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Azimuth Spread Estimation for Slightly Distributed Scatterers Using the Generalized Array Manifold Model

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***Abstract*— In this paper, an azimuth spread estimator (ASE) for slightly distributed scatters (SDSs) is derived based on the generalized-array-manifold (GAM) approximation model [1]. To improve the performance of this estimator we propose an array size adaptation (ASA) technique that adjusts the array aperture selectively for each SDS by modifying the number of antennas. This technique is applied to uniform linear arrays but can be extended to two or three dimensional arrays. Simulation results demonstrate the improvement achieved with the combined ASA-ASE scheme. In particular, it is shown that this scheme outperforms the conventional Spread-ESPRIT technique.**

I. Introduction

In propagation environments, situations frequently occur where scatterers have a certain geometrical extent which is small in the view of the receiver (Rx) or local scattering exists around a transmitter (Tx) located far away from the Rx. In both cases, the received signal contributed by each of these scatterers or clusters of local scatterers can be conceived as the sum of the contributions of multiple sub-scatterers with slightly different azimuths of arrival (AoAs) where only horizontal propagation is considered. We refer to such scatterers or clusters of local scatterers as slightly distributed scatterers (SDSs) [2], [3] and [4]. The signal contribution of an SDS can be described by the nominal AoA (NAoA) and the azimuth spread (AS) of the SDS.

Recently, different techniques for estimation of nominal azimuth and AS of SDSs have been proposed. These techniques can be grouped into two categories: i) methods based on post-processing of the parameter estimates of specular scatterers (SSs) and ii) standard estimation techniques using approximation models for the SDSs.

Category i) includes the methods proposed in [5], [6] and [7]. These methods rely on visual inspection or grouping algorithms for identification of the SDSs based on parameter estimates of SSs. They are affected by the influence of subjective grouping of these estimates. Furthermore, the heavy-tailed distribution of azimuth estimates derived based on the SS model [2] strongly affects the accuracy of the AS estimates.

Category ii) includes the Spread-F technique derived based on a two-ray model [3], the COMET-EXIP [8] and the SIOD approaches [9] using a stochastic distribution model, as well as the subspace-based methods [1] and the SAGE algorithms [10] [11] derived based on the Generalized Array Manifold (GAM) model proposed in [1]. The Spread-F technique is less complex than the COMET-EXIP and SIOD methods. According to [3] the Spread-F technique requires a pre-generated look-up table which compensates for the inherent bias of the AS estimates. In addition, the Akaike Information Criteria (AIC) [12] or the Minimum Description Length (MDL) method [13] need to be applied before using the Spread-F technique in order to estimate the number of SSs. The Spread-F technique is applicable when this number is twice the number of SDSs. However this condition is only satisfied when the AS and the spacing between the nominal azimuths of SDSs are larger than certain values. For instance, simulation studies show that when an 8-element uniform linear array (ULA) with half-a-wavelength spaced elements is used, the AS of the SDSs must be larger than 3° and their nominal azimuth spacing must be larger than 20° in order for the Spread-F technique to be applicable. When the AS and the nominal azimuth spacing are less than these values, the Spread-F technique is inapplicable. The methods based on the GAM model proposed in [1], [10] and [11] are mainly for nominal direction estimation. In this paper we propose an AS estimator (ASE) based on the estimates of the parameters in the GAM model. Furthermore, to improve the performance of this estimator a technique is introduced which selectively modifies the aperture of the antenna array by adjusting the number of elements in the As estimation for each SDS. This technique is applied for ULAs but can be easily extended for use with two or three dimensional arrays.

The organization of the paper is as follows. Section II describes the signal model. Section III and IV introduce respectively the ASE and the array size adaptation (ASA) technique. Section V reports the simulation results. Concluding remarks are addressed in Section VI.

