad (2.3)$$

As defined in the introduction we study three versions of the technological frontier; the contemporary  $w_t^{\text{CTF}}(r, \boldsymbol{E}_t)$ , the rolling  $w_t^{\text{RTF}}(r, \boldsymbol{E}_1, \boldsymbol{E}_2, \dots, \boldsymbol{E}_t)$ , and the intertemporal  $w^{\text{ITF}}(r, \boldsymbol{E})$ , where  $\boldsymbol{E}_t$  denotes the set of techniques used at time  $t$  and  $\boldsymbol{E}$  the total set of techniques. An obvious analytical property of these three versions of the technological frontier is that:<sup>10</sup>

$$w_t^{\text{CTF}}(r, \boldsymbol{E}_t) \leq w_t^{\text{RTF}}(r, \boldsymbol{E}_1, \boldsymbol{E}_2, \dots, \boldsymbol{E}_t) \leq w^{\text{ITF}}(r, \boldsymbol{E}) \quad \forall t = 1, 2, \dots, T \quad (2.4)$$

<sup>9</sup>See Kurz and Salvadori (1995, p. 26–28) for a discussion of the origin and implications of this peculiar result.

<sup>10</sup>Since  $\{\boldsymbol{E}_t\} \subseteq \{\boldsymbol{E}_1, \boldsymbol{E}_2, \dots, \boldsymbol{E}_t\} \subseteq \{\boldsymbol{E}\} \quad \forall t = 1, 2, \dots, T$

For convenience and completeness we restate the analytical properties associated with the technological frontier. For proofs and further discussion of these properties, see Pasinetti (1977 p. 158–59).

1. At the switch point between two techniques, each commodity has the same price.
2. If, for a given rate of profit, one technique dominates another, then it will yield prices, in terms of the wage rate, that are strictly lower than those yielded by the other technique.
3. The switch points are independent of the choice of *numéraire*.
4. The technological frontier is strictly decreasing as the rate of profit increases.
5. (Corollary) At the switch points between two techniques, the change will occur in one, and only one, industry, i.e., piecemeal.

For the purpose of computing and interpreting the technological frontier, property number three is very convenient, since it implies that the set of technologies forming the technological frontier is independent of the choice of *numéraire*. Since an objective of this study is to construct indices for comparative studies it is imperative that these indices are (more or less) independent of the *numéraire* chosen to compute it. However, while the set of techniques constituting the technological frontier is independent of the choice of *numéraire*, the shape of the frontier is not. Consequently, the choice of *numéraire* will to some extent influence our results, but the influence will be suppressed by the stability of the switch points. In the analysis, the number of switch points on the envelopes will be reported to provide a first approximation of the robustness of our results.

Furthermore, as will be clear later, property number five turns out, from a computational point of view, to be extremely convenient.

### 3 The Velupillai-Fredholm-Zambelli Index

The technological frontier can be interpreted as an *access to technology constraint*, since it provides a proxy for the maximum potential level of productivity. The technological frontier allows us to reformulated, in a more general terms, the well known *catching up hypothesis*, i.e., that the growth rate in productivity varies inversely with the productivity level. Replacing the US (the leader) with the intertemporal technological frontier, allows us to study the same problems, but with a benchmark extracted from the entire sample. Furthermore, we are now able to address the question 'is the US catching up?' Why should the US not be able to catch up to something more efficient already potentially available in the system, defined by the technological frontier? Naturally, this also provides a convenient framework in which to study *overtaking*, i.e., if one country should overtake the leader.

To study this and more, we construct what we shall call the country specific *Velupillai-Fredholm-Zambelli* (VFZ) index that provides a measure

of the average efficiency relative to the intertemporal technological frontier. For the  $j$ th country at time  $t$  the VFZ-index is computed as.

$$VFZ_{j,t} = 1 - \frac{1}{R_{j,t}} \sum_{r=0}^{R_{j,t}} \left[ w^{\text{ITF}}(r, \mathbf{E}) - w(r, \mathbf{A}_{j,t}, \mathbf{l}_{j,t}) \right] \quad (3.1)$$

$$j = 1, 2, \dots, N, \quad t = 1, 2, \dots, T$$

In words, the VFZ-index is computed as one minus the average vertical distance between the individual countries wage-profit frontiers and the intertemporal 1970–2005 technological frontier. The range of the index is between zero and one. The closer the index is to unity the more efficient is the technology used in the single country relative to the theoretical maximum computed from the entire set of production activities.<sup>11</sup>

An analogue global version of the VFZ-index is computed from the vertical distances between the contemporary technological frontiers and the intertemporal technological frontier, *viz.*

$$VFZ_t^{\text{global}} = 1 - \frac{1}{R_t^{\text{CTF}}} \sum_{r=0}^{R_t^{\text{CTF}}} \left[ w^{\text{ITF}}(r, \mathbf{E}) - w_t^{\text{CTF}}(r, \mathbf{E}_t) \right] \quad (3.2)$$

$$t = 1, 2, \dots, T \quad (3.3)$$

Where  $R_t^{\text{CTF}}$  is the maximum profit rate associated with the contemporary technological frontier at time  $t$ . The global VFZ-index provides a measure of the technological progress for the global economy as a whole.

The advantages of the VFZ-indices over conventional ones are:

1. The method is non-parametric and non-stochastic.
2. Technology, value, and aggregation are fully integrated through the prices of production, hence to some extent circumvents standard index number and value problems.
3. The indices are time-invariant, i.e., they are fully determined within single accounting period.<sup>12</sup>
4. The stability of the switch points greatly limits the sensitivity of changes in the *numéraire*.
5. The interdependence among industries is endogenously captured by changes in the prices of production.

<sup>11</sup>An alternative index could be computed using some proxy for the actual distribution among wages and profits. Hence, using only one point on (or a segment of) each frontier.

<sup>12</sup>However, updating the entire dataset with new data, say the 2010 OECD tables, will almost certainly change the intertemporal technological frontier, but the within-period ranking will remain unaffected.

6. The indices will not change as a consequence of simple changes in the scale of production in the single industries, but only if real technological innovations are observed in one or more industries.<sup>13</sup>
7. In the study of convergence, the benchmark/reference point is determined from the system as a whole and not simple a 'leading country'.

## 4 Data and the Choice of *Numéraire*

For the actual computation of the technological frontiers we have chosen the OECD 1970–2005 input–output tables from the US, Germany, the UK, France, Canada, Denmark, Japan, and Australia. All based on the the ISIC 2 or ISIC 3 classifications with respectively 35 and 48 industries.<sup>14</sup> The tables contain both the domestic interindustrial flow and industry-specific imports of capital goods.

Some problems of comparability exist between the two methods of classification, but steps have been taken to minimize these problems. The initial 48 and 35 industries have been aggregated into 23 industries following standards of national accounting. The main reason for so doing is that it ensures comparability over time and non-singular matrices for the whole dataset.

Unfortunately, tables are not available for all countries for all time periods. To further increase comparability we have chosen to substitute the missing tables with the most commensurable table, typically the table from the previous accounting period in the same country. For details, see Table A.1 in Appendix A.

As labour inputs we use data from the OECD on the industry-level 'compensation of employees' and use this to distribute the total employment in hours to the single industries. When available we use detailed industry-level employment data from The Groningen Growth and Development Centre.<sup>15</sup> Note, that both over time and across industries labour is treated as a homogeneous input. This is a very strong assumption, but necessary given the data availability.

There is a fundamental problem related to the units of accounting, since the tables are denominated in current values of the national currency. Macro-industry deflators have been computed as the differences between macro-industry GDP denomination in respectively current and base period prices, and used to deflate the value denominated tables. This is probably the best available proxy for the physical flow among industries found in the OECD input–output tables. For a discussion on monetary vs. physical denominated input–output data, see Han and Schefold (2006, p. 750).

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<sup>13</sup>By real technological innovations we mean changes in the matrix of technological coefficients and/or in the corresponding (normalised) vector of labour inputs.

<sup>14</sup>See [www.OECD.org](http://www.OECD.org).

<sup>15</sup>See [www.GGDC.net](http://www.GGDC.net).

Appendix A contains additional information on the data used.

As a *numéraire* we choose the vector of domestic net products from the base year 2000 normalised with the total hours worked. We use the domestic net products, because if imported means of production were subtracted we would not necessarily obtain a vector of non-negative entries. Section 6.4 discusses the relationship between the choice of *numéraire* and the stability of our results based on the number of switch points on the envelopes.

## 5 Algorithms

Formally speaking, the problem of computing a technological frontier is *computable*, i.e., there exist an algorithmic procedure that in a finite number of steps can compute it, or equivalent under the Church-Turing thesis there exists a Turing-machine that always halts.<sup>16</sup> The formal proof of this is given in terms of the following brute-force algorithm.

### 5.1 A brute-force algorithm

1. import data and convert the data into matrices of technical coefficients
2. loop through all possible systems,  $k = 1 : 1 : N^n$ 
  - (a) compute the maximum eigenvalue  $\lambda(k)$  for the  $k$ th system
3. use the minimum  $\lambda(k)$  to compute  $\bar{R}$  associated with the technological frontier
4. loop through all possible systems
  - (a) compute the wage rate for incremental steps of  $0 < r < \bar{R}$
5. for each  $r$  select the system associated with the maximum wage rate

However, when using the above algorithm the computational complexity of the problem implies that it is practical impossible to compute the technological frontier for even small datasets, since for each rate of profit all possible combinations of techniques must be evaluated. Using the *Big-O notation* the time-complexity is (at least)  $O(N^n)$ . This implies that no matter how powerful a computer that will be developed within, say the next century, it will always be possible to include additional available data, such that the algorithm will not halt within any reasonable time frame.<sup>17</sup> What makes this

<sup>16</sup>Note that the domain of the technological frontier, see Equation 2.2 and 2.3, has been defined on the rational line.

<sup>17</sup>A *back-on-the-envelope* (!) estimate of the computer power needed to compute the intertemporal technological frontier based on  $(T \cdot N)^n = (8 \cdot 8)^{23} \approx 3.5 \cdot 10^{41}$  unique systems; for just one rate of profit, running a whole year, the computer must evaluate  $1.1 \cdot 10^{34}$  systems per second, each including several matrix operations. Not anything near such a computer exists today or will within any reasonable time frame.

problem serious for even rather small values of  $N$  and  $n$ , is that the algorithmic procedures must contain computations of eigenvalues and inversions of  $n \times n$  matrices.

The computational complexity can however be drastically reduced (in the order of  $N^n$  to  $N \cdot n$ ) by exploiting property number five listed in Section 2. Using any point on any frontier the following procedure, so to say, climbs the individual wage-profit frontiers using the switch points as stepping stones.

## 5.2 The Piecemeal algorithm

1. import data and convert it into matrices of technical coefficients
2. choose an initial point on any frontier
3. from this point, while  $r > 0$ , lower the profit rate one increment<sup>18</sup> and compute the wage rate without changing the techniques, save this as  $\bar{w}$ 
  - (a) one by one, change the techniques (piecemeal), i.e.,  $n \cdot (N - 1)$  times, and for each system
    - i. if the profit rate is smaller than the maximum profit rate, compute the wage rates
    - ii. if this wage rate is greater than  $\bar{w}$ , then we have passed a switch point. Fix the new set of techniques and the associated wage rate. Else use  $\bar{w}$
4. Now reverse the procedure, while  $w > 0$ , increase the profit rate one increment and compute the wage rate without changing the techniques, save this as  $\bar{w}$ 
  - (a) one by one, change the techniques and for each system:
    - i. if the profit rate is smaller than the maximum profit rate, compute the wage rates.
    - ii. if this wage rate is greater than  $\bar{w}$ , then we have passed a switch point. Fix the new set of techniques and the associated wage rate. Else use  $\bar{w}$
5. go to point # 3 as long as loop # 3 and 4 do not produce identical results, else terminate and collect the results

Both algorithms can be implemented with no serious demand for the available memory, but unlike the brute-force algorithm the Piecemeal algorithm cannot be run parallel.

An easy way to verify the outcome from the Piecemeal algorithm is to apply the two algorithms on a tractable subset and check that they yield identical results. This has been done with positive results.<sup>19</sup>

<sup>18</sup>In the actual computation the stepsize is fixed at  $\frac{1}{1000}$ . Between  $\frac{1}{500}$  and  $\frac{1}{1000}$  the number of switch point increased, which implies that the algorithm missed some switch points. No changes in the results are found when narrowing the stepsize to  $\frac{1}{2000}$ .

<sup>19</sup>There exist one potential problem; it is theoretical possible, by some fluke, that the envelope is not connected by intersections with the initially chosen frontier. However, the probability of this occurring tends to zero as the number of techniques tends to infinity.

The full set of results based on the eight OECD countries for eight time periods can be computed within a few hours, with the Piecemeal algorithm using a standard desktop computer.

