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On the Impact of TDM in Estimation of MIMO Channel Capacity from Phase-Noise Impaired Measurements

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***Abstract*— Due to the significantly reduced cost and effort for system calibration time-division multiplexing (TDM) is a commonly used technique to switch between the transmit and receive antennas in multiple-input multiple-output (MIMO) radio channel sounding. Nonetheless, Baum *et al.* [1], [2] have shown that phase noise of the transmitter and receiver local oscillators, when it is assumed to be a white Gaussian random process, can cause up to around 100 % errors of the estimated channel capacity of a low-rank MIMO channel when using the standard channel matrix estimator. Experimental evidence shows that consecutive phase noise samples affecting measurement samples collected with real TDM-MIMO channel sounders are correlated. In addition the spatio-temporal aperture induced by the selected switching schemes has an impact on the ordering of the phase noise samples in the estimation of the channel matrix estimate. This paper investigates how both effects affect the channel capacity estimator based on the standard channel matrix estimator. We show by means of Monte Carlo simulations that by using an experimentally obtained ARMA model of phase noise the predicted error of the ergodic capacity estimate is reduced compared to the case where phase noise is white and Gaussian. We also show that the estimated ergodic capacity is highly influenced by the choice of the spatio-temporal aperture.**

I. Introduction

To save hardware cost and alleviate the needed calibration procedures, most advanced multiple-input multiple-output (MIMO) radio channel sounders rely on a time-division multiplexing (TDM) technique. In such a system, which is represented schematically in Fig. 1, a single sounding waveform generator is connected to a number of transmit antennas via a switch. Similarly, the output terminals of the receive array are sensed via another switch. Thereby channel observations are made via a spatio-temporal aperture [3].

It has been shown recently that the concatenated phase noise of the two oscillators in the transmitter and the receiver affects the estimation of MIMO channel capacity when using the standard channel matrix estimator to obtain a capacity estimate [1], [4]. For short we call this concatenated noise the phase noise of the sounding system. The effect of phase noise on MIMO capacity estimation is studied in [4] assuming that phase noise is a random walk process. Theoretical investigations reported in [1], [2] show that, provided phase noise is white and Gaussian, it leads to large measurement errors in terms of estimated channel capacity of a low-rank MIMO channel. In [2] a number of analytical results are given under the assumptions that the TDM, i.e. the spatio-temporal array [3], fulfills a separability condition and that the phase noise process is white. However, experimental studies [5] show that phase noise cannot be assumed white on the time-scale of a measurement period, which is the observation period critical for capacity estimation. In addition, the spatio-temporal aperture induced by the used switching schemes has an impact on the ordering of the phase noise samples in the estimation of the traditional channel matrix estimate. Both effects significantly affect the performance of capacity estimation based on this matrix estimator. Finally, it is worth mentioning that non-separable spatio-temporal arrays exist that are more efficient than separable spatio-temporal arrays, in the sense that they lead to better performance of bi-direction and Doppler frequency estimators [6], [3].

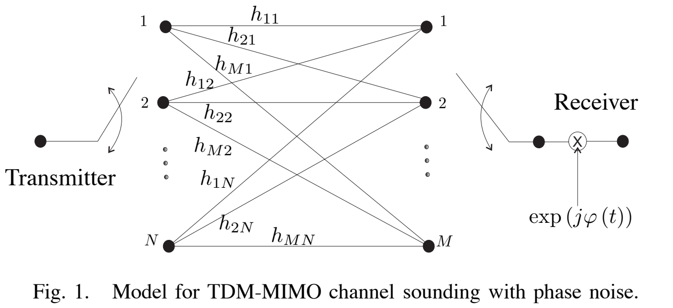

Fig. 1. Model for TDM-MIMO channel sounding with phase noise.

In this paper we analyze the combined impact of phase noise correlation and spatio-temporal aperture of a TDM-MIMO sounding system on the capacity estimation based on the traditional channel matrix estimator using the experimental phase noise model developed in [5]. We compare the performance of separable and non-separable spatio-temporal arrays for the purpose of capacity estimation.