II. Signal model

In a propagation scenario with a single SDS, the output signal of a M-element Rx array can be viewed as composed of the contributions of multiple sub-scatterers distributed with respect to the azimuth of arrival (AoA):

$$\boldsymbol{y}(t) \;=\; [\sum_{\ell=1}^{L} a_\ell(t)\boldsymbol{c}(\bar{\phi}+\tilde{\phi}_\ell)]\cdot s(t)+\boldsymbol{w}(t), \qquad t=t_1,\ldots,t_N. \tag{1}$$

The components of the M-dimensional (M-D) complex vector $\boldsymbol{y}(t)$ denote the M output signals of the Rx array at time t, $s(t)$ represents the complex envelope of the transmitted signal and the noise vector $\boldsymbol{w}(t)$ is a spatially and temporally white M-D Gaussian process with component variance σ_w^2. We assume that totally N observation samples are collected at time instances t_n, $n=1,\ldots,N$. Moreover, the total number of sub-scatterers equals L and

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the complex weight of the signal contribution from the ℓth sub-scatterer is denoted by $a_\ell(t)$. The AoA of the ℓth sub-scatterer is decomposed as the sum of the NAoA $\bar{\phi}$ of the SDS and a small deviation $\tilde{\phi}_\ell$ from $\bar{\phi}$. Finally, $\boldsymbol{c}(\cdot) = [c_1(\cdot), \ldots, c_M(\cdot)]^{\mathrm{T}}$ with $[\cdot]^{\mathrm{T}}$ denoting transposition, is the array response. We assume that $\boldsymbol{c}(\phi)$ is such that the space spanned by the L vectors in the sum in (1) has dimension M with probability one. This supposes that $L \geq M$. Usually $L \gg M$. Additionally we assume that $s(t)$ is known to the Rx. Without loss of generality, $s(t) = 1$.

The vector-valued function $\boldsymbol{c}(\phi)$ in (1) can be approximated by its first-order Taylor series expansion at $\bar{\phi}$. Inserting the first-order Taylor approximation for each $\boldsymbol{c}(\bar{\phi} + \tilde{\phi}_\ell)$ in (1) yields the so-called GAM model [1]

$$\begin{aligned}\boldsymbol{y}(t) &= \sum_{\ell=1}^{L} a_\ell(t)[\boldsymbol{c}(\bar{\phi}) + \tilde{\phi}_\ell \boldsymbol{c}'(\bar{\phi})] + \boldsymbol{w}(t), \\ &= \alpha(t)\boldsymbol{c}(\bar{\phi}) + \beta(t)\boldsymbol{c}'(\bar{\phi}) + \boldsymbol{w}(t),\ t = t_1, \ldots, t_N, \quad (2)\end{aligned}$$

where $\alpha(t) \doteq \sum_{\ell=1}^{L} a_\ell(t)$, $\beta(t) \doteq \sum_{\ell=1}^{L} a_\ell(t)\tilde{\phi}_\ell$, and $\boldsymbol{c}'(\bar{\phi}) = \frac{\partial \boldsymbol{c}(\bar{\phi})}{\partial \bar{\phi}}$. Using matrix notation, (2) can be written as $\boldsymbol{y}(t) = \boldsymbol{F}(\bar{\phi})\boldsymbol{\xi}(t) + \boldsymbol{w}(t)$ with $\boldsymbol{F}(\bar{\phi}) = [\boldsymbol{c}(\bar{\phi})\ \boldsymbol{c}'(\bar{\phi})]$ and $\boldsymbol{\xi}(t) = [\alpha(t), \beta(t)]^{\mathrm{T}}$.

We assume that the deviations $\tilde{\phi}_1, \ldots, \tilde{\phi}_L$ are zero-mean uncorrelated random variables with identical variance $\sigma_{\tilde{\phi}}^2$. The weight processes $a_1(t), \ldots, a_L(t)$ are uncorrelated complex (zero-mean) circularly-symmetric wide-sense stationary (WSS) processes with autocorrelation function $R_{a_1}(\tau), \ldots, R_{a_L}(\tau)$ respectively. The azimuth deviations and the weight processes are uncorrelated. As a consequence of these assumptions, $\alpha(t)$ and $\beta(t)$ are uncorrelated complex circularly-symmetric zero-mean WSS processes with correlation functions $R_\alpha(\tau) = \sum_{\ell=1}^{L} R_{a_\ell}(\tau)$ and $R_\beta(\tau) = \sigma_{\tilde{\phi}}^2 R_\alpha(\tau)$ respectively. The azimuth variance $\sigma_{\tilde{\phi}}^2$ can be calculated by solving the equation