## 6 Efficiency, Technological Change, and Convergence

This analysis is both from a theoretical and empirical point of view 'average', as oppose to 'marginal', since it deals with average costs, returns, revenues, etc. while mainstream (marginal) theory focus on the corresponding marginal magnitudes. Given that marginal magnitudes can never be observed, but average magnitudes can, this is the appropriate approach to empirical studies. However, in one case where the orthodox theory is average, this approach is specific; we do not assume a representative firm.

A general problem associated with the measurement of technological progress is related to the fact that different production activities use different sets of factors of production. For example one can consider the production of energy; nuclear energy, wind mills, hydro-power, solar-energy, oil, coal, gas, etc. It would be rather difficult to asses which production process that is most efficient. Moreover, it is not always the case that the adoption of a new method indicates that the method is superior. There might be other reasons different from technological superiority, and the expected costs could differ from from the actual costs.

The economic system as a whole most likely adopts a combination of the different methods of production. Consequently, the observed output from an industry and the corresponding vectors of industry inputs are not only average over the accounting period (as it should be), but also average across the techniques used. However, this problem should diminish as the number of industries in the national accounting increases.

### 6.1 The empirical technological frontiers

Figure 6.1 shows the complete collection of contemporary and rolling technological frontiers. Analogue to the study of the wage-profit frontiers for the individual countries, an outward shift of the frontier implies unambiguously technological progress. If two frontiers intersect, it cannot unambiguously be determined whether of not a higher level of productivity is reached.

The contemporary technological frontiers show a clockwise and steady shift outwards, while the rolling technological frontiers show a more parallel shift. This difference provides a first-hand insight into the nature of the global technological progress. But however tempting as it might be, it is not unambiguous, to interpret the shifts of the contemporary technological frontiers as a global labour-saving technological progress, since the value

of the circulating capital not necessarily changes monotonic with the profit rate.

The problem of intersection(s) between frontiers does however not exist for the rolling technological frontier, since these by construction will never intersect. Consequently, together with the other frontiers, this property makes the rolling technological frontier a strong analytical tool. An observed

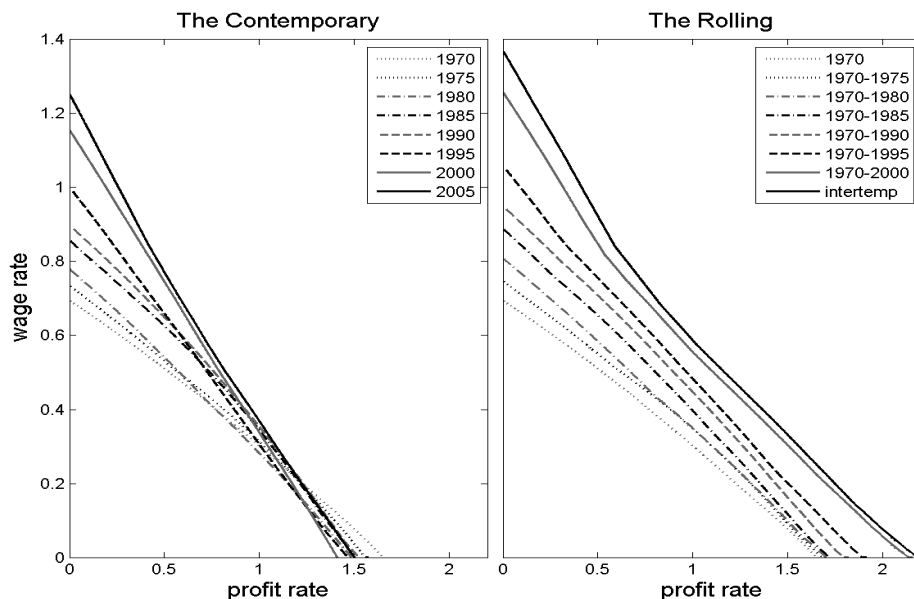


Fig. 6.1: *The contemporary and rolling and technological frontiers*

difference between the contemporary and rolling frontiers implies that there exist some combinations of the old and new production techniques, which are more productive than all combinations of the techniques currently used.

However, it could be argued that some old techniques of production should be discarded from the set of techniques forming the rolling (and intertemporal) technological frontier. This could be techniques that are both (under some circumstances) superior to contemporary techniques, but practical obsolete, e.g., because of severe negative externalities or depletions of raw materials. And hence *de facto* no longer exist in the *book of available blueprints*.

Figure 6.2 and 6.3 show the wage-profit frontiers for the individual countries together with the contemporary and rolling technological frontiers. Figure 6.2 for the period 1970–1985 and Figure 6.3 for 1990–2005. As expected the US is from the 1970s the leading country, but the US wage-profit frontiers do not shift as much as the other countries' frontiers in the 1970s, i.e., evidence of a slowdown in the US and catching up by the other countries.<sup>20</sup>

<sup>20</sup>See Degasperis and Fredholm (2010) for a discussion of the US productivity slowdown.



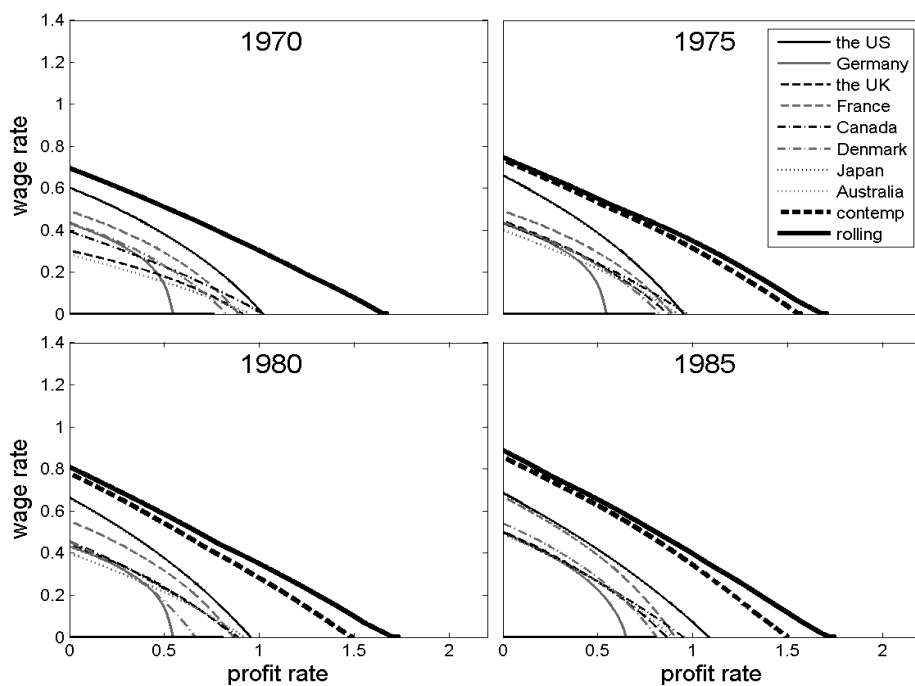


Fig. 6.2: *Wage-profit, contemporary, and intertemporal technological frontiers: 1970–1985*

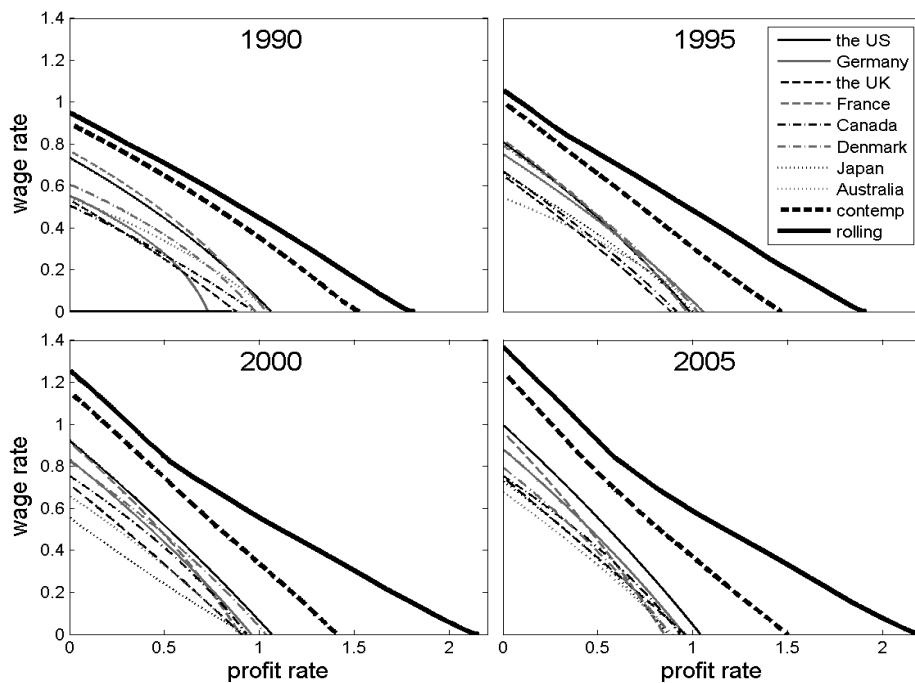


Fig. 6.3: *Wage-profit, contemporary, and intertemporal technological frontiers: 1990–2005*

See also Figure B.2 and B.3 in the statistical companion, where the frontiers are presented country-by-country.

## 6.2 The Velupillai-Fredholm-Zambelli index

The *Fredholm-Zambelli-Velupillai* index computed for the eight countries and the economy as a whole is collected in Figure 6.4 and Table 6.1. The global VFZ-index tells a story of stepwise technological development for the economy as a whole. During the 1970s the index stayed at a fairly stable level about 40–45 percent of the intertemporal maximum represented by the intertemporal technological frontier. From the mid 1980s to the mid 1990s

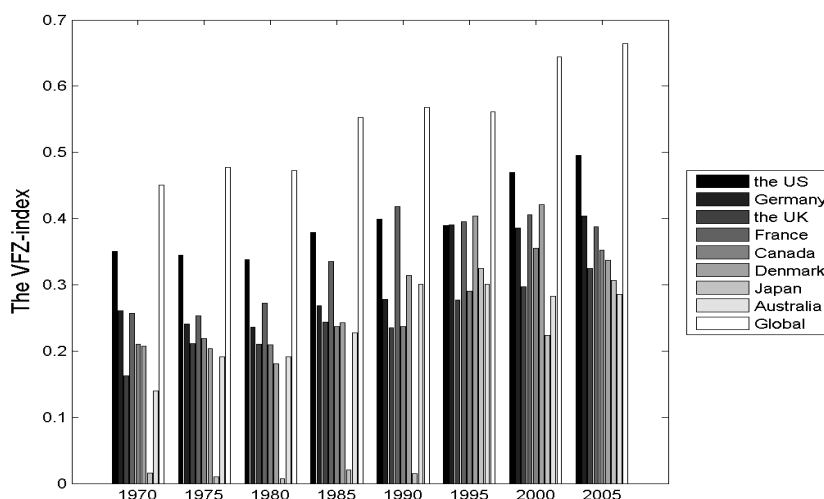


Fig. 6.4: *The Velupillai-Fredholm-Zambelli index*

the global productivity level stabilised at a new level about 55 percent, and finally in 2000 and 2005 reached a level close to 65 percent.<sup>21</sup>

It is surprising that over a period of 35 years the global VFZ-index has only increased from 0.45 to 0.66. This corresponds to a compounded growth rate at 1.1 percent per year, which is far less than the often reported 1.5–2 percent. Over a period of 35 years the difference between a growth rate of 1.1 and 2.0 percent corresponds to an increase of a factor 1.5 and 2, respectively.

For the single countries the difference between the level of 1970 and 2005 corresponds to a compounded growth rate of; the US 1.0, Germany 1.2, the UK 2.0, France 1.2, Canada 1.5, Denmark 1.4, Japan 8.1, and Australia 2.1 percent per year. As with the global index these growth rates are, except

<sup>21</sup>Of course much more could be said, if data had been available at a yearly basis, and the stepwise technological development must to some extent a consequence of the five year intervals.

for Japan, surprisingly small. Especially, the 1.0 percent annual growth for US.