II. System Model

We consider the TDM sounding system depicted schematically in Fig. 1 with N transmit antennas and M receive antennas. As depicted in this figure the observed signal is modulated with a time varying phasor $\exp(j\varphi(t))$.

A. Phase Noise Model

In the model proposed in [5], which we adopt here, the phase noise $\varphi(t)$ is split into a non-stationary long-term component $\varphi_{\mathrm{L}}(t)$, and a wide-sense-stationary short-term component $\varphi_{\mathrm{S}}(t)$ such that

$$\varphi(t_k) = \varphi_{\mathrm{L}}(t_k) + \varphi_{\mathrm{S}}(t_k), \quad k = 1, 2, \ldots, \tag{1}$$

where t_k is the kth sample time instant. The short-term component is modelled as an auto-regressive moving-average (ARMA) process. The long-term component is modelled as an auto-regressive integrated moving average (ARIMA) process. We refer to [5] for the specifications of these two processes. On the scale of one measurement cycle, i.e. the period needed

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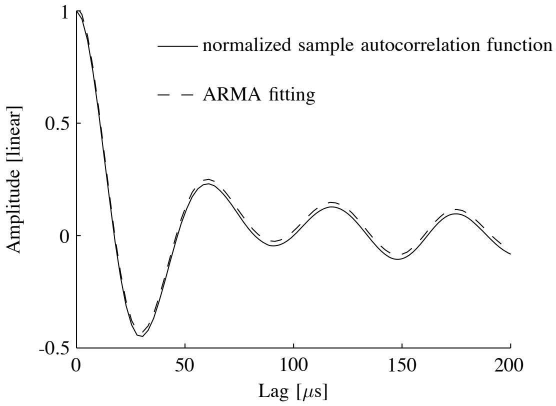

Fig. 2. Normalized sample autocorrelation function and autocorrelation function of the ARMA process fitted to the short-term component of phase noise [5].

to sense all MN sub-channels of the MIMO system the long-term component of phase noise can be considered as constant. Without loss of generality we equate it to zero: $\varphi_L(t_k) = 0$. Fig. 2 depicts the normalized sample autocorrelation function of the short term component of a measured phase noise series, together with the normalized autocorrelation function of an ARMA process fitted to this component. The sampling period T of the measured phase noise is $T = 2.54\,\mu\mathrm{s}$. It corresponds to twice the duration of a 127-chip long sequence with a chip rate of $100\,\mathrm{MHz}$. The same sampling period is used in the selected phase noise model, i.e. $t_k = kT$ in (1).

B. Signal Model for TDM Sounding

The coefficient h_{mn} of the sub-channel consisting of the mth transmit array element, the propagation channel, and the nth receive array element is measured with the transmitter switch in position n and the receiver switch in position m (see Fig. 1). At time t_k a measurement is acquired with the transmitter and receiver switches in position $n(k)$ and $m(k)$ respectively. The sequence $\{(t_k, m(k), n(k))\}$ defines the spatio-temporal array of the sounding system [6], [3]. The process of acquiring one measurement of the full $M \times N$ channel matrix $\mathbf{H}, [\mathbf{H}]_{mn} = h_{mn}$, is called a *measurement cycle*. The kth measurement belongs to the cycle with index $i(k)$. A spatio-temporal array is *separable* if it fulfills [2]

$$t_k = i(k)T_c + [\mathbf{t}_{\mathrm{Tx}}]_{n(k)} + [\mathbf{t}_{\mathrm{Rx}}]_{m(k)}, \tag{2}$$

where $\mathbf{t}_{\mathrm{Tx}}$ and $\mathbf{t}_{\mathrm{Rx}}$ are vectors of dimensions N and M respectively, and $T_c = MNT$ is the cycle duration.

Four examples of spatio-temporal arrays [3] are reported in Fig. 3. Array A is the traditionally used identity array [6], [3]. Array B is a cycle-dependent spatio-temporal array optimized for joint Doppler frequency and direction estimation [6], [3]. Array C is a modified version of Array A where the receiver switching scheme has been changed to achieve non-separability. Array D is a modified version of A, where the receiver switching sequence has been modified in such a way that for every transmit antenna, the receive antennas are switched in a different, randomly selected, order. Arrays B, C, and D are not separable.