$$\sigma_\beta^2 = \sigma_\alpha^2 \cdot \sigma_{\tilde{\phi}}^2, \quad (3)$$

where $\sigma_\beta^2 = R_\beta(0)$ and $\sigma_\alpha^2 = R_\alpha(0)$. In this paper we consider the case where the impinging signal power is highly confined around the NAoA and $R_{a_1}(0) = \ldots = R_{a_L}(0)$. In this particular case, $\sigma_{\tilde{\phi}}$ expressed in radian provides a close approximation of the direction spread of the SDS [4]. We refer to $\sigma_{\tilde{\phi}}$ as the AS in the sequel. Furthermore, we consider a time-variant environment and assume that $R_\alpha(|t_{n'} - t_n|) = 0,\ n \neq n',\ n, n' = 1, \ldots, N$, or equivalently that, $\alpha(t)$ and $\beta(t)$ are uncorrelated white random sequences.

In a scenario with D SDSs, (2) becomes

$$\boldsymbol{y}(t) = \sum_{d=1}^{D} \alpha_d(t)\boldsymbol{c}(\bar{\phi}_d) + \beta_d(t)\boldsymbol{c}'(\bar{\phi}_d) + \boldsymbol{w}(t), \quad t = t_1, \ldots, t_N. \quad (4)$$

The signal models (2) and (4) can be easily extended to include elevations of arrival of the sub-scatterers. In this case, the direction spread of an SDS can be characterized with close accuracy by the AS, elevation spread and azimuth-elevation correlation coefficient of the SDS. The reader is referred to [11] for detailed information on the estimators of the above parameters and the required model extension.

III. The AS Estimator

Identity (3) suggests the following ASE of $\sigma_{\tilde{\phi}}$

$$\hat{\sigma}_{\tilde{\phi}} = \sqrt{\widehat{\sigma_\beta^2}/\widehat{\sigma_\alpha^2}}. \quad (5)$$

The estimates $\widehat{\sigma_\beta^2}$ and $\widehat{\sigma_\alpha^2}$ can be obtained using standard estimation techniques, such as the maximum-likelihood estimator (MLE) derived based on the stochastic GAM model [10]. Here the term "stochastic" means that the unknown parameters $\alpha(t)$ and $\beta(t)$ in (2) are assumed to be stochastic. Alternatively $\widehat{\sigma_\beta^2}$ and $\widehat{\sigma_\alpha^2}$ can be estimated from the MLEs of the instantaneous values of $\alpha(t)$ and $\beta(t)$, $t = t_1, \ldots, t_N$, derived based on the deterministic GAM model where $\alpha(t)$ and $\beta(t)$ are assumed to be deterministic [10]. In this case these estimates read

$$\begin{aligned}\widehat{\sigma_\beta^2} &= \frac{1}{N}\sum_{t=t_1}^{t_N} |\hat{\beta}(t) - <\hat{\beta}(t)>|^2, \\ \widehat{\sigma_\alpha^2} &= \frac{1}{N}\sum_{t=t_1}^{t_N} |\hat{\alpha}(t) - <\hat{\alpha}(t)>|^2,\end{aligned}$$

with $<\hat{\beta}(t)>=\frac{1}{N}\sum_{t=t_1}^{t_N}\hat{\beta}(t)$ and $<\hat{\alpha}(t)>=\frac{1}{N}\sum_{t=t_1}^{t_N}\hat{\alpha}(t)$.

The ASE (5) can be applied for arbitrary array configurations with non-isotropic array elements and when the transmitted signal $s(t)$ is unknown. In the latter case $\alpha(t)$ and $\beta(t)$ in (2) need merely to be redefined as $\alpha(t) \doteq \sum_{\ell=1}^{L} a_\ell(t)s(t)$ and $\beta(t) \doteq \sum_{\ell=1}^{L} a_\ell(t)\tilde{\phi}_\ell s(t)$.