If these results are—as implicitly claimed—more reliable than the usual indices of technological progress, then the results are indeed interesting. In particular, since technological progress determines the limit for a sustainable increase in the standard of living. Sustainable as oppose to the extreme increases in asset prices and consumption preceding the current economic turmoil. Reliable indices of technological change, determined from the phys-

	1970	1975	1980	1985	1990	1995	2000	2005
the US	0.35	0.34	0.34	0.38	0.40	0.39	0.40	0.50
Germany	0.26	0.24	0.24	0.27	0.28	0.39	0.39	0.40
the UK	0.16	0.21	0.21	0.24	0.24	0.28	0.30	0.32
France	0.26	0.25	0.27	0.34	0.42	0.40	0.41	0.39
Canada	0.21	0.22	0.21	0.24	0.24	0.29	0.36	0.35
Denmark	0.21	0.20	0.18	0.24	0.31	0.40	0.42	0.34
Japan	0.02	0.01	0.01	0.02	0.02	0.32	0.22	0.31
Australia	0.14	0.19	0.19	0.23	0.30	0.30	0.28	0.29
global	0.45	0.48	0.47	0.55	0.57	0.56	0.64	0.66

Table 6.1: *The Velupillai-Fredholm-Zambelli index*

ical flow among industries, could prove instrumental to better assess the technological constraints for our long-run standard of living.<sup>22</sup>

Apart from the differences in levels, the evolution in the global index is partly mimicked by the US index which also evolves in uneven steps. The difference between the US index and the global index increased slightly over the period considered with a large spike around 2000. It is here worth noting, that the global index from 1995 to 2000 increased from 56 to 64 percent while the US index only increased with one basis point. Hence, it is unlikely that it was the US that was driving the global development in the late 1990s. More will be said on this in Section 6.3, where the analysis is carried to the industry level.

Another interesting point is that the US is not facing an impending access to technology constraint. Naturally, this depends on the availability of the foreign production techniques. Some techniques might be country-specific, i.e, cannot be transferred; a great deal will probably not be 'public goods', but internal to multinational corporations, which at least limits the transferability; and some (if not most) production techniques require a great deal of human capital which in one way or another also must be transferred. In any case, we observe that the US from the 1980s has been approaching

<sup>22</sup>As oppose to some, however deflated, market price denominated proxy for (net) output per unit of labour. Not to mention indices build on an *ad hoc* and partly stochastic measure of fixed capital (aka the *perpetual inventory method*).

the intertemporal technological frontier, but also that there still potentially is a long way to go.<sup>23</sup>

The other countries, apart from Japan, show a more steady technological development, with a slow convergence towards both the US and the intertemporal frontier. In 1990, France actually reached the level of the US which it maintained until the US took off between 2000 and 2005. The same

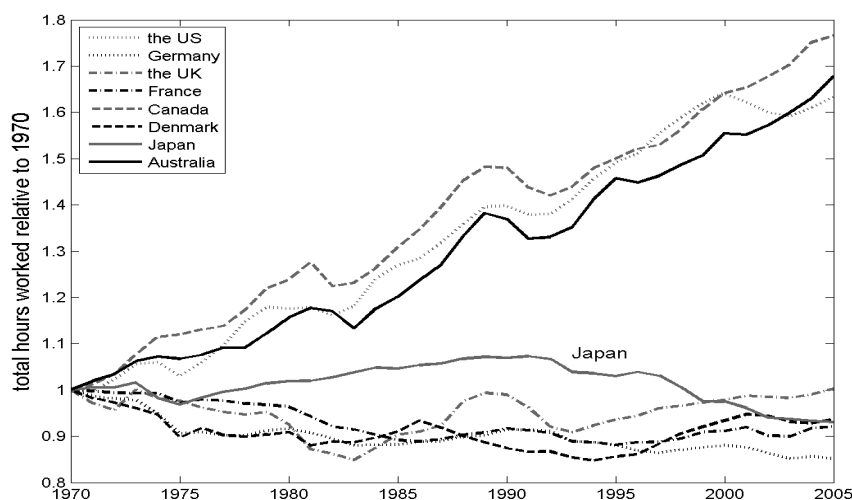


Fig. 6.5: Index of 'total hours worked', 1970 = index 1.  
(Source: Official OECD employment data)

goes for Germany and Denmark from 1995, while the UK and Australia remained behind.

Japan is showing an extraordinary development, until 1990 it is far behind all the other countries, but around 1995 Japan attained the level of the UK, Canada, and Australia. Without going into details, part of the explanation is probably found in the deregulations between 1990 and 1995 and subsequent drastic decrease in 'total hours worked' as shown in Figure 6.5.<sup>24</sup>

Figure 6.5 shows a very uneven development in the total hours worked. The US, Canada, and Australia show steady increases in the total hours worked (much of which is likely a consequence of immigration), while the European countries show a relatively stable or slightly decreasing development. Japan is the odd one out, it increased steadily during the 1970s and 1980s, where after it decreased with an average annual rate corresponding to about 0.8 percent between 1990 and 2005. Furthermore, Japan is the

<sup>23</sup>When the next set of OECD tables are published, will this increase the intertemporal frontier more than the US wage-profit frontier?

<sup>24</sup>For details on the deregulation initiatives in Japan in the 1990, see the homepage of The Japan Institute of Labour and Training, [www.jil.go.jp](http://www.jil.go.jp).

only country where there is no apparent periodic business cycle to be found in the employment data.<sup>25</sup>

Combining our surprising result of the 1.0 percent US year-to-year increase in our measure of technological progress between 1970 and 2005 with the aggregate employment data reported in Figure 6.5, points towards an economic development in the US driving more by increased work effort than increased productivity. Between 1970 and 2005 the US employment measured in hours increased by a factor 1.6 corresponding to a yearly increase of about 1.4 percent.

Going into the causes behind the convergence in productivity levels lie outside the scope of the paper. As concluded by Abramovitz (1986, p. 405):

differences among countries in productivity levels create a strong potentiality for subsequent convergence of levels, provided that countries have a "social capability" adequate to absorb more advanced technologies. [However,] the institutional and human capital components of social capability develop only slowly as education and organization respond to the requirements of technological opportunity and to experience in exploiting it.

Hence, we might be able to observe the actual effects from the processes of catching up, but the deep causes explaining country-specific differences must be understood in terms of social and institutional characteristics. However, it is possible to venture a step deeper into a descriptive study of the country-specific development, *viz.* the industry-level development.

### 6.3 Industry-level development

Figure 6.6 shows the (unweighted) average industry-level frequency of the single countries contribution to the contemporary and rolling technological frontiers.

Even though the US is considered the leading country, it is only in few cases the country that is contributing most to the technological frontiers (including the intertemporal which is the rightmost of the rolling frontiers). This indicates that the US in a few industries strongly dominates, i.e., all or most segments of the envelope include a particular US technique, and that these industries must play a vital role for the economy as a whole. By inspecting Figures B.4–B.26 for the single industries in the statistical companion, it is found that the US dominates in 'Construction'; 'Machinery and equipment, nec'; and 'Business activities (finance, real estate, and R&D)'.<sup>25</sup>

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<sup>25</sup>However, it must be noted that between 1990 and 1995 the input–output tables change from the ISIC 2 to the present ISIC 3 standard of accounting. Whether or not this greatly influence our results is *pro tempore* unknown.

Together with Germany the US also dominates in 'Manufacturing, nec'. Germany dominates in 'Electrical machinery and apparatus'; 'Transport equipment'; and 'Manufacturing nec; recycling (include Furniture)'. Canada in

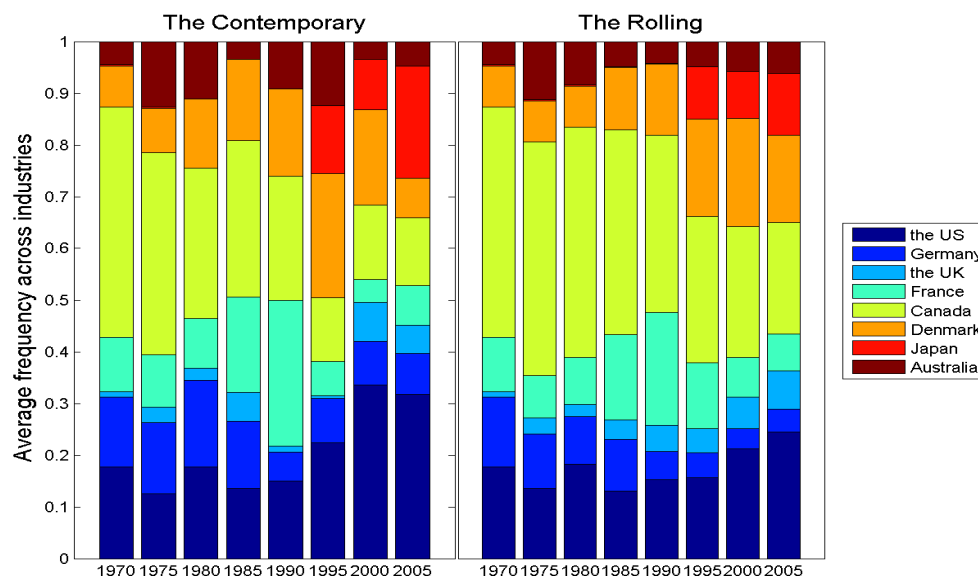


Fig. 6.6: *The composition of the contemporary and rolling technological frontiers*

'Other non-metallic mineral products'; 'Metals'; 'Fabricated metal products, except machinery and equipment'. Denmark, however insignificant on the world market, dominates in 'Mining and quarrying' and 'Food products, beverages, and tobacco'.

In many cases, about 30–40 percent of the production techniques entering the envelopes are Canadian. This is surprising given that Canada, measured by, e.g., the VFZ-index, does not rank as one of the most productive countries.

An important general point is that no country at a single point in time dominates the entire technological frontier (contemporary or rolling). Hence, all countries could, at any point in time, potentially gain from further global integration, either through increased trade or transfer of production techniques (including human capital).

The statistical companion contains detailed empirical evidence on the country/industry specific contributions to the different technological frontiers. From these results it is possible to go deeper into an analysis of the displacement of the production techniques over time. In particular, it would be interesting to study the difference between the displacement of techniques in the contemporary and rolling technological frontiers, since this can tell us to what extent new more productive techniques have been introduced as oppose to new combinations of old techniques of production.

## 6.4 The number of switch points

Since the switch points on the envelopes are independent of the choice of *numéraire* a large number of switch points would imply that our results would be relatively unaffected by the choice of *numéraire*.

For the contemporary frontiers the number of switch points starting with the 1970 frontier are 25, 22, 22, 21, 22, 25, 26, and 31, and for the rolling frontiers 25, 36, 44, 46, 42, 44, 53, and 49. It is assessed that the number of switch points in general is of a magnitude, that ensures a fairly *numéraire* independent envelope. The intuition behind this conclusion follows directly from the properties listed in Section 2. If the *numéraire* is changed the subsequent results must also change, but a strictly decreasing envelope, together with 20 to 50 fixed points, do not leave much room for disturbance.

In hindsight not surprising, is the fact that the number of switch points tends to increase with the number of techniques on which the envelope is computed. For the contemporary frontiers the number of unique systems is  $8^{23}$  and for the rolling it increase as  $8^{23}, 16^{23}, 24^{23}, \dots, 64^{23}$ .

For the record, no *reverse capital deepening* or *reswitching* are found on any of the technological frontiers computed for this study.<sup>26</sup>

## 7 Concluding Remarks

The value-added of this paper is two-fold. First, an algorithm, the Piece-meal algorithm, has been developed that is capable of computing actual technological frontiers from huge collections of production techniques. The algorithm computes the entire technological frontier, the envelope, without going through partial or stochastic *ad hoc* short cuts.

Second, by exploiting the power the Piecemeal algorithm three different versions of the technological frontier have been computed and analysed; the contemporary, the rolling, and the intertemporal. From these frontiers a set of indices, the VFZ-indices, has been extracted to provide global as well as country-specific condensed measures of technological progress.

The global VFZ-index, computed from the contemporary technological frontiers and intertemporal technological frontier, provides a measure of the technological progress for the global economy as a whole. The index based on the eight OECD countries tells a story of a stepwise technological development, with major jumps in the periods 1980–1985 and 1995–2000. But also a global development that shows a more moderate rate of productivity growth compared with the conventional stylized facts of a 1.5–2.0 percent year-to-year increase. The global VFZ-index increased from 0.45 in 1970 to 0.66 in 2005, i.e., an increase of a factor 1.5 against the factor 2 implied by a 2.0 percent per year compounded growth over 35 years.

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<sup>26</sup>See Han and Schefold (2006).

The VFZ-indices for the single countries, constructed from the individual countries' wage-profit frontiers and the intertemporal technological frontier, have provided new insight into the country-specific technological progress and convergence. Among the results are evidence of an economic development in the US driven at least as much by an increased work effort as an increased productivity.

Furthermore, by identifying which production techniques that enter the different technological frontiers, we have carried out a preliminary analysis of the country/industry specific contributions to the overall development. It has been shown that even though the US is the leading country, it is only in few cases the US that is contributing most to the technological frontiers. The few industries where the US strongly dominates on the technological frontiers are 'Construction'; 'Machinery and equipment, nec'; and 'Business activities (finance, real estate, and R&D)'. Moreover, we see that no country at any point in time dominates anything near an entire technological frontier, i.e., the potential gains from further global integration have not been exhausted for any country at any point in time.

Common for these frontiers and the indices based hereupon is a considerable resilience to the theoretical problems that hitherto have haunted the construction of index numbers and thereby any form of productivity accounting. In particular, the technological frontiers and the associated indices are; non-parametric and non-stochastic; the interdependence among industries is endogenously captured by changes in the prices of production; and will not change as a consequence of simple changes in the scale of production in the single industries.