The channel matrix $\mathbf{G}_i$ measured during cycle i is of the form [1]

$$\mathbf{G}_i = \mathbf{H} \circ \mathbf{exp}(j\mathbf{\Phi}_i), \quad i = 1, \ldots, I, \tag{3}$$

where $[\mathbf{\Phi}_{i(k)}]_{m(k)n(k)} = \varphi(t_k)$, $\mathbf{exp}(\cdot)$ is the element-wise exponential, $\circ$ denotes the Hadamard product, and I stands for the number of cycles. To simplify the notation we introduce the phase noise matrices

$$\mathbf{\Theta}_i = \mathbf{exp}(j\mathbf{\Phi}_i), \quad i = 1, \ldots, I. \tag{4}$$

It should be noticed that the ordering of the phase noise samples in $\mathbf{\Phi}_i$ is determined by the spatio-temporal array. Thus the matrices $\mathbf{\Theta}_i$ and $\mathbf{G}_i$ also depend on the spatio-temporal array.

C. Estimation of Capacity

When the channel is not known at the transmitter, but fully known at the receiver, its capacity at signal-to-noise ratio ρ reads [7]

$$C(\mathbf{HH}^{\mathrm{H}}) = \log_2 \det(\mathbf{I}_M + \tfrac{\rho}{N}\mathbf{HH}^{\mathrm{H}}), \tag{5}$$

where $\mathbf{H}^{\mathrm{H}}$ denotes the Hermitian transpose of $\mathbf{H}$. A straightforward estimate of $C(\mathbf{HH}^{\mathrm{H}})$ is $C(\widehat{\mathbf{HH}^{\mathrm{H}}})$, where $\widehat{\mathbf{HH}^{\mathrm{H}}}$ is an estimate of $\mathbf{HH}^{\mathrm{H}}$. In the sequel we consider the standard estimate of $\mathbf{HH}^{\mathrm{H}}$ computed based on measurements of $\mathbf{H}$ obtained with the considered TDM-MIMO channel sounder under the assumption that channel noise is zero:

$$\widehat{\mathbf{HH}^{\mathrm{H}}} = \tfrac{1}{I}\textstyle\sum_i \mathbf{G}_i\mathbf{G}_i^{\mathrm{H}}. \tag{6}$$

In Section V we comment further on the choice of the capacity estimator.

III. A Scenario When Capacity Estimation is Unaffected by Phase Noise

We consider the case where $I = 1$ and give the necessary and sufficient condition on the phase noise matrix such that $C(\mathbf{HH}^{\mathrm{H}}) = C(\mathbf{GG}^{\mathrm{H}})$ is fulfilled when $\mathbf{H}$ has rank one.

Theorem: Let $\mathbf{H} = \mathbf{ab}^{\mathrm{T}}$ where $\mathbf{a}$ and $\mathbf{b}$ are vectors with non-zero elements and let $\mathbf{G} = \mathbf{H} \circ \mathbf{\Theta}$. Then

$$C(\mathbf{HH}^{\mathrm{H}}) = C(\mathbf{GG}^{\mathrm{H}}) \Leftrightarrow \mathbf{\Theta} = \tilde{\mathbf{U}}\mathbf{1}_M\mathbf{1}_N^{\mathrm{T}}\tilde{\mathbf{V}}, \tag{7}$$

where $\tilde{\mathbf{U}}$ and $\tilde{\mathbf{V}}$ are unitary matrices and $\mathbf{1}_p$ is an all-one vector of dimension p.

Proof: For any matrix $\mathbf{H}$ we have the condition

$$C(\mathbf{HH}^{\mathrm{H}}) = C(\mathbf{GG}^{\mathrm{H}}) \Leftrightarrow \mathbf{G} = \mathbf{UHV}, \tag{8}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices. By the assumptions of the theorem the right-hand identity in (8) reads

$$\mathbf{ab}^{\mathrm{T}} \circ \mathbf{\Theta} = \mathbf{UHV} \tag{9}$$

$$= \mathbf{Uab}^{\mathrm{T}}\mathbf{V}. \tag{10}$$

Using the identity $\mathbf{ab}^{\mathrm{T}} \circ \mathbf{\Theta} = \mathrm{diag}(\mathbf{a})\mathbf{\Theta}\,\mathrm{diag}(\mathbf{b})$ [1, Lemma 1], with $\mathrm{diag}(\cdot)$ denoting the diagonal matrix with diagonal elements equal to the elements of the vector given as an argument, in (10) yields

$$\mathrm{diag}(\mathbf{a})\mathbf{\Theta}\,\mathrm{diag}(\mathbf{b}) = \mathbf{Uab}^{\mathrm{T}}\mathbf{V}. \tag{11}$$