IV. The Array Size Adaptation Technique

The discussion focuses first on a single-SDS scenario. Simulation studies show that the ASE (5) is biased for all AS values but a certain value, which depends on the number of array elements and the signal-to-noise ratio (SNR) (see Fig. 2). This bias is due to the mismatch between the approximation model (2) and the full model described by (1). In (2) the signal space is spanned by the two vectors $\boldsymbol{c}(\bar{\phi})$ and $\boldsymbol{c}'(\bar{\phi})$, while the signal space spanned by the L vectors in the sum in (1) has usually M dimensions with probability one. The covariance matrix of $\boldsymbol{y}(t)$ generated using the GAM model (2) has two signal eigenvectors. It can be shown by simulations that the ratio between the largest and the smallest eigenvalues associated with these two signal eigenvectors is larger than a certain threshold ϵ which depends on the SNR and the array aperture, i.e. the number m of antennas in the ULA. The covariance matrix of $\boldsymbol{y}(t)$ generated using the full model (1) has usually M signal eigenvectors with probability one. The ratio between the largest and the second largest eigenvalues increase when m decreases and vice versa. When this ratio is larger than or equal to ϵ, the signal generated by the full model (1) can be closely approximated by the GAM model (2). The above observation indicate that the array size can be adaptively selected in such a way that the ratio between the largest and second largest eigenvalues is larger than or equal to ϵ.

In a D-SDS scenario where the signal contribution of each SDS is described by the full model (1), the ratio between the mean of the D largest eigenvalues and the D next largest eigenvalues is calculated. A good approximation using the GAM model to the received signal is obtained when m is reduced if necessary, until the ratio is larger than or equal to ϵ. Notice that in both single-SDS and multiple-SDS scenarios, it may occur that the calculated ratio is smaller than ϵ for all possible values of m. In this case, $m = M$ is selected for the AS estimation.

The above array size adaptation (ASA) technique that adjusts m specifically for each SDS is applied to ULAs in this contribution, but it can be easily generalized to two or three dimensional arrays. In the sequel, we refer to the ASE combined with the ASA technique as "ASA-ASE". Similarly the MLE and the SAGE algorithm derived with the GAM model [10], are called the GAM "ASA-MLE" (GAM-ASA-MLE) and the GAM "ASA-SAGE" (GAM-ASA-SAGE) algorithm respectively.

A. ASA technique using Gerschgorin Radii

The ASA technique using the calculated eigenvalues is difficult to implement from a practical point of view because the following reasons. First, the threshold ϵ is a function of SNR as the eigenvalues in the calculation of ϵ contain both signal and noise contributions. Thus, a three-dimensional look-up table which relates the threshold and the corresponding SNR and m need to be pre-generated. Second, in order to use the look-up table, the SNR of the received signal generated using the full model (1) has to be known in advance. Moreover the number of SDSs is assumed to be known as well. The latter two conditions cannot be satisfied usually.

To solve these problems, we use Gerschgorin Radii (GR) calculated from the signal covariance matrix using the unitary transformation described in [14] in the ASA technique. It can be shown that when the number of samples is large, *i)* the GR associated with a noise eigenvector equals zero and *ii)* the GR associated with a signal eigenvector is independent of noise. Since the GR does not depend on the noise components, the look-up table can be generated with respect to m only. Thus, the SNR is not necessary to be known in advance. In addition, as will be shown later this technique can be also used to estimate the number of SDSs when this number is unknown.

We first focus on the scenario where the number D of SDSs is known. In this scenario, the GR $r_i^{(m)}$ are computed, where the superscript (m) denotes the number of the used antennas and the values $r_i^{(m)}$, $i = 1, \ldots, m-1$ are sorted in descending order. The ratio between the sum of the first D GR and the sum of the second D GR is calculated. The array size is selected in such a way that the ratio calculated is greater than or equal to a pre-defined threshold $\tau_{\text{th}}^{(m)}$.

When the number of SDSs is unknown, we may first estimate this number by calculating

$$\tau^{(m)}(k) = \sum_{i=1}^{k} r_i^{(m)} \Big/ \sum_{i=k+1}^{2k} r_i^{(m)}, \qquad (6)$$

where $k \in [1, \lfloor (m-1)/2 \rfloor]$ is the possible number of SDSs and the notation $\lfloor \cdot \rfloor$ means the largest integer not larger than the argument. The smallest value of k satisfying $\tau^{(m)}(k) \geq \tau_{\text{th}}^{(m)}$ with m being the largest possible value is viewed as an estimate of the number of SDSs. The array size used to obtain this estimate is selected for the AS estimation. Investigations not shown here demonstrate that this method performs good in estimating the number of SDSs where the SDSs differ in power by less than 8 dB.