While the envelope is *numéraire* dependent, the stability provided by the 20–50 switch points greatly limits the sensitivity to changes in the *numéraire*.

A huge work remains to be carried out going deeper into an analysis of the displacement of the production techniques on the contemporary and rolling technological frontiers.

## Acknowledgements

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## A Data Description

Table A.1 shows which OECD input–output tables that are available from the period 1970–2005. Tables are not necessarily available from the exact five

	1970	1975	1980	1985	1990	1995	2000	2005
the US	×	×	×	×	×	×	×	×
Germany			×	×	×	×	×	×
the UK	×		×	×	×	×	×	×
France		×	×	×	×	×	×	×
Canada	×	×	×	×	×	×	×	×
Denmark		×	×	×	×	×	×	×
Japan	×	×	×	×	×	×	×	×
Australia	×	×		×	×		×	×

Table A.1: *Available input–output tables*

year intervals, e.g., the US tables here labelled 1970 and 1975 are actually the 1972 and 1977 tables, respectively.<sup>27</sup>

The list below shows how the original tables have been aggregated down to the  $23 \times 23$  used in this study. The numbers in the brackets refer to their respective ISIC 2 and ISIC 3 classification, *viz.* {[ISIC 3],[ISIC 2]}.

1. Agriculture, hunting, forestry, and fishing {[1],[1]}
2. Mining and quarrying {[2–3],[2]}
3. Food products, beverages, and tobacco {[4],[3]}
4. Textiles, textile products, leather, and footwear {[5],[4]}
5. Wood and products of wood and cork {[6],[5]}
6. Pulp, paper, paper products, printing, and publishing {[7],[6]}
7. Coke, refined petroleum products, and nuclear fuel {[8],[9]}
8. Chemicals {[9–10],[7–8]}
9. Rubber and plastics products {[11],[10]}
10. Other non-metallic mineral products {[12],[11]}
11. Metals {[13–14],[12–13]}
12. Fabricated metal products, except machinery and equipment {[15],[14]}
13. Machinery and equipment, nec {[16],[15]}
14. Electrical machinery and apparatus {[17–20],[16–18]}
15. Transport equipment {[21–25],[19–22]}
16. Manufacturing nec; recycling (include furniture) {[25],[23–24]}
17. Production and distribution of electricity, gas, and water {[26–29],[25]}
18. Construction {[30],[26]}
19. Wholesale and retail trade {[31],[27]}
20. Service activities (transport, hotels and restaurants) {[32–36],[28–29]}
21. Post and telecommunications {[37],[30]}
22. Business activities (finance, real estate, and R&D) {[38–43],[31–32]}
23. Public administration, education and health {[44–48],[33–35]}

<sup>27</sup>The full list of available tables are: the US {1972, 1977, 1982, 1985, 1990, 1995, 2000, 2005}, Germany {1978, 1986, 1990, 1995, 2000, 2005}, the UK {1968, 1979, 1984, 1990, 1995, 2000, 2003}, France {1977, 1980, 1985, 1990, 1995, 2000, 2005}, Canada {1971, 1976, 1981, 1986, 1990, 1995, 2000}, Denmark {1977, 1980, 1985, 1990, 1995, 2000, 2004}, Japan {1970, 1975, 1980, 1985, 1990, 1995, 2000, 2005}, and Australia {1968, 1974, 1986, 1989, 1999, 2005}.

Table A.2–A.9 show the macro-industry deflators used to convert the tables denominated in current prices (possible domestic currency) into tables denominated in fixed US 2000 prices. The transition to the EURO has been taken into account in the tables below. The deflators are computed as the ratio between GDP in constant prices and GDP in current prices and when necessary also divided by the dollar-domestic currency exchange rate ([www.sourceoecd.org](http://www.sourceoecd.org)). The missing values marked with a ‘–’ correspond with the unavailable OECD tables.

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	1.23	0.84	0.71	0.74	0.65	0.67	1.00	0.91
II Industry including energy	2.83	1.89	1.14	1.09	1.02	0.96	1.00	0.93
III Construction	4.97	3.28	2.03	1.79	1.43	1.26	1.00	0.71
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	2.13	1.53	1.12	1.05	0.97	0.87	1.00	0.98
V Financial intermediation; real estate, business activities	4.79	3.45	2.34	1.88	1.47	1.23	1.00	0.88
VI Other service activities	5.15	3.57	2.37	1.98	1.53	1.24	1.00	0.82

Table A.2: *Macro-industry deflators for the US 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	–	–	0.53	0.60	0.64	1.00	1.03	1.39
II Industry including energy	–	–	0.81	0.60	0.57	1.02	1.03	1.01
III Construction	–	–	1.15	0.84	0.71	1.02	1.03	0.98
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	–	–	0.78	0.63	0.60	1.01	1.03	1.03
V Financial intermediation; real estate, business activities	–	–	1.01	0.69	0.61	1.02	1.03	0.94
VI Other service activities	–	–	0.98	0.75	0.67	1.08	1.03	0.97

Table A.3: *Macro-industry deflators for Germany 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	5.97	–	2.26	2.43	1.40	1.06	1.57	1.37
II Industry including energy	11.3	–	3.65	2.21	1.86	1.64	1.57	1.57
III Construction	18.4	–	4.44	3.14	2.15	2.00	1.57	1.35
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	13.2	–	4.41	2.87	1.91	1.72	1.57	1.51
V Financial intermediation; real estate, business activities	14.8	–	4.50	2.87	2.01	1.71	1.57	1.34
VI Other service activities	21.5	–	5.81	3.84	2.33	1.94	1.57	1.38

Table A.4: *Macro-industry deflators for the UK 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	—	0.25	0.23	0.17	0.15	0.98	1.06	1.09
II Industry including energy	—	0.34	0.26	0.17	0.16	1.04	1.06	1.12
III Construction	—	0.56	0.38	0.26	0.21	1.19	1.06	0.86
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	—	0.39	0.30	0.20	0.17	1.05	1.06	0.95
V Financial intermediation; real estate, business activities	—	0.53	0.42	0.29	0.22	1.22	1.06	0.94
VI Other service activities	—	0.58	0.41	0.26	0.21	1.18	1.06	0.91

Table A.5: *Macro-industry deflators for France 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	3.53	1.63	1.00	0.96	0.95	0.78	0.81	—
II Industry including energy	3.95	2.37	1.38	1.18	1.04	0.94	0.81	—
III Construction	3.33	1.89	1.31	1.18	0.92	0.87	0.81	—
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	2.63	1.76	1.20	0.96	0.85	0.82	0.81	—
V Financial intermediation; real estate, business activities	4.26	2.50	1.58	1.17	0.94	0.87	0.81	—
VI Other service activities	4.40	2.58	1.71	1.29	1.02	0.90	0.81	—

Table A.6: *Macro-industry deflators for Canada 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	—	.104	.094	.074	.078	.101	.119	.143
II Industry including energy	—	.305	.246	.178	.149	.140	.119	.113
III Construction	—	.360	.290	.222	.173	.147	.119	.104
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	—	.244	.210	.134	.120	.117	.119	.109
V Financial intermediation; real estate, business activities	—	.329	.253	.179	.150	.128	.119	.109
VI Other service activities	—	.360	.285	.197	.151	.136	.119	.103

Table A.7: *Macro-industry deflators for Denmark 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	.0135	.0083	.0067	.0063	.0060	.0056	.0065	.0071
II Industry including energy	.0111	.0078	.0063	.0058	.0058	.0059	.0065	.0074
III Construction	.0313	.0152	.0104	.0087	.0073	.0066	.0065	.0066
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	.0123	.0082	.0067	.0062	.0062	.0061	.0065	.0068
V Financial intermediation; real estate, business activities	.0177	.0119	.0091	.0078	.0069	.0064	.0065	.0068
VI Other service activities	.0288	.0137	.0104	.0086	.0075	.0066	.0065	.0067

Table A.8: *Macro-industry deflators for Japan 1970–2005*

	1970	1975	1980	1985	1990	1995	2000	2005
I Agriculture, hunting and forestry; fishing	3.32	2.09	—	0.98	0.73	—	0.88	0.75
II Industry including energy	3.78	2.76	—	0.94	0.88	—	0.80	0.55
III Construction	4.45	2.72	—	1.05	0.88	—	0.77	0.65
IV Wholesale and retail trade, repairs; hotels and restaurants; transport	5.35	3.42	—	1.02	0.84	—	0.77	0.69
V Financial intermediation; real estate, business activities	7.30	4.98	—	1.34	0.99	—	0.80	0.65
VI Other service activities	5.04	2.96	—	1.20	1.01	—	0.79	0.61

Table A.9: *Macro-industry deflators for Australia 1970–2005*

## B Additional Results

### B.1 The Wage-profit and Intertemporal Technological Frontiers

Figure B.1 shows the wage-profit frontiers forming the intertemporal technological frontier, and Figure B.2 and B.3 show the wage-profit frontiers for the individual countries together with the intertemporal technological frontier.

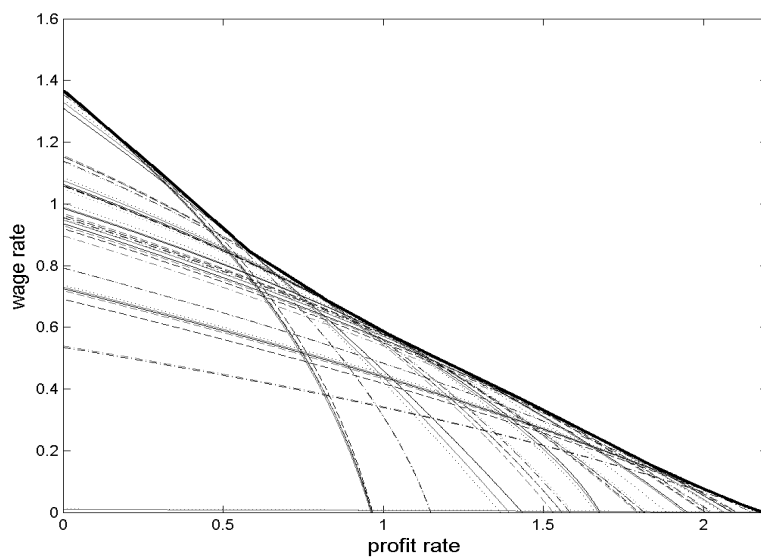


Fig. B.1: *The intertemporal technological frontier*

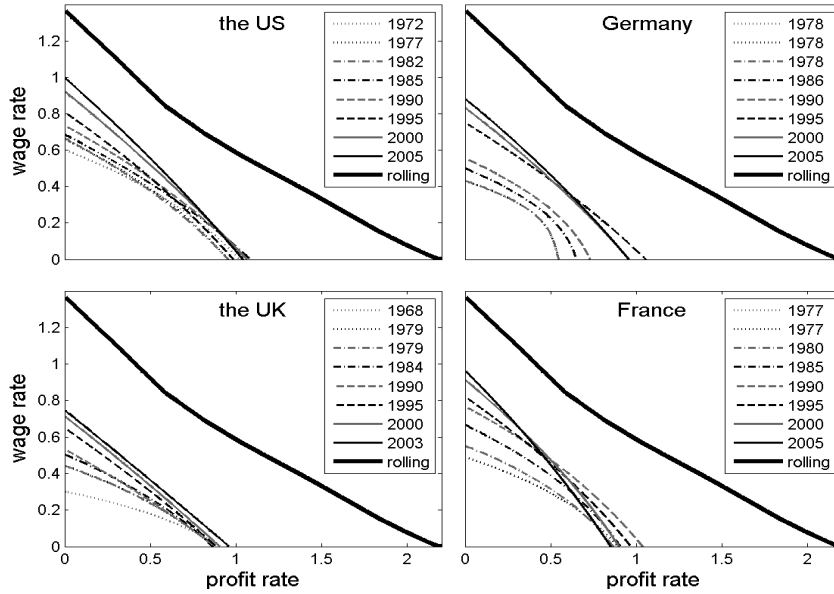


Fig. B.2: *Wage-profit frontiers and the intertemporal technological frontier: the US, Germany, the UK, and France*

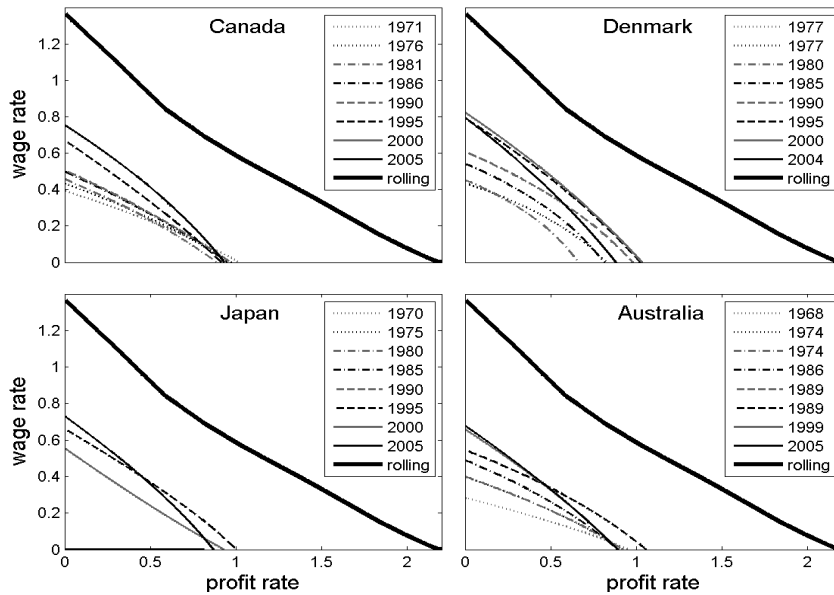


Fig. B.3: *Wage-profit frontiers and the intertemporal technological frontier: Canada, Denmark, Japan, and Australia*

## B.2 Industrylevel Frequency of the Single Countries Contribution to the Contemporary Technological Frontiers

The following 23 figures show the industry-level frequency of the single countries contribution to the contemporary and rolling technological frontiers.