Solving for $\mathbf{\Theta}$ we obtain

$$\mathbf{\Theta} = \mathrm{diag}(\mathbf{a})^{-1}\mathbf{Uab}^{\mathrm{T}}\mathbf{V}\,\mathrm{diag}(\mathbf{b})^{-1}. \tag{12}$$

Noticing that for a diagonal matrix $\mathbf{D}$ and a unitary matrix $\mathbf{S}$, there exists a unitary matrix $\tilde{\mathbf{S}}$ such that $\mathbf{SD} = \mathbf{D}\tilde{\mathbf{S}}$ we can recast (12) as

$$\mathbf{\Theta} = \tilde{\mathbf{U}}\,\mathrm{diag}(\mathbf{a})^{-1}\mathbf{a}\mathbf{b}^{\mathrm{T}}\,\mathrm{diag}(\mathbf{b})^{-1}\tilde{\mathbf{V}} \tag{13}$$
$$= \tilde{\mathbf{U}}\mathbf{1}_M\mathbf{1}_N^{\mathrm{T}}\tilde{\mathbf{V}}, \tag{14}$$

which is the sought identity. ■

Example: We consider the case where the phasor $\exp(j\phi(t))$ can be assumed constant during the time needed to switch all receive antennas once. This is the case when the normalized autocorrelation function is assumed close to unity for a time lag less that MT or, expressed in standard terminology, when the coherence time of the short-term component of the phase noise is larger than MT. Then $[\mathbf{\Theta}]_{mn} = \theta_n$ holds for all receive antenna indices m. We see that in this case

$$\mathbf{\Theta} = [\theta_1\mathbf{1}_M\ \theta_2\mathbf{1}_M\ \ldots\ \theta_N\mathbf{1}_M] \tag{15}$$
$$= \mathbf{I}_M\mathbf{1}_M\mathbf{1}_N^{\mathrm{T}}\,\mathrm{diag}(\theta_1,\ldots,\theta_N) \tag{16}$$

and therefore, by Theorem 1, $C(\mathbf{HH}^{\mathrm{H}}) = C(\mathbf{GG}^{\mathrm{H}})$.

IV. Numerical Results

Fig. 4 reports the results of a Monte Carlo simulation of the ergodic capacity estimate using the four spatio-temporal arrays defined in Fig. 3 and the experimental phase noise model described in Subsection II-A. In each Monte Carlo run a rank-1 channel matrix $\mathbf{H}$ (i.e. a key-hole channel) with a single non-zero eigenvalue of $\mathbf{HH}^{\mathrm{H}}$ equal to M is generated. Phase noise is generated according to the model given in [5]. The estimate of the ergodic capacity resulting from one spatio-temporal array at a specific signal-to-noise ratio is obtained by averaging over the capacity estimates computed from 100 Monte Carlo runs with this setting. The ergodic capacity estimates for the case without phase noise and for the case with uncorrelated Gaussian phase noise [1] are also given for comparison purpose.

As can be seen from Fig. 4 all four simulation curves lie between the "No phase noise" and "Uncorrelated phase noise" curves. Obviously, the lower the curve is, the better the performance of the estimator is. We conclude that the experimental phase noise model leads to a lower ergodic capacity estimate compared to the uncorrelated phase noise case. The error reduction is a result of the correlation among consecutive phase noise samples. Furthermore, the performance of the ergodic capacity estimator is significantly affected by the choice of the spatio-temporal array. Arrays A and B yield equal ergodic capacity estimates, while the ergodic capacity estimate is slightly lower for Array C. Among the tested arrays, Array D yields the highest ergodic capacity estimate.