The threshold $\tau_{\text{th}}^{(m)}$ can be determined numerically as follows. We first randomly generate the signal of the GAM model (2) in the noiseless case and calculate the GR. The obtained GR consist of two non-zero values, $\hat{r}_1^{(m)}$ and $\hat{r}_2^{(m)}$, calculated respectively using the eigenvector associated with the largest eigenvalue and the eigenvector corresponding to the smallest eigenvalue. Then the ratio $\hat{\tau}^{(m)} = \hat{r}_1^{(m)}/\hat{r}_2^{(m)}$ is computed. It can be shown that $\hat{\tau}^{(m)}$ depends on the values of m, the NAoA and the AS of the SDS, but it is independent of the power of the SDS. Fig. 1 depicts the graphs of $\hat{\tau}^{(m)}$ versus the AS $\sigma_{\tilde{\phi}}$ with m as a parameter when the NAoA is equal to $20°$ with respect to (w.r.t.) the boreside of the array. All the curves exhibit minima $\hat{\tau}_{\min}^{(m)}$ at a certain AS depending on m. Studies also show that the values of the minima $\hat{\tau}_{\min}^{(m)}$ are nearly constant for a fixed m and the NAoA within the array beam-width. Based on these observations, we may conclude that the relationship $\tau^{(m)} \geq \hat{\tau}_{\min}^{(m)}$ must hold if the GAM model (2) approximates the full signal model (1) accurately. Thus, the values of the thresholds $\tau_{\text{th}}^{(m)}$ are set equal to the values of $\hat{\tau}_{\min}^{(m)}$.

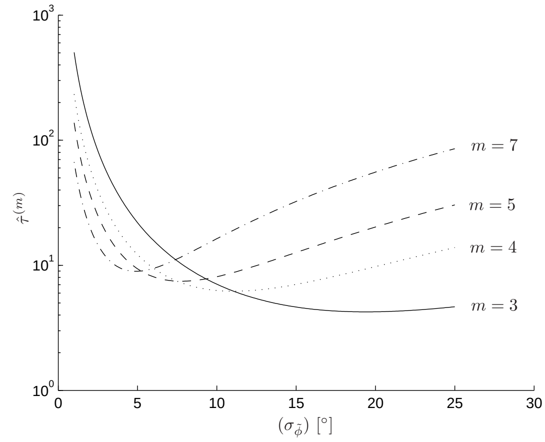

Fig. 1. Estimated $\tau^{(m)}$ versus $\sigma_{\tilde{\phi}}$ with array size m as a parameter.

B. Implementation of the ASA technique

As already mentioned, the AS value for which the ASE is unbiased, say $\tilde{\sigma}_{\tilde{\phi}}^{(m)}$, decreases when m increases. The value of m can be selected within the range $[M_{\min}, M]$, where $M_{\min}$ denotes the minimum array size required to estimate the unknown parameters in the GAM model. Thus, if the array size is selected appropriately the bias of the ASE can be maintained within a small range, e.g. $[-0.5°, +0.5°]$, provided the true AS $\sigma_{\tilde{\phi}}$ ranges in $[\tilde{\sigma}_{\tilde{\phi}}^{(M)}\ \tilde{\sigma}_{\tilde{\phi}}^{(M_{\min})}]$. Notice that $M_{\min}$ depends on the used ASEs. For example, when the MLE and the SAGE algorithm derived with the deterministic GAM model [10] are used, the projection matrix $\Pi_{\boldsymbol{F}(\bar{\phi})}$ computed in the loglikelihood function becomes an identity matrix when $m = 2$. Therefore, $M_{\min}$ has to be larger than 2 to avoid this situation. The above restriction does not apply for the MLE and the SAGE algorithm derived with the stochastic GAM model [10] and $M_{\min} = 2$ is selected.

It is worth mentioning that reducing m results in a lower intrinsic azimuth resolution of the array. As a consequence the variances of the GAM parameter estimates increase. In order to maintain the resolution as high as possible, in a single-SDS scenario we choose the array size to be the maximum value of m which provide the ratio calculated as in Section IV-A larger than the predefined thresholds. Moreover, if the selected m is less than M, the original array can be partitioned into sub-arrays with the selected size. Then the GAM parameter estimators and the ASE