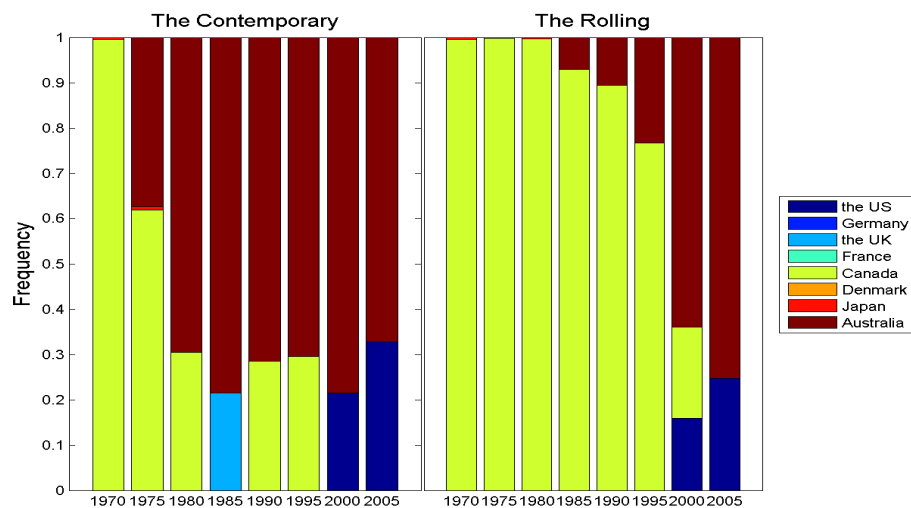


Fig. B.4: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Agriculture, hunting, forestry, and fishing*

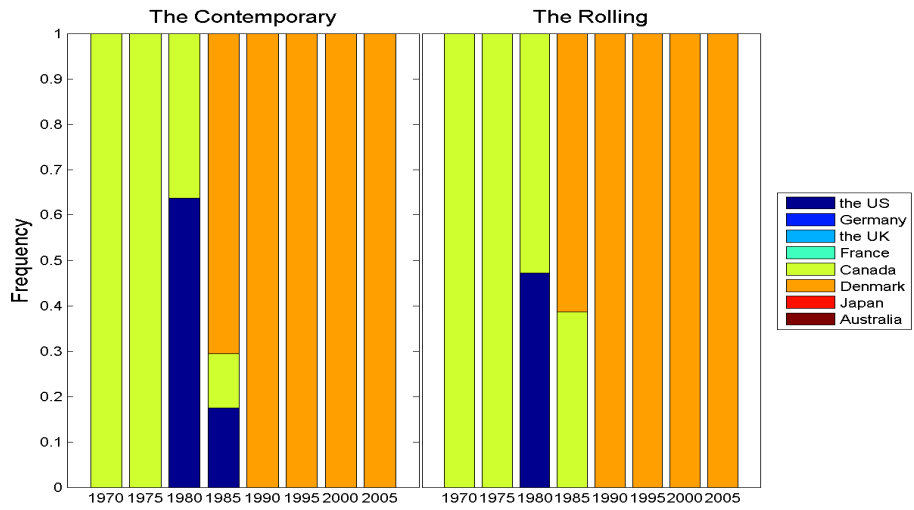


Fig. B.5: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Mining and quarrying

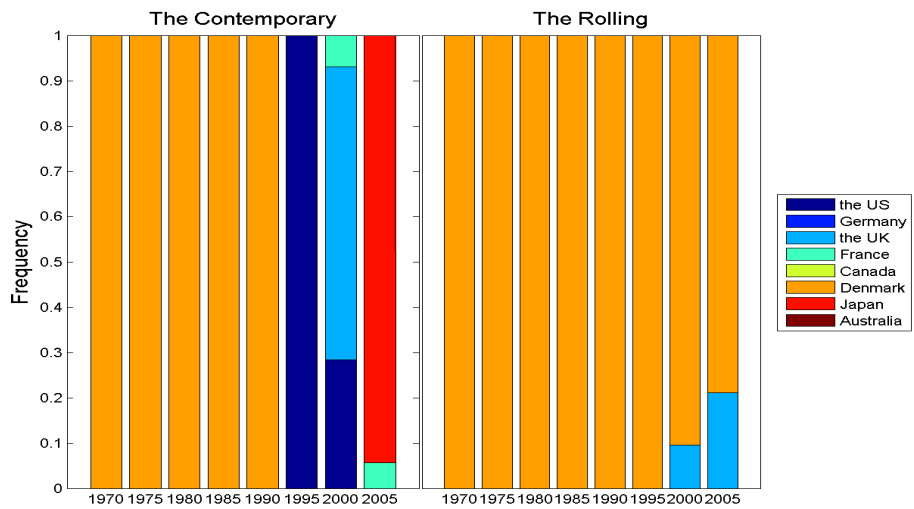


Fig. B.6: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Food products, beverages, and tobacco



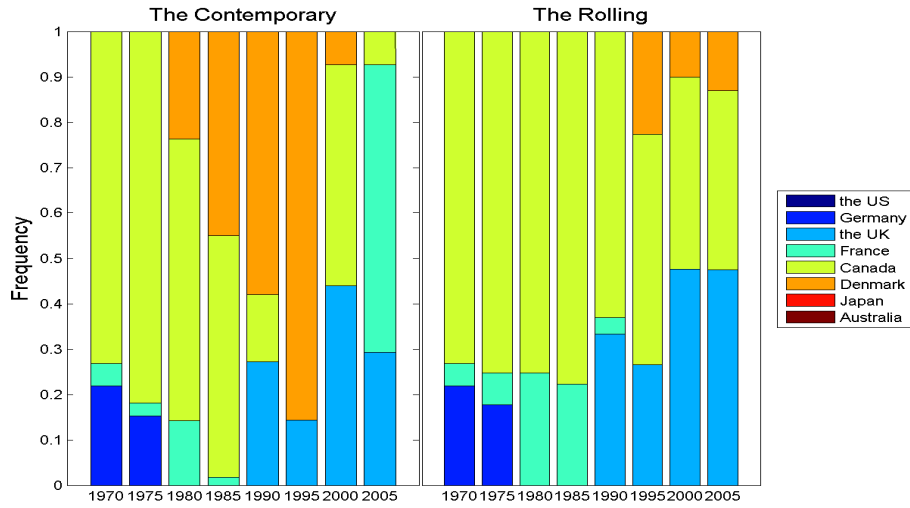


Fig. B.7: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Textiles, textile products, leather, and footwear*

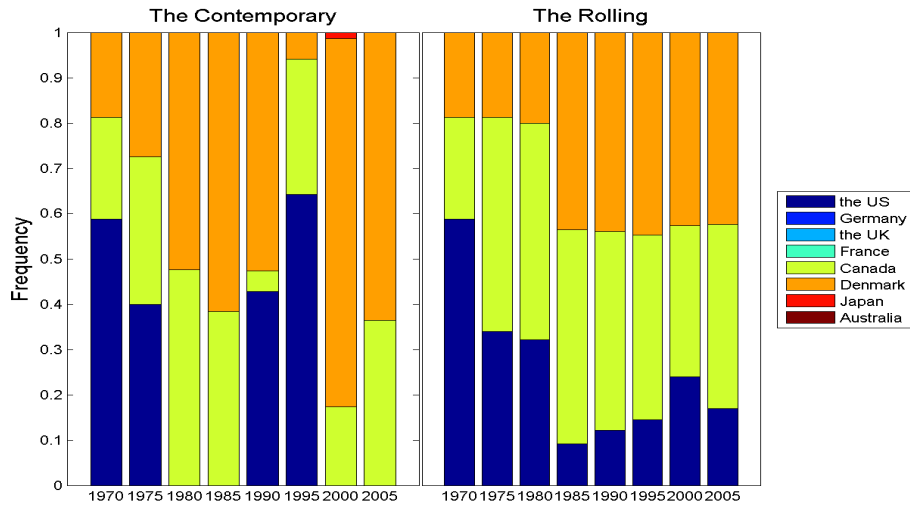


Fig. B.8: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Wood and products of wood and cork*

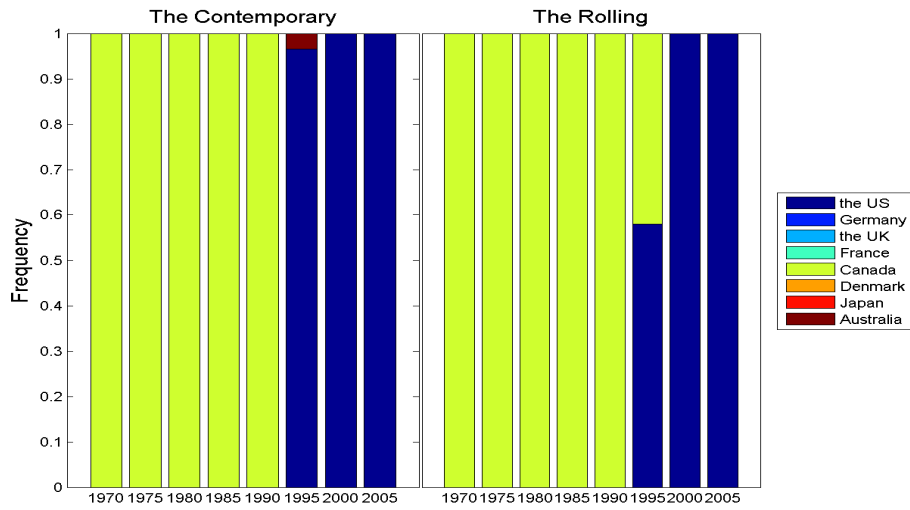


Fig. B.9: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Pulp, paper, paper products, printing, and publishing

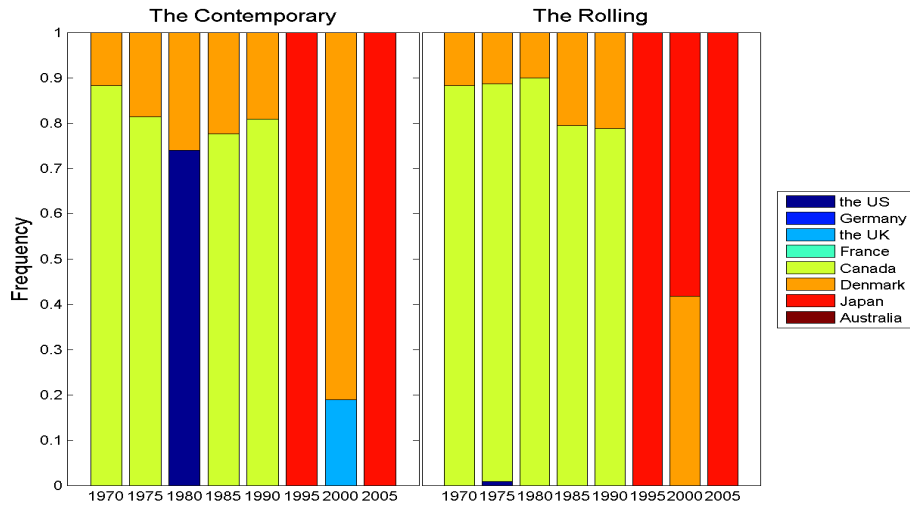


Fig. B.10: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Coke, refined petroleum products, and nuclear fuel