The reason for the gross difference in ergodic capacity estimate for Array D compared to Arrays A, B, and C, is that the columns (and the rows) of the phase noise matrix $\mathbf{\Theta}_i$ are whitened due to the sample ordering induced by Array D. It should be remarked that despite the similar performance of the ergodic capacity estimators obtained with Arrays A, B and C, Array B is superior in terms of higher accuracy and robustness of joint Doppler and bi-direction estimates of path parameters [3], [6].

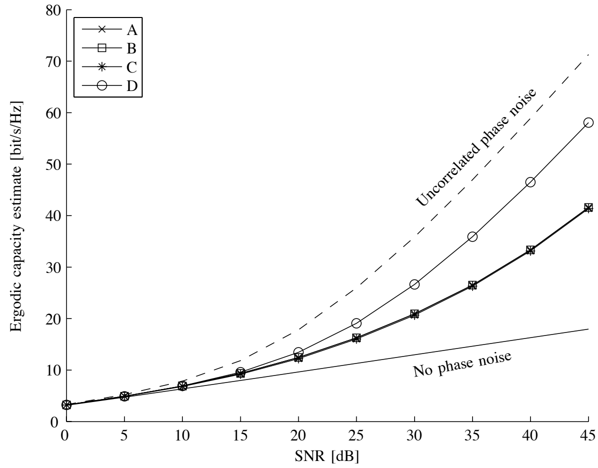

Fig. 4. Ergodic capacity estimate versus signal-to-noise ratio for a rank-1 channel as described in the text. The spatio-temporal arrays marked A, B, C, and D are defined in Fig. 3.

V. Discussion

The numerical results presented in the previous section have shown that short-term correlation of phase noise combined with appropriate choice of the spatio-temporal array aperture enable to significantly reduce the impact of phase noise on the capacity estimation based on the traditional channel matrix estimator. Another straightforward way to reduce this impact is to consider more than one cycle in (6), provided the channel can be assumed time-invariant over the duration of all the considered cycles [8]. The example in Section III provides with some indication on an additional alternative: Select the bandwidth of the feedback loop in the phase-locked loop of the local oscillators in such a way that the resulting short-term phase noise exhibits a coherence time larger than MT. In this method, the selected bandwidth depends on both the number of elements in the receive array and the duration of the sounding sequence. Interestingly, the number of elements in the transmit array is not critical here.

However, the above approaches do not avoid the additional problem that, in practice, the measured matrices $\{\mathbf{G}_i\}$ are also impaired by additive noise, an effect which also impairs on the accuracy of the capacity estimator $C(\widehat{\mathbf{HH}^{\mathrm{H}}})$. This problem, and in fact the sensitivity to phase noise as well, is a consequence of the fact that the traditionally used estimator in (6) does not take into account these two noises. Estimators of $\mathbf{HH}^{\mathrm{H}}$ and $\mathbf{H}$ can be derived that exploit the statistical properties of these noises in order to mitigate their effects. Estimates $\hat{\mathbf{H}}$ constructed from estimates of the parameters of a parametric model of $\mathbf{H}$ seems to offer a promising solution. An example is the recently published phase noise compensated SAGE estimator for the estimation of path parameters [9]. This work shows that the effect of phase noise can be mitigated by taking its statistical property into consideration in the signal model underlying the derivation of the path parameter estimators. However, an open issue is how the mismatch between the physical world and the approximation of it provided the parametric model affects the capacity estimate.

VI. Conclusions

This paper has presented some results on the impact of TDM-MIMO channel sounding on the estimation of MIMO