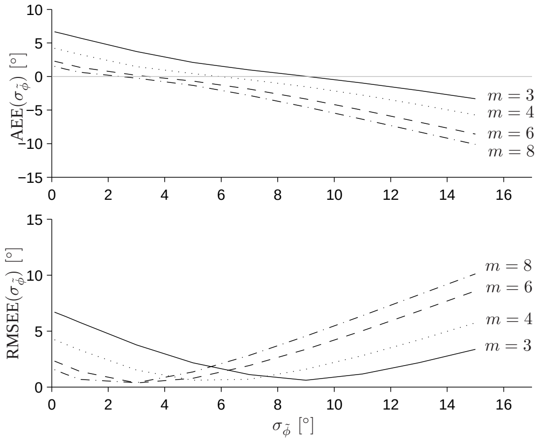

Fig. 2. AEE($\sigma_{\tilde{\phi}}$) (Upper plot) and RMSEE($\sigma_{\tilde{\phi}}$) (Lower plot) vs. the true AS value $\sigma_{\tilde{\phi}}$ with the array size m as a parameter.

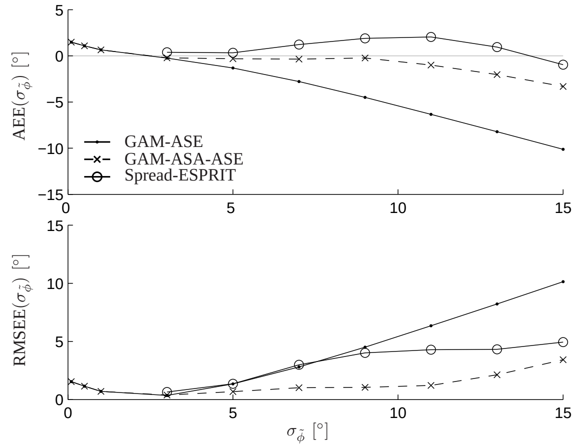

Fig. 3. AEE($\sigma_{\tilde{\phi}}$) (Upper plot) and RMSEE($\sigma_{\tilde{\phi}}$) (Lower plot) vs. the true AS value $\sigma_{\tilde{\phi}}$ at input SNR equal to 10dB.

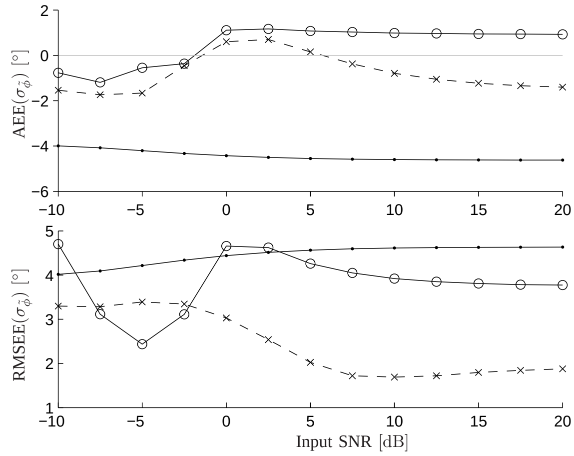

Fig. 4. AEE($\sigma_{\tilde{\phi}}$) (Upper plot) and RMSEE($\sigma_{\tilde{\phi}}$) (Lower plot) vs. the input SNR with AS $\sigma_{\tilde{\phi}} = 8°$. The legend for the curves is given in Fig. 3.

are applied with these individual sub-arrays. The obtained estimates can then be combined into the final estimate, e.g. by simple averaging.

The ASA technique can be used jointly for AS estimation in multiple-SDS scenarios. We describe here the application of this technique with the GAM-SAGE algorithms [10]. In the initialization step of the SAGE algorithms, when the parameters of SDS$_d$, $d = 1, \ldots, D$ are estimated, the array size is selected in such a way that the ratio between the sum of the $D-d+1$ GR and the sum of the $D-d+1$ next GR is larger than the predefined threshold. The number D can be either known in advance, or estimated using the proposed method. In the iterations following the initialization step, the ASA technique is implemented with the same procedure as described in the single-SDS scenario.

V. Simulation Studies

Monte-Carlo simulations are performed considering first a single-SDS scenario and then a two-SDS scenario. Each individual SDS consists of $L = 50$ sub-scatterers. The AoAs of the sub-scatterers are independent, identically von-Mises distributed random variables centered around the NAoA of the SDS. The complex gains of the propagation paths via the sub-scatterers have equal amplitude and independent $[0, 2\pi)$-uniformly-distributed random phases. In addition, the path gain phases and the AoAs are uncorrelated. Under these assumptions, $\alpha(t)$ and $\beta(t)$ in (2) are uncorrelated. The environment is assumed to be time variant. Totally $N = 50$ realizations are considered in one simulation run. The Rx array is a 8-element ULA with half-a-wavelength spaced elements. The figures shown in the subsequent are generated with 100 runs.