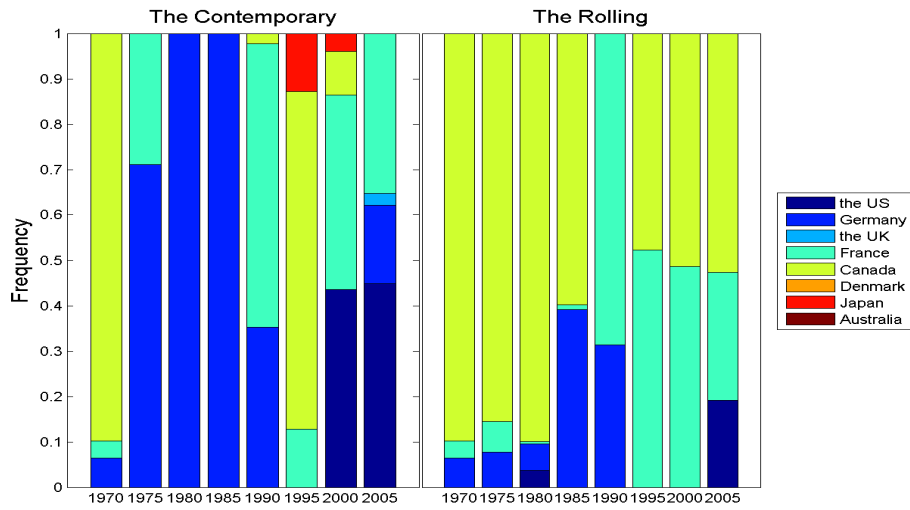


Fig. B.11: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Chemicals

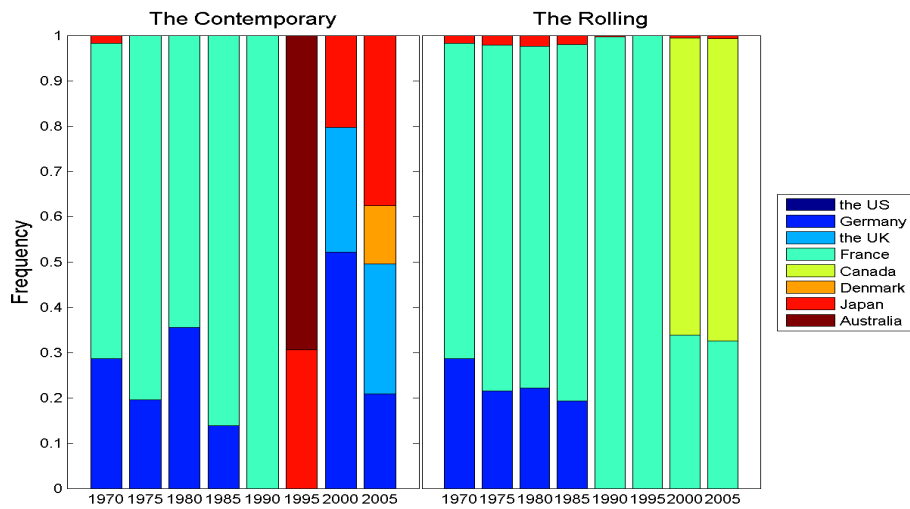


Fig. B.12: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Rubber and plastics products

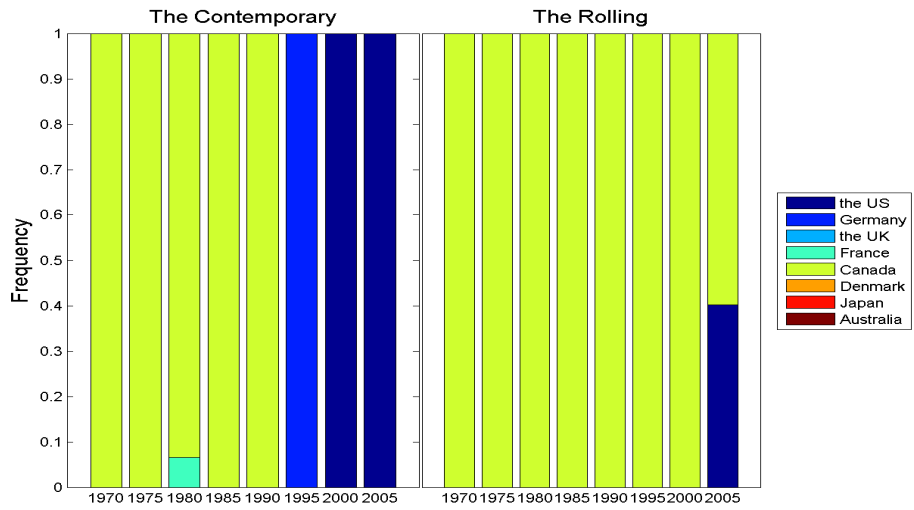


Fig. B.13: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Other non-metallic mineral products

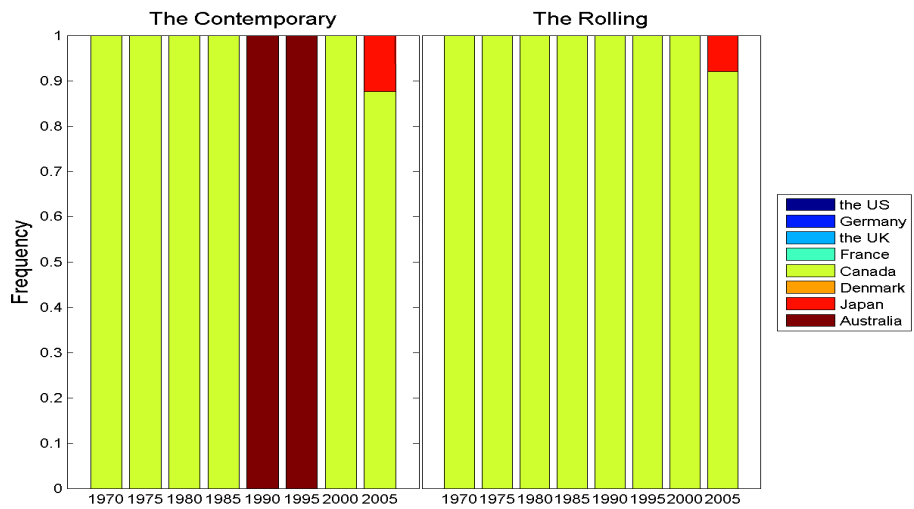


Fig. B.14: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Metals

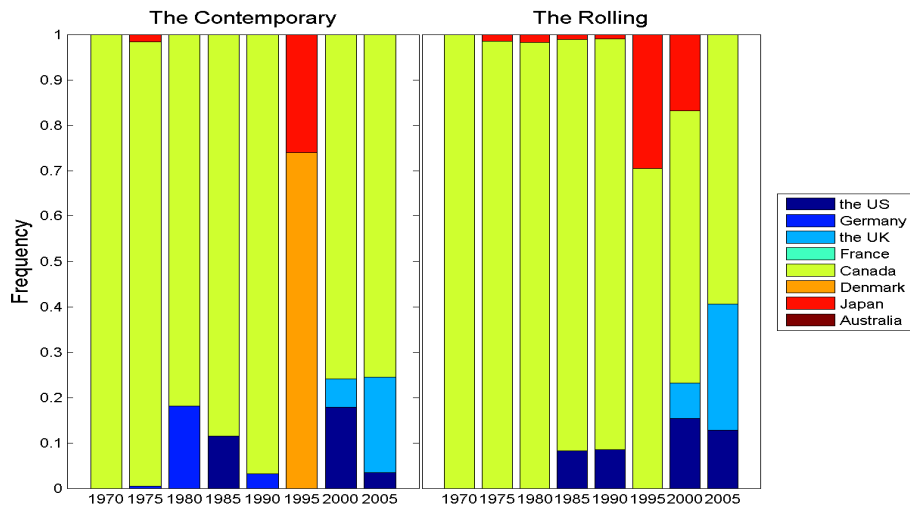


Fig. B.15: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Fabricated metal products, except machinery and equipment

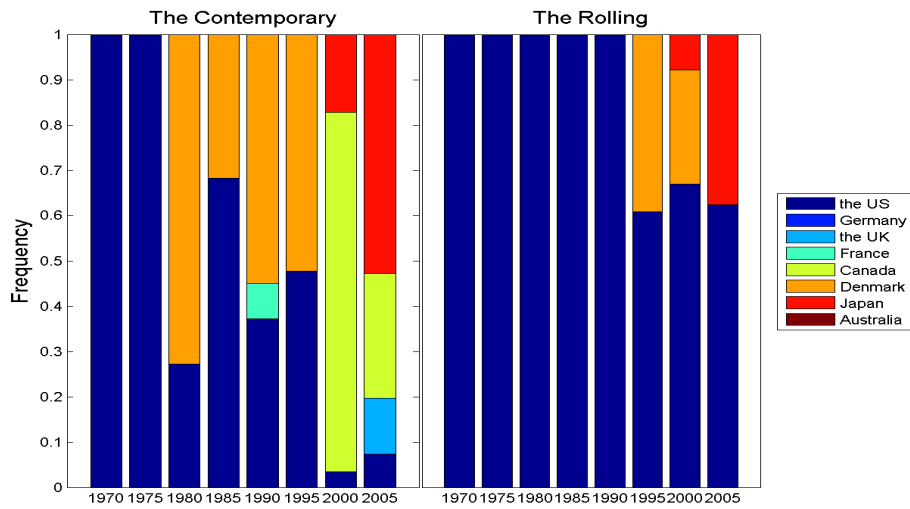


Fig. B.16: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Machinery and equipment, nec

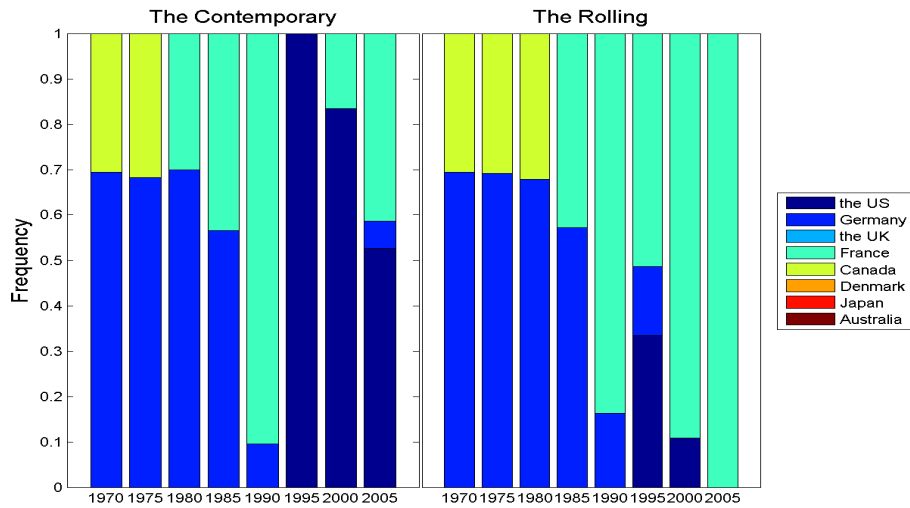


Fig. B.17: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Electrical machinery and apparatus

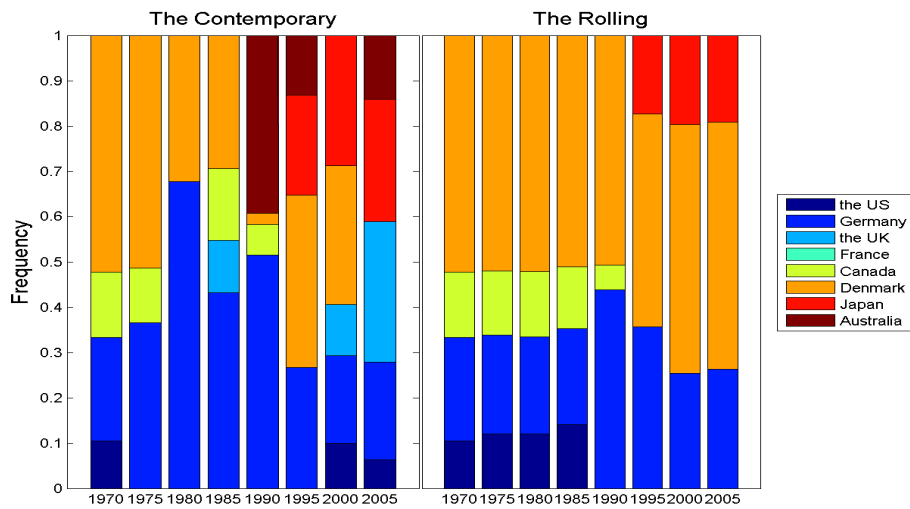


Fig. B.18: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Transport equipment