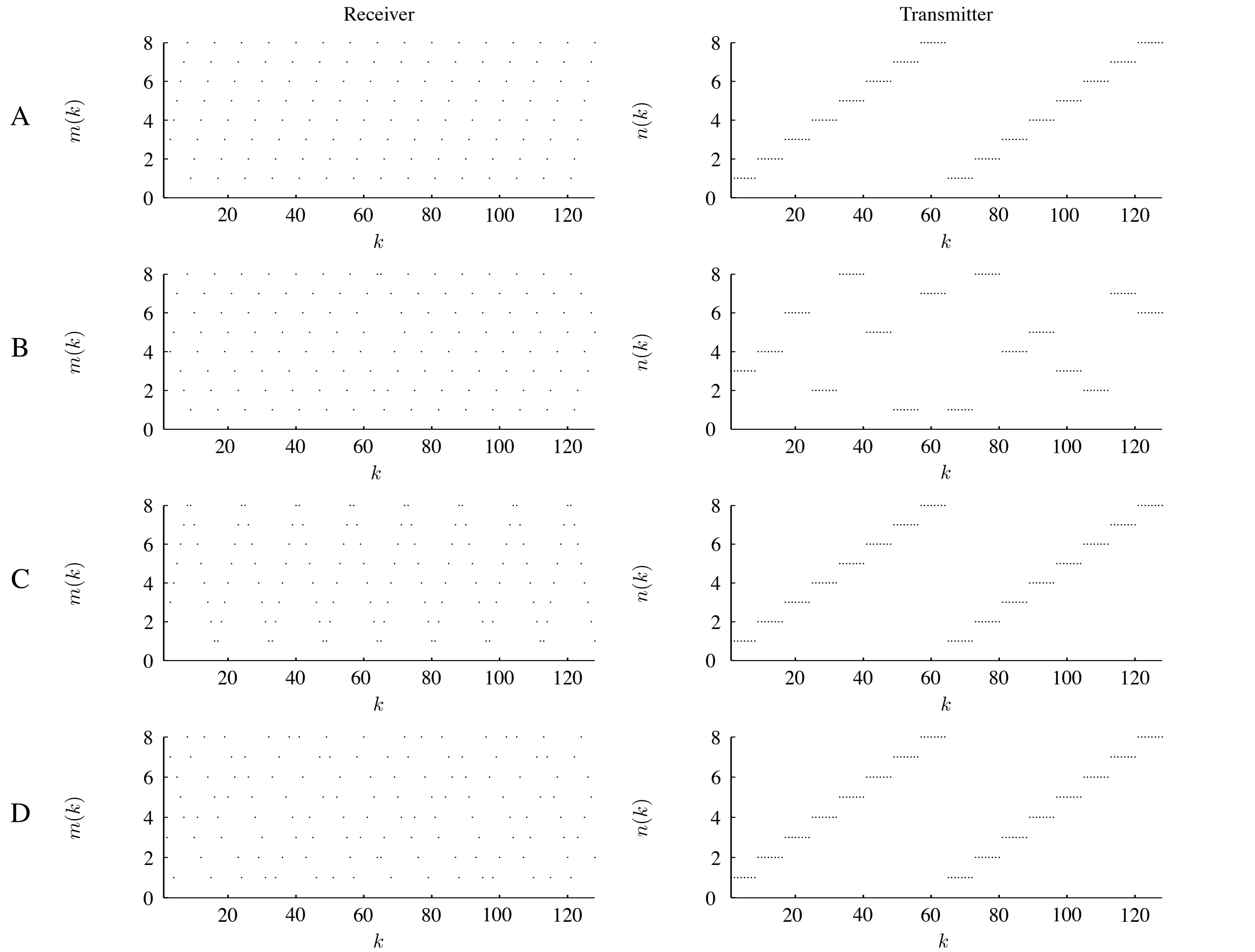

Fig. 3. Four spatio-temporal arrays with $M = N = 8$, $I = 2$ and the timing scheme defined as $t_k = kT$, where the sampling T is 2.54 µs. Array A is the commonly used identity array [6]; Array B is a cycle-dependent and non-separable array [6]; Array C is a modified version of Array A where the receiver switching pattern is modified to achieve a non-separable array; Array D is a non-separable array.

channel capacity using the traditional channel matrix estimator. The necessary and sufficient condition on the phase noise matrix for the capacity estimate to be unaffected by phase noise is given. It is shown by means of Monte Carlo simulations that the choice of spatio-temporal array heavily impacts on the accuracy of the capacity estimator in the presence of correlated phase noise. It was found that non-separable arrays exist that lead to the same capacity estimation error as separable arrays. As shown in [6], [3], the use of non-separable arrays leads to a lower mean square error and better ambiguity resolution abilities when used for estimation of Doppler frequency and bi-direction.

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