The performance of the ASE with different array sizes is assessed in a single-SDS scenario firstly. Fig. 2 depicts the AS average estimation error (AEE($\sigma_{\tilde{\phi}}$)) and root mean square estimation error (RMSEE($\sigma_{\tilde{\phi}}$)) versus the true AS value $\sigma_{\tilde{\phi}}$ with the array size m as a parameter. The input SNR, i.e. the SNR at individual antennas, equals 10 dB. It can be observed that the ASE is unbiased only for a certain AS value that increases when m decreases. The minima of RMSEE($\sigma_{\tilde{\phi}}$) graphs coincide pretty well with the zero-crossings of the respective AEE($\sigma_{\tilde{\phi}}$) graphs, indicating that when the array size is selected appropriately, both the absolute AEE($\sigma_{\tilde{\phi}}$) and the RMSEE($\sigma_{\tilde{\phi}}$) can be kept at a reasonably low level.

The performance in a single-SDS scenario of the GAM-ASE and that of the GAM-ASA-ASE are reported in Fig. 3 and Fig. 4 respectively. For comparison purpose, the Spread-ESPRIT technique, one of the Spread-F techniques proposed in [3], is also implemented for AS $\sigma_{\tilde{\phi}} \geq 3°$. Fig. 3 depicts the AEE($\sigma_{\tilde{\phi}}$) and RMSEE($\sigma_{\tilde{\phi}}$) versus the true AS $\sigma_{\tilde{\phi}}$ with the input SNR equal to 10 dB. It can be observed that the GAM-ASA-ASE outperforms the GAM-ASE in terms of lower absolute AEE($\sigma_{\tilde{\phi}}$) and RMSEE($\sigma_{\tilde{\phi}}$). The improvement using the ASA technique is more pronounced for large ASs. The GAM-ASA-ASE also returns lower RMSEE($\sigma_{\tilde{\phi}}$) than the Spread-ESPRIT technique. It can be observed from Fig. 3 that for AS $\sigma_{\tilde{\phi}} \leq 2°$, the GAM-ASE and the GAM-ASA-ASE have positive AEE($\sigma_{\tilde{\phi}}$), which increase when the AS decreases. This behavior is due to the fact that when the AS is very small, i.e. the deviations $\tilde{\phi}_\ell$ in (2) are close to zero, the signal space in the full model (1) has effectively one dimension. The 2-dimensional signal space of the GAM model (2) fails to provide accurate approximations of the signal space in the full model. As a consequence, the variance σ_β^2 is estimated to be larger than its true value, resulting in a positive bias in the AS estimates.

Fig. 4 depicts respectively AEE($\sigma_{\tilde{\phi}}$) and RMSEE($\sigma_{\tilde{\phi}}$) versus the input SNR with true AS equal to $8°$. It can be observed that the GAM-ASA-ASE performs the best among the three estimators. In addition the Spread-ESPRIT technique shows poor performance in the low SNR region.

Simulation results not shown here also demonstrate that by simple averaging of the NAoA estimates obtained using individual sub-arrays, the GAM-ASA-MLE shows perfor-

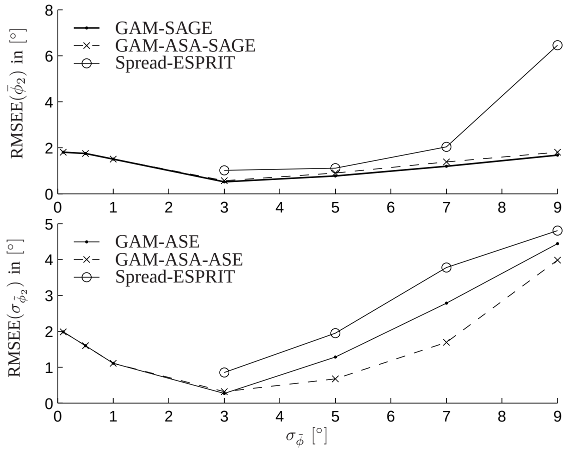

Fig. 5. RMSEE of the NAoA (Upper) and RMSEE($\sigma_{\tilde{\phi}}$) (Lower) for SDS_2, i.e. the SDS with weaker power.

mance similar to that of the GAM-MLE. When the AS is large and the SNR is high, the GAM-ASA-MLE even performs slightly better than the GAM-MLE. Moreover, the Spread-ESPRIT technique returns large RMSEE for NAoA estimates in the case of small ASs and low SNRs. Both the GAM-MLE and GAM-ASA-MLE perform better than the Spread-ESPRIT technique in estimating the NAoA.