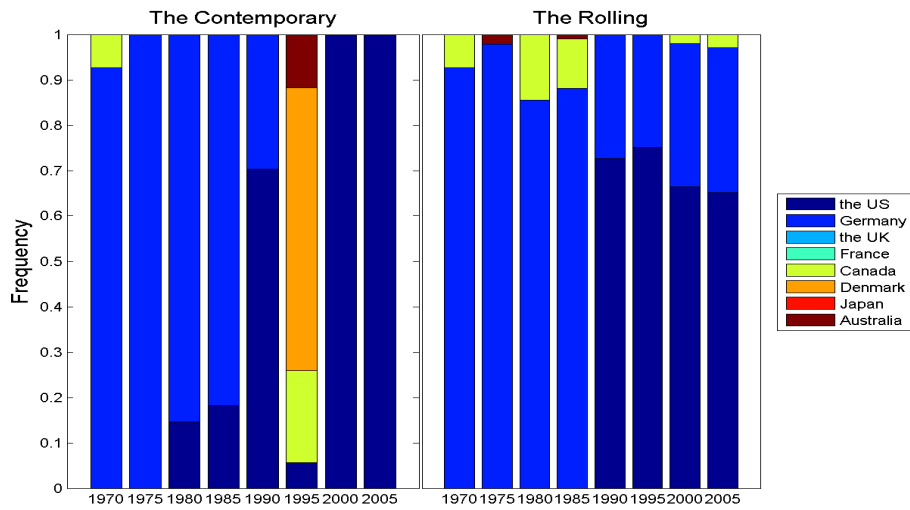


Fig. B.19: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Manufacturing nec; recycling (include Furniture)

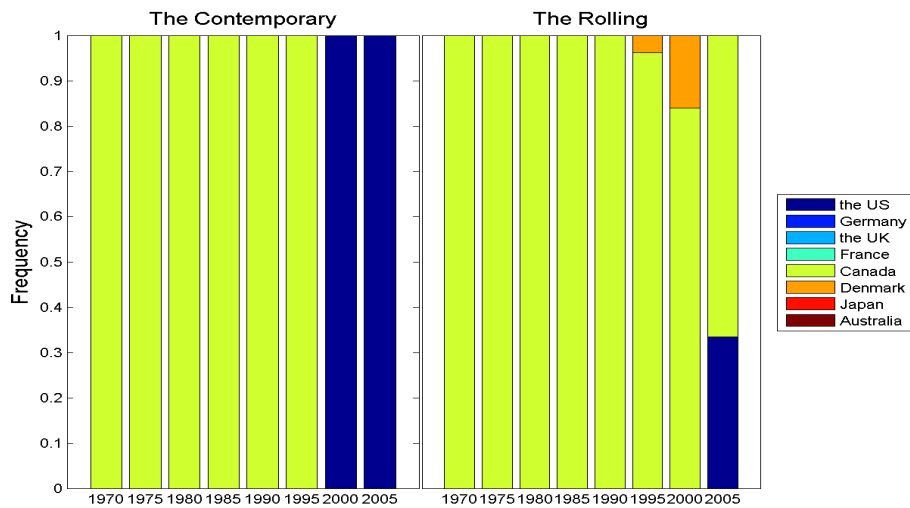


Fig. B.20: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Production and distribution of electricity, gas, and water

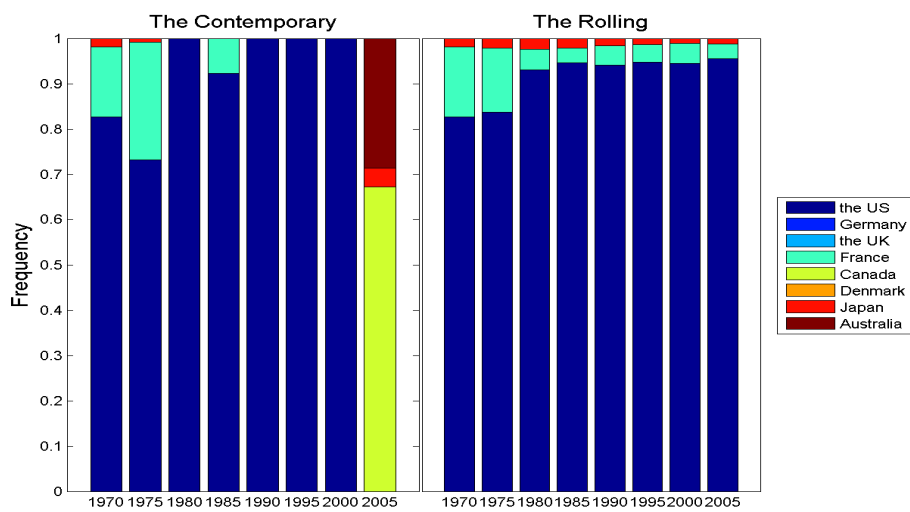


Fig. B.21: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Construction*



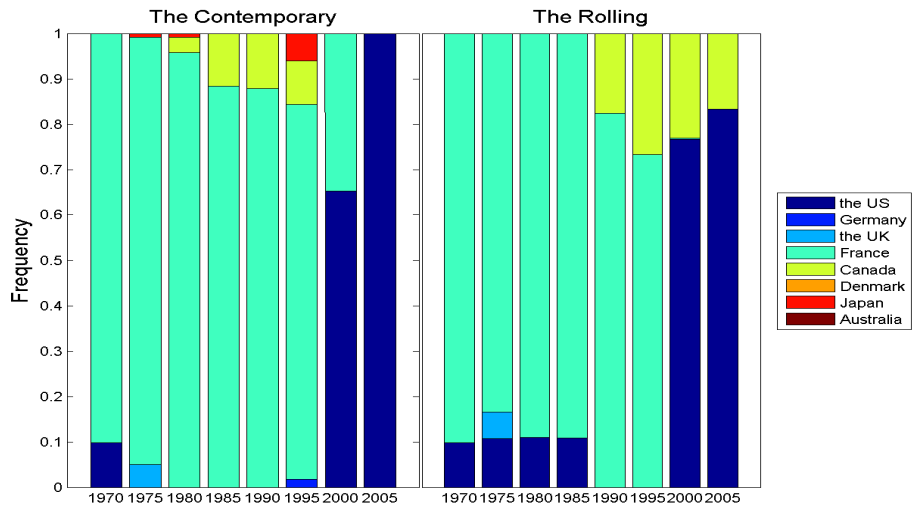


Fig. B.22: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Wholesale and retail trade

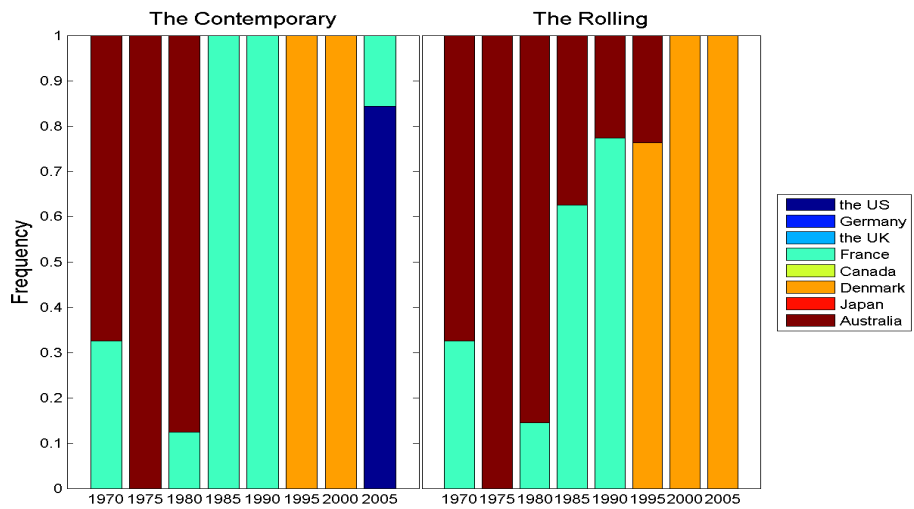


Fig. B.23: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Service activities (transport, hotels and restaurants)

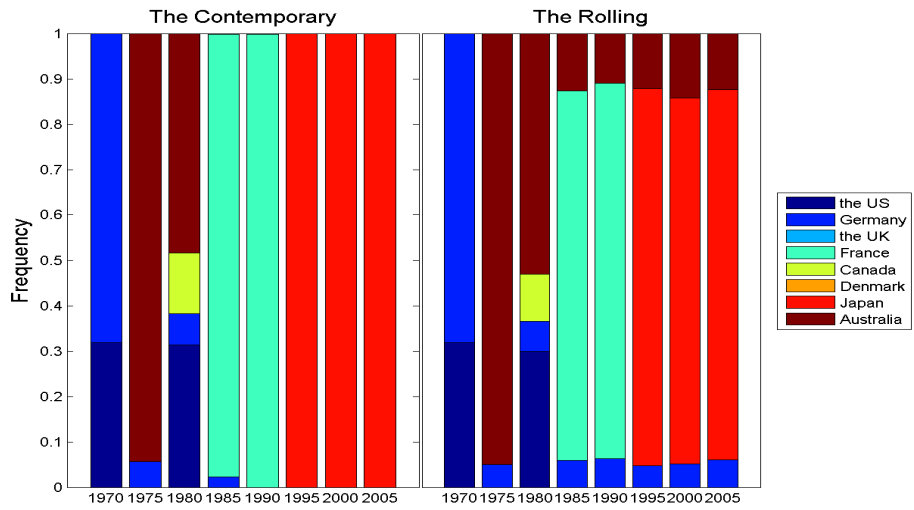


Fig. B.24: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Post and telecommunications

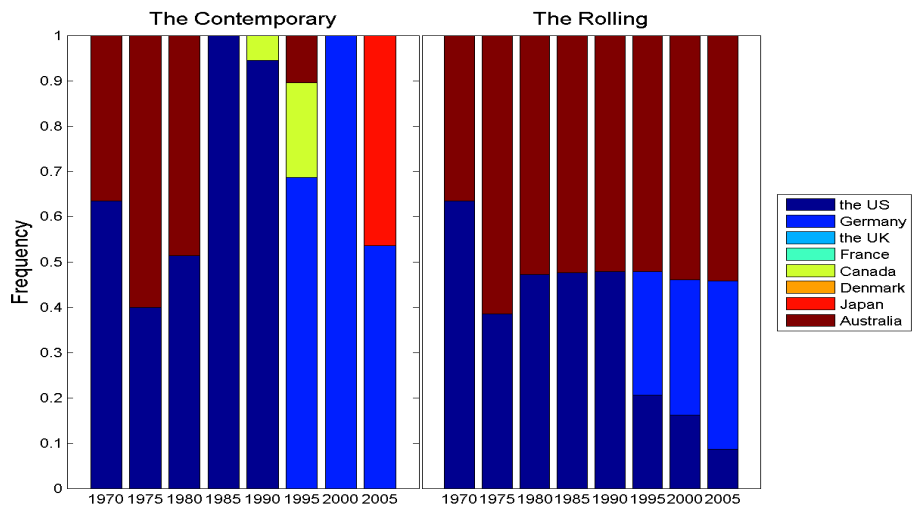


Fig. B.25: Countries and industry specific contributions to the contemporary and rolling technological frontiers, Business activities (finance, real estate, and R&D)

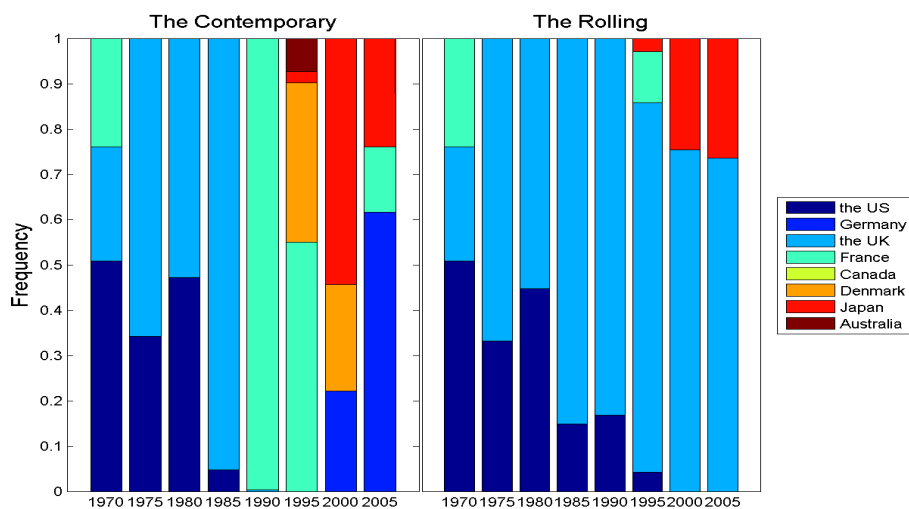


Fig. B.26: *Countries and industry specific contributions to the contemporary and rolling technological frontiers, Public administration, education and health*

### B.3 Country Specific Contributions to the Contemporary and Rolling Technological Frontiers

The following 8 figures show the country specific contributions to the contemporary and rolling technological frontiers. The maximum value is equal to the number of industries, 23, and would imply that the given country's wage-profit frontier coincided with the technological frontier.

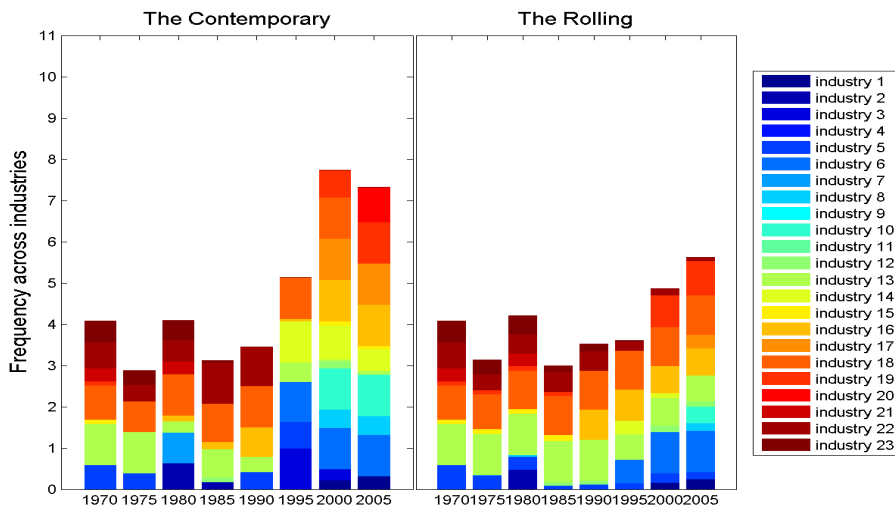


Fig. B.27: *Country specific contributions to the contemporary technological frontiers, the US*

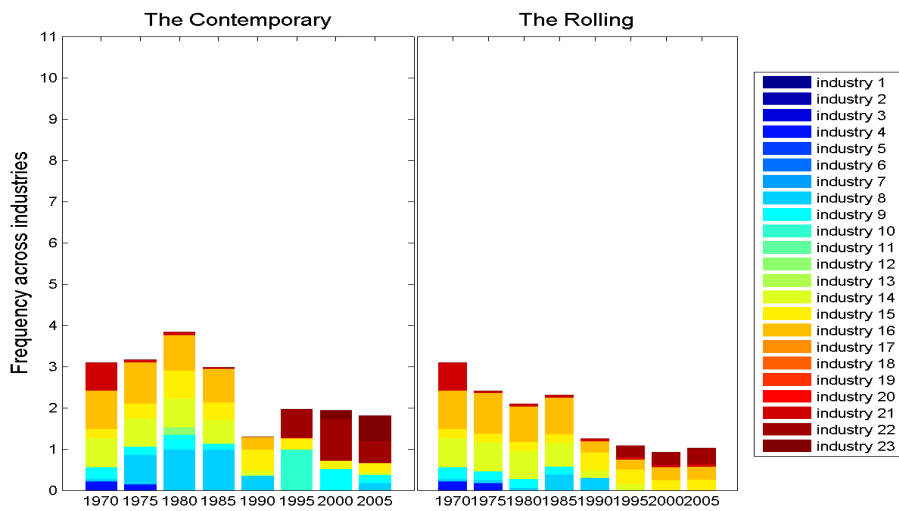


Fig. B.28: *Country specific contributions to the contemporary technological frontiers, Germany*

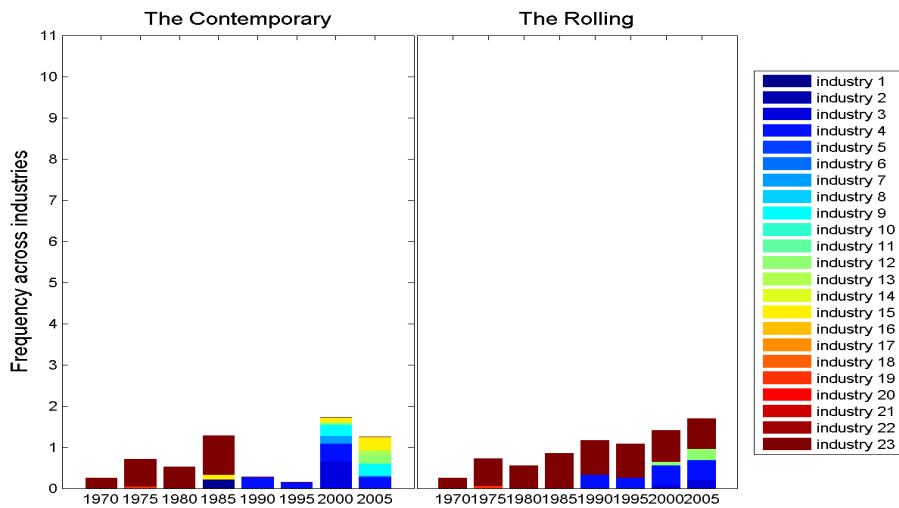


Fig. B.29: *Country specific contributions to the contemporary technological frontiers, the UK*

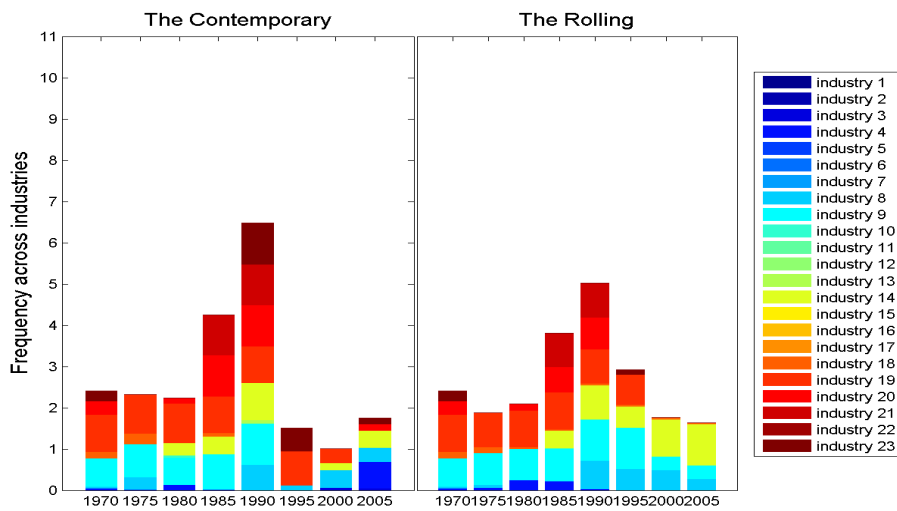


Fig. B.30: *Country specific contributions to the contemporary technological frontiers, France*

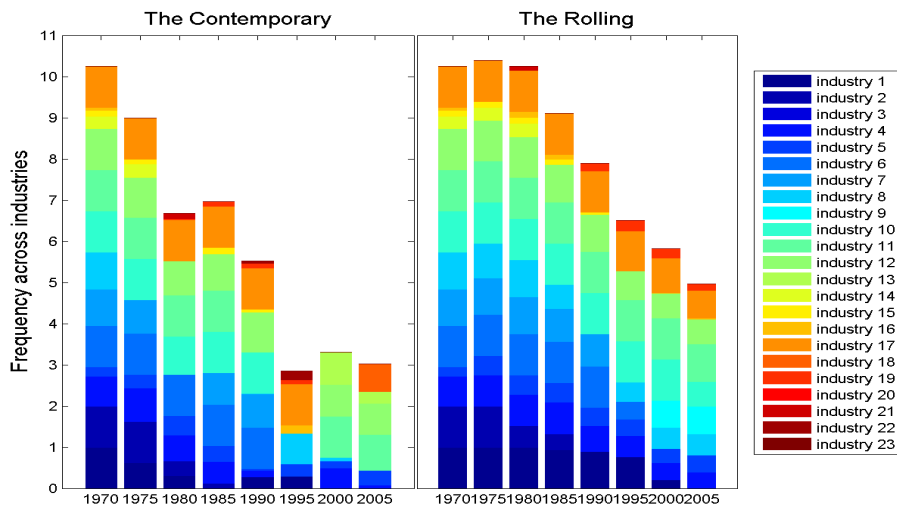


Fig. B.31: *Country specific contributions to the contemporary technological frontiers, Canada*

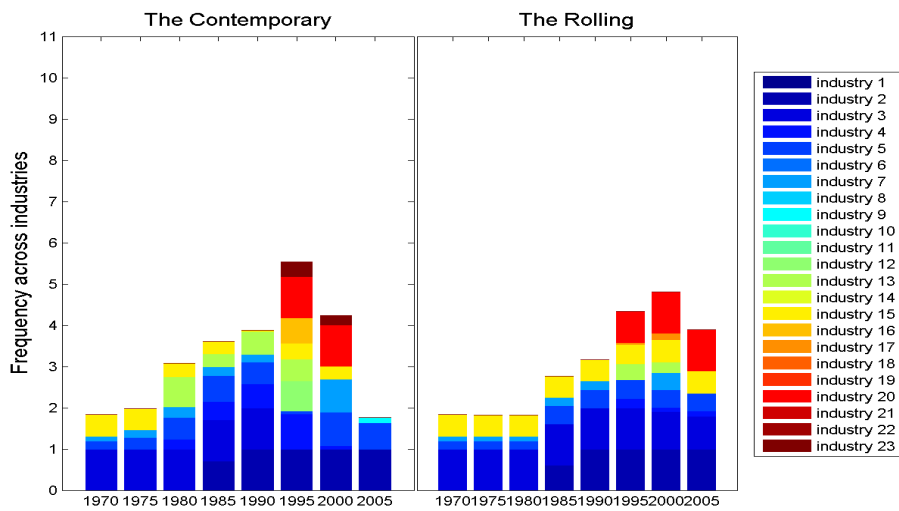


Fig. B.32: *Country specific contributions to the contemporary technological frontiers, Denmark*

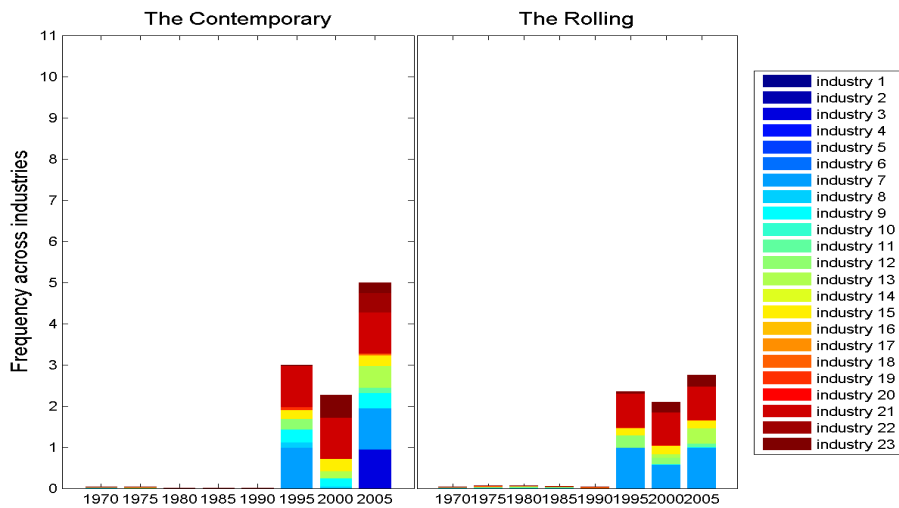


Fig. B.33: *Country specific contributions to the contemporary technological frontiers, Japan*

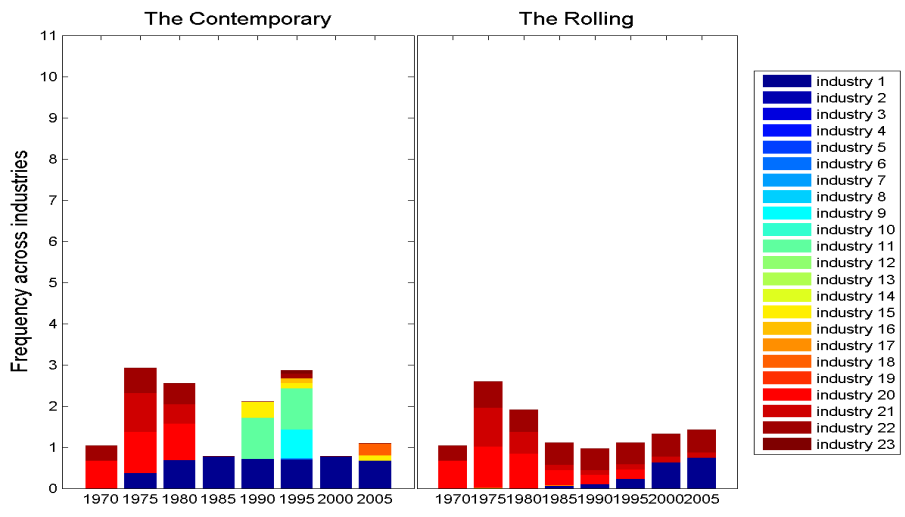


Fig. B.34: *Country specific contributions to the contemporary technological frontiers, Australia*