In the two-SDS scenario, the NAoAs of the first SDS (SDS_1) and the second SDS (SDS_2) equal respectively $\bar{\phi}_1 = 30°$ and $\bar{\phi}_2 = -30°$ w.r.t. the array boreside. The two SDSs have identical AS ranging from $0.1°$ to $9°$. The input SNRs for SDS_1 and SDS_2 are 13 dB and 10 dB respectively, i.e. we assume a difference of 3 dB in power. The GAM-SAGE algorithm derived with the deterministic GAM model [10] and the GAM-ASA-SAGE algorithm are applied to estimate the NAoAs and the ASs for the two SDSs using 4 iterations. The performance of the Spread-ESPRIT technique is also reported. Each element in the pair of the computed NAoA estimates, say $(\hat{\bar{\phi}}', \hat{\bar{\phi}}'')$, is assigned to one of the two SDSs according to

$$(\hat{\bar{\phi}}_1, \hat{\bar{\phi}}_2) = \arg \min_{\substack{(\phi', \phi'') \in \\ \{(\hat{\bar{\phi}}', \hat{\bar{\phi}}''), (\hat{\bar{\phi}}'', \hat{\bar{\phi}}')\}}} \|(\phi', \phi'') - (\bar{\phi}_1, \bar{\phi}_2)\|,$$

where $\|\cdot\|$ is the Euclidean norm.

Fig. 5 depicts the RMSEE of the NAoA (RMSEE($\bar{\phi}_2$)) and RMSEE($\sigma_{\tilde{\phi}_2}$) versus the true AS for the weaker SDS (SDS_2). The result is similar for the stronger SDS. It can be observed that for $\sigma_{\tilde{\phi}} \geq 3°$ the GAM-SAGE and GAM-ASA-SAGE algorithms perform similarly in estimating the NAoA, while the latter performs better than the former in estimating the AS. Both schemes outperform the Spread-ESPRIT technique. Notice that the improvement by using the ASA technique is less significant than in the single-SDS scenario. This is because the model mismatch between the GAM model (2) and the full model (1) cannot be completely removed even though the GAM approximation is more accurate when the ASA technique is used. The hidden data estimated in the E-step of the GAM-ASA-SAGE algorithm [10] still contains part of the signal contribution from the other SDS. This residual interference deteriorates the performance of the AS estimation, leading to a higher RMSEE($\sigma_{\tilde{\phi}}$) compared to that resulting in the single-SDS scenario.

VI. Conclusions

In this paper, we proposed an azimuth spread (AS) estimator (ASE) for slightly distributed scatters (SDSs) based on the generalized-array-manifold (GAM) approximation model proposed in [1]. This ASE is biased for all AS values but one value depending on the number of antennas and the SNR. This bias is due to the mismatch between the signal described by the GAM model and the "real" signal contributed by a SDS. To improve the performance of the ASE, we proposed a technique which adapts the aperture of the used uniform linear array (ULA) to each SDS selectively by changing the array size, i.e. the number of antennas. In this method, the array size is selected for each SDS in such a way that the GAM model provides a close approximation of the signal contribution of this SDS. As a result, the ASE exhibits smaller bias and lower root mean squared estimation error. This array size adaptation (ASA) technique is applied to ULAs in this contribution but it can be generalized to two or three dimensional arrays. It can be implemented jointly with standard estimation schemes, like the SAGE algorithm, for estimating the ASs of multiple SDSs.

Simulation studies demonstrate that using the ASA technique improves the performance of the ASE. This improvement is more pronounced in a single-SDS scenario when the AS is large. In particular, the method outperforms the conventional Spread-ESPRIT technique in both single-SDS and two-SDS scenarios.

The proposed technique has low complexity and can be easily generalized to estimate the spreads in multiple dispersive dimensions (delay, azimuth and elevation of departure, azimuth and elevation of arrival, Doppler frequency) of the SDSs.